collinear polarization.

I thank Professors N. V. Mitskevich and N. R. Sibgatullin for discussion and critical comments.

LITERATURE CITED

1. P. Bell and P. Szekeres, Gen. Relativ. Gravit., 5, 275 (1974).
2. S. Chandrasekhar and B. S. Xanthopoulos, Proc. R. Soc. London, Ser. A, 398, 223 (1985); 410, 311 (1987).
3. N. R. Sibgatullin, Oscillations and Waves in Strong Gravitational and Electromagnetic Fields [in Russian], Nauka, Moscow (1984).
4. W. Kinnersley and D. M. Chitre, J. Math. Phys., 18, 1538 (1977); 19, 1926, 2037 (1978).
5. K. Harrison, J. Math. Phys., 9, 1744 (1968).
6. A. Garcia, J. Math. Phys., 29, 1442 (1988).
7. V. Ferrari and J. Ibañez, Gen Relativ. Gravit., 19, 405 (1987).
8. D. Kramer, H. Stephani, M. MacCallum, and E. Herit, Exact Solutions of the Einstein's Field Equations, Cambridge University Press, Cambridge (1980).
9. C. M. Cosgrove, J. Math. Phys., 22, 2624 (1981).
10. K. Khan and R. Penrose, Nature (London), 229, 185 (1971).
11. J. Ehlers and W. Kundt, in: Gravitation: An Introduction to Current Research (L. Witten, ed.), Wiley, New York (1962).
12. S. Chandrasekhar and B. S. Xanthopoulos, Proc. R. Soc. London, Ser. A, 410, 311 (1987).
13. Yu. G. Sbytov, Zh. Eksp. Teor. Fiz., 63, 737 (1972) (Sov. Phys. JETP, 36, 386 (1972)); Zh. Eksp. Teor. Fiz., 71, 2001 (1976) (Sov. Phys. JETP, 44, 1051 (1976)).
14. A. A. Logunov and M. A. Mestvirishvili, The Relativistic Theory of Gravitation [in Russian], Nauka, Moscow (1989).
15. F. J. Ernst, D. A. Garcia, and I. Hauser, J. Math. Phys., 28, 2951 (1987).

EXCITATION SPECTRUM OF A HEISENBERG ANTIFERROMAGNET AT FINITE TEMPERATURES

A. L. Kuzemskii and D. Marvakov*

It is shown that the spectrum of spin-wave excitations in a Heisenberg antiferromagnet can be described consistently in the framework of the method of irreducible Green's functions. The anomalous contributions to the renormalization of the mean field are taken into account from the very beginning. The spin wave damping is calculated self-consistently.

1. Introduction

The Heisenberg model of the exchange interaction of localized spins [1] is used to describe the magnetic properties of ferro-, ferri-, and antiferrodielectrics. For a ferromagnet the ground state, in which all the spins are aligned parallel, is known exactly. Small deviations from the ground state, or spin waves, can be described quite well by dynamical variables of oscillator type [2,3]. A dependence of the frequency of the normal modes on the wave vector, i.e., dispersion, arises as a result of interaction between the oscillators. At temperatures near zero, a linear approximation for the spin waves is used.

For an antiferromagnet an exact ground state is not known. Néel [4] introduced the concept of two mutually interpenetrating sublattices to explain the behavior of the susceptibility of antiferromagnets. However, the ground state in the form of two sublattices (Néel state) is only a classical approximation. The spin-wave theory for an

*Physics Faculty, University of Sofia, Bulgaria.

Joint Institute for Nuclear Research, Dubna. Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 83, No. 1, pp. 147-157, April, 1990. Original article submitted December 22, 1988; revision submitted June 22, 1989.

antiferromagnet [1-7] is, from the very beginning, a greater approximation than in the case of a ferromagnet, since the system has more resemblance to a set of anharmonic oscillators. In contrast to ferromagnets, in which the mean molecular field is homogeneous and proportional to the magnetization, in ferri- and antiferromagnets the mean molecular field is strongly inhomogeneous. The local molecular fields of Néel [4] are a more general concept. Multisublattice magnets with inequivalent sublattices and complicated magnetic ordering cannot be described by using a homogeneous mean field.

From the point of view of quantum statistical mechanics, the problem of adequate introduction of mean fields for systems of many interacting particles can be most consistently investigated in the framework of the method of irreducible Green's functions [1,8-13]. A correct calculation of the quasiparticle spectra and their damping, particularly for systems with complicated spectrum and strong interaction [11,13-16], shows that the generalized mean fields can have very complicated structures that cannot be described by a functional of the mean density. Even for a Heisenberg ferromagnet, calculation of the damping of the spin-wave spectrum in the framework of the method of irreducible Green's functions [1,10] requires the inclusion of the correlation functions of the longitudinal and transverse spin components in the mean fields. There is no spin-wave damping in a mean field proportional to the mean magnetization.

In this paper, we calculate the spin-wave spectrum and damping of a Heisenberg antiferromagnet in the framework of the method of irreducible Green's functions. An initial formulation was given in [17,18].

2. Hamiltonian of the Model

We shall proceed from a model of an isotropic two-sublattice antiferromagnet with Hamiltonian of the form

$$H=-\frac{1}{2}\sum_{q\alpha\alpha'}J_q^{\alpha\alpha'}\mathbf{S}_{q\alpha}\mathbf{S}_{-q\alpha'}=-\frac{1}{2}\sum_{q\alpha\alpha'}J_q^{\alpha\alpha'}\left\{\frac{1}{2}(S_{q\alpha}^{+}S_{-q\alpha'}^{-}+S_{q\alpha}^{-}S_{-q\alpha'}^{+})+S_{q\alpha}^{z}S_{q\alpha'}^{z}\right\}. \tag{1}$$

Here, α and α' take two values {a, b}. It is assumed that the spontaneous magnetization of sublattice a is directed along the z axis, and of sublattice b along the $-z$ axis. The simple two-sublattice model (1) with two interpenetrating sublattices is valid only for systems with simple cubic and bcc lattices.

For what follows, it is convenient to rewrite (1) in the form

$$H=-\frac{1}{2}\sum_{q\alpha\alpha'}I_q^{\alpha\alpha'}(S_{q\alpha}^{+}S_{-q\alpha'}^{-}+S_{q\alpha}^{z}S_{-q\alpha'}^{z}), \tag{2}$$

where

$$I_q^{\alpha\alpha'}=1/2(J_q^{\alpha\alpha'}+J_{-q}^{\alpha'\alpha})=I_{-q}^{\alpha'\alpha}. \tag{3}$$

It is known [1,2,19-21] that the additional anisotropic contributions play an important part in the Hamiltonian (1). Since allowance for them would significantly complicate not only the expression but also the form of the ground state and the spectrum, these contributions are omitted in the present paper in order to demonstrate more transparently the advantages of the chosen method. We intend to take into account the anisotropy and an external field later.

3. Method of Calculation

In this paper we shall use the method of irreducible Green's functions [1,8-13]. The essence of the method is as follows [13]. The introduction of the irreducible parts of the Green's functions results in separation of all renormalizations of the mean field. As a result, without having to make any truncation of the hierarchy of equations for the Green's functions, one can write down a Dyson equation and obtain an exact analytic representation for the mass operator in terms of the many-particle Green's functions. Approximate solutions are constructed as definite approximations for the mass operator. This is the difference from the ordinary method of equations of motion for the Green's functions. At the same time, the splitting procedure can be controlled formally by analogy with the diagram approach. The method of irreducible Green's functions is intimately related to the projection operator method, which expresses the idea of a "reduced description" of the

system in the most general form. The projection operation [12] makes it possible to reduce the infinite hierarchy of coupled equations to a few relatively simple equations, that "effectively" take into account the essential information about the system that determines the specific nature of the given problem. Since a regular structure of the solution for the single-particle Green's function is used (in the form of a formal solution of the Dyson equation), approximate solutions can be constructed systematically. The method makes it possible to calculate the damping due to inelastic collisions for systems with complicated spectrum and strong interaction [13].

The connection between adequate introduction of mean fields and the nature of the broken symmetry of the system [2] was emphasized in [13,18]. It should be emphasized that although the Néel state is a fairly good approximation to the exact ground state, there is still no consistent microscopic theory of long-range antiferromagnetic order. From the point of view of the concept of broken symmetry [1,22], the system tends to go over, as a rule, to a less symmetric phase when the temperature is reduced. For example, in a superconductor the breaking of gauge invariance results in the appearance of phase coherence over large distances. In [23], Bogolyubov showed that the broken symmetry in a superconductor can be described by adding to the Hamiltonian infinitesimally small sources. As a result of this, nonvanishing anomalous pairings arise. Developing Bogolyubov's idea, Nambu [24] showed that a vacuum of the required form, corresponding to the nature of the broken symmetry, can be uniquely fixed by means of the spinor representation.

In this paper, a similar approach is used to describe a Heisenberg antiferromagnet. It is shown that the use of "anomalous averages," which fix the vacuum, makes it possible to determine uniquely generalized mean fields and calculate in a very compact manner the spectrum of spin-wave excitations and their damping due to inelastic magnon–magnon scattering processes. A transition from the spin operators to Bose operators is not required. A method for calculating the damping for arbitrary value of the spin S is proposed.

4. Derivation of the Dyson Equation

To calculate the spin-wave spectrum of a two-sublattice Heisenberg antiferromagnet, we consider the two-time retarded Green's function

$$G(t-t')=\langle\langle A(t),\ B(t')\rangle\rangle=-i\Theta(t-t')\langle[A(t),\ B(t')]_-\rangle. \tag{4}$$

As operators A and B in the Heisenberg ferromagnet, the operators S^+ and S^- have been chosen. In the case of an antiferromagnet, as will be shown below, it is convenient to choose the following representation of "spinor" type:

$$A=\begin{bmatrix} S_{ka}^{+} \\ S_{kb}^{+} \end{bmatrix},\quad B=[S_{-ka}^{-}, S_{-kb}^{-}]. \tag{5}$$

The equation of motion for the Fourier transform of the Green's function can be written in the form

$$\omega\left\langle\!\left\langle S_{ka}^{+}\middle|\begin{bmatrix} S_{-ka}^{-} \\ S_{-kb}^{-} \end{bmatrix}\right\rangle\!\right\rangle_\omega=\begin{bmatrix} 2\langle S_a^z\rangle \\ 0 \end{bmatrix}+\left\langle\!\left\langle\Phi_{k,ab}\middle|\begin{bmatrix} S_{-ka}^{-} \\ S_{-kb}^{-} \end{bmatrix}\right\rangle\!\right\rangle_\omega. \tag{6}$$

Here,

$$\Phi_{k,ab}=N^{-1/2}\sum_q I_q^{ab}S_{kq}^{ab}+N^{-1/2}\sum_q I_q^{aa}S_{kq}^{aa},\quad S_{kq}^{ab}=S_{k-q,a}^{+}S_{qb}^{z}-S_{qb}^{+}S_{k-q,a}^{z}. \tag{7}$$

We introduce irreducible Green's functions (or irreducible operators, from which these Green's functions are constructed) in accordance with the definition

$$(S_{kq}^{ab})^{\mathrm{ir}}=S_{kq}^{ab}-D_q^{ab}S_{ka}^{+}+D_{k-q}^{bq}S_{kb}^{+}, \tag{8}$$

$$(S_{q\alpha}^{z})^{\mathrm{ir}}=S_{q\alpha}^{z}-N^{1/2}\langle S_a^z\rangle\delta_{q,0}. \tag{9}$$

The choice of the irreducible part of the Green's functions is uniquely determined by the conditions

$$\left\langle\left[(S_{kq}^{ab})^{\mathrm{ir}},\begin{bmatrix} S_{-ka}^{-} \\ S_{-kb}^{-} \end{bmatrix}\right]_-\right\rangle=0, \tag{10}$$

which lead to the vanishing of the inhomogeneous terms in the equations for the irreducible Green's functions. From (10) we find that

$$D_q^{ab} = \frac{2\langle (S_{-qa}^z)^{ir} (S_{qb}^z)^{ir}\rangle + \langle S_{-qa}^- S_{qb}^+\rangle}{2\sqrt{N}\langle S_a^z\rangle}. \tag{11}$$

Taking into account (7)-(11), we represent Eq. (6) in the form

$$(\omega - \omega_{aa})\langle\langle S_{ka}^+ | \begin{bmatrix} S_{-ka}^- \\ S_{-kb}^- \end{bmatrix}\rangle\rangle + \omega_{ab}\langle\langle S_{kb}^+ | \begin{bmatrix} S_{-ka}^- \\ S_{-kb}^- \end{bmatrix}\rangle\rangle = \begin{bmatrix} 2\langle S_a^z\rangle \\ 0 \end{bmatrix} + \langle\langle \Phi_a^{ir}(k) | \begin{bmatrix} S_{-ka}^- \\ S_{-kb}^- \end{bmatrix}\rangle\rangle. \tag{12}$$

In (12) we have introduced the notation

$$\omega_{aa} = \left\{ (I_0^{aa} - I_k^{aa})\langle S_a^z\rangle + I_0^{ab}\langle S_b^z\rangle + \sum_q [(I_q^{aa} - I_{k-q}^{aa}) D_{Nq}^{aa} + I_q^{ab} D_{Nq}^{ab}] \right\}, \tag{13}$$

$$D_{Nq}^{\alpha\beta} = \frac{1}{\sqrt{N}} D_q^{\alpha\beta}, \quad \alpha, \beta = a, b$$

$$\omega_{ab} = \left[I_k^{ab}\langle S_a^z\rangle + \sum_q I_{k-q}^{ab} D_{Nq}^{ba} \right], \tag{14}$$

$$\Phi_a^{ir}(k) = N^{-1/2} \sum_{q,\gamma=a,b} I_q^{a\gamma} [S_{k-q,a}^+ (S_{q\gamma}^z)^{ir} - S_{q\gamma}^+ (S_{k-q,a})^{ir}]^{ir}. \tag{15}$$

Similarly, we obtain

$$(\omega - \omega_{bb})\langle\langle S_{kb}^+ | \begin{bmatrix} S_{-kb}^- \\ S_{-ka}^- \end{bmatrix}\rangle\rangle + \omega_{ba}\langle\langle S_{ka}^+ | \begin{bmatrix} S_{-kb}^- \\ S_{-ka}^- \end{bmatrix}\rangle\rangle = \begin{bmatrix} 2\langle S_b^z\rangle \\ 0 \end{bmatrix} + \langle\langle \Phi_b^{ir}(k) | \begin{bmatrix} S_{-kb}^- \\ S_{-ka}^- \end{bmatrix}\rangle\rangle. \tag{16}$$

To calculate the irreducible Green's functions on the right-hand sides of Eqs. (12) and (16), we use the device of differentiating with respect to the second time [8]. After introduction of the corresponding irreducible parts in the resulting equations, the system of equations can be represented in the matrix form

$$\hat{G}(k, \omega) = \hat{G}_0(k, \omega) + \hat{G}_0(k, \omega)\hat{\Pi}(k, \omega)\hat{G}_0(k, \omega). \tag{17}$$

We have here introduced the Green's function G_0 in the generalized mean field approximation and the scattering operator Π. If follows from the Dyson equation

$$\hat{G}(k, \omega) = \hat{G}_0(k, \omega) + \hat{G}_0(k, \omega)\hat{M}(k, \omega)\hat{G}(k, \omega) \tag{18}$$

that

$$\hat{\Pi}(k, \omega) = \hat{M}(k, \omega) + \hat{M}(k, \omega)\hat{G}_0(k, \omega)\hat{\Pi}(k, \omega). \tag{19}$$

Thus, on the basis of the relation (19) we can speak of the mass operator M as the intrinsic part of the operator Π by analogy with the diagram technique, in which the mass operator is the connected part of the scattering operator (for a more detailed discussion, see [13]). It can be shown that the explicit form of the mass operator M is

$$\hat{M} = (\hat{\Pi})^p = \frac{1}{4\langle S_a^z\rangle^2} \begin{bmatrix} \langle\langle \Phi_q^{ir}(k) | \Phi_a^{ir+}(k)\rangle\rangle & \langle\langle \Phi_a^{ir}(k) | \Phi_b^{ir+}(k)\rangle\rangle \\ \langle\langle \Phi_b^{ir}(k) | \Phi_a^{ir+}(k)\rangle\rangle & \langle\langle \Phi_b^{ir}(k) | \Phi_b^{ir+}(k)\rangle\rangle \end{bmatrix}. \tag{20}$$

The formal solution of the Dyson equation for the retarded Green's function (18) can be represented in the form

$$\hat{G}(k, \omega) = [\hat{G}_0^{-1}(k, \omega) - \hat{M}(k, \omega)]^{-1}. \tag{21}$$

Thus, the finding of the total Green's function has been reduced to calculation of the Green's function in the generalized mean field approximation and the calculation of the mass operator.

6. Quasiparticle Excitations in the Mean Field

In accordance with the definition (18), the Green's function in the generalized mean field approximation has the form

$$\hat{G}_0(k,\omega)=\frac{2\langle S_a^z\rangle}{\det\hat{\Omega}}\begin{bmatrix}(\omega-\omega_{aa}) & \omega_{ab}\\ \omega_{ab} & -(\omega-\omega_{bb})\end{bmatrix}, \tag{22}$$

where

$$\det\hat{\Omega}=(\omega-\omega_{aa})(\omega-\omega_{bb})-\omega_{ab}\omega_{ba}. \tag{23}$$

We find the poles of the Green's function (22) from the equation

$$\det\hat{\Omega}=0, \tag{24}$$

from which it follows that

$$\omega^{\pm}=\pm\sqrt{\omega_{aa}^2-\omega_{ab}^2}. \tag{25}$$

By analogy with [5], we introduce the quantities [1]

$$u_k^2=\tfrac{1}{2}[(1-\gamma_k^2)^{-1/2}+1],\quad v_k^2=\tfrac{1}{2}[(1-\gamma_k^2)^{-1/2}-1],\quad \gamma_k=\frac{1}{z}\sum_l e^{ikl},\quad I_q^{aa}=0,\quad I_q^{bb}=0. \tag{26}$$

By means of (26), we represent the Green's function (22) in the form

$$G_0^{aa}(k,\omega)=2\langle S_a^z\rangle\left[\frac{u_k^2}{\omega-\omega^+(k)}-\frac{v_k^2}{\omega-\omega^-(k)}\right]=G_0^{bb}(k,-\omega), \tag{27}$$

$$G_0^{ab}(k,\omega)=2\langle S_a^z\rangle\left[\frac{-u_kv_k}{\omega-\omega^+(k)}+\frac{u_kv_k}{\omega-\omega^-(k)}\right]=G_0^{ba}(k,\omega). \tag{28}$$

The Green's functions (27) and (28) show that the propagation of quasiparticle excitations in the antiferromagnet is associated with definite superpositions of the two branches of spin waves.

The spin-wave dispersion law in the generalized mean field approximation for arbitrary spin S has the form

$$\omega(k)=I\cdot z\cdot\langle S_a^z\rangle\left[1-\frac{1}{\sqrt{N}\langle S_a^z\rangle}\sum_q\gamma_qD_q^{ab}\right](1-\gamma_k^2)^{1/2}, \tag{29}$$

where $I_q = z\cdot I\cdot\gamma_q$, and z is the number of nearest neighbors in the lattice. The first term in (29) corresponds to the Tyablikov approximation [1]. The second term in (29) describes elastic scattering of the spin-wave excitations. At low temperatures, the fluctuations of the longitudinal spin components are small, and, therefore, for (29) we obtain

$$\omega(k)\cong I\cdot S\cdot z[1-C(T)](1-\gamma_k^2)^{1/2}. \tag{30}$$

The function C(T) determines the temperature dependence of the spin-wave spectrum:

$$C(T)=\frac{1}{2NS^2}\sum_q(\langle S_{-qa}^-S_{qa}^+\rangle+\gamma_q\langle S_{-qa}^-S_{qb}^+\rangle). \tag{31}$$

In the case when C(T) → 0 we obtain the result of the Tyablikov decoupling:

$$E(k)=I\langle S_a^z\rangle\cdot z(1-\gamma_k^2)^{1/2}. \tag{32}$$

From (32) we obtain the well-known expression for the Néel temperature:

$$T_N=\frac{I\cdot z}{2k_B}\left(N^{-1}\sum_k\frac{1}{1-\gamma_k^2}\right)^{-1}. \tag{33}$$

For small values of the wave vector k, when ω(k) in Eq. (32) is a linear function of k, it can be estimated that for an antiferromagnet the spin-wave rigidity coefficient behaves with the temperature as

$$C(T)\sim T^4. \tag{34}$$

We emphasize that the dependence (34) is due to the elastic scattering processes. To estimate the contribution of the inelastic processes, it is necessary to take into account the corrections due to the mass operator. We note that the spectrum of an antiferromagnet in a generalized mean field (or generalized Hartree-Fock approximation) was investigated in detail in [19] under significantly more general assumptions concerning the model (see also the supplement to [1] and the subsequent studies [20,21] of these authors).

6. Damping of Quasiparticle Excitations

An antiferromagnet is a system with complicated quasiparticle spectrum. The calculation of the damping due to inelastic scattering processes in such systems has some important aspects [13–16]. When calculating the damping, it is necessary to take into account the contributions from all matrix elements of the mass operator (20). In other words, the damping of the quasiparticle excitations is determined on the basis of a Green's function of the form

$$G_{11}(k,\omega)=\frac{2\langle S_a^z\rangle}{\omega-\omega(k)-2\langle S_a^z\rangle\,\Sigma(k,\omega)}. \tag{35}$$

Here, the self-energy operator $\Sigma(k, \omega)$ is determined by the expression

$$\Sigma(k,\omega)=M_{11}(k,\omega)-\frac{2\langle S_a^z\rangle M_{12}(k,\omega)M_{21}(k,\omega)}{\omega+\omega(k)+2\langle S_a^z\rangle M_{22}(k,\omega)}. \tag{36}$$

In the case when $k, \omega \to 0$, a restriction may be made to the approximation

$$\Sigma(k,\omega)\sim M_{11}(k,\omega). \tag{37}$$

It follows from (20) that to calculate the damping it is necessary to find the Green's function $\langle\langle \Phi_\alpha^{ir}(k) \mid \Phi_\beta^{ir+}(k)\rangle\rangle$. As an example, we consider the calculation of one of them. By means of the spectral theorem [1], we have

$$\langle\langle \Phi_a{}^{ir}(k) \mid \Phi_a{}^{ir+}(k)\rangle\rangle=\int_{-\infty}^{\infty}\frac{d\omega'}{\omega-\omega'}(e^{\beta\omega'}-1)\int_{-\infty}^{\infty}\frac{dt}{2\pi}e^{i\omega' t}\langle\Phi_a{}^{ir+}(k)\,\Phi_a{}^{ir}(k,t)\rangle. \tag{38}$$

Thus, it is necessary to find a felicitous approximation for the correlation function on the right-hand side of (38). We consider an approximation of the following form (the physical justification of the choice of the approximation (39) was discussed in detail in [11,13–18]):

$$\langle(S_{-qb}^z)^{ir}S_{-(k-q')a}^-S_{k-q'a}^+(t)\,(S_{q'b}^z(t))^{ir}\rangle\approx\frac{1}{4NS^2}\sum_p(K_{k-p,aa}^{-+}(t)K_{q+p,bb}^{-+}(t)K_{p,bb}^{+-}(t)+$$

$$K_{k-q,ab}^{-+}(t)K_{q+p,ab}^{-+}(t)K_{p,bb}^{+-}(t))\delta_{q,q'}, \tag{39}$$

where

$$K_{q,ab}^{-\pm}(t)=\langle S_{-qa}^-S_{qb}{}^+(t)\rangle. \tag{40}$$

By analogy with the diagram technique, we can say that the approximation (39) corresponds to neglect of the vertex corrections to the magnon–magnon inelastic collisions. Using (39) in (38), we obtain

$$\langle\langle \Phi_a{}^{ir}(k) \mid \Phi_a{}^{ir+}(k)\rangle\rangle\approx\frac{1}{16NS^4}\sum_{qp}\int d\omega_1\,d\omega_2\,d\omega_3\,\frac{\nu(\omega_2)[\nu(\omega_3)-\nu(\omega_1)]+[1+\nu(\omega_1)]\nu(\omega_3)}{\omega-\omega_1-\omega_2+\omega_3}\times$$

$$\left\{-\frac{1}{\pi}\operatorname{Im}G_{aa}(k-q,\omega_1)\right\}\left\{-\frac{1}{\pi}\operatorname{Im}G_{bb}(q+p,\omega_2)\right\}\left\{-\frac{1}{\pi}\operatorname{Im}G_{bb}(p,\omega_3)\right\}, \tag{41}$$

where $\nu(\omega)$ is the Bose distribution function.

Equations (18) and (41) form a self-consistent system of equations. To solve this system of equations, we can, in principle, use any convenient initial representation for the Green's function, substituting it on the right-hand side of Eq. (41). The system can then be solved iteratively. To estimate the damping, it is usually sufficient as the first iteration to use the simplest single-pole approximation (see Eq. (30))

$$-\frac{1}{\pi}\operatorname{Im}G(k,\omega)\sim\delta(\omega-\omega(k)). \tag{42}$$

As a result, for the damping of the spin-wave excitations we obtain

$$\Gamma(k,\omega)=-2S\operatorname{Im}\Sigma(k,\omega), \tag{43}$$

$$\Gamma(k,\omega)=\frac{\pi}{N}(zI)^2(1-e^{-\beta\omega})\sum_{qp}\nu_p(1+\nu_{q+p})(1+\nu_{k-q})\times$$

$$\delta(\omega-\omega(k-q)+\omega(p))M_{11}(k,p;k-q,p+q). \quad (44)$$

In the Appendix, we give M_{11} explicitly. The result (44) exactly reproduces the expression for the damping found in [7]. However, our method is much simpler and shorter. Moreover, in our approach it is not difficult to take into account inelastic scattering of the spin waves due to scattering by longitudinal spin fluctuations. For this, instead of the approximation (39), we can use

$$\langle (S^z_{-qb})^{1-} S^-_{-(k-q)a} S^+_{k-q'a}(t) (S^z_{q'b}(t))^{1-}\rangle \approx K^{-+}_{k-q,aa}(t) K^{zz}_{q,bb}(t)\delta_{q,q'}. \quad (45)$$

If we restrict ourselves to the static approximation,

$$K^{-+}_{k-q,aa}(t)K_{q,bb}(t) \approx K^{+}_{k-q,aa}(t)K_{q,bb}(0), \quad (46)$$

then on the basis of (38) we find

$$M_{aa}(k,\omega)=\frac{(zI)^2}{2S}\sum_q \frac{1}{\omega^2-\omega^2_{k-q}}\left[(\gamma_q^2+\gamma^2_{k-q})\left(\omega+\frac{\omega_{k-q}}{\sqrt{1-\gamma^2_{k-q}}}\right)K^{zz}_{q,aa}+\frac{\gamma_q\gamma_{k-q}\omega_{k-q}}{2\sqrt{1-\gamma^2_{k-q}}}K^{zz}_{q,ab}\right]. \quad (47)$$

In conclusion, we emphasize that on the basis of the proposed approach we can very readily calculate the spectrum and damping of quasiparticle excitations for arbitrary value of the spin S. For this, it is necessary to use the well-known representation [1]

$$S^z=-S+(2S)^{-1}S^+S^-+[(2S)^2(2S-1)]^{-1}S^+S^+S^-S^-+\ldots. \quad (48)$$

Using (48) in (39), we can obtain explicit expressions for $\Gamma(k,\omega)$ (44) for arbitrary value of the spin. For spin $S=\frac{1}{2}$, we retain the first two terms in (48), for spin $S=1$ it is necessary to retain the first three terms in (48), etc.

There has been much recent study of the spin-wave damping of Heisenberg antiferromagnets due to multimagnetic processes. On the basis of the expression (44), it is possible to describe broadening of an antiferromagnetic resonance line [25] and the width of a spin-wave line in the case of inelastic scattering of slow neutrons [26]. It is also important to take into account processes of magnon–magnon scattering in optical investigations of antiferrodielectrics [27]. On the basis of these investigations, it can be concluded that the damping is weak $\omega(k)/\Gamma\sim 10^2\div 10^3$, i.e., antiferromagnons are well-defined excitations. Following [14,15], it is possible to determine the temperature dependence of the damping (cf. [7])

$$\Gamma(k,\omega)\sim\alpha\frac{\omega(k)}{S^2}\left(\frac{k_BT}{JSz}\right)^4 \quad (49)$$

in the regime $\tau \ll \varepsilon(k) \ll 1$. Here, $\tau = k_BT/JzS$ and $\varepsilon(k) = \omega(k)/2JzS$. In general, the estimates of the temperature dependence of the damping in the general case are very complicated, and in this question there are still some obscure points [7,25–28].

7. Conclusions

In this paper we have shown that on the basis of the method of irreducible Green's functions it is relatively easy to calculate the spectrum and damping due to magnon–magnon scattering for a two-sublattice Heisenberg antiferromagnet in a wide range of temperatures. At the same time, all the calculations can be made in the representation of spin operators for arbitrary value of the spin S. In contrast, in the framework of the diagram approach allowance for the dependence on the value of the spin leads to significantly more complicated calculations. The theory we have proposed can be directly extended to the case of a large number of magnetic sublattices with inequivalent spins, i.e., it can be used to describe complicated ferrimagnets.

In the framework of our approach, one can show that the mean fields in an antiferromagnet must include "anomalous" means, this reflecting the local nature of the Néel molecular fields. Even in an ordinary cubic Heisenberg ferromagnet allowance for next-neighbor interaction forces us to go beyond the framework of the mean field approximation [29]. This is all the more true for an antiferromagnet [30,31]. Thus, the mean field in an antiferromagnet, like the mean field in a superconductor, has a more complicated structure. This circumstance has a number of interesting consequences. However, this group of questions goes beyond the scope of the present study and warrants a separate discussion. It would also be interesting to apply our method to more complicated configurations, for example, the spin-flop phase, for which consistent calculation of the damping

is a very topical problem.

Appendix

We give here the explicit expression for one of the elements of the matrix of the mass operator:

$$-\operatorname{Im} M_{11}(k,\omega)=\frac{\pi}{2SN}(1-e^{-\beta\omega})\sum_{qp}\nu_p(1+\nu_{p+q})(1+\nu_{k-q})\,\delta(\omega-\omega(k-q)-\omega(p+q)-\omega(p))M_{11}(k,p;k-q,p+q),$$

$$M_{11}(k,p;k-q,p+q)=z^2I^2\{[\gamma_q{}^2u^2_{k-q}v^2_{q+p}v_p{}^2+\gamma_q{}^2u^2_{k-q}u^2_{p+q}u_p{}^2+\gamma^2_{k-p}u^2_{k-q}v^2_{p+q}v_p{}^2]+$$

$$[\gamma_q{}^2u_{k-q}v_{k-q}u_{p+q}v_{p+q}v_p{}^2+\gamma_q{}^2u_{k-q}v_{k-q}u^2_{p+q}u_pv_p+\gamma^2_{k-p}u^2_{k-q}u_{p+q}v_{p+q}u_pv_p]+$$

$$2(2\gamma_q\gamma_{k-q}+\gamma_p\gamma_{k-p})u_{k-q}v_{k-q}u_{p+q}v_{p+q}u_pv_p+[\gamma_q\gamma_{k-q}(u^2_{k-q}v^2_{p+q}u_pv_p+u^2_{k-q}u_{p+q}V_{p+q}u_p{}^2)+\gamma_p\gamma_{k-p}u_{k-q}v^2_{p+q}u_p{}^2v_p]+$$

$$[\gamma_q\gamma_pu^2_{k-q}v^2_{p+q}u_pv_p+\gamma_q\gamma_{q+p}u^2_{k-q}u_{p+q}v_{p+q}u_p{}^2+\gamma_{k-q}\gamma_{k-p}u_{k-q}v_{k-q}v^2_{p+q}u_p{}^2]+$$

$$[\gamma^2_{k-q}v^2_{k-q}u^2_{p+q}u_p{}^2+\gamma^2_{k-q}v^2_{k-q}v^2_{p+q}v_p{}^2+\gamma_p{}^2v^2_{k-q}v^2_{p+q}u_p{}^2]+$$

$$[\gamma_{k-q}\gamma_p(u_{k-q}v_{k-q}u_{p+q}v_{p+q}u_p{}^2+v^2_{k-q}u_{p+q}v_{p+q}u_pv_p)+\gamma_{k-q}\gamma_{p+q}u_{k-q}v_{k-q}v^2_{p+q}u_pv_p]\}.$$

The remaining elements M_{12}, M_{21}, and M_{22} have a similar form.

LITERATURE CITED

1. S. V. Tyablikov, Methods in the Quantum Theory of Magnetism [in Russian], Nauka, Moscow (1975). (English translation published by Plenum Press, New York.)
2. K. J. Van Kranendon and J. H. Van Vleck, Rev. Mod. Phys., 30, 1 (1958).
3. V. L. Ginzburg and V. M. Fain, Zh. Eksp. Teor. Fiz., 39, 1323 (1960).
4. L. Néel, Usp. Fiz. Nauk, 107, 185 (1972).
5. N. N. Bogolyubov and S. V. Tyablikov, Zh. Eksp. Teor. Fiz., 19, 256 (1949).
6. V. G. Bar'yakhtar and V. A. Popov, Fiz. Tverd. Tela (Leningrad), 10, 773 (1968).
7. A. B. Harris, D. Kumar, B. I. Halperin, and P. C. Hohenberg, Phys. Rev. B, 3, 961 (1971).
8. Yu. A. Tserkovnikov, Dokl. Akad. Nauk SSSR, 169, 1064 (1966).
9. N. M. Plakida, Teor. Mat. Fiz., 5, 147 (1979).
10. N. M. Plakida, Phys. Lett. A, 43, 481 (1973).
11. A. L. Kuzemskii, Teor. Mat. Fiz., 36, 208 (1978).
12. Yu. A. Tserkovnikov, Teor. Mat. Fiz., 49, 219 (1981).
13. A. L. Kuzemsky, Commun. E17-88-677 [in English], JINR, Dubna (1988).
14. V. Christoph, A. L. Kuzemsky, and T. Frauenheim, in: Crystalline Electric Field Effects in f-Electron Magnetism, Plenum Press, New York (1982), pp. 219-227.
15. D. Marvakov, J. Vlahov, and A. L. Kuzemsky, J. Phys. C, 18, 2871 (1985).
16. D. Marvakov, A. L. Kuzemsky, and J. Vlahov, Physica (Utrecht) B, 138, 129 (1986).
17. D. Marvakov and A. L. Kuzemsky, Preprint ICTP IC/87/234, Trieste (1987).
18. A. L. Kuzemskii and D. Marvakov, in: Proc. of the Fourth International Symposium on Selected Problems of Statistical Mechanics, D-17-88-95 [in Russian], JINR, Dubna (1988), pp. 236-240.
19. V. I. Lymar' and Yu. G. Rudoi, Teor. Mat. Fiz., 21, 86 (1974).
20. V. I. Lymar' and Yu. G. Rudoi, Teor. Mat. Fiz., 24, 391 (1975).
21. V. I. Lymar' and Yu. G. Rudoi, Teor. Mat. Fiz., 30, 249 (1977).
22. N. N. Bogolyubov and N. N. Bogolyubov (Jr), Introduction to Quantum Statistical Mechanics [in Russian], Nauka, Moscow (1984).
23. N. N. Bogolyubov, Physica (Utrecht), 26, 1 (1960).
24. J. Nambu, Phys. Rev., 117, 648 (1960).
25. S. M. Rezende and R. M. White, Phys. Rev. B, 14, 2339 (1976).
26. C. G. Windsor, D. H. Saunderson, and E. Schedler, Phys. Rev. Lett., 37, 855 (1976).
27. U. Balucani and V. Tognetti, Rivista Nuovo Cimento, 6, 39 (1976).
28. S. W. Lovesey, Theory of Neutron Scattering from Condensed Matter, Vol. 2, Clarendon Press, Oxford (1984), p. 344.
29. R. H. Swendsen, Phys. Rev. Lett., 32, 1439 (1974).
30. R. H. Swendsen, Phys. Rev. B, 11, 1935 (1975).
31. R. H. Swendsen, Phys. Rev. B, 13, 3912 (1976).