Fundamental characteristics of elementary particles and spectroscopy of light exotic atoms

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• antiprotonic helium

magnetic moment of the antiproton, known to 0.3%

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• muonic hydrogen

Zemach radius of the proton

charge radius of the proton

pseudo-scalar coupling constant g_P

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charge radius of the proton

pseudo-scalar coupling constant g_P – muCap exp.

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Zemach radius of the proton

charge radius of the proton – Lamb shift exp.

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Zemach radius of the proton

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Zemach radius of the proton

Difficult spectroscopy: very weak M1-transitions, \rightarrow much power required

Other difficulties:



HF transitions in antiprotonic ⁴He Sensitivity S to variations of μ_p in (35,1)

E₁= 6.6113 GHz; $\delta E_1/\delta \mu_p = 28.2$ MHz E₂= 6.4784 GHz; $\delta E_2/\delta \mu_p = -30.2$ MHz E₃=-6.2846 GHz; $\delta E_3/\delta \mu_p = 36.2$ MHz E₄=-6.4456 GHz; $\delta E_4/\delta \mu_p = -32.5$ MHz

$$\begin{split} \nu_{s_{-}} &= 132.94 \text{ MHz;} \quad \delta\nu_{s_{-}}/\delta\mu_{p} = -58.5 \text{ MHz,} \\ \nu_{s_{+}} &= 160.93 \text{ MHz;} \quad \delta\nu_{s_{+}}/\delta\mu_{p} = -68.5 \text{ MHz,} \\ \nu_{m_{-}} &= 12923.96 \text{ MHz;} \quad \delta\nu_{m_{-}}/\delta\mu_{p} = -8.0 \text{ MHz} \\ \nu_{m_{+}} &= 12895.98 \text{ MHz;} \quad \delta\nu_{m_{+}}/\delta\mu_{p} = 2.0 \text{ MHz} \\ \nu_{s_{\circ}} &= 13056.90 \text{ MHz;} \quad \delta\nu_{s_{\circ}}/\delta\mu_{p} = -66.5 \text{ MHz,} \end{split}$$



~4.10-4

HF structure of antiprotonic ⁴He

Effective spin Hamiltonian of ⁴Hep⁻

$$\begin{aligned} H^{eff} &= H_1(\vec{s}_e.\vec{J}) + H_2(\vec{s}_{\overline{p}}.\vec{J}) + H_3(\vec{s}_{\overline{p}}.\vec{s}_e) \\ &+ H_4\Big(3(\vec{s}_{\overline{p}}.\vec{J})(\vec{s}_e.\vec{J}) - J(J+1)(\vec{s}_{\overline{p}}.\vec{s}_e)\Big) \end{aligned}$$

Analysis of the theoretical uncertainty and the sensitivity of the transition frequencies to μ_p required.

At present, $\Delta_q H_i \sim 10^{-4}$ (where $\Delta x = \delta x/x$) Response $R = \Delta_q v / \Delta_q H_i$, of the transition frequencies to variations of H_i : $\Delta v = \max R.|\Delta H|$

i	S	\mathbf{S}_+	m_	m ₊	m_0	Х
1	-0.031	-0.025	0.995	1.000	0.988	0.000
2	1.125	0.929	0.000	0.000	0.011	0.000
3	1.141	-0.903	-0.011	0.012	0.000	-10.61
4	-1.235	0.999	0.013	-0.012	0.000	11.61

Most appropriate HF lines in (33,1)^a and (35,3)^b

HF transition	S_	\mathbf{S}_+	m_	m ₊	m ₀	X
Upper limit on the	0.90	0.72	0.06	0.12	0.84	0.18
experimental error (a)						
accuracy gain in μ_p	35.3	36.9	0.0	0.1	0.6	5.0
experimental error (b)	0.27	0.54	0.16	0.11	0.42	0.27
accuracy gain	40.6	34.4	0.1	0.1	0.1	8.9



- 5(+1) allowed M1-transitions:
 - 1: $v_{s_{-}} = (E_1 E_2)/h$
 - 2: $\nu_{s_+} = (E_3 E_4)/h$
 - 3: $\nu_{m_{-}} = (E_1 E_3)/h$
 - 4: $v_{m_+} = (E_2 E_4)/h$
 - 5: $v_{so} = (E_1 E_4)/h$

 $6: \boldsymbol{\nu}_{\mathrm{X}} = \boldsymbol{\nu}_{\mathrm{m_{-}}} - \boldsymbol{\nu}_{\mathrm{m_{+}}}$

Tripple laser-VHFlaser resonance

• First laser pulse with λ =726 nm and width 100 MHz. The laser is detuned from the center of F₋ and depopulates predominantly E1



Tripple laser-VHFlaser resonance

• Level E1 of the doublet F_ is depopulated.



Tripple laser-VHFlaser resonance

• VHF-stimulated M1 transitions of frequency v_{s-} fill E1 from E2 for 160 ns.



Tripple laser-VHFlaser resonance

• E1 is partially refilled, while E2 is partially depopulated



Tripple laser-VHFlaser resonance

 Second laser pulse with λ=726 nm and width 100 MHz. The annihilation rate depends on the rate of refilling E1









Asymmetric depopulation rate ratio q:					120%		150%		
(L,v)→(L',v')	λ	Γ_{c}	Γ_d	d	q_x	D	f	D	f
(34,2)→(33,2)	470	24	499	57	1.6	136	.84	359	.29
(33,1)→(32,1)	372	9	630	75	1.9	146	.87	375	.39
(35,1)→(34.3)	726	75	323	26	1.2	247	.40		

Muonic hydrogen atom Theory



General expression of the hfs of hydrogenlike atoms $\Delta E = \Delta E^{F} (1 + \delta^{QED} + \delta^{str})$

- $\delta^{\text{QED}} = a_e + \alpha^2 (\ln 2 1) + \dots$
- $\delta^{\text{str}} = \delta^{\text{rigid}} + \delta^{\text{pol}} + \delta^{\text{hvp}} + \dots$
- $\delta^{\text{rigid}} = \delta^{\text{Zemach}} + \delta^{\text{recoil}}$
- $\delta^{\text{Zemach}} = -2\alpha m R_p + O(\alpha^2)$

Muonic hydrogen atom Theory



The quantities in red depend on the proton radius R_p : $\Delta E = \Delta E^F (1 + \delta^{QED} + \delta^{str})$

•
$$\delta^{\text{QED}} = a_e + \alpha^2 (\ln 2 - 1) + \dots$$

•
$$\delta^{\text{str}} = \delta^{\text{rigid}} + \delta^{\text{pol}} + \delta^{\text{hvp}} + \dots$$

- $\delta^{\text{rigid}} = \delta^{\text{Zemach}} + \delta^{\text{recoil}}$
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Proton Zemach radius

• First moment of the convolution of the charge and magnetic form factors

$$R_{p} = -\frac{1}{\pi^{2}} \int \frac{d^{3}p}{(p^{2} + \alpha^{2}m^{2})^{2}} \left(\frac{1}{\mu_{p}} G_{E}(-p^{2}) G_{M}(-p^{2}) - 1 \right)$$

• Depends on both G_E and G_M !

Why muonic hydrogen?

Zemach radius from the HFS of H

$$\begin{split} &R_p = 1.037(16) \text{ (Dupays et al.- known } \delta^{\text{pol}}, \delta^{\text{QED}} \text{)} \\ &R_p = 1.047(19) \text{ (Volotka et al. - similar } \delta^{\text{QED}} \text{)} \\ &R_p = 1.013(18) \text{ (Brodsky et al. - muonium)} \\ &R_p = 1.086(12) \text{ (Friar & Sick - elastic scattering)} \end{split}$$

HFS of muonic hydrogen atoms Comparison of H with μ⁻p

	hydr	ogen	Muonic hydrogen		
E ^{Fermi}	1.4 GHz	$\pm 10^{-8}$	0.182 eV	$\pm 10^{-7}$	
ΔE^{Zemach}	39×10 ⁻⁶	$\pm 2 \times 10^{-6}$	7.5×10 ⁻³	$\pm 10^{-4}$	
ΔE^{recoil}	6×10 ⁻⁶	$\pm 10^{-8}$	1.7×10 ⁻³	$\pm 10^{-6}(*)$	
ΔE^{pol}	1.4×10 ⁻⁶	$\pm .6 \times 10^{-6}$.46×10 ⁻³	$\pm .8 \times 10^{-4}$	
ΔE^{hvp}	10-8	$\pm 10^{-9}$	2×10 ⁻⁵	$\pm 2 \times 10^{-6}$	
ΔE^{QED}	1.1×10 ⁻³	±10 ⁻⁹	~10 ⁻³	$\pm 10^{-6}(*)$	

Measuring the HFS of $(\mu^{-}p)_{1s}$

Muon exchange



 $(\mu p) + Z --> p + (\mu Z)$

Measuring the HFS of $(\mu^{-}p)_{1s}$

Energy dependence of the muon exchange rate λ (general case)



Measuring the HFS of $(\mu^{-}p)_{1s}$

Energy dependence of the muon exchange rate λ (resonance case)



Muon exchange rate (theory)

Hyper-radial approach

- Diabatic per sector representation
- No Coriolis terms
- Higher partial waves taken into account

Muon exchange rate (theory)

Hyper-radial approach

- Diabatic per sector representation
- No Coriolis terms
- Higher partial waves taken into account
- Calculated: rates of exchange with O, Ne and C for a broad range of collision energies
- Good agreement with the few experimental points

Measuring the HFS of $(\mu^{-}p)_{1s}$ Rates of muon exchange with O and Ne, calculated in the hyper-radial approach



Muon exchange rate (theory)

For O: the step-like behaviour not reproduced!

- Accounting for the molecular excitations in O₂ in progress
- Accounting for the dominant Fermi spin-spin interaction of the muon with the proton in preparation
- More gases to be studied

Measuring the HFS of $(\mu p)_{1s}$



Monte Carlo simmulations of the experiment

- Good efficiency
- Successful optimization

Measuring the HFS of $(p\mu p)_{10}$

- Two Coulomb states of ppµ ground (para) with L=0, excited (ortho) with L=1.
- Formed in L=1.
- Ortho-para transitions in 20% of the molecules, detectable by the X-rays



Measuring the HFS of $(p\mu p)_{10}$

- Ortho-para from s=1/2 only.
- Laser-stimulated transitions $s=1/2 \leftrightarrow s=3/2$ reduce the observable ortho-para rate.
- Tunable lasers with λ~9µm
 available
- High efficieny, low Doppler broadening



Measuring the HFS of $(p\mu p)_{10}$?

- More accurate experimental value of ΔE^{hfs} expected compared to (µ⁻p)
- Challenge for theory: Accuracy of 10⁻⁵ or better required