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Study of cascade processes in $p\mu^-$ and pK^- atoms based on a new approach

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Atomic formation (initial stage):

$$\{p\mu^-, pK^-\} (n_0, l_0, E_0)$$

$$n_0 \sim \sqrt{\frac{m}{m_e}}, \quad n_0^{(K)} \approx 30, \quad n_0^{(\mu)} \approx 14, \quad W \approx \frac{2l_0 + 1}{n_0^2}, \quad E_0 \sim 1 \text{ eV}.$$

Cascade processes (de-excitation stages):

$$(px^-)_{nl} + H_2 \rightarrow (px^-)_{n'l'} + H_2 \quad \left\{ \begin{array}{l} \text{elastic scattering } (n' = n, l' = l), \\ \text{Stark transitions } (n' = n, l' \neq l), \\ \text{Coulomb transitions } (n' \neq n), \end{array} \right.$$

$$x^- = K^-, \mu$$

$$(px^-)_{nl} + H_2 \rightarrow (px^-)_{n'l'} + H_2^+ + e^- - \text{Auger transitions } (n' < n),$$

$$(px^-)_{nl} \rightarrow (px^-)_{n'l'} + \gamma \quad - \quad \text{radiative transitions } (n' < n).$$

Accompanying processes:

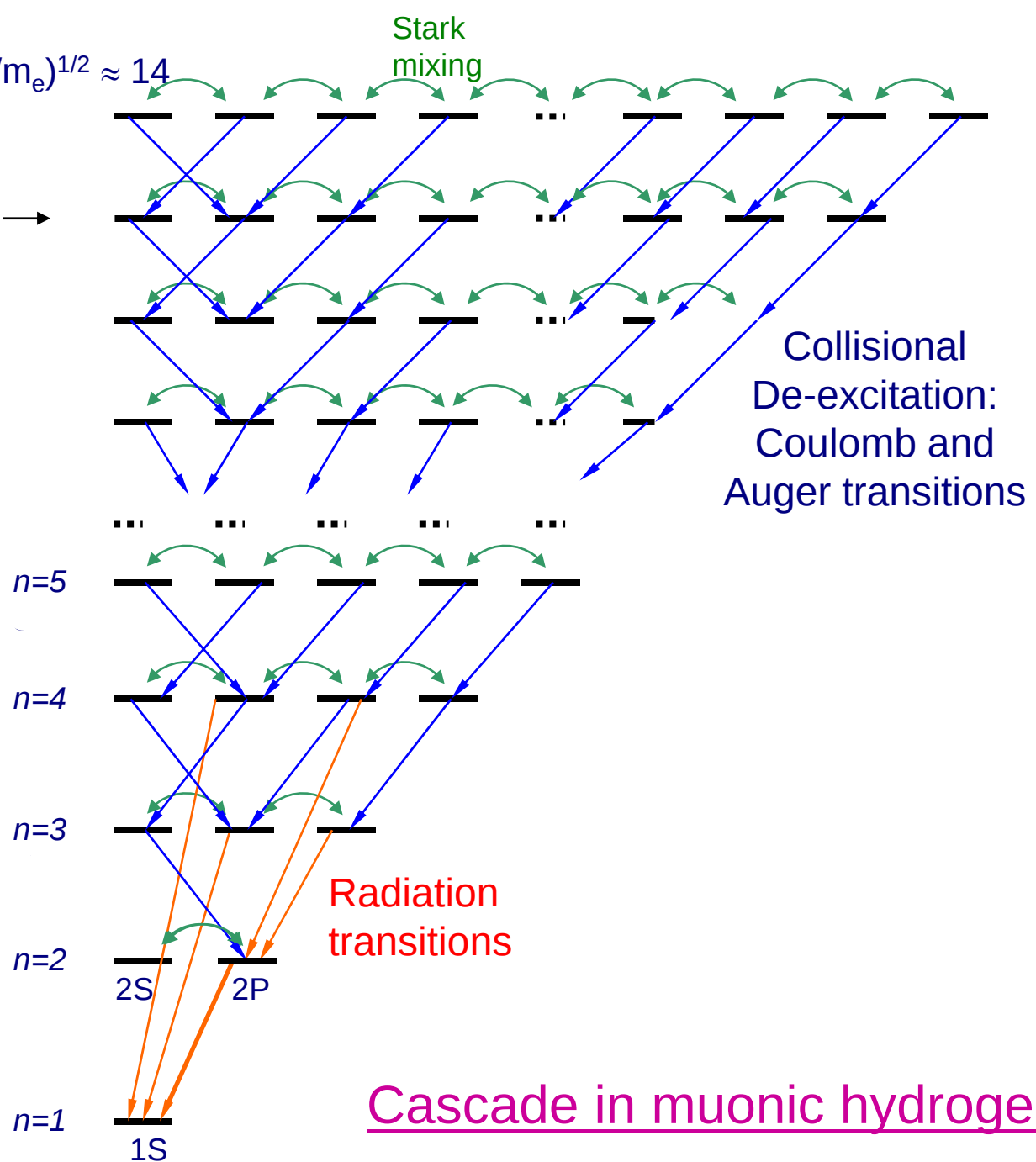
weak decay,

nuclear absorption.



$$n_{\text{ini}} \sim (m_{\mu}/m_e)^{1/2} \approx 14$$

$n l \rightarrow$
 $n' l'$



Cascade in muonic hydrogen



Mechanisms of $(px^-)_{nl}$ exotic atoms acceleration

$$(px^-)_{nl}(E) + p \rightarrow (px^-)_{n'l'}(E') + p \quad - \text{Coulomb transitions } (n' < n, E' > E),$$

$$(px^-)_{nl}(E) + H \rightarrow (px^-)_{n'l'}(E') + p + e^- - \text{Auger transitions } (n' < n, \Delta = n_1 - n_2 \geq 0),$$

$$(px^-)_{nl} + H \rightarrow C' + e^-, \quad C' \equiv [(px)_{n'l'} p] \quad - \text{Auger capture } (n' = n - 1, \Delta = n_1 - n_2 < 0),$$

$$C' \rightarrow C'' + \gamma, \quad C'' \equiv [(px)_{n''l''} p] \quad - \text{radiative deexcitation } (n' < n),$$

$$C' \rightarrow (px)_{n''l''} + p \quad - \text{predissociation } (n'' < n' = n - 1).$$



The general problem: $(px)_{nI} + H \rightarrow \text{all final states}$

The existing approaches to solve this problem:

Quantum Mechanics (QM) methods:

- four-body problem;
- multi-channel Coulomb problem ($n^2 \sim 100 \div 1000$ muonic/kaonic states);
- total and differential cross sections, the most part of which are lacking.

Classical Mechanics (CM) description:

- four planets problem (classical collisions);
- Coulomb charged planets, including the muon motion;
- natural description of multi-quantum Coulomb transitions ($\Delta = n - n' > 1$);
- possibility to take into account protons chemical binding in H_2 molecule.

Good argument for solution of the QM problem by CM methods is successful description of electron charge exchange in collisions of multi-charged ions with hydrogen atoms (R. Olson and A. Salop, 1976): differences between calculated and experimental cross-sections are about $\sim 20\%$.

Another argument is the Bohr Correspondence Principle: CM results coincide with QM ones at large n .



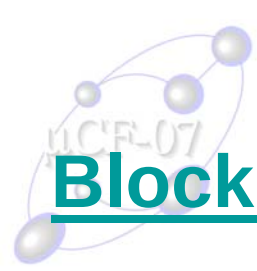
New Quantum-Classical Monte Carlo method

Proposed scheme (NQCMC) of cascade calculations:

- Radiative transitions are considered by **QM** methods;
- Collisions are considered by methods of **CM**;
- Auger processes are treated semiclassically.

The processes of Auger capture are negligible for heavy exotic atoms (e.g., pK^-), which become more and more energetic during the cascade due to multi-quantum Coulomb transitions.

How Auger processes is important for light exotic atoms $p\mu$?



Block-scheme of muonic atom cascade in hydrogen

Output:

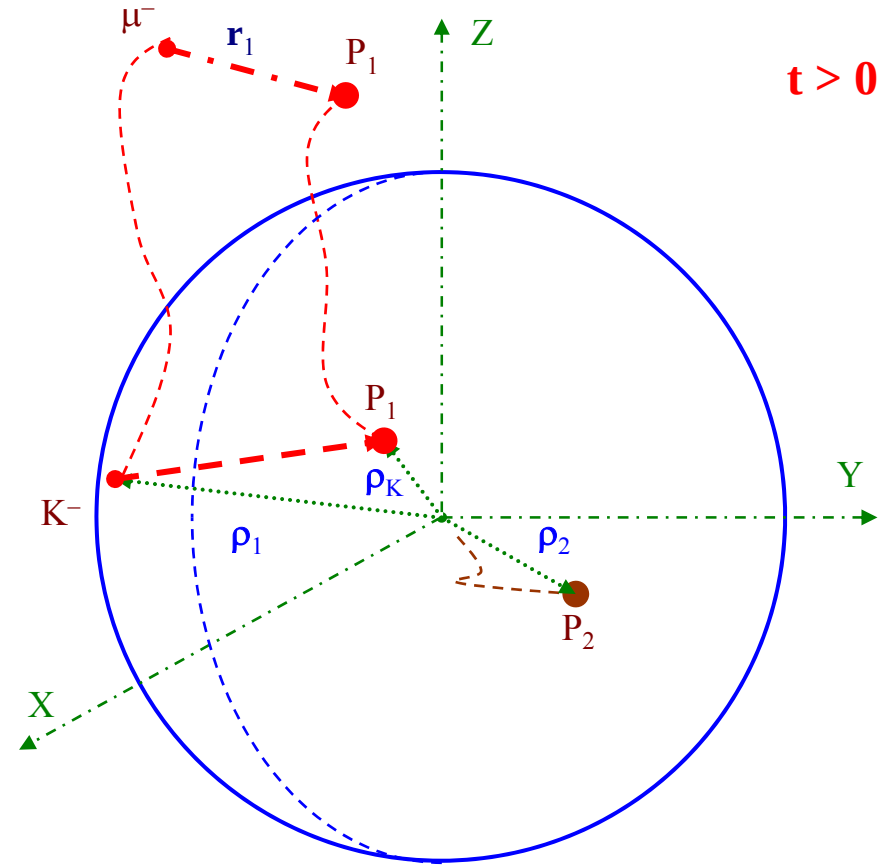
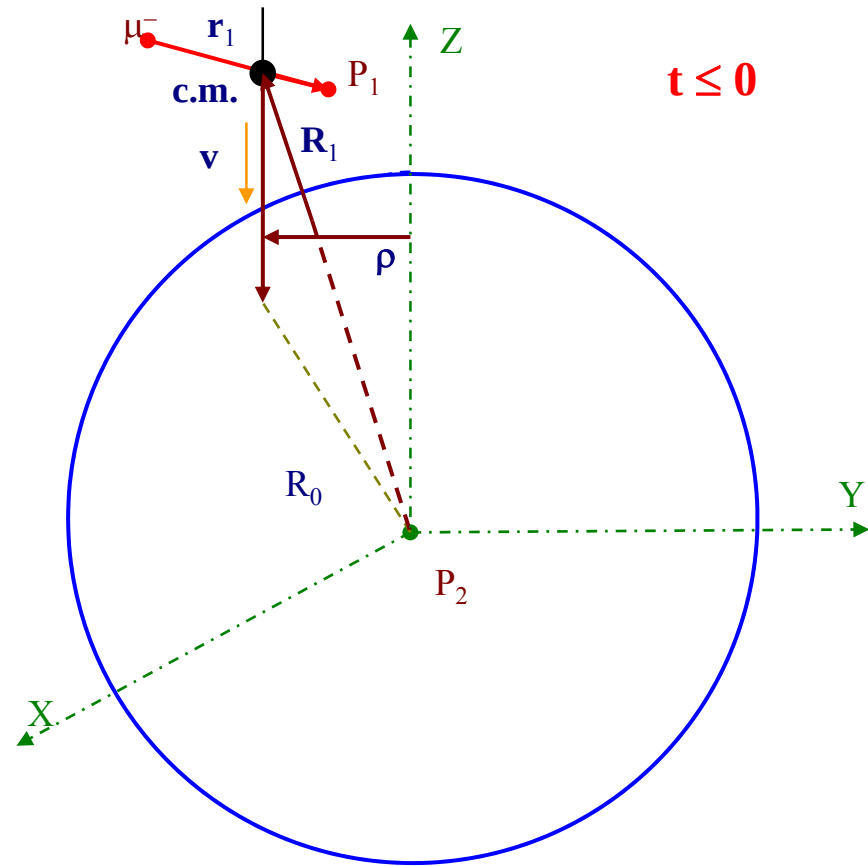
- cross-sections of Coulomb, Stark and Auger transitions;
- kinetic energy distributions;
- decay characteristics of the exotic molecular complex;
- cascade time in the exotic atom;
- Doppler broadening of the atomic $\{nl\}$ -state;
- X-ray yields.



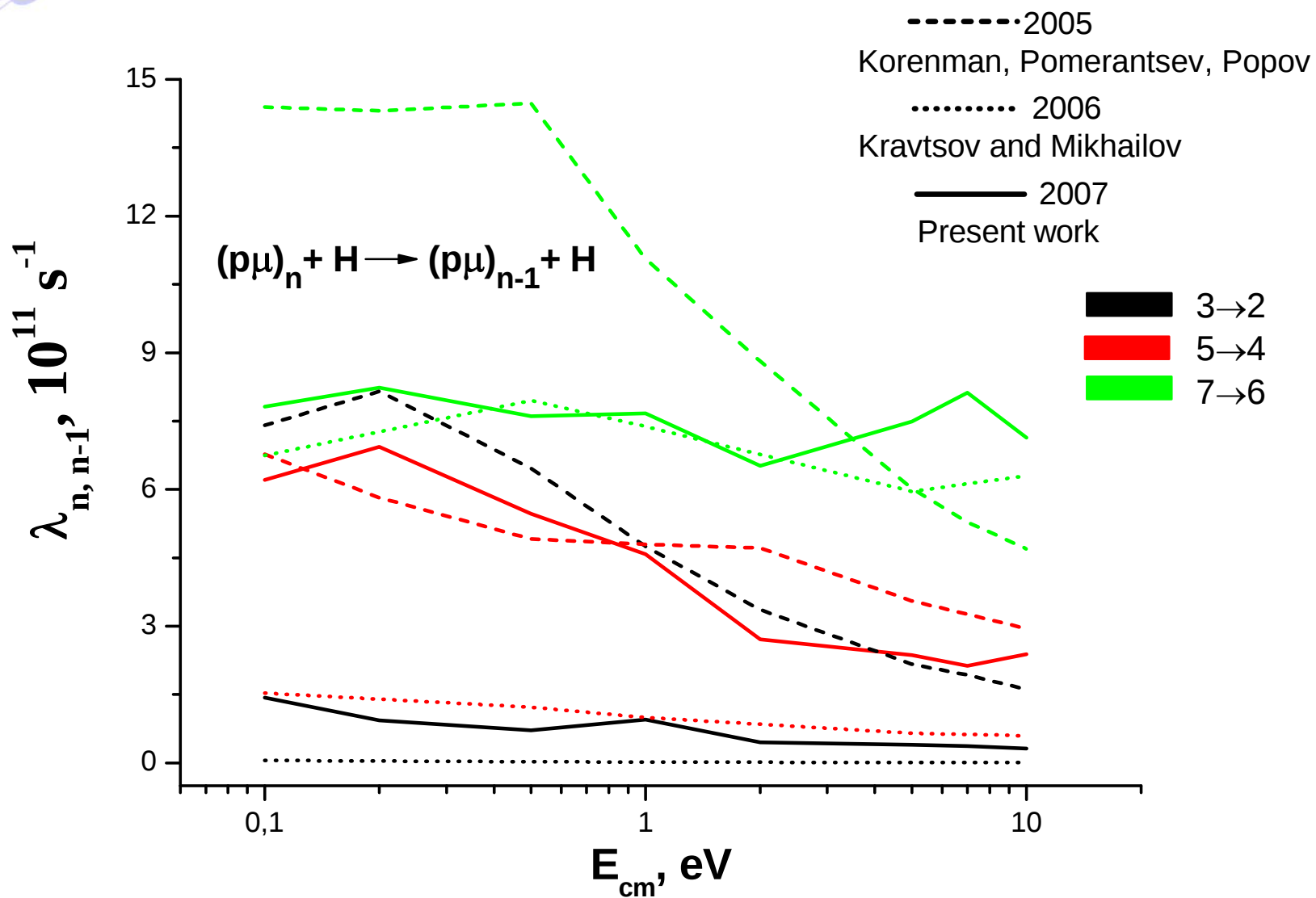
“Initial data” sphere

$\rho_1(t), \rho_2(t), \rho_K(t)$ – vector-coordinates of two protons and muon;

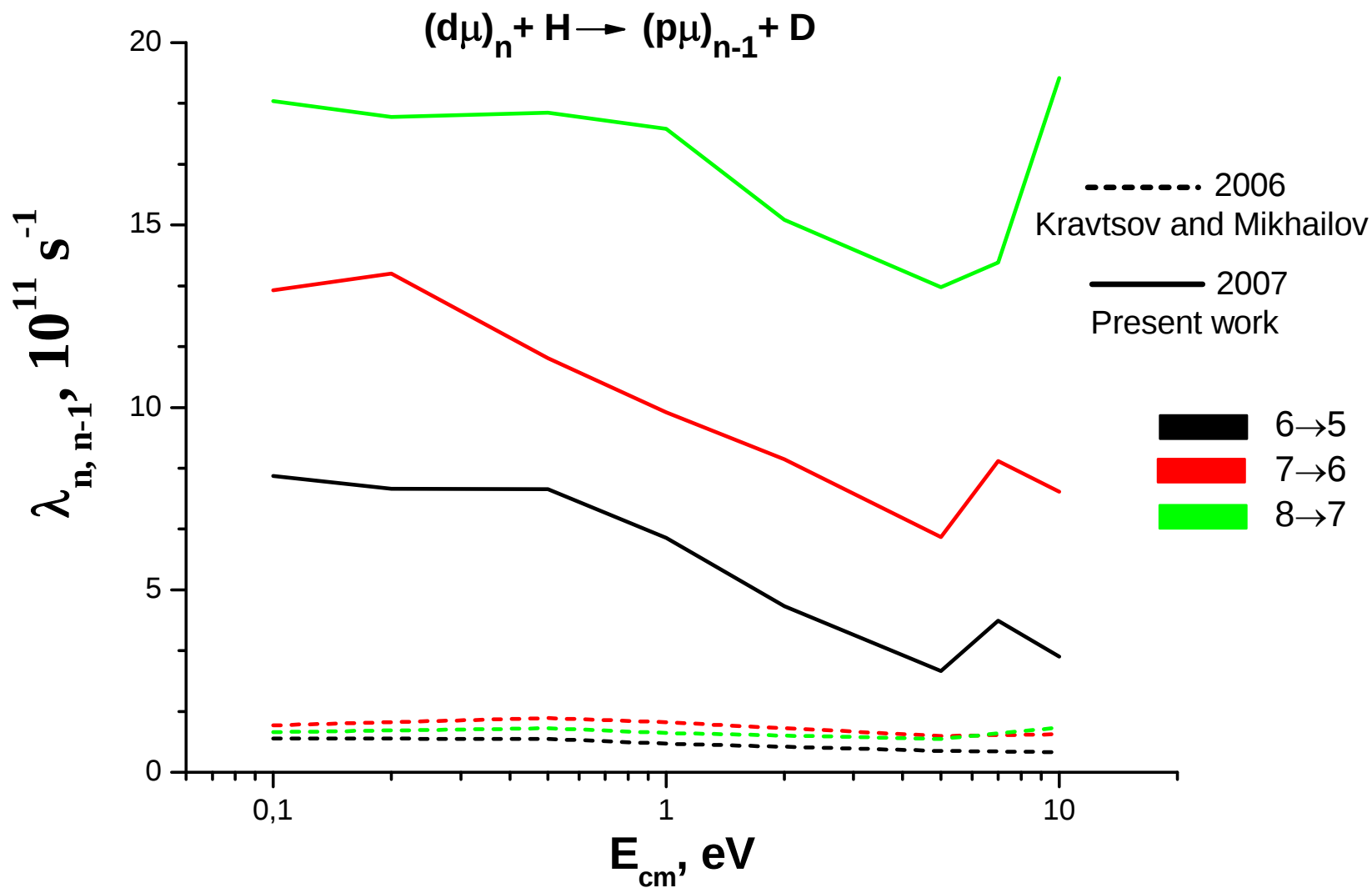
ρ – impact parameter; $\mathbf{R}_1 = \mathbf{R}_{\text{c.m.}} - \rho_2$.

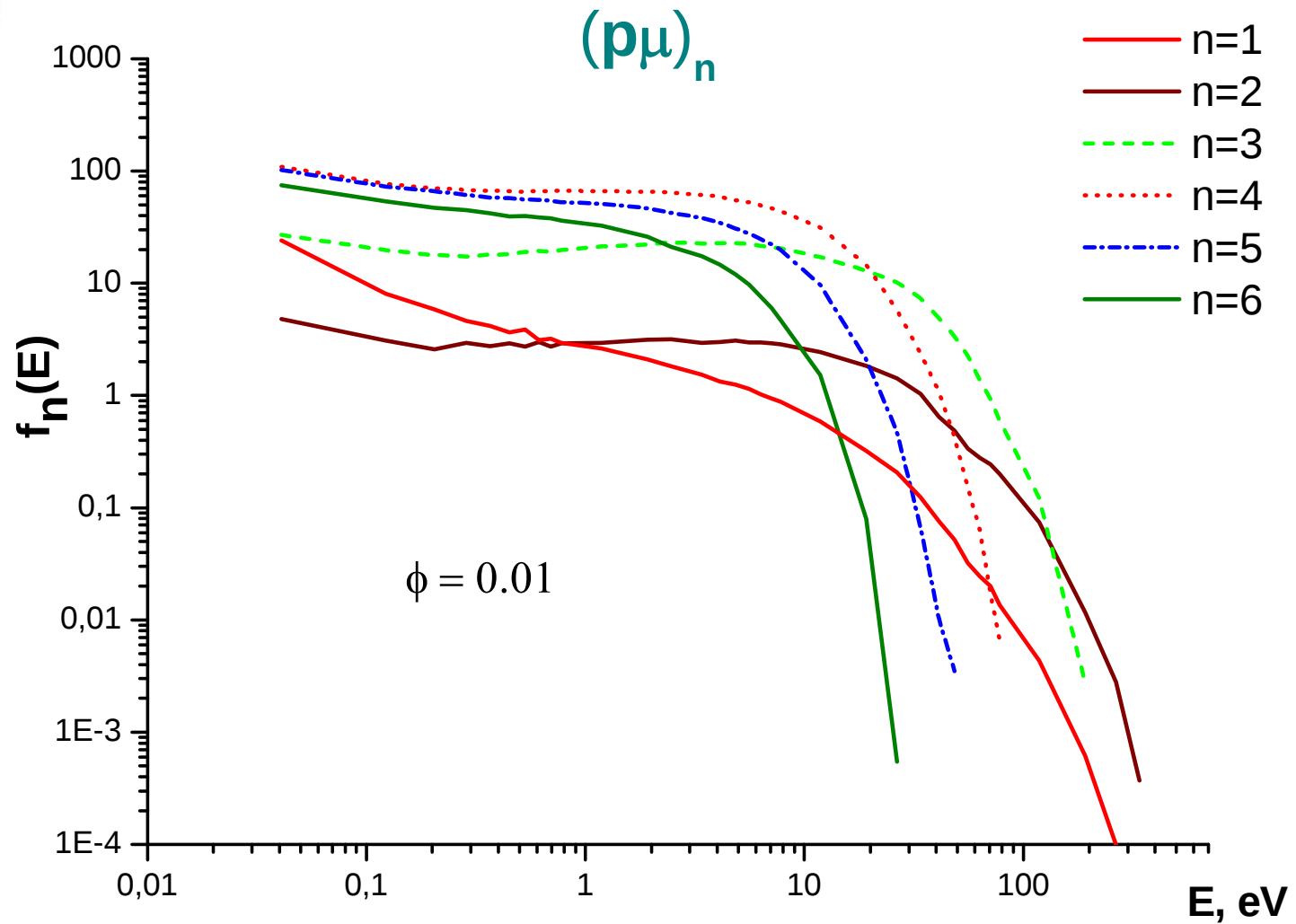


Coulomb de-excitation

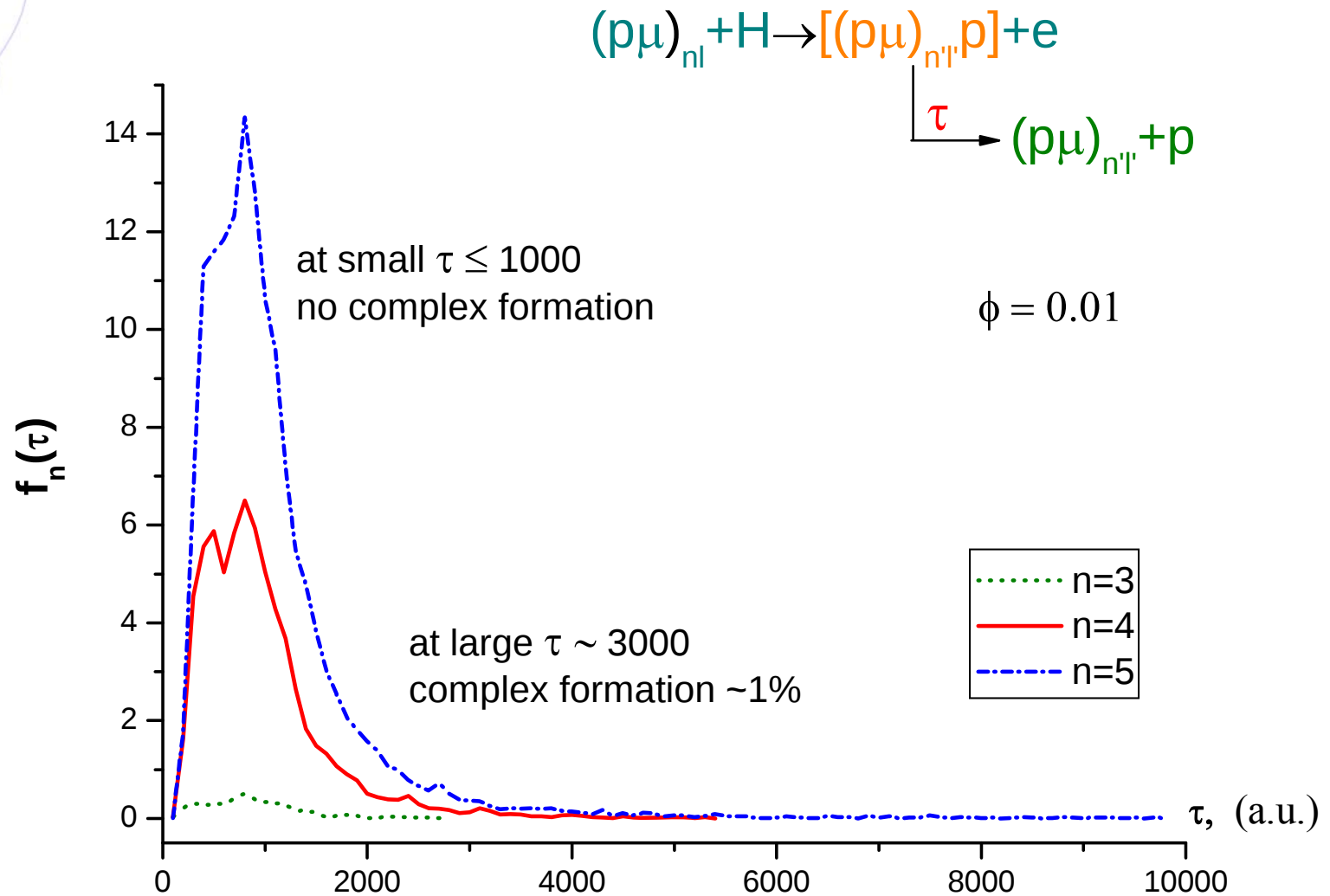


Charge exchange reaction

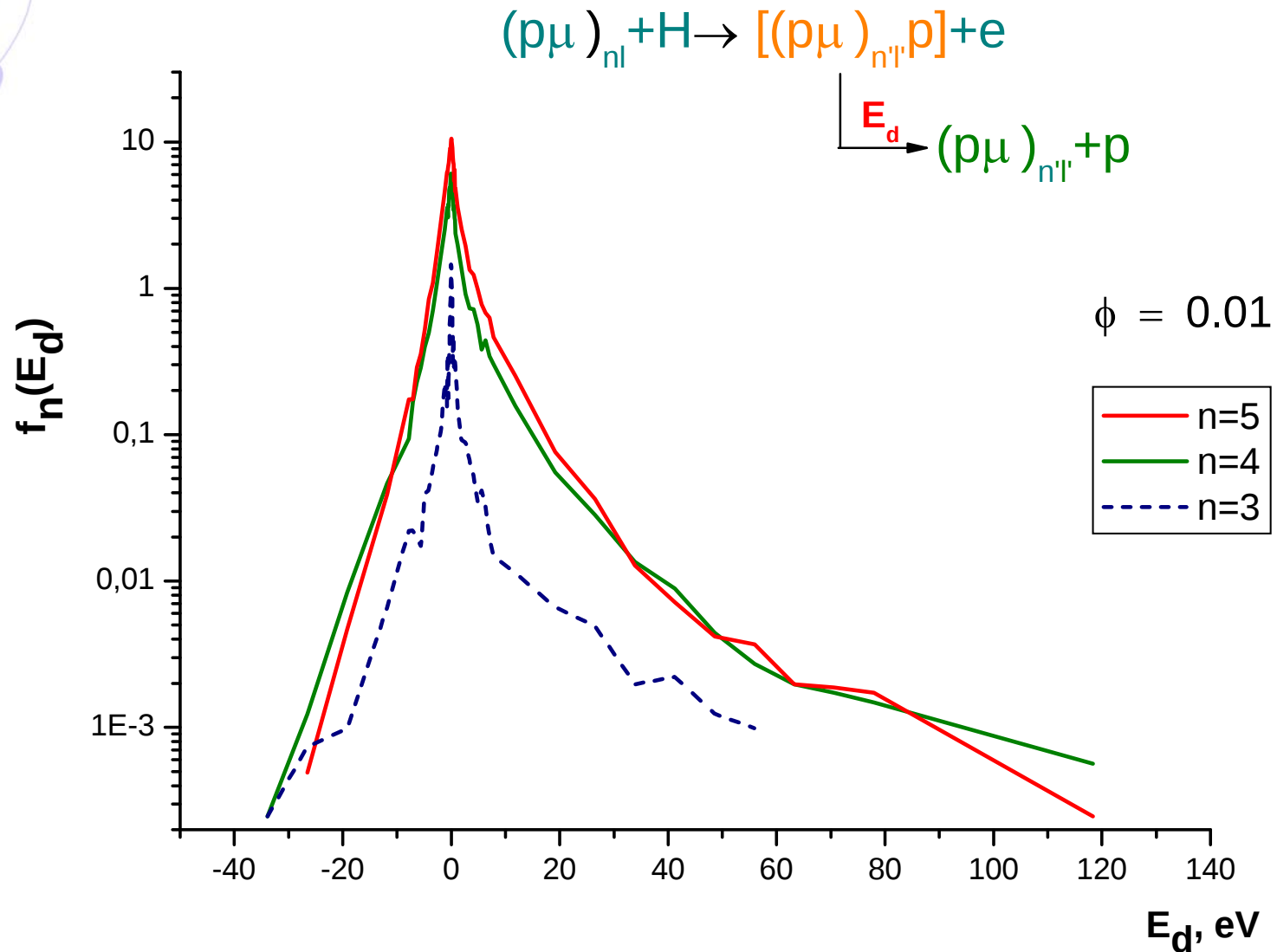




Kinetic energy distributions of $(p\mu)$ atoms
in n -state at density $\phi=0.01$ LHD.

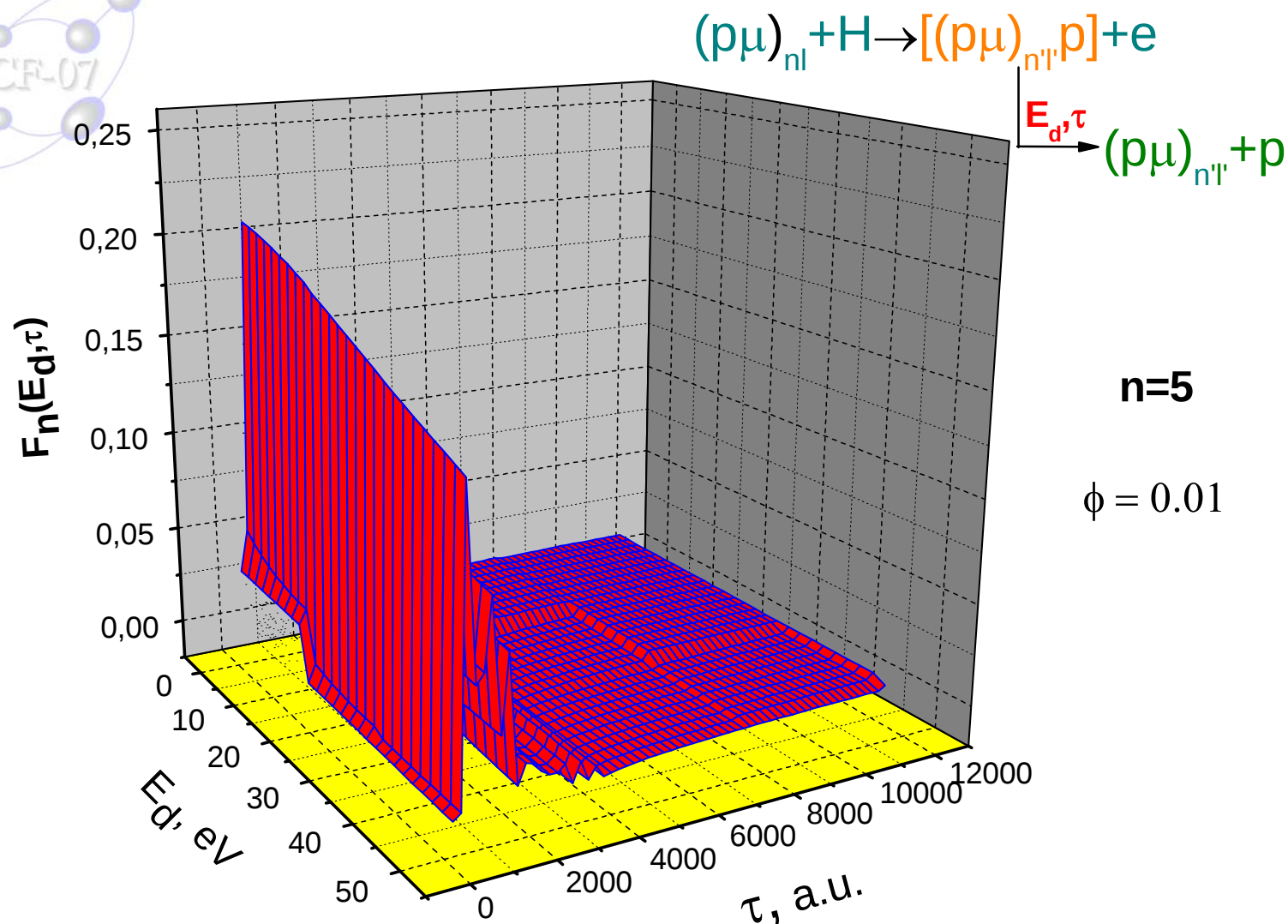
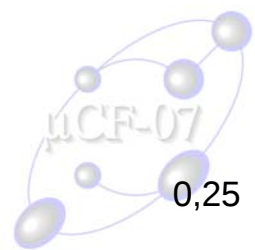


Time decay distributions of the muonic molecular complexes formed in Auger capture processes.



Distributions of $(p\mu)$ atoms over kinetic energy difference

$$E_d = E_{p\mu(nl)} - E_{p\mu(n'l')} \text{ gained after Auger process.}$$



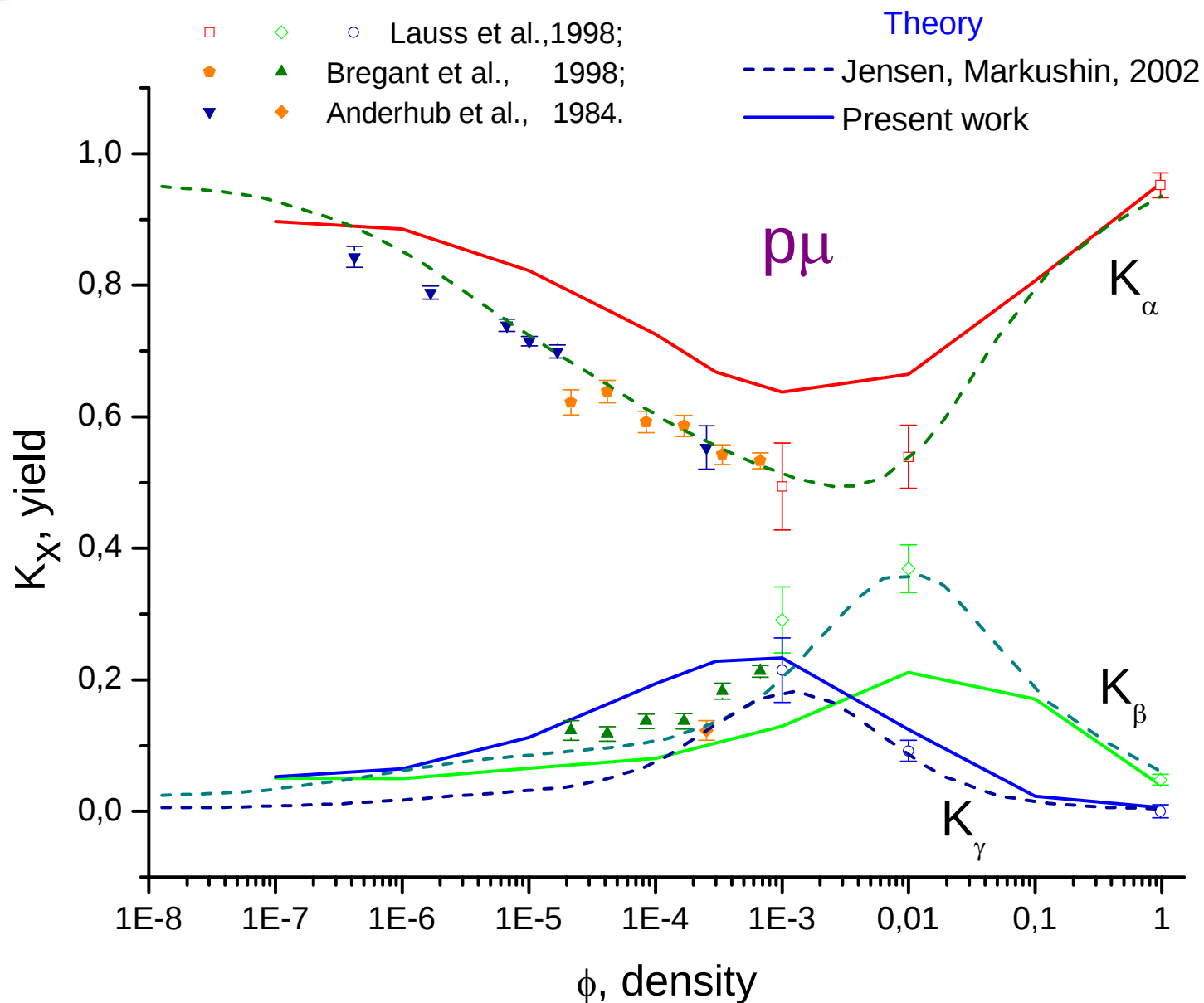
Distribution of $(p\mu)$ atoms over kinetic energy difference E_d and time decay τ after decay of the muonic molecular complex:

small τ , large E_d – Coulomb transitions;

large τ and E_d – predissociation (~1% of events)

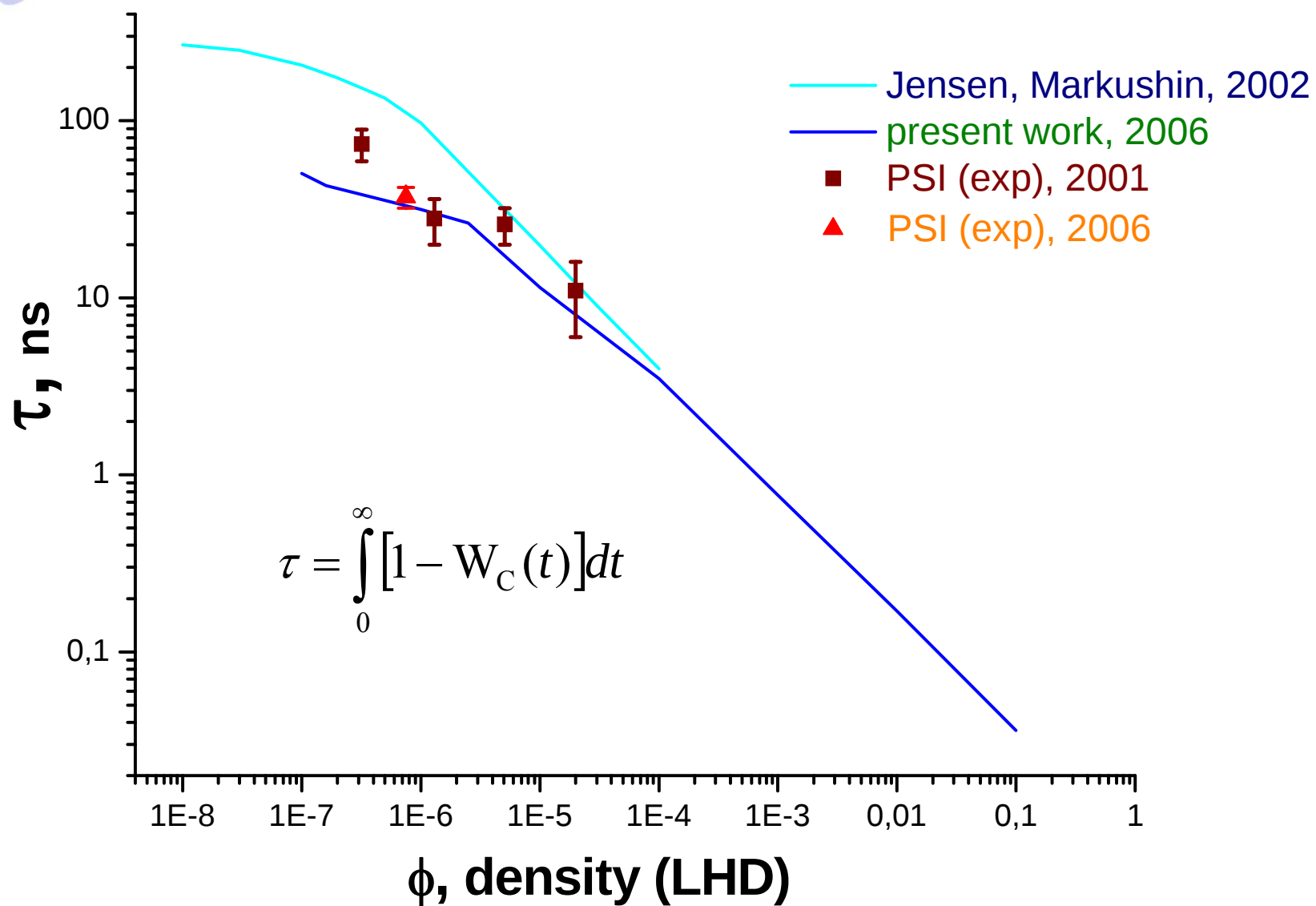


X-ray yields in the muonic hydrogen



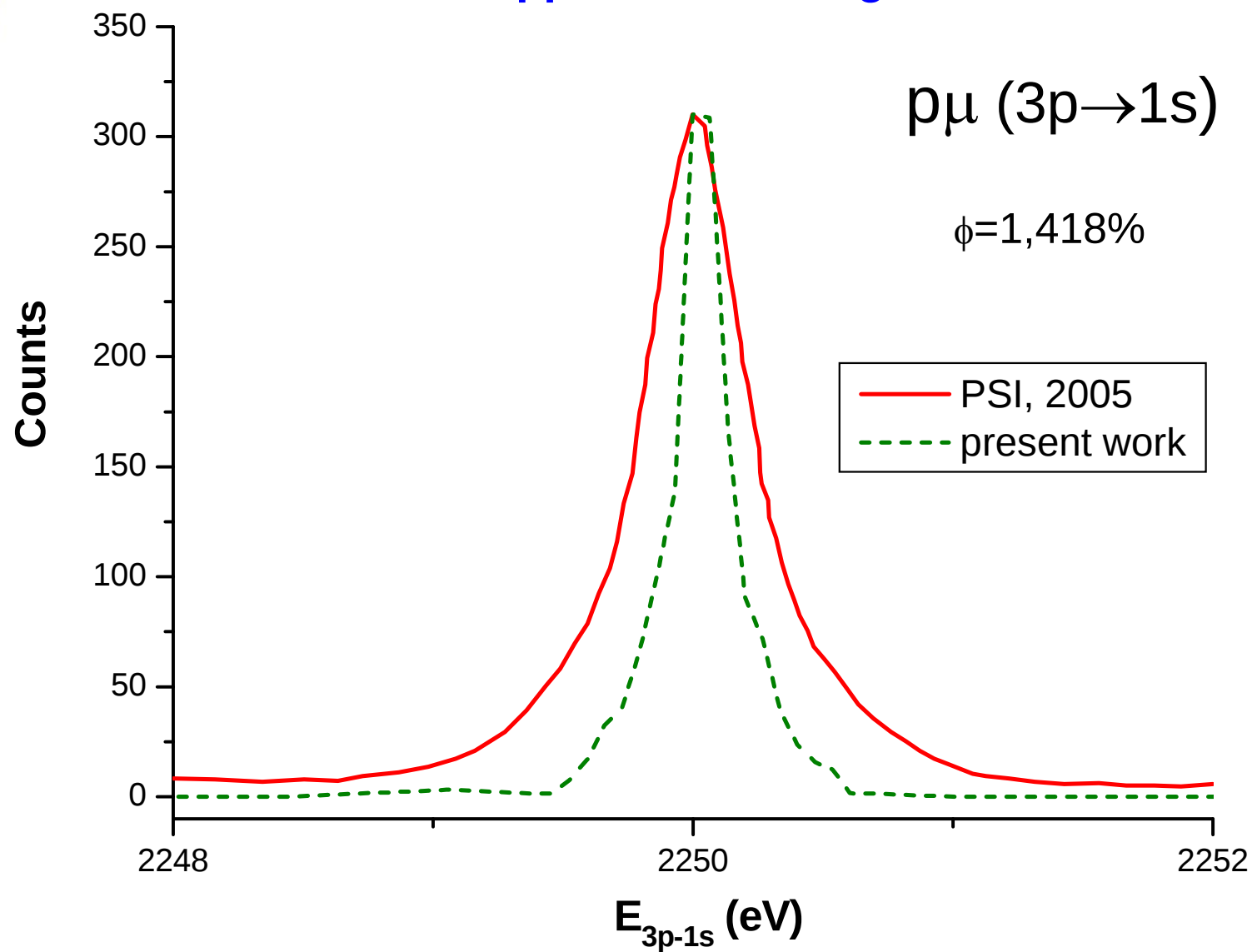


$p\mu$ atomic cascade time



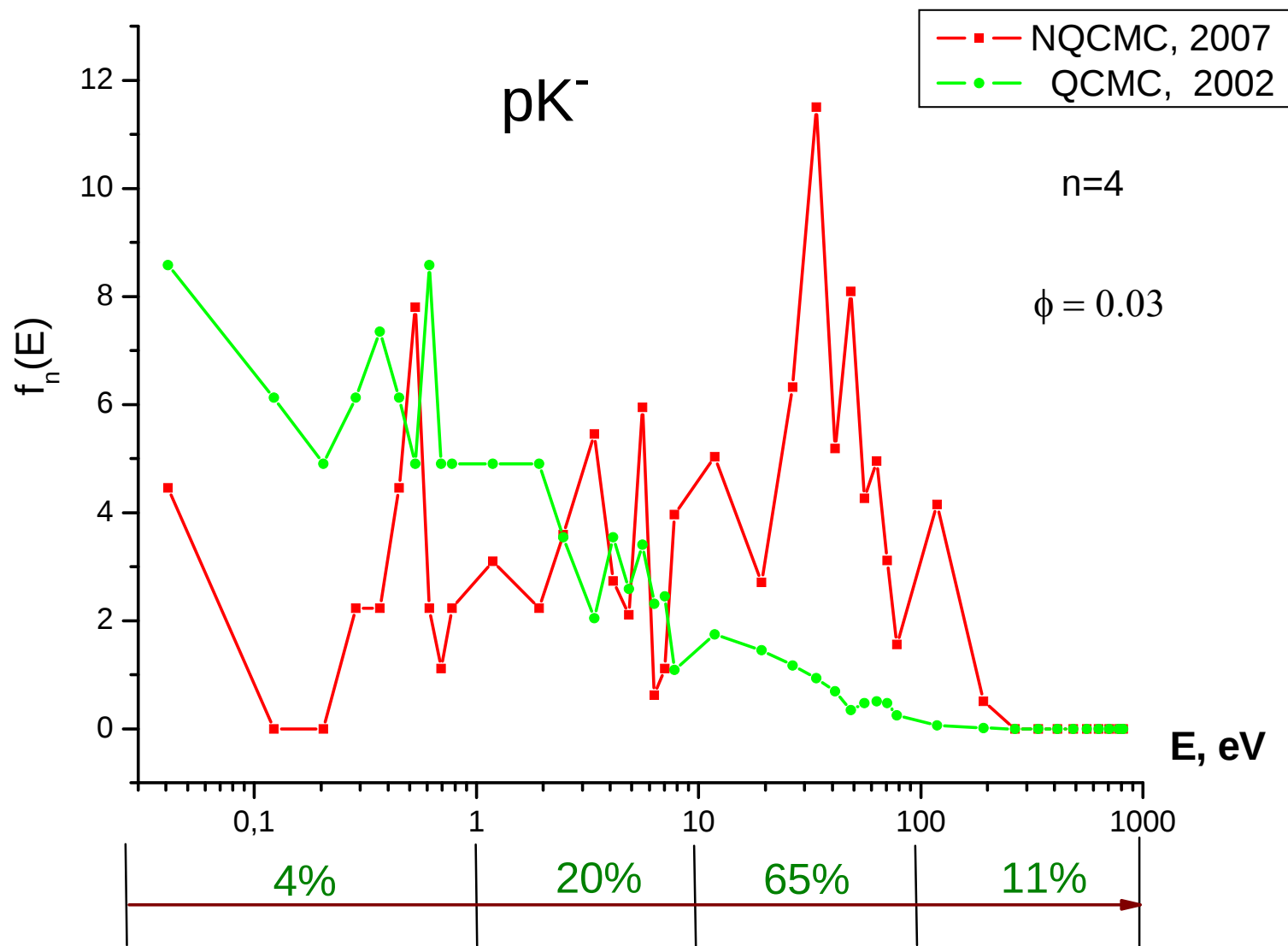


Doppler broadening

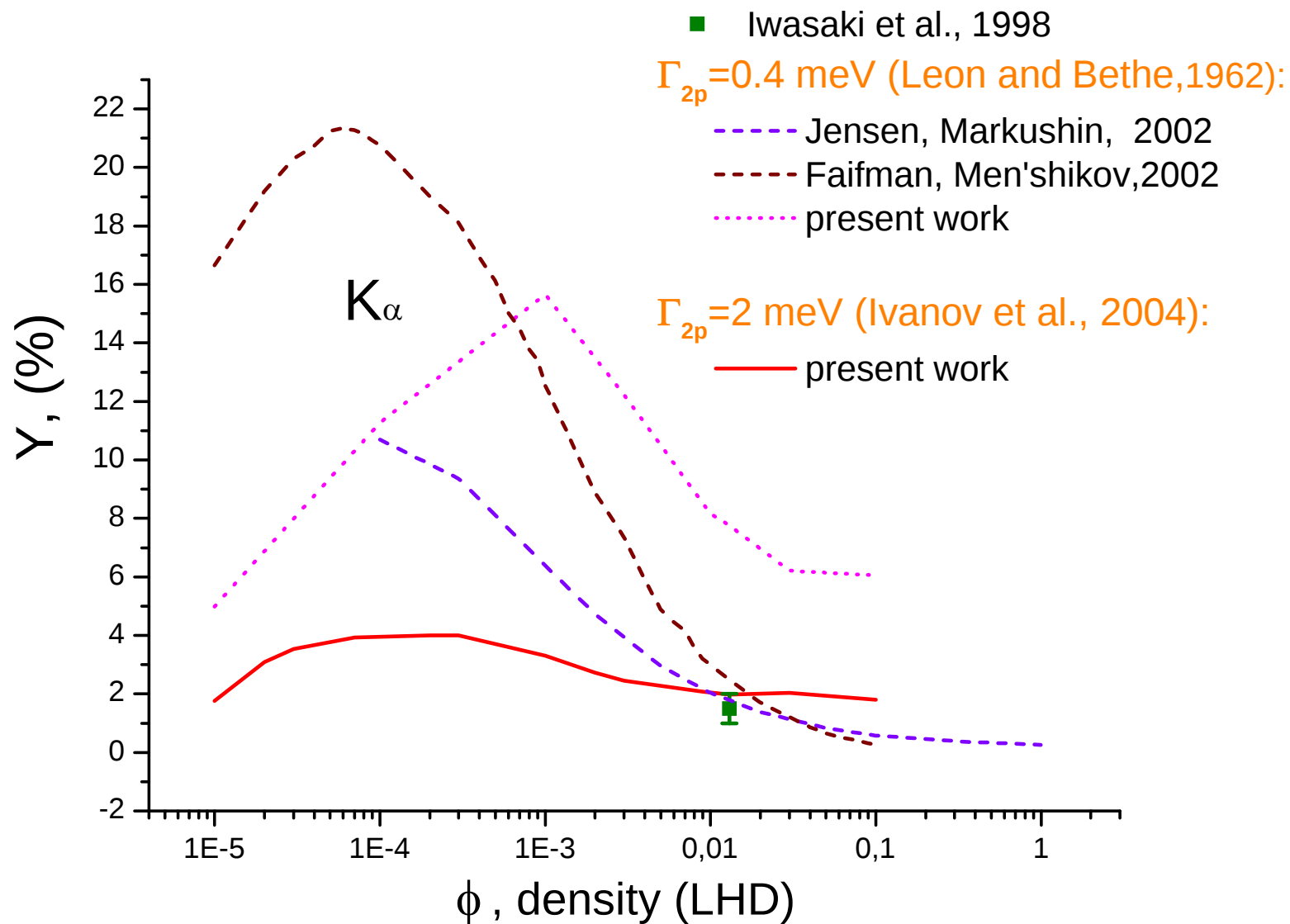




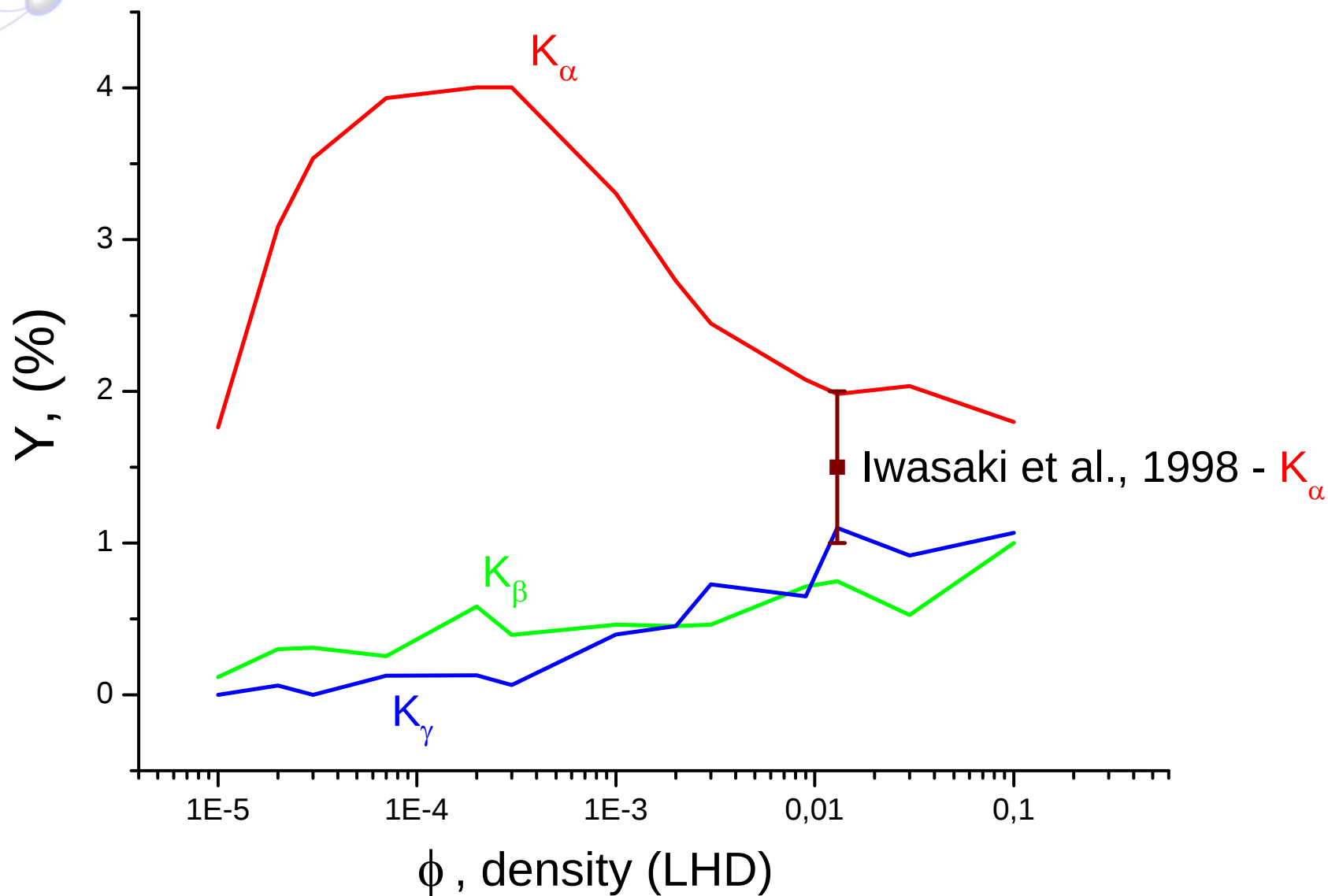
Fractions of pK^- atoms in the selected energy intervals



Calculated K_α -ray yields of pK^- atoms at $\Gamma_{2p}=0.4$ meV and $\Gamma_{2p}=2$ meV



X-ray yields of kaonic hydrogen atoms





Summary

- ❖ A new NQCMC code for *ab initio* calculations of cascade in exotic hydrogen atoms is developed.
- ❖ The analysis of the kinetics of cascade processes in muonic and kaonic hydrogen atoms leads to conclusion, which is important for simplifying the cascade calculations:
Auger acceleration is negligible for all exotic hydrogen atoms.
- ❖ The obtained results have demonstrated good agreement between theory and experiment.
- ❖ The developed code enables to carry out calculations (with sufficient accuracy ~ 30% and less) of main characteristics of cascade processes:
 - **cross-sections of Coulomb, Stark and Auger transitions;**
 - **kinetic energy distributions;**
 - **cascade time in the exotic atom;**
 - **Doppler broadening of the atomic states;**
 - **X-ray yields.**