Muonic atoms scattering from hydrogen molecules using the Morse potential

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Muon Catalyzed Fusion and Related Topics, 2007

Outline

Considered Processes

Muonic hydrogen scattering from hydrogenic molecules

- elastic scattering: $a\mu(F) + BC \rightarrow a\mu(F) + BC$
- isotopic exchange: $a\mu + BC \rightarrow b\mu + AC$
- spin-flip: $a\mu(F) + AB \rightarrow a\mu(F') + AB$

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- This work:

$$d\mu + \mathrm{H}_2 \rightarrow d\mu + \mathrm{H}_2$$
$$t\mu + \mathrm{H}_2 \rightarrow t\mu + \mathrm{H}_2$$

for:

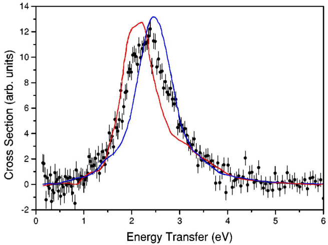
- muonic atom in the ground state $(1S)$
- different initial rotational states
- the collision energies $\varepsilon \leq$ 30 eV (CMS)

Motivation

- Muonic hydrogen atom is a small, neutral object $\longrightarrow$ the methods developed for neutron scattering can be adapted for muonic hydrogen scattering
- Previous models:
  - $E < E_{\mathrm{diss}}$ harmonic approximation
  - $E \gg E_{\mathrm{diss}}$ free-nuclei approximation

Motivation

Deep Inelastic Neutron Scattering



– Molecular regime

– Atomic regime

U. Bafile *et al.*, Phys.Rev. **B** 58 (1998)

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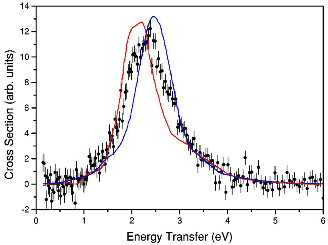

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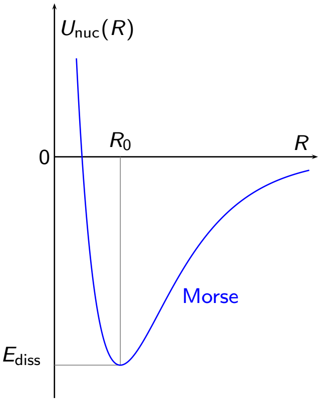
- New model:
  - Crossover region $\rightarrow$
    $\rightarrow$ The Morse potential model
- Planning and interpreting experiments:
  - μ^- nuclear capture in $p\mu$
  - measurements of the Lamb shift in $p\mu$

Comparison With Previous Models



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- The Morse potential

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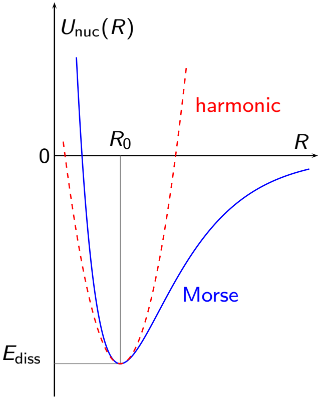

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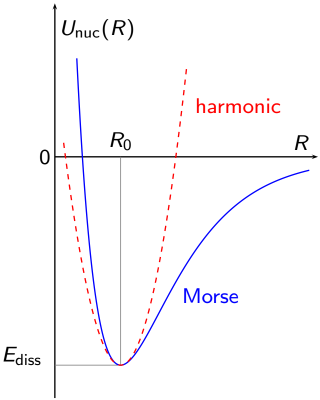

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  - vibrational anharmonicities
  - centrifugal distortions
  - molecular dissociation

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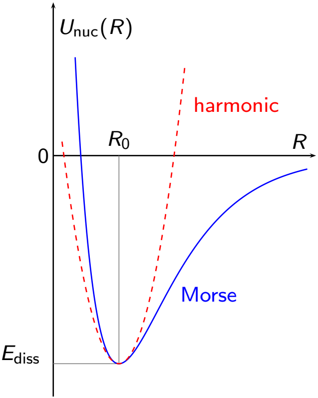

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- The presented model reproduces results:
  - harmonic approximation - low energy limit
  - free-nuclei approximation - high energy limit

The Morse Potential Model - Assumptions

- Born approximation
- Intramolecular interaction → The Morse potential
- Cross sections directly expressed by the corresponding nuclear amplitudes
- Effects of internal motion of nuclei inside molecules
- Electron-screening corrections

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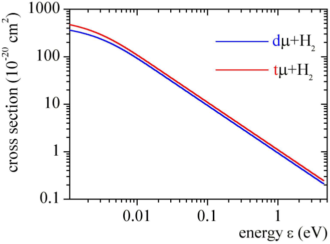

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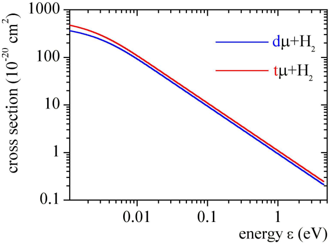

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- Our results: The first full quantum-mechanical calculations based on the Morse potential
- DINS: The Morse potential spectra have been obtained using semi-classical WKB approximation

The Morse Potential Spectrum For Hydrogen Molecule

- Discrete
  - $E' < 0$
  - Analytical solution only for $K' = 0$
  - 332 rotovibrational states
    - $K' \leq 31$
    - $\nu' \leq 16$

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- Discrete
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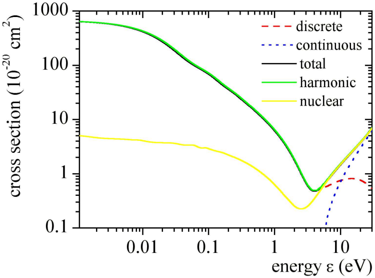
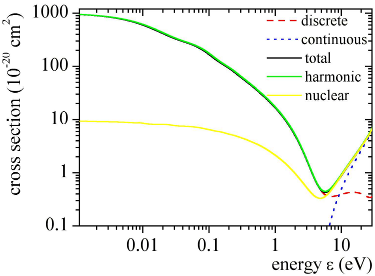
- Continuous
  - $E' > 0$
  - infinite degeneracy in K'
  - for the collision energies $\varepsilon \leq 30$ eV $\longleftrightarrow$ $K' \leq 70$

Total Cross Sections

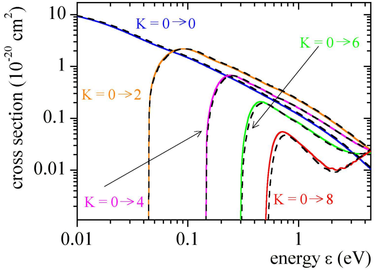


$$d\mu + H_2(K=0) \rightarrow d\mu + H_2$$



$$t\mu + H_2(K=1) \rightarrow t\mu + H_2$$

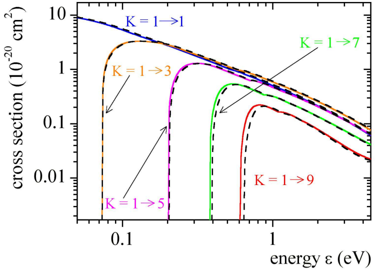
Rotational Transitions



$d\mu + H_2(K = 0) \rightarrow \; d\mu + H_2$

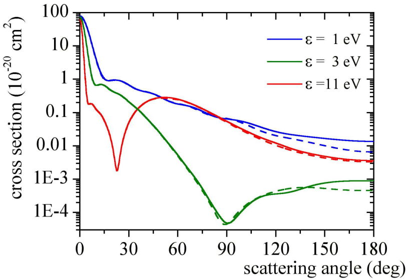
color lines → the Morse potential model

black lines → harmonic approximation



$t\mu + H_2(K = 1) \rightarrow \; t\mu + H_2$

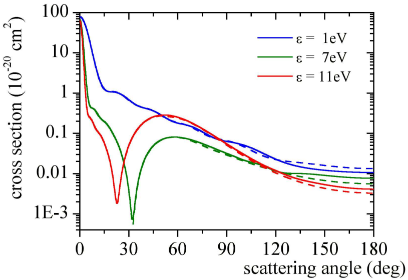
Differential Cross Sections



$t\mu + H_2(K=0) \rightarrow \; t\mu + H_2$

solid lines $\rightarrow$ the Morse potential model

dash lines $\rightarrow$ harmonic approximation



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The Morse potential describes muonic hydrogen scattering from hydrogen molecules in a more realistic and accurate way

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Outlook

- Differential cross sections in LAB
- Monte Carlo simulation