Gravitational states of antihydrogen atoms bouncing on the solid surface

P. Froelich[1,2]

- Antimatter
  - Fundamental Symmetries of Nature
  - Antihydrogen, CPT and WEP
  - Trapping and cooling antihydrogen at CERN's AD
- Antihydrogen - wall interactions
  - Quantum reflection
  - Ultracold antihydrogen between two walls
  - Confinement, cooling and guiding
  - Quantum states of antihydrogen bouncing above the surface in the gravitational field of Earth
  - Time evolution of the bouncing motion - wave packet dynamics
- Conclusions

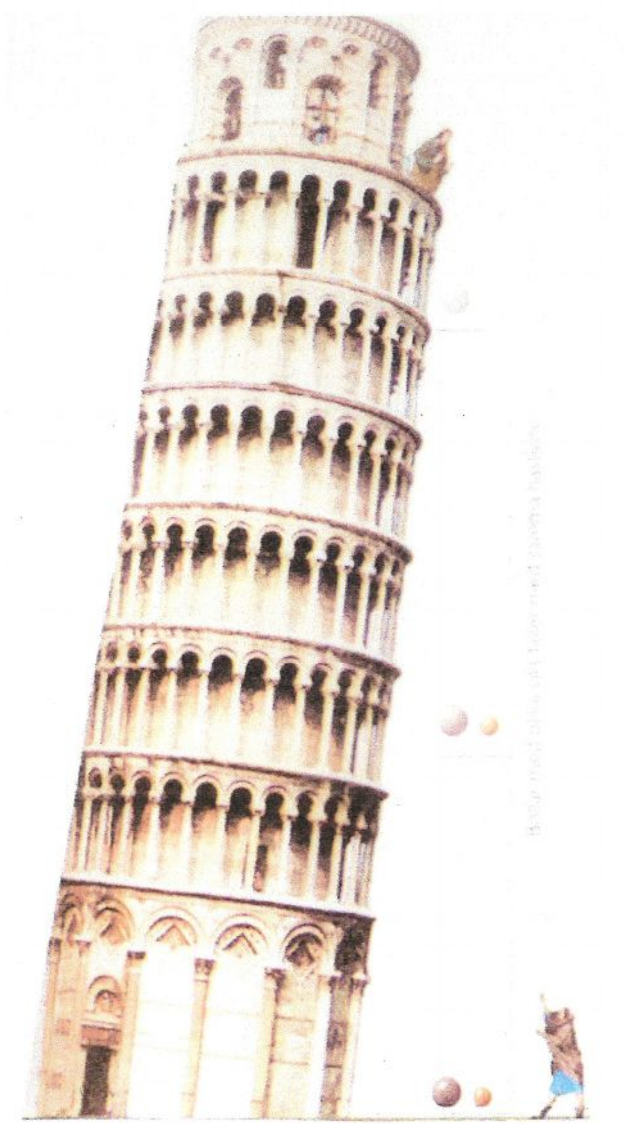
[1] Uppsala University, [2]collaboration: Alexei Voronin, Lebedev Institute.

Antihydrogen - a microscopic laboratory for testing the fundamental symmetries of Nature

Physical questions behind experiments with antihydrogen:

- Does the charge-parity-time (CPT) symmetry hold?
- Why is the Universe matter-antimatter asymmetric?
- How does antimatter interact gravitationally with matter? Does the weak equivalence principle (WEP) hold?
- How does antimatter interact in contact with matter?

How does antimatter interact with gravity?

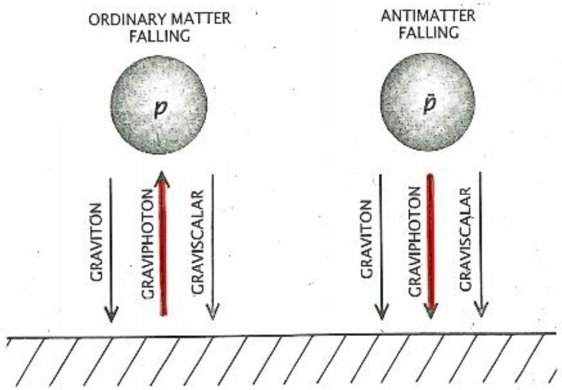


- Aristotle (350 BC): an iron ball falls faster than feather
- Newton (1687): the gravitational charge / mass ratio is the same for all sorts of matter.
- Einstein (1907): general relativity and WEP. There is no distinction between the inertial and gravitational mass.
- Modern theories of gravitation, following from unification of quantum mechanics and general relativity theory, allow violations of WEP
- Ted Hänsch (2004): tests of WEP by interferometric measurments of the Earth acceleration for various isotops of Rb atoms ($\frac{\delta g}{g} = 1.2 \pm 1.7 \cdot 10^{-7}$)

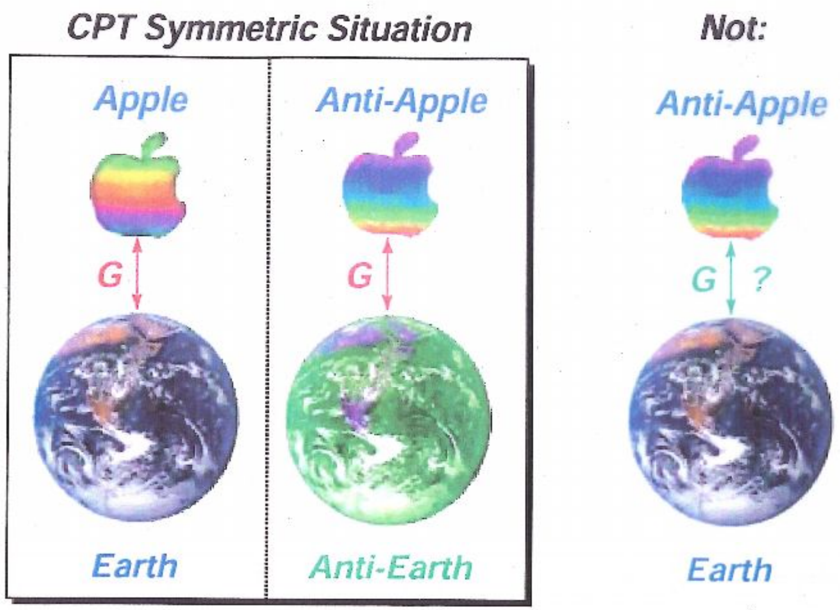
  S. Fray C.A. Diez, T. Hänsch and M Weitz, PRL **93**, 240404, 2004.
- Most interesting case (largest deviations): gravitational interaction of matter and antimatter.

- Galileo (1604): all bodies fall with the same acceleration (principle of universality of free fall).
  Is that true for antimatter?

- Preservation of the CPT symmetry does not preclude violation of the WEP.
- The contemporary theories of gravitation, following from unification of general relativity with quantum theory, allow violations of the WEP.



How does it fall?



Matter - Antimatter Interactions

Quantum Antichemistry

$$
\mathrm{H} + \overline{\mathrm{H}} \longrightarrow \begin{cases} \mathrm{H} + \overline{\mathrm{H}} & \text{elastic scattering} \\ \mathrm{Pn} + \mathrm{Ps} & \text{rearrangement} \\ e^{+} + e^{-} + yy & p\bar{p} \text{ annihilation in flight} \\ p + \bar{p} + \gamma, + 2\gamma & e^{+}e^{-} \text{ annihilation in flight} \\ \mathrm{H}\overline{\mathrm{H}} + h\nu & \text{radiative association} \end{cases}
$$

$\mathrm{H}(1)+\overline{\mathrm{H}}(n) \rightarrow \mathrm{H}(n)+\overline{\mathrm{H}}(1)$ excitation transfer

$\overline{\mathrm{H}} + \mathrm{He} \rightarrow \ldots$

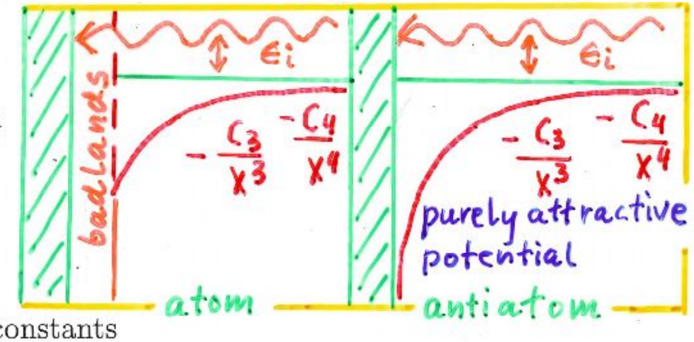
$\overline{\mathrm{H}} + \mathrm{SURFACE}$

- Calculations include Coulomb + strong nuclear force + long range interactions + gravitational field

- Presence of external fields ($\mathbf{B}, \mathbf{E}$) [trapping, tuning, spectroscopy]
- 4-body calculations

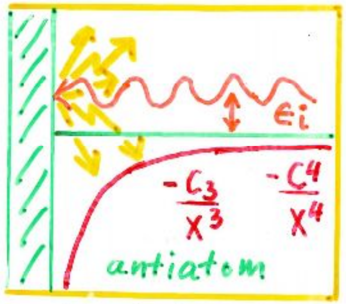
Antihydrogen - surface interactions

- An atom (antiatom) in the presence of the wall - scatters on the purely attractive long range potential obtained by Casimir and Polder (PR **73**, 360, 1948)



- Van der Waals region:
  $1 \ll x \ll \lambda_w \qquad V(x) \sim -\frac{C_3}{x^3}$
- Casimir region:
  $x \gg \lambda_w \qquad V(x) \sim -\frac{C_4}{x^4}$
- Physics behind the strength constants
  - $\frac{C_3}{x^3}$: van der Waals interaction, $C_3 = \frac{1}{4pi}\int_0^\infty \alpha(i\omega)d\omega$ with $\alpha(i\omega)$ being the atomic dynamic electric-dipole polarizability at imaginary frequency $i\omega$
  - C_4: retardation effects (QED), $C_4 = \frac{3\alpha(0)}{8\pi\alpha_c}$
- $\frac{C_3}{x^3} \longrightarrow \frac{C_4}{x^4}$: retardation, purely quantum electrodynamical effect.
- Scattering on the wall $\Rightarrow$ check of QED ... if not the short range interaction!
- What happens at the short range?
  - atom: reflects via complicated physics from an imperfect surface, stics, etc.
  - antiatom: a) no repulsion between nuclei
    b) no "Coulomb hole"
    c) no "Fermi hole"

  $\Rightarrow$ purely attractive potential for $R \in [0, \infty]$



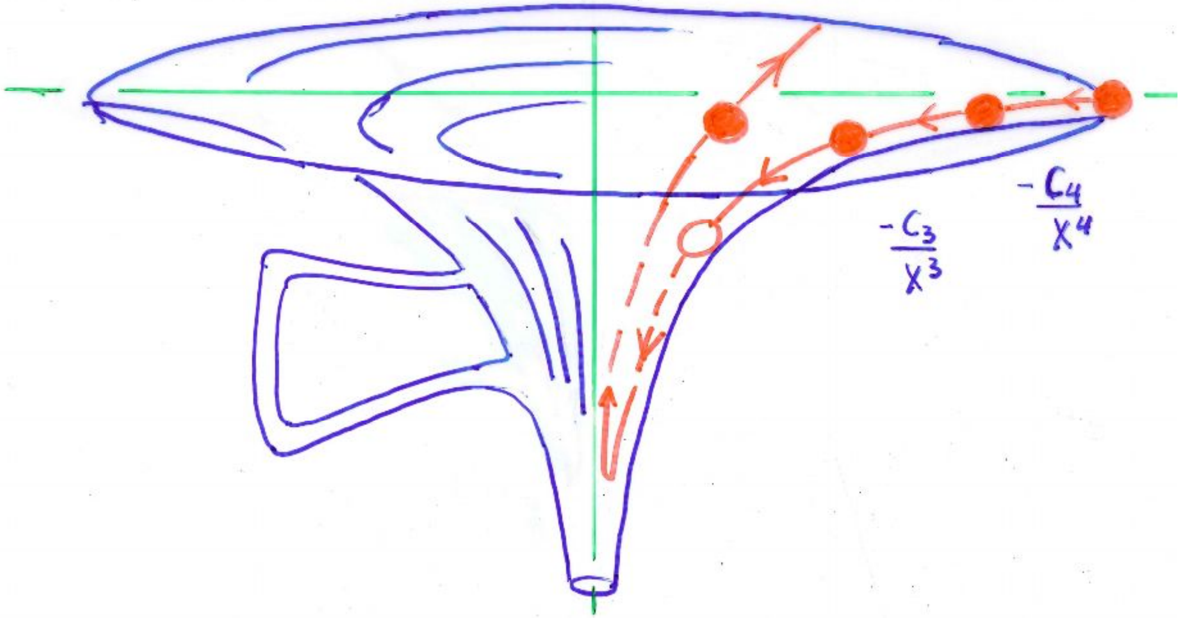
- What happens at the VERY short range?
  - strong force $\Rightarrow$ annihilation. Is it good or bad?
- Annihilation purifies the scattering experiment: in case of total absorption at $R \simeq 0$ there is no reflection from the contact with the surface, the scattering occurs on the well known long range potential

  $\Rightarrow$ QED could be tested

  $\Rightarrow$ Antiatom - matter interactions and perheaps WEP could be tested

Quantum reflection

- For potentials vanishing faster than $-\frac{1}{x^2}$: quantum reflection!



- Reflection probability increases with the decrease of collisional energy

$$R = 1 - bk$$

where $k = \frac{mv}{\hbar}$, b = length scale parameter characterizing the asymptotic tail, e.g. for homogeneous potentials $-\frac{C_n}{x^n}$: $b_n = \left(\frac{2mC_n}{\hbar^2}\right)^{\frac{1}{n-2}}$; $b_n(C_n)$

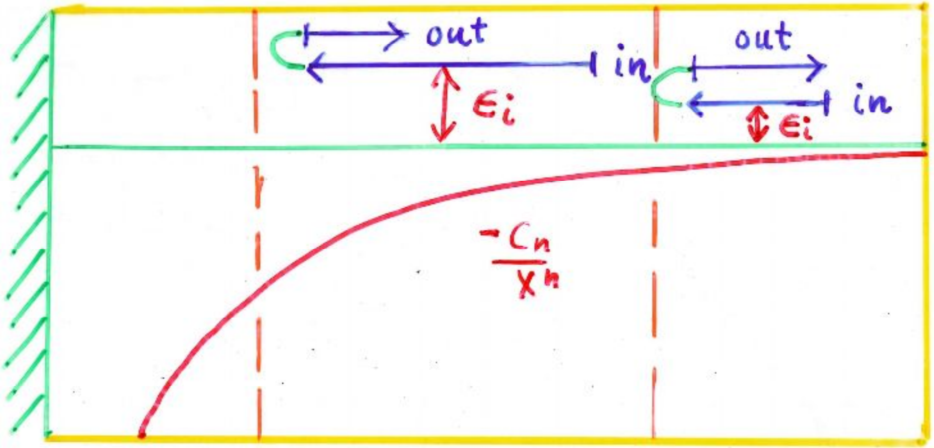
- Condition for quantum reflection for homogeneous potentials: the local de Broglie wave length $\lambda(x) = \frac{2\pi\hbar}{p(x)} = \frac{2\pi\hbar}{\sqrt{2m[\epsilon_i - V(x)]}}$ must change fast as function of distance:

$$\left|\frac{d\lambda}{dx}\right| = \left|\frac{\hbar p'}{p^2}\right| \geq 1 \qquad (\text{bad WKB})$$

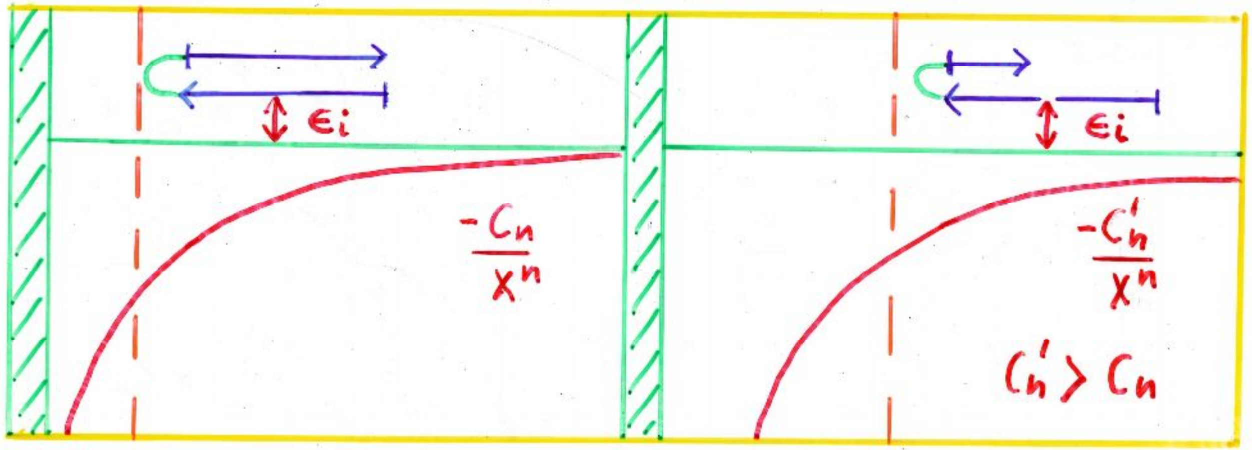
- For homogeneous potentials $V_n(x) \sim -\frac{C_n}{x^n}$ (and for $\epsilon_i \to 0$) reflection occurs when $x \geq \left(2\sqrt{2mC_n}/n\right)^{\frac{2}{n-2}}$

Remarkable features of quantum reflection

- Reflection occures in spite of the purely attractive potential
- Reflection increases with the *decrease* of the collision energy: $R = 1 - b_n k$



- Reflection increases with the *decrease* of $b_n = \left(\frac{2mC_n}{\hbar^2}\right)^{\frac{1}{n-2}}$ i.e. weaker potential (smaller C_n) reflects better!
- The decrease of the potential strength shortens the "reflection distance", $x_n \simeq k^{-\frac{2}{n}} \left[\frac{2mC_n}{\hbar^2}\right]^{\frac{1}{n}}$.
- Weaker potential reflects better and brings the reflection closer to the surface.



Quantum reflection - general case

- Homogeneous potential $V(x) = -\frac{C_n}{x^n}$ $\Rightarrow$ $R(b_n)$, $b(C_n)$
- General case $V(x)$ $\Rightarrow$ $R(a)$, where a is the scattering length
- The length-scale parameter b can be expressed in terms of the scattering length
- Reflectivity is given by the S matrix element for the elastic scattering, $R = |S_{ii}|^2$
- S_{ii} is obtained by solving the Schrödinger equation for the antiatom scattering off the wall

$$\left[-\frac{1}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E_i\psi(x)$$

  where $V(x)$ is the exact potential for an atom in the presence of a conductive surface (Casimir-Polder potential)

- The solution in the van der Waals region ($V(x) \sim -\frac{C_3}{x^3}$) and for $E_i \simeq 0$ is

$$\psi(x) \sim \sqrt{x}\left[H_1^{(1)}(\rho) + e^{2i\delta}H_1^{(2)}(\rho)\right], \qquad \rho = 2\sqrt{2mC_3/x}$$

  where $H_1^{(1)}(\rho)$, $H_1^{(2)}(\rho)$ are Hankel functions of order 1, $\delta = \delta_1 + i\delta_2$ is a complex phase shift produced by the *short range part* of the interaction

- Our short range interaction is due to strong forces and causes annihilation. We use full absorption boundary condition:

$$\delta_2 \gg 1 \qquad \Rightarrow \qquad \psi(x) \sim \sqrt{(}x)H_1^{(1)}(\rho)$$

  i.e. there is no outgoing component due to the strong absorption by annihilation

- The solution does not depend on the details of the short-range interaction (is independent on δ)
- Schrödinger equation is integrated with $V_{CP}(x)$ (using $\psi(x) \sim \sqrt{(}x)H_1^{(1)}(\rho)$ as the boundary condition for $x \to 0$ and $\psi(x) \sim e^{-ikx} - Se^{ikx}$ for $x \to +\infty$) to determine the scattering matrix S and the scattering length a
- Scattering length (effective range) approximation is applied: $S = 1 - 2ika$.

$\Rightarrow R(a)$

$\bar{\text{H}}$ reflection / absorption on the wall

- The scattering length $a = \alpha - i\beta$ is complex due to absorption
- The reflection coefficient (reflected flux) is given by $R = |S|^2 = 1 + 2iIm(2ika) + 4k^2a^*a \simeq 1 - 4k\beta \equiv 1 - kb$, with $b = 4Im(a)$.
- The absorption coefficient (annihilated flux) is $P = 1 - |S|^2 \simeq kb$
- Numerical calculation of the scattering length $a \Rightarrow$ length parameter $b \Rightarrow$ reflectivity $R = 1 - kb$

Results - absorption (P) / reflection (R) as function of energy

$\log(E/a.u)$	T	P	R	$P \simeq 1 - \exp(-kb)$	$Im(a) = \lvert\ln(1-P)\rvert/(4k)$, a.u.
-9		0.95		0.99	365.5
-10		0.69		0.74	479.8
-11	$10^{-6}K$	0.33	0.77	0.34	529.3
-12		0.12		0.13	540.9
-13		0.041		0.04	542.8
-14	$10^{-9}K$	0.013	0.99	0.013	543.1
-15		0.0042		0.0042	543.1
-16		0.0013		0.0013	543.2
-17		0.00042		0.00042	543.2
-18		0.00013		0.00013	543.2

Table 1: The annihilation probability P for the ultra-cold antihydrogen impinging on the wall

- Validity of the scattering approximation: $k|a| \ll 1$ satisfied for $E_i < 10^{-11}$ a.u.
- Scattering length for the exact Casimir-Polder potential:
  $a_{CP} = -81.7 - i543.2$ a.u. (our numerical calculation) $\Rightarrow b = 4Im(a) = 0.1\mu$
- Scattering length for the purely homogeneous case $V(x) \sim -\frac{C_4}{x^4}$:
  $a_4 = -i\sqrt{2mC_4} = -i519.9$ a.u. (purely imaginary, Voronin PRA **67**, 062706, 2003)
- We note $Im(a_{CP}) \simeq a_4 \Rightarrow$ scattering happens predominantly on the Casimir tail

Reflection probability as a function of distance

- What is the "reflection distance"? The amplitude of the reflected wave is generated at all distances.

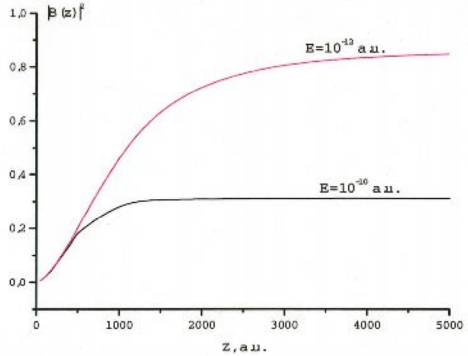


Figure: Contribution of different antiatom-wall distances to the reflection probability. $|B(z)|^2$ expresses the reflection probability accumulated in the interval between z_0 and z.

- Analysis of the wave function $\Rightarrow |B(z)|^2$, fraction of the reflected wave generated between z_0 and z

  - $|B(\infty)|^2 = P(E)$ ("full reflection probability")

  - $B(z < z_0) = 0$ ($\Phi(z < z_0)$ is a purely incoming wave)

- contribution to the amplitude of the reflected wave from the distances $z < 100$ a.u. is very small

- $E = 10^{-12}$ a.u., $P = |B(\infty)|^2 = 0.88$

  - 75% of the reflection is generated between 500 a.u. and 5 000 a.u. (pure Casimir tail)

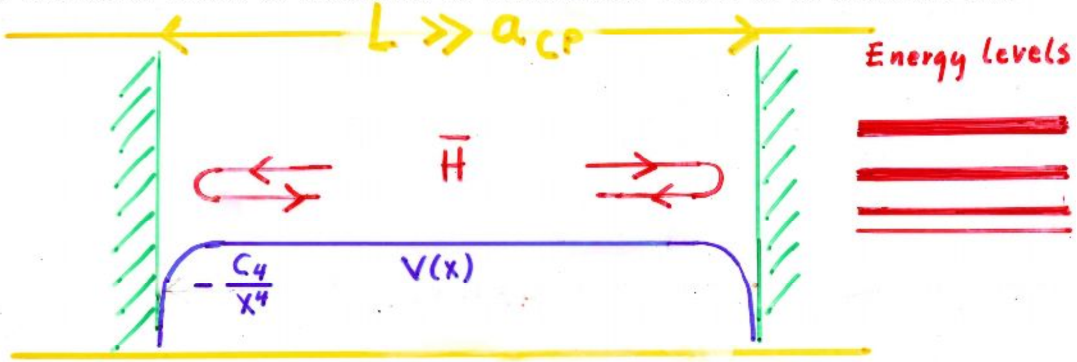
  - 22% of the reflection is generated between 100 a.u. and 500 a.u. (van der Waals range)

- $E = 10^{-10}$ a.u., $P = |B(\infty)|^2 = 0.31$

  -92% of the reflection is generated within $\Delta z = [100 - 1000]$ a.u.,
  $z_r \in \Delta z$ is the "reflection distance".

Ultracold $\bar{\mathrm{H}}$ between two walls

- Quantum reflection leads to existence of metastable states of $\bar{\mathrm{H}}$ between two walls



- $\psi_{left}(x) \sin[kx + \delta_{CP}]$, $\qquad a_{CP} \ll x \ll L - |a_{CP}|$
- $\psi_{right}(x) \sin[k(L - x) + \delta_{CP}]$, $\qquad a_{CP} \ll x \ll L - |a_{CP}|$
- Matching $\quad \Rightarrow \quad$ quantization condition: $kL + 2\delta_{CP} = \pi n$
- Scattering length approximation: $\delta_{CP} = -ka_{CP} \quad \Rightarrow \quad k = \frac{\pi n}{(L-2a_{CP})}$
- Quantized box-state energies:

$$E_n = \frac{\pi n^2}{2m(L - 2a_{CP})} \simeq \frac{\pi^2 n^2}{2mL^2}\left(1 + 4\frac{Re(a_{CP})}{L}\right) - i|Im(a_{CP})|\frac{4\pi^2 n^2}{2mL^3}$$

- Modification induced by quantum reflection

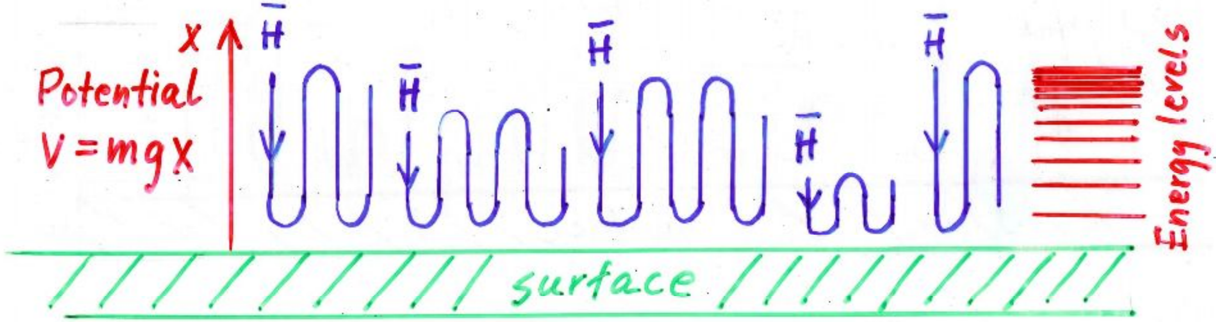
$$E_n = E_n^{(0)}\left(1 + 4\frac{\alpha_{CP}}{L}\right) - i\underbrace{\frac{4\beta_{CP}}{L}E_n^{(0)}}_{\Gamma/2}$$

- Example: $L = 10\ \mu\mathrm{m} \quad \Rightarrow \quad E_0 = 7.5 \cdot 10^{-14}$ a.u., $\Gamma_0 = 1.7 \cdot 10^{-15}$ a.u. $\Rightarrow$ lifetime $\tau = 0.014$ s.
- To increase the lifetime: enlarge the box, $\tau \sim L^3$.
- To increase the lowest energy level: shrink the box, $E_n \sim E_n^{(0)}(\frac{1}{L^2}) + \Delta E(\frac{1}{L^3})$

Gravitational effects

Quantum states of $\bar{\mathrm{H}}$ in the gravitational field of Earth

- Quantum reflection might be used for measuring the gravitational interaction of $\bar{\mathrm{H}}$ atoms, perheaps probing WEP
- Falling $\bar{\mathrm{H}}$ atoms will bounce on the surface



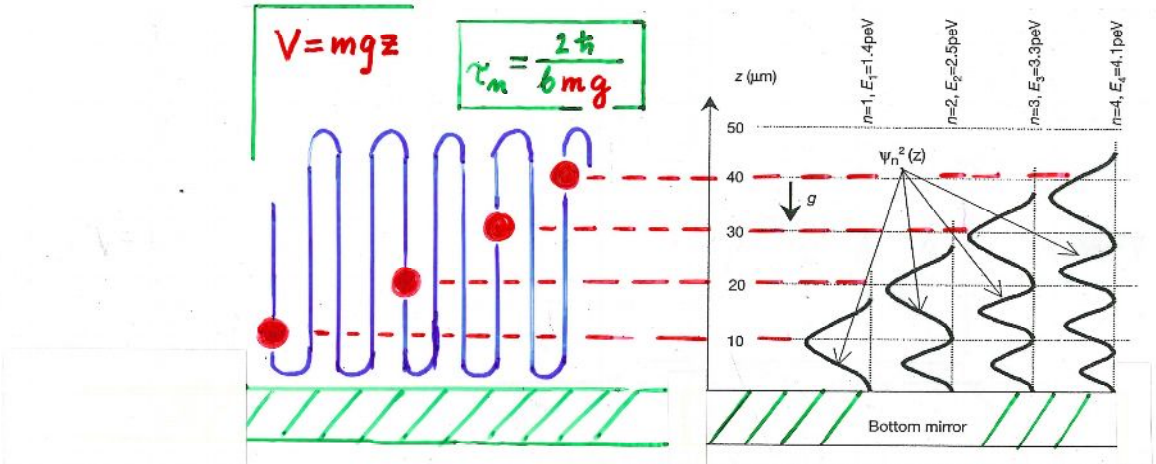
- Quantization is achieved by the confinement
  - from below: by quantum reflection via Casimir interaction
  - from above: by the gravitational field
- The wave function of the particle bouncing on the *perfect* mirror:
  $\psi_n(x) = C \cdot Ai(\frac{x}{l_0} - \lambda_n)$; $l_0 = \sqrt[3]{\hbar^2/(2m^2 g)} = 5.87\ \mu\text{m}$.
  inertial mass → mM ← gravitational mass
- The corresponding eigenvalues $E_n = \epsilon\lambda_n$, ($\epsilon = \sqrt[3]{\hbar^2 m g^2/2} = 2.2 \cdot 10^{-14}$ a.u.) are determined from $Ai(-\lambda_n) = 0$
  $\frac{M^2}{m}$
- Example:
  - $\lambda_1 = 2.338$, $\lambda_2 = 4.088$, $\lambda_3 = 5.521$, ...
  - $E_1 = 5.14 \cdot 10^{-14}$ a.u., ...

Quantum states of $\bar{\mathrm{H}}$ in the gravitational field of Ear

- Modification of the eigenvalues due to the Casimir interaction: obtained by perturbation theory in conjunction with the scattering length approximation ⇒ $\lambda_n \to \lambda_n + \frac{a_{CP}}{l_0}$; $\quad E_n \to E_n + \epsilon \frac{Re(a_{CP})}{l_0}$

$$\Gamma_n = 2\epsilon \frac{Im(a_{CP})}{l_0} = \epsilon \frac{b}{2l_0} \quad \Rightarrow \quad \boxed{\tau_n = \frac{2\hbar}{bmg}}$$

- Example: $E_1 = 5.17 \cdot 10^{-14}$ a.u., $\quad \tau_1 = \frac{\hbar}{\Gamma_1} \simeq 0.1$ s $\quad$ M: gravitational mas
- C.f. the experiment with neutrons: discovery of the lowest quantum state (Nesvizhevsky *et al.* PRD **67**, 102002, 2003; Nature **415**, 297, 2002)



$E = mgh, \quad t_f = \sqrt{\frac{2E}{mg^2}}$

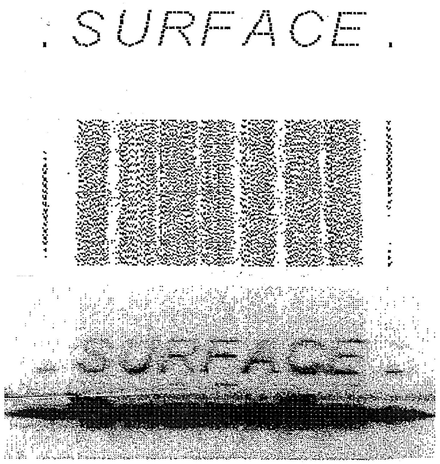
Figure 1 Wavefunctions of the quantum states of neutrons in the potential well formed by the Earth's gravitational field and the horizontal mirror. The probability of finding neutrons at height z, corresponding to the nth quantum state, is proportional to the square of the neutron wavefunction $\psi_n^2(z)$. The vertical axis z provides the length scale for this phenomenon. E_n is the energy of the nth quantum state.

- The width is *independent* on energy (for $E_n < 10^{-11}$ a.u.): $\Gamma \sim \omega \cdot P \sim \frac{1}{\sqrt{E}} \cdot \sqrt{E}$ = const. $\quad$ ($\omega \sim t_f^{-1}$, $P \sim k$)
- The measurment of the lifetime of $\bar{\mathrm{H}}$ bouncing on the surface allows determination of the force mg, i.e. one gets access to the gravitational properties of antimatter (WEP?)

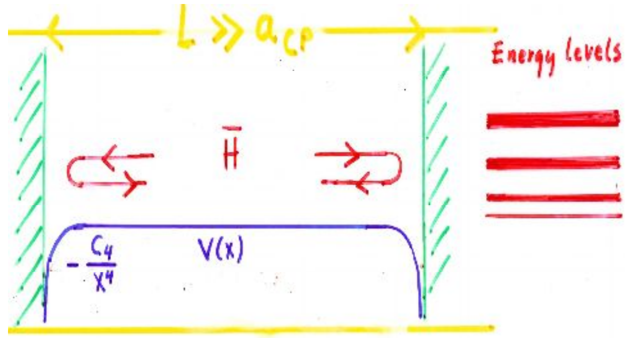
M: gravitational mass

From mayonnaise to quantum mirrors

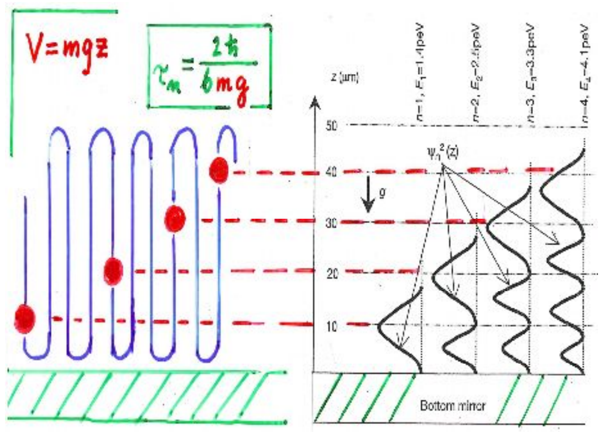
- **Interaction between antiatoms and a solid surface**
- Quantum reflection of ultracold antiatoms occurs predominantly on the Casimir tail of the dispersive atom-surface interaction
- The prospects of confining, storage and/or quiding antiatoms
- Measurments of gravitational interaction between matter and antimatter.
- Interesting offspins of interest for atom optics and nanotechnology, e.g. atom holography, atom mirrors and lenses, interferometric devices, guiding and trapping in mesoscopic devices, atomic microscope, reflection of BEC-clouds (Ketterle and Cornell, 2006), ...



Quantum reflection of antiatoms



Gravitational states of antiatoms



Conclusions on $\bar{\mathrm{H}}$ - wall interaction

- Ultracold $\bar{\mathrm{H}}$ is reflected from the wall by quantum reflection
- Annihilation "purifies" the $\bar{\mathrm{H}}$ - wall scattering making it independent on the details of the short range interaction. Reflection occurs on the tail of the Casimir potential, without contact with the surface
- The reflection probability $R = 1 - 4Im(a_{CP})$ depends on the scattering length a_{CP} determined by the tail of the Casimir interaction for the $\bar{\mathrm{H}}$-wall system
- Reflection becomes effective ($> 50\%$) for $E < 10^{-5}$ K
- Measurment of the rate of annihilation on the wall gives information about the scattering length for the Casimir potential, and might be used for probing QED
- Quantum reflection supports the existence of long-lived metastable states of $\bar{\mathrm{H}}$ confined between the walls ($\tau \sim L^3$)
- Perspectives of:
  - trapping
  - cooling
  - quiding
- The gravitational motion of $\bar{\mathrm{H}}$ bouncing above the surface is quantized. The lifetime of the quantum states is $\tau = \frac{2\hbar}{bmg} \simeq 0.1$ s, regardless the energy
- Wave packet analysis reveals "decaying revivals" in the bouncing motion
- Measurment of the liftime of $\bar{\mathrm{H}}$ bouncing on the surface might allow determination of the gravitational force between matter and antimatter, perheaps tests of WEP.