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LOW ENERGY MAGNONS IN ANTIFERROMAGNETIC SEMICONDUCTORS

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Abstract. A reduced *s-f* Hamiltonian is introduced to describe the low energy acoustic magnons in two-sublattice antiferromagnetic semiconductor. A mean-field approximation has been constructed using the irreducible Green functions. The contribution of the conduction electrons to the energy and the damping of the acoustic antiferromagnetic magnons have been evaluated.

Резюме. Для описания низкоэнергетических акустических магнонов в двуподрешеточном антиферромагнитном полупроводнике введен редуцированный *s-f* гамильтониан. При помощи неприводимых функций Грина построено приближение среднего поля. Оценен вклад электронов проводимости в энергию и затухание акустических антиферромагнитных магнонов.

1. Introduction

The theoretical study of the spin wave spectrum within the framework of the exchange *s-f* model for the magnetic semiconductors [1] is an important problem in the study of magnetic materials from the non-Heisenberg type. A distinct feature of the magnetic semiconductors is the presence of free carriers in the conduction band and localized magnetic moments in the lattice sites, which define the magnetic properties of the undoped material. The role of the conduction electrons is reduced to inducing indirect exchange interaction between the magnetic ions which produces an additional temperature dependence in the spectrum and the damping of the magnons.

The detailed study of the magnon spectrum in ferromagnetic semiconductors has been reported [2—6]. It is shown that the conduction electron subsystem acquires a magnetic moment under the effect of the local spins and plays the role of a second magnetic lattice. In this way the magnon spectrum of a ferromagnetic semiconductor includes parallel to the acoustic magnons also an optical mode and Stoner continuum [3—6]. The role of inelastic electron-magnon interaction processes on magnon lifetime has been studied [4, 6]. The mechanism of damping due to the electron-hole magnon decay with spin flip has been examined in detail [3, 4].

On the other hand, the magnon spectrum and the damping in antiferromagnetic semiconductors is practically lacking in the literature. The effect of weak *s-f* exchange interaction on the magnon dispersion in non-degenerate and degenerate antiferromagnetic semiconductors has been reported [7, 8]. A decoupling procedure has been applied to the equations of motion for the spin Green function in boson representation, the resulting dispersion law of the acoustic spin waves is not linear in the long wavelength region.

In the present work we consider the low energy magnons in a two sublattice antiferromagnetic semiconductor using the irreducible Green function [6, 9]. A reduced Hamiltonian is introduced for the exchange *s-f* model for the antiferromagnetic semiconductor, which provides the possibility to determine correctly the acoustic spin wave spectrum. Within the framework of mean field approximation we analyse the spectrum and the damping of the low energy magnons in a two sublattice antiferromagnetic semiconductor.

2. Model

The exchange *s-f* model is the generally accepted model for an antiferromagnetic semiconductor [10, 11]. The magnetic moments formed for example in EuTe by the 4f electrons are ordered in two interpenetrating sublattices *a* and *b*. Each lattice is ferromagnetic but the full magnetic moment of the semiconductor is zero at any temperature. As a rule, due to the strong 4f electron delocalization, the system of magnetic moments at the sites of the two sublattices is described by the Heisenberg Hamiltonian

$$H_f = \sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}a} \cdot \mathbf{S}_{-\mathbf{q}b}, \tag{1}$$

where the exchange integral $J_{\mathbf{q}}$ gives the interaction between two spins from the different sublattices in the near neighbour approximation.

The second subsystem of the antiferromagnetic semiconductor namely the conduction electrons is usually described by the Hamiltonian

$$\mathbf{H}_s = \sum_{\mathbf{k}\sigma} t_{\mathbf{k}} \left[a^+(\mathbf{k}\sigma)\, b\,(\mathbf{k}\sigma) + b^+(\mathbf{k}\sigma)\, a\,(\mathbf{k}\sigma)\right] \tag{2}$$

and these electrons are considered as *s*-electrons. Here the operators $a^+(\mathbf{k}\sigma)$, $a(\mathbf{k}\sigma)$ and $b^+(\mathbf{k}\sigma)$, $b(\mathbf{k}\sigma)$ create or annihilate an electron with a wave vector $\mathbf{k}$ and spin σ in sublattice *a* or *b*, respectively. The form of Hamiltonian (2) proves that the electron motion in a given sublattice is realized through the second sublattice of the antiferromagnetic semiconductor.

The conduction electrons and the local magnetic moments are related through the local spin-spin interaction

$$H_{s-f} = -\frac{I}{\sqrt{N}} \sum_{\mathbf{kq}\sigma} \sum_{\alpha=a,b} \{S^{-\sigma}_{-\mathbf{q}\alpha}\, \alpha^+(\mathbf{k}\sigma)\, \alpha\,(\mathbf{k}+\mathbf{q},\ -\sigma) + z_\sigma S^z_{-\mathbf{q}\alpha}\, \alpha^+(\mathbf{k}\sigma)\, \alpha\,(\mathbf{k}+q,\ \sigma)\}, \tag{3}$$

where the exchange interaction I has a typical value of 0.1 eV for EuTe [1]. Thus the total Hamiltonian of the two sublattice antiferromagnetic semiconductor is defined by the sum

$$H = H_f + H_s + H_{s-f}. \tag{4}$$

As shown elsewhere [11] using (4) in a wideband antiferromagnetic semiconductor two quasiparticle bands are formed with a dispersion law (for each electron spin projection)

$$\varepsilon_{ki} = \pm\tau_k = \pm\sqrt{t_k^2 + I^2 S_z^2}, \tag{5}$$

where S_z is the sublattice magnetization. It is supposed from the two degenerate bands $t_k = -(W/2)\gamma_k$ (γ_k is the structure factor) that the width is $W > IS_z$ and the structure is simple cubic. Since in the ordered state there is always a gap between the quasiparticle electron bands (5) only the lower energy band will be partially filled for any reasonable doping of the semiconductor. For this reason when considering the low energy acoustic magnons the electron interband transitions may be neglected and the spin wave dynamics may be described by the reduced Hamiltonian

$$\tilde{H} = H_f + \tilde{H}_s = \tilde{H}_{s-f}, \tag{6}$$

where

$$\tilde{H}_s = -\sum_{k\sigma} \tau_k d^+(k\sigma) d(k\sigma),$$

$$\tilde{H}_{s-f} = -\frac{I}{\sqrt{N}} \sum_{kq\sigma} \sum_{\alpha=a,b} \{S_{-q\alpha}^{-\sigma} d^+(k\sigma) d(k+q, -\sigma) + z_\sigma S_{-q\alpha}^z d^+(k\sigma) d(k+q, \sigma)\}. \tag{7}$$

The operators $d^+(k\sigma)$ and $d(k\sigma)$ create or annihilate a quasiparticle in the low energy conduction band.

The reduced Hamiltonian (6—7) is the reference point for describing the acoustic magnons in a two-sublattice antiferromagnetic semiconductor.

3. Method

When calculating the spin wave spectrum we shall follow the approach described in [12]. It comprises the use of "anomalous averages" fixing the vacuum and providing a possibility to determine the generalized mean fields. For this purpose we consider the matrix Green function

$$\hat{G}(k, \omega) = \begin{pmatrix} \langle\langle S_{ka}^+ | S_{-ka}^- \rangle\rangle & \langle\langle S_{ka}^+ | S_{-kb}^- \rangle\rangle \\ \langle\langle S_{kb}^+ | S_{-ka}^- \rangle\rangle & \langle\langle S_{kb}^+ | S_{-kb}^- \rangle\rangle \end{pmatrix} \tag{8}$$

by taking effectively into account the role of the itinerant electrons in forming the magnon spectrum.

The equation of motion of $\langle\langle S_{k\alpha}^+ | h \rangle\rangle$, where $\alpha = a, b$ and $h = S_{-ka}^-, S_{-kb}^-$ after introducing the irreducible functions in a way described in refs [6, 12] has the form

$$\sum_\gamma [(\omega + \omega_0^\alpha)\delta_{\alpha\gamma} - \omega_k^{\gamma\alpha}(1 - \delta_{\alpha\gamma})] \langle\langle S_{k\gamma}^+ | h \rangle\rangle + \frac{I}{\sqrt{N}} \langle S_\alpha^z \rangle \langle\langle \sigma_k^+ | h \rangle\rangle = \langle [S_{k\alpha}^+, h] \rangle + \langle\langle C_{k\alpha}^{ir} h \rangle\rangle. \tag{9}$$

We use here the following notations:

$$\omega_0^a = 2\left[\langle S_b^z \rangle J_0 + \frac{1}{\sqrt{N}} \sum_q J_q A_q^{ab}\right] = -\omega_0^b,$$

$$\omega_k^{ba} = 2\left[\langle S_a^z \rangle J_k + \frac{1}{\sqrt{N}} \sum_q J_{k-q} A_q^{ba}\right] = -\omega_k^{ab}, \tag{10}$$

$$A_{\mathbf{q}}^{ab}=\frac{2\langle (S_{-\mathbf{q}a}^{z})^{ir}\,(S_{\mathbf{q}b}^{z})^{ir}\rangle+\langle S_{-\mathbf{q}a}^{-}\,S_{\mathbf{q}b}^{+}\rangle}{2\sqrt{N}\langle S_{a}^{z}\rangle}.$$

The irreducible Green functions $\langle\langle C_{\mathbf{k}a}^{ir}\,|\,h\rangle\rangle$ are constructed using the operators

$$C_{\mathbf{k}a}^{ir}=A_{\mathbf{k}a}^{ir}+B_{\mathbf{k}a}^{ir},$$

$$A_{\mathbf{k}a}^{ir}=\frac{2}{\sqrt{N}}\sum_{\mathbf{q}} J_{\mathbf{q}}\{S_{\mathbf{q}b}^{+}(S_{\mathbf{k}-\mathbf{q}a}^{z})^{ir}-S_{\mathbf{k}-\mathbf{q}a}^{+}(S_{\mathbf{q}b}^{z})^{ir}\}^{ir}, \tag{11}$$

$$B_{\mathbf{k}a}^{ir}=-\frac{I}{\sqrt{N}}\sum_{\mathbf{pq}}(S_{\mathbf{k}-\mathbf{q}a}^{z})^{ir}\,d_{\mathbf{p}\uparrow}^{+}\,d_{\mathbf{p}+\mathbf{q}\downarrow}+\frac{I}{2N}\sum_{\mathbf{pq}\sigma} z_{\sigma}\,S_{\mathbf{k}-\mathbf{q}a}^{+}\,(d_{\mathbf{p}\sigma}^{+}\,d_{\mathbf{p}+\mathbf{q}\sigma})^{ir},$$

where

$$(S_{\mathbf{q}a}^{z})^{ir}=S_{\mathbf{q}a}^{z}-\langle S_{a}^{z}\rangle\sqrt{N}\delta_{\mathbf{q},0}, \tag{12}$$

$$(d_{\mathbf{p}\sigma}^{+}\,d_{\mathbf{p}+\mathbf{q}\sigma})^{ir}=d_{\mathbf{p}\sigma}^{+}\,d_{\mathbf{p}+\mathbf{q}\sigma}-\langle d_{\mathbf{p}\sigma}^{+}\,d_{\mathbf{p}+\mathbf{q}\sigma}\rangle\delta_{\mathbf{q},0}$$

and we have taken into account that the equality $\langle S_{b}^{z}\rangle=-\langle S_{a}^{z}\rangle$ holds in antiferromagnetic state, i. e. the magnetization of the two sublattices is in opposite directions.

The electron subsystem dynamics is described by the Green functions $\langle\langle\sigma_{\mathbf{k}}^{+}\,|\,h\rangle\rangle$ defined by the operators

$$\sigma_{\mathbf{k}}^{+}=\sum_{\mathbf{q}} d_{\mathbf{q}\uparrow}^{+}\,d_{\mathbf{k}+\mathbf{q}\downarrow},\quad \sigma_{-\mathbf{k}}^{-}=(\sigma_{\mathbf{k}}^{+})^{+} \tag{13}$$

which include the electron spin flip processes. As in ref. [6] the equation of motion yields

$$\langle\langle\sigma_{\mathbf{k}}^{+}\,|\,h\rangle\rangle=\frac{I\sqrt{N}}{2}\chi_{\mathbf{k}}(\omega)\sum_{\gamma}\langle\langle S_{\mathbf{k}\gamma}^{+}\,|\,h\rangle\rangle+\frac{I}{2\sqrt{N}}\sum_{\mathbf{p}}\frac{\langle\langle P_{\mathbf{p},\mathbf{k}}^{ir}\,|\,h\rangle\rangle}{\omega_{\mathbf{p},\mathbf{k}}}. \tag{14}$$

Here

$$\chi_{\mathbf{k}}(\omega)=\frac{1}{N}\sum_{\mathbf{q}}\frac{n_{\mathbf{q}+\mathbf{k}\downarrow}-n_{\mathbf{q}\uparrow}}{\omega+\tau_{\mathbf{q}+\mathbf{k}}-\tau_{\mathbf{q}}}=\frac{1}{N}\sum_{\mathbf{q}}\frac{n_{\mathbf{q}+\mathbf{k}\downarrow}-n_{\mathbf{q}\uparrow}}{\omega_{\mathbf{q},\mathbf{k}}}$$

is the electron susceptibility in the considered quasiparticle band with respect to the spin flip of the system and

$$D_{\mathbf{p},\mathbf{k}}^{\gamma,ir}=\sum_{\mathbf{q}}\{S_{-\mathbf{q}\gamma}^{+}[(d_{\mathbf{p}-\mathbf{q}\downarrow}^{+}\,d_{\mathbf{p}+\mathbf{k}\downarrow})^{ir}-(d_{\mathbf{p}\uparrow}^{+}\,d_{\mathbf{p}+\mathbf{q}+\mathbf{k}\uparrow})^{ir}] \tag{15}$$

$$+(S_{-\mathbf{q}\gamma}^{z})^{ir}\,[d_{\mathbf{p}-\mathbf{q}\uparrow}^{+}\,d_{\mathbf{p}+\mathbf{k}\downarrow}+d_{\mathbf{p}\uparrow}^{+}\,d_{\mathbf{p}+\mathbf{q}+\mathbf{k}\downarrow}]\}.$$

After substituting (14) in (9) the equation for the matrix Green function (8) has the following form:

$$\hat{\Omega}\,\hat{G}(\mathbf{k},\omega)=\hat{I}+\hat{G}_{1}(\mathbf{k},\omega). \tag{16}$$

Here

$$\widehat{\Omega}(\mathbf{k},\omega)=\begin{pmatrix}\omega+\omega_0+\frac{I^2S_z}{2}\chi_{\mathbf{k}}(\omega), & \gamma_{\mathbf{k}}\omega_0+\frac{I^2S_z}{2}\chi_{\mathbf{k}}(\omega)\\ -\left\{\gamma_{\mathbf{k}}\omega_0+\frac{I^2S_z}{2}\chi_{\mathbf{k}}(\omega)\right\}, & \omega-\omega_0-\frac{I^2S_z}{2}\chi_{\mathbf{k}}(\omega)\end{pmatrix} \tag{17}$$

the structure factor of the lattice being $\gamma_{\mathbf{k}}=(1/z)\sum_l e^{i\mathbf{k}\cdot R_l}$ (z is the number of near neighbours);

$$\widehat{I}=\begin{pmatrix}2S_z & 0\\ 0 & -2S_z\end{pmatrix},$$

$$\widehat{G}_1(\mathbf{k},\omega)=\begin{pmatrix}\ll C^{ir}_{\mathbf{k}a}+I^2S_z\sum_{\mathbf{p}}\frac{D^{a,ir}_{\mathbf{p},\mathbf{k}}}{\omega_{\mathbf{p},\mathbf{k}}}\,|\,S^-_{-\mathbf{k}a}\gg & \ll C^{ir}_{\mathbf{k}a}+I^2S_z\sum_{\mathbf{p}}\frac{D^{a,ir}_{\mathbf{p},\mathbf{k}}}{\omega_{\mathbf{p},\mathbf{k}}}\,|\,S^-_{-\mathbf{k}b}\gg\\ \ll C^{ir}_{\mathbf{k}b}+I^2S_z\sum_{\mathbf{p}}\frac{D^{b,ir}_{\mathbf{p},\mathbf{k}}}{\omega_{\mathbf{p},\mathbf{k}}}\,|\,S^-_{-\mathbf{k}a}\gg & \ll C^{ir}_{\mathbf{R}b}+I^2S_z\sum_{\mathbf{p}}\frac{D^{b,ir}_{\mathbf{p},\mathbf{k}}}{\omega_{\mathbf{p},\mathbf{k}}}\,|\,S^-_{-\mathbf{k}b}\gg\end{pmatrix}. \tag{18}$$

Equation (16) is the reference point for deriving the Dason equation in the irreducible Green function method [9]. For this purpose it is necessary to write the equation of motion for the function $\widehat{G}_1(\mathbf{k},\omega)$ and to introduce the corresponding irreducible parts. Then

$$\widehat{G}(\mathbf{k},\omega)=\widehat{G}_0(\mathbf{k},\omega)+\widehat{G}_0(\mathbf{k},\omega)\widehat{M}(\mathbf{k},\omega)\widehat{G}(\mathbf{k},\omega), \tag{19}$$

where

$$\widehat{G}_0(\mathbf{k},\omega)=\widehat{\Omega}^{-1}(\mathbf{k},\omega)=\widehat{I} \tag{20}$$

and the mass operator is given with accuracy up to I^2 by the expression

$$\widehat{M}(\mathbf{k},\omega)=\frac{1}{4S_z^2}\begin{pmatrix}\ll C^{ir}_{\mathbf{k}a}\,|\,C^{+,ir}_{\mathbf{k}a}\gg & \ll C^{ir}_{\mathbf{k}a}\,|\,C^{+,ir}_{\mathbf{k}b}\gg\\ \ll C^{ir}_{\mathbf{k}b}\,|\,C^{+,ir}_{\mathbf{k}a}\gg & \ll C^{ir}_{\mathbf{k}b}\,|\,C^{+,ir}_{\mathbf{k}b}\gg\end{pmatrix}. \tag{21}$$

The formal solution of equation (19) may be written in the form

$$\widehat{G}(\mathbf{k},\omega)=[\widehat{G}_0^{-1}(\mathbf{k},\omega)-\widehat{M}(\mathbf{k},\omega)]^{-1}. \tag{22}$$

In this way to find the total Green function is reduced to determining the Green function in a mean field generalized approximation $\widehat{G}_0(\mathbf{k},\omega)$ and the calculation of the mass operator $\widehat{M}(\mathbf{k},\omega)$, defining the additional renormalization of the magnon operator and the magnon damping due to inelastic interactions. In the next Section we shall consider the magnon spectrum and damping using the function $\widehat{G}_0(\mathbf{k},\omega)$.

4. Magnon spectrum and damping

The Green function $\widehat{G}_0(\mathbf{k},\omega)$ determines in a generalized Hartree-Fock approximation the spectrum of the elementary excitations [6, 9]. The poles of function (20) are defined by the condition

$$\det \widehat{\Omega}(\mathbf{k}, \omega) = 0. \tag{23}$$

Taking into account that for $W > IS_z$ the itinerant electron contribution is a correction to the main exchange interaction, we obtain for the magnon energy

$$\omega_{\mathbf{k}}^{\pm} = \pm\omega_{\mathbf{k}} = \pm\left\{\omega_0\sqrt{1-\gamma_{\mathbf{k}}^2} \mp \frac{I^2 S_z}{2}\bar{\chi}_{\mathbf{k}}\sqrt{\frac{1-\gamma_{\mathbf{k}}}{1+\gamma_{\mathbf{k}}}}\right\}. \tag{24}$$

We use here the notation $\bar{\chi}_{\mathbf{k}} = \chi_{\mathbf{k}}(\omega_{\mathbf{k}})$. As seen from (24) the acoustic magnon dispersion law becomes linear for $k \to 0$

$$\omega_{\mathbf{k}}^{\pm} = \pm D(T)\,|\mathbf{k}| \tag{25}$$

with a characteristic constant

$$D(T) = zJS_z\left(1 - \frac{1}{\sqrt{NS_z}}\sum_{\mathbf{q}}{}' \gamma_{\mathbf{q}} A_{\mathbf{q}}^{ab}\right) - \frac{I^2 S_z}{4N}\lim_{k\to 0}\sum_{\mathbf{q}}{}' \frac{n_{\mathbf{q}+\mathbf{k}\downarrow} - n_{\mathbf{q}\uparrow}}{\omega_{\mathbf{k}} + \tau_{\mathbf{q}+\mathbf{k}} - \tau_{\mathbf{q}}}. \tag{26}$$

The first term in (26)$\sim J$ expresses the magnon stiffness due to the direct exchange interaction between the local moments and the second term shows that the submagnetic itinerant electrons in each sublattice produce an increase in the magnon energy in the wide band antiferromagnetic semiconductor by

$$\Delta\omega = I^2 S_z \rho(\varepsilon_F),$$

where $\rho(\varepsilon_F)$ is the density of quasiparticle states at the Fermi level.

The presence of free electrons lays an essential role in forming the magnon damping. In the mean field approximation it is due to the presence of the two spin degenerate electron bands (5) between which electron transition takes place with spin flip. Then

$$\gamma_{\mathbf{k}} \simeq \frac{\pi I^2 S_z}{4N}|\mathbf{k}|\sum_{\mathbf{q}}{}' (n_{\mathbf{q}+\mathbf{k}\downarrow} - n_{\mathbf{q}\uparrow})\,\delta(\omega_{\mathbf{k}} + \tau_{\mathbf{q}+\mathbf{k}} - \tau_{\mathbf{q}}) \tag{27}$$

and in the long wavelength limit $\mathbf{k} \to 0$ the damping shows the characteristic behaviour $\gamma_{\mathbf{k}} \sim |\mathbf{k}|^2$ [13], when $k > k_c \sim k_F$.

Additional energy renormalization and damping of the antiferromagnons due to the inelastic scattering between magnons and electron is given by the imaginary part of the mass operator (21). Complete analysis of these corrections will be discussed in a separate paper.

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