Bogoliubov's Foresight and Development of the Modern Theoretical Physics

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Abstract: A brief survey of the author's works on the fundamental conceptual ideas of quantum statistical physics developed by N. N. Bogoliubov and his school was given. The development and applications of the method of quasiaverages to quantum statistical physics and condensed matter physics were analyzed. The relationship with the concepts of broken symmetry, quantum protectorate and emergence was examined, and the progress to date towards unified understanding of complex many-particle systems was summarized. Current trends for extending and using these ideas in quantum field theory and condensed matter physics were discussed, including microscopic theory of superfluidity and superconductivity, quantum theory of magnetism of complex materials, Bose-Einstein condensation, chirality of molecules, etc.

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The theory of symmetry is a basic tool for understanding and formulating the fundamental notions of physics. Symmetry considerations show that symmetry arguments are very powerful tool for bringing order into the very complicated picture of the real world. Many fundamental laws of physics in addition to their detailed features possess various symmetry properties. These symmetry properties lead to certain constraints and regularities on the possible properties of matter.
Thus the principles of symmetries belong to the underlying principles of physics. Moreover, the idea of symmetry is a useful and workable tool for many areas of the quantum field theory, statistical physics and condensed matter physics The fundamental works of N.N. Bogoliubov on many-body theory and quantum field theory [1, 2], on the theory of

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phase transitions, and on the general theory of symmetry provided a new perspective. Works and ideas of N.N. Bogoliubov and his school continue to influence and vitalize the development of modern physics [1, 3]. In recently published review article by A.L. Kuzemsky [4], which is a substantially extended version of his talk on the last Bogoliubov's Conference [3], the detailed analysis of a few selected directions of researches of N.N. Bogoliubov and his school was carried out. This interdisciplinary review focuses on the applications of symmetry principles to quantum and statistical physics in connection with some other branches of science. Studies of symmetries and the consequences of breaking them have led to deeper understanding in many areas of science. The role of symmetry in physics is well-known [5, 6, 7, 8, 9, 10]. Symmetry was and still is one of the major growth areas of scientific research, where the frontiers of mathematics and physics collide. Symmetry has always played an important role in condensed matter physics [5], from fundamental formulations of basic principles to concrete applications. Last decades show clearly its role and significance for fundamental physics. This was confirmed by awarding the Nobel Prize to Y. Nambu et al. in 2008. In fact, the fundamental ideas of N.N. Bogoliubov influenced Y. Nambu works greatly.

A symmetry can be exact or approximate. Symmetries inherent in the physical laws may be dynamically and spontaneously broken, i.e., they may not manifest themselves in the actual phenomena. It can be as well broken by certain reasons. It was already pointed by many authors, that non-Abelian gauge field become very useful in the second half of the twentieth century in the unified theory of electromagnetic and weak interactions, combined with symmetry breaking. Within the literature the term *broken symmetry* is used both very often and with different meanings. There are two terms, the spontaneous breakdown of symmetries and dynamical symmetry breaking, which sometimes have been used as opposed but such a distinction is irrelevant. However, the two terms may be used interchangeably. It should be stressed that a symmetry implies degeneracy. In general there are a multiplets of equivalent states related to each other by congruence operations. They can be distinguished only relative to a weakly coupled external environment which breaks the symmetry. Local gauged symmetries, however, cannot be broken this way because such an extended environment is not allowed (a superselection rule), so all states are singlets, i.e., the multiplicities are not observable except possibly for their global part. In other words, since a symmetry implies degeneracy of energy eigenstates, each multiplet of states forms a representation of a symmetry group G. Each member of a multiplet is labeled by a set of quantum numbers for which one may use the generators and Casimir invariants of the chain of subgroups, or else some observables which form a representation of G. It is a dynamical question whether or not the ground state, or the most stable state, is a singlet, a most symmetrical one.

Peierls [11, 12] gives a general definition of the notion of the spontaneous breakdown of symmetries which is suited equally well for the physics of particles and condensed matter physics. According to Peierls [11, 12], the term *broken symmetries* relates to situations in which symmetries which we expect to hold are valid only approximately or fail completely in certain situations.

The intriguing mechanism of spontaneous symmetry breaking is a unifying concept that lie at the basis of most of the recent developments in theoretical physics, from statistical mechanics to many-body theory and to elementary particles theory. It is known that when the Hamiltonian of a system is invariant under a symmetry operation, but the ground state is not, the symmetry of the system can be spontaneously broken. Symmetry breaking is termed *spontaneous* when there is no explicit term in a Lagrangian which manifestly breaks the symmetry.

The existence of degeneracy in the energy states of a quantal system is related to the invariance or symmetry properties of the system. By applying the symmetry operation to the ground state, one can transform it to a different but equivalent ground state. Thus the ground state is degenerate, and in the case of a continuous symmetry, infinitely degenerate. The real, or relevant, ground state of the system can only be one of these degenerate states. A system may exhibit the full symmetry of its Lagrangian, but it is characteristic of infinitely large systems that they also may condense into states of lower symmetry.

The article [4] examines the Bogoliubov's notion of quasiaverages, from the original papers [13, 14], through to modern theoretical concepts and ideas of how to describe both the degeneracy, broken symmetry and the diversity of the energy scales in the many-particle interacting systems. Current trends for extending and using Bogoliubov's ideas to quantum field theory and condensed matter physics problems were discussed, including microscopic theory of superfluidity and superconductivity, quantum theory of magnetism of complex materials, Bose-Einstein condensation, chirality of molecules, etc. It was demonstrated there that the profound and innovative idea of quasiaverages formulated by N.N. Bogoliubov, gives the so-called macro-objectivation of the degeneracy in domain of quantum statistical mechanics, quantum field theory and in the quantum physics in general. The complementary unifying ideas of modern physics, namely: spontaneous symmetry breaking, quantum protectorate and emergence were discussed also.

The interrelation of the concepts of symmetry breaking, quasiaverages and quantum protectorate was analyzed in the context of quantum theory and statistical physics. The leading idea was the statement of F. Wilczek [10]: "primary goal of fundamental physics is to discover profound concepts that illuminate our understanding of nature". The works of N.N. Bogoliubov on microscopic theory of superfluidity and superconductivity as well as on quasiaverages and broken symmetry belong to this class of ideas. Bogoliubov's notion of quasiaverage is an essential conceptual advance of modern physics, as well as the later concepts of quantum protectorate and emergence. These concepts manifest the operational ability of the notion of symmetry; they also demonstrate the power of the unification of various complicated phenomena and have certain predictive ability. Broadly speaking, these concepts are unifying and profound ideas "that illuminate our understanding of nature". In particular, Bogoliubov's method of quasiaverages gives the deep foundation and clarification of the concept of broken symmetry. It makes the emphasis on the notion of degeneracy and plays an important role in equilibrium statistical mechanics of many-particle systems. According to that concept, infinitely small perturbations can trigger macroscopic responses in the system if they break some symmetry and remove the related degeneracy (or quasi-degeneracy) of the equilibrium state. As a result, they can produce macroscopic effects even when the perturbation magnitude is tend to zero, provided that happens after passing to the thermodynamic limit. This approach has penetrated, directly or indirectly, many areas of the contemporary physics. Practical techniques covered include quasiaverages, Bogoliubov theorem on the singularity of $1/q^2$, Bogoliubov's inequality, and its applications to condensed matter physics.

Condensed matter physics is the field of physics that deals with the macroscopic physical properties of matter. In particular, it is concerned with the condensed phases that appear whenever the number of constituents in a system is extremely large and the interactions between the constituents are strong. The most familiar examples of condensed phases are solids and liquids. More exotic condensed phases include the superfluid and the Bose-Einstein condensate found in certain atomic systems. In condensed matter physics, the symmetry is important in classifying different phases and understanding the phase transitions between them. The phase transition is a physical phenomenon that occurs in macroscopic systems and consists in the following. In certain equilibrium states of the system an arbitrary small influence leads to a sudden change of its properties: the system passes from one homogeneous phase to another. Mathematically, a phase transition is treated as a sudden change of the structure and properties of the Gibbs distributions describing the equilibrium states of the system, for arbitrary small changes of the parameters determining the equilibrium [15]. The crucial concept here is the order parameter. In statistical physics the question of interest is to understand how the order of phase transition in a system of many identical interacting subsystems depends on the degeneracies of the states of each subsystem and on the interaction between subsystems. In particular, it is important to investigate a role of the symmetry and uniformity of the degeneracy and the symmetry of the interaction. Statistical mechanical theories of the system composed of many interacting identical subsystems have been developed frequently for the case of ferro- or antiferromagnetic spin system, in which the phase transition is usually found to be one of second order unless it is accompanied with such an additional effect as spin-phonon interaction. Second order phase transitions are frequently, if not always, associated with spontaneous breakdown of a global symmetry. It is then possible to find a corresponding order parameter which vanishes in the disordered phase and is nonzero in the ordered phase. Qualitatively the transition is understood as condensation of the broken symmetry charge carriers. The critical region is reasonably described by a local Lagrangian involving the order parameter field. Combining many elementary particles into a single interacting system may result in collective behavior that qualitatively differs from the properties allowed by the physical theory governing the individual building blocks. This is the essence of the emergence phenomenon.

It is known that the description of spontaneous symmetry breaking that underlies the connection between classically ordered objects in the thermodynamic limit and their individual quantum-mechanical building blocks is one of the cornerstones of modern condensed-matter theory and has found applications in many different areas of physics. The theory of spontaneous symmetry breaking, however, is inherently an equilibrium theory, which does not address the dynamics of quantum systems in the thermodynamic limit. Any state of matter is classified according to its order, and the type of order that a physical system can possess is profoundly affected by its dimensionality. Conventional long-range order, as in a ferromagnet or a crystal, is common in three-dimensional systems at low temperature. However, in two-dimensional systems with a continuous symmetry, true long-range order is destroyed by thermal fluctuations at any finite temperature. Consequently, for the case of identical bosons, a uniform two-dimensional fluid cannot undergo Bose-Einstein condensation, in contrast to the three-dimensional case. The two-dimensional system can be effectively investigated on the basis of Bogoliubov' inequality. Generally inter-particle interaction is responsible for a phase transition. But Bose-Einstein condensation type of phase transition occurs entirely due to the Bose-Einstein statistics. The typical situation is a many-body system made of identical bosons, e.g. atoms carrying an integer total angular momentum. To proceed one must construct the ground state. The simplest possibility to do so occurs when bosons are non-interacting. In this case, the ground state is simply obtained by putting all bosons in the lowest energy single particle state, as the brilliant Bogoliubov's theory describes.

The method of quasiaverages is a constructive workable scheme for studying systems with spontaneous symmetry breakdown. A quasiaverage is a thermodynamic (in statistical mechanics) or vacuum (in quantum field theory) average of dynamical quantities in a specially modified averaging procedure, enabling one to take into account the effects of the influence of state degeneracy of the system. The method gives the so-called macro-objectivation of the degeneracy in the domain of quantum statistical mechanics and in quantum physics. In statistical mechanics, under spontaneous symmetry breakdown one can, by using the method of quasiaverages, describe macroscopic observable within the framework of the microscopic approach.

In considering problems of findings the eigenfunctions in quantum mechanics it is well known that the theory of perturbations should be modified substantially for the degenerate systems. In the problems of statistical mechanics we have always the degenerate case due to existence of the additive conservation laws. The traditional approach to quantum statistical mechanics [16, 17] is based on the unique canonical quantization of classical Hamiltonians for systems with finitely many degrees of freedom together with the ensemble averaging in terms of traces involving a statistical operator ρ. For an operator $\hat{A}$ corresponding to some physical quantity A the average value of A will be given as

$$\langle A \rangle_H = \mathrm{Tr} \rho A; \quad \rho = \exp^{-\beta H} / \mathrm{Tr} \exp^{-\beta H}, \tag{1}$$

where H is the Hamiltonian of the system, $\beta = 1/k_B T$ is the reciprocal of the temperature. In general, the statistical operator [16] or density matrix ρ is defined by its matrix elements in the φ_m-representation:

$$\rho_{nm} = \frac{1}{N} \sum_{i=1}^{N} c_n^i (c_m^i)^*. \tag{2}$$

In this notation the average value of A will be given as

$$\langle A \rangle = \frac{1}{N} \sum_{i=1}^{N} \int \Psi_i^* A \Psi_i d\tau. \tag{3}$$

The averaging in Eq.(3) is both over the state of the ith system and over all the systems in the ensemble. The Eq.(3) becomes

$$\langle A \rangle = \mathrm{Tr}\rho A; \quad \mathrm{Tr}\rho = 1. \tag{4}$$

Thus an ensemble of quantum mechanical systems is described by a density matrix [16, 18]. In a suitable representation, a density matrix ρ takes the form

$$\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$$

where p_k is the probability of a system chosen at random from the ensemble will be in the microstate $|\psi_k\rangle$. So the trace of ρ, denoted by $\mathrm{Tr}(\rho)$, is 1. This is the quantum mechanical analogue of the fact that the accessible region of the classical phase space has total probability 1. It is also assumed that the ensemble in question is stationary, i.e. it does not change in time. Therefore, by Liouville theorem, $[\rho, H] = 0$, i.e., $\rho H = H\rho$, where H is the Hamiltonian of the system. Thus the density matrix describing ρ is diagonal in the energy representation.
Suppose that

$$H = \sum_i E_i |\psi_i\rangle\langle\psi_i|,$$

where E_i is the energy of the i-th energy eigenstate. If a system i-th energy eigenstate has n_i number of particles, the corresponding observable, the number operator, is given by

$$N = \sum_i n_i |\psi_i\rangle\langle\psi_i|.$$

It is known [16], that the state $|\psi_i\rangle$ has (unnormalized) probability

$$p_i = e^{-\beta(E_i - \mu n_i)}.$$

Thus the grand canonical ensemble is the mixed state

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = \tag{5}$$
$$\sum_i e^{-\beta(E_i - \mu n_i)} |\psi_i\rangle\langle\psi_i| = e^{-\beta(H - \mu N)}.$$

The grand partition, the normalizing constant for $\mathrm{Tr}(\rho)$ to be 1, is

$$\mathcal{Z} = \mathbf{Tr}[e^{-\beta(H-\mu N)}].$$

Thus we obtain [16]

$$\langle A \rangle = \mathrm{Tr}\rho A = \mathrm{Tr}e^{\beta(\Omega - H + \mu N)} A. \tag{6}$$

Here $\beta = 1/k_B T$ is the reciprocal temperature and Ω is the normalization factor.
It is known [16] that the averages $\langle A \rangle$ are unaffected by a change of representation. The most important is the representation in which ρ is diagonal $\rho_{mn} = \rho_m \delta_{mn}$. We then have $\langle \rho \rangle = Tr\rho^2 = 1$. It is clear then that $Tr\rho^2 \leq 1$ in any representation. The core of the problem lies in establishing the existence of a thermodynamic limit [19] (such as $N/V =$ const, $V \to \infty$, $N =$ number of degrees of freedom, $V =$ volume) and its evaluation for the quantities of interest.
The evolution equation for the density matrix is a quantum analog of the Liouville equation in classical mechanics. A related equation describes the time evolution of the expectation values of observables, it is given by the Ehrenfest theorem. Canonical quantization yields a quantum-mechanical version of this theorem. This procedure, often used to devise quantum analogues of classical systems, involves describing a classical system using Hamiltonian mechanics. Classical variables are then re-interpreted as quantum operators, while Poisson brackets are replaced by commutators. In this case, the resulting equation is

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[H, \rho] \tag{7}$$

where ρ is the density matrix. When applied to the expectation value of an observable, the corresponding equation is given by Ehrenfest theorem, and takes the form

$$\frac{d}{dt}\langle A \rangle = \frac{i}{\hbar}\langle [H, A] \rangle \tag{8}$$

where A is an observable. Thus in the statistical mechanics the average $\langle A \rangle$ of any dynamical quantity A is defined in a single-valued way [16, 18].
In the situations with degeneracy the specific problems appear. In quantum mechanics, if two linearly independent state vectors (wavefunctions in the Schroedinger picture) have the same energy, there is a degeneracy. In this case more than one independent state of the system corresponds to a single energy level. If the statistical equilibrium state of the system possesses lower symmetry than the Hamiltonian of the system (i.e. the situation with the spontaneous symmetry breakdown), then it is necessary to supplement the averaging procedure (6) by a rule forbidding irrelevant averaging over the values of macroscopic quantities considered for which a change is not accompanied by a change in energy.
This is achieved by introducing quasiaverages, that is, averages over the Hamiltonian $H_{\nu\vec{e}}$ supplemented by infinitesimally-small terms that violate the additive conservations laws $H_{\nu\vec{e}} = H + \nu(\vec{e} \cdot \vec{M})$, $(\nu \to 0)$. Thermodynamic averaging may turn out to be unstable with respect to such a change of the original Hamiltonian, which is another indication of degeneracy of the equilibrium state.
According to Bogoliubov [13, 14], the quasiaverage of a dynamical quantity A for the system with the Hamiltonian $H_{\nu\vec{e}}$ is defined as the limit

$$\preccurlyeq A \succcurlyeq = \lim_{\nu \to 0} \langle A \rangle_{\nu\vec{e}}, \tag{9}$$

where $\langle A \rangle_{\nu\vec{e}}$ denotes the ordinary average taken over the Hamiltonian $H_{\nu\vec{e}}$, containing the small symmetry-breaking terms introduced by the inclusion parameter ν, which vanish as $\nu \to 0$ after passage to the thermodynamic limit $V \to \infty$. Thus the existence of degeneracy is reflected directly in the quasiaverages by their dependence upon the arbitrary unit vector $\vec{e}$. It is also clear that

$$\langle A \rangle = \int \preccurlyeq A \succcurlyeq d\vec{e}. \tag{10}$$

According to definition (10), the ordinary thermodynamic average is obtained by extra averaging of the quasiaverage over the symmetry-breaking group [13, 17]. Thus to describe the case of a degenerate state of statistical equilibrium quasiaverages are more convenient, more physical, than ordinary averages [16, 13]. The latter are the same quasiaverages only averaged over all the directions $\vec{e}$.

It is necessary to stress, that the starting point for Bogoliubov's work [13] was an investigation of additive conservation laws and selection rules, continuing and developing the approach by P. Curie for derivation of selection rules for physical effects. Bogoliubov demonstrated that in the cases when the state of statistical equilibrium is degenerate, as in the case of the Heisenberg ferromagnet, one can remove the degeneracy of equilibrium states with respect to the group of spin rotations by including in the Hamiltonian H an additional noninvariant term $\nu M_z V$ with an infinitely small ν. Thus the quasiaverages do not follow the same selection rules as those which govern the ordinary averages. For the Heisenberg ferromagnet the ordinary averages must be invariant with regard to the spin rotation group. The corresponding quasiaverages possess only the property of covariance. It is clear that the unit vector $\vec{e}$, i.e., the direction of the magnetization $\vec{M}$ vector, characterizes the degeneracy of the considered state of statistical equilibrium. In order to remove the degeneracy one should fix the direction of the unit vector $\vec{e}$. It can be chosen to be along the z direction. Then all the quasiaverages will be the definite numbers. This is the kind that one usually deals with in the theory of ferromagnetism.

The value of the quasi-average (9) may depend on the concrete structure of the additional term $\Delta H = H_\nu - H$, if the dynamical quantity to be averaged is not invariant with respect to the symmetry group of the original Hamiltonian H. For a degenerate state the limit of ordinary averages (10) as the inclusion parameters ν of the sources tend to zero in an arbitrary fashion, may not exist. For a complete definition of quasiaverages it is necessary to indicate the manner in which these parameters tend to zero in order to ensure convergence [16]. On the other hand, in order to remove degeneracy it suffices, in the construction of H, to violate only those additive conservation laws whose switching lead to instability of the ordinary average. Thus in terms of quasiaverages the selection rules for the correlation functions [16] that are not relevant are those that are restricted by these conservation laws.

By using H_ν, we define the state $\omega(A) = \langle A \rangle_\nu$ and then let ν tend to zero (after passing to the thermodynamic limit). If all averages $\omega(A)$ get infinitely small increments under infinitely small perturbations ν, this means that the state of statistical equilibrium under consideration is nondegenerate [16]. However, if some states have finite increments as $\nu \to 0$, then the state is degenerate. In this case, instead of ordinary averages $\langle A \rangle_H$, one should introduce the quasiaverages (9), for which the usual selection rules do not hold.

The method of quasiaverages is directly related to the principle weakening of the correlation [16] in many-particle systems. According to this principle, the notion of the weakening of the correlation, known in statistical mechanics [16], in the case of state degeneracy must be interpreted in the sense of the quasiaverages.
The quasiaverages may be obtained from the ordinary averages by using the cluster property which was formulated by Bogoliubov [14]. This was first done when deriving the Boltzmann equations from the chain of equations for distribution functions, and in the investigation of the model Hamiltonian in the theory of superconductivity [16]. To demonstrate this let us consider averages (quasiaverages) of the form

$$F(t_1, x_1, \dots t_n, x_n) = \langle \dots \Psi^\dagger(t_1, x_1) \dots \Psi(t_j, x_j) \dots \rangle, \tag{11}$$

where the number of creation operators $\Psi^\dagger$ may be not equal to the number of annihilation operators Ψ. We fix times and split the arguments $(t_1, x_1, \dots t_n, x_n)$ into several clusters $(\dots, t_\alpha, x_\alpha, \dots), \dots, (\dots, t_\beta, x_\beta, \dots)$. Then it is reasonably to assume that the distances between all clusters $|x_\alpha - x_\beta|$ tend to infinity. Then, according to the cluster property, the average value (11) tends to the product of averages of collections of operators with the arguments $(\dots, t_\alpha, x_\alpha, \dots), \dots, (\dots, t_\beta, x_\beta, \dots)$

$$\lim_{|x_\alpha - x_\beta| \to \infty} F(t_1, x_1, \dots t_n, x_n) = F(\dots, t_\alpha, x_\alpha, \dots) \dots F(\dots, t_\beta, x_\beta, \dots). \tag{12}$$

For equilibrium states with small densities and short-range potential, the validity of this property can be proved [16]. For the general case, the validity of the cluster property has not yet been proved. Bogoliubov formulated it not only for ordinary averages but also for quasiaverages, i.e., for anomalous averages, too. It works for many important models, including the models of superfluidity and superconductivity [17].
To illustrate this statement consider Bogoliubov's theory of a Bose-system with separated condensate, which is given by the Hamiltonian [16]

$$H_\Lambda = \int_\Lambda \Psi^\dagger(x)(-\frac{\Delta}{2m})\Psi(x)dx - \mu \int_\Lambda \Psi^\dagger(x)\Psi(x)dx \tag{13}$$
$$+\frac{1}{2}\int_{\Lambda^2} \Psi^\dagger(x_1)\Psi^\dagger(x_2)\Phi(x_1 - x_2)\Psi(x_2)\Psi(x_1)dx_1dx_2.$$

This Hamiltonian can be written also in the following form

$$H_\Lambda = H_0 + H_1 = \int_\Lambda \Psi^\dagger(q)(-\frac{\Delta}{2m})\Psi(q)dq \tag{14}$$
$$+\frac{1}{2}\int_{\Lambda^2} \Psi^\dagger(q)\Psi^\dagger(q')\Phi(q - q')\Psi(q')\Psi(q)dqdq'.$$

Here, $\Psi(q)$, and $\Psi^\dagger(q)$ are the operators of annihilation and creation of bosons. They satisfy the canonical commutation relations

$$[\Psi(q), \Psi^\dagger(q')] = \delta(q - q'); \quad [\Psi(q), \Psi(q')] = [\Psi^\dagger(q), \Psi^\dagger(q')] = 0. \tag{15}$$

The system of bosons is contained in the cube A with the edge L and volume V. It was assumed that it satisfies periodic boundary conditions and the potential $\Phi(q)$ is spherically symmetric and proportional to the small parameter. It was also assumed that, at temperature zero, a certain macroscopic number of particles having a nonzero density is situated in the state with momentum zero.
The operators $\Psi(q)$, and $\Psi^{\dagger}(q)$ are represented in the form

$$\Psi(q) = a_0/\sqrt{V}; \quad \Psi^{\dagger}(q) = a_0^{\dagger}/\sqrt{V}, \tag{16}$$

where a_0 and $a_0^{\dagger}$ are the operators of annihilation and creation of particles with momentum zero. To explain the phenomenon of superfluidity, one should calculate the spectrum of the Hamiltonian, which is quite a difficult problem. Bogoliubov suggested the idea of approximate calculation of the spectrum of the ground state and its elementary excitations based on the physical nature of superfluidity. His idea consists of a few assumptions. The main assumption is that at temperature zero the macroscopic number of particles (with nonzero density) has the momentum zero. Therefore, in the thermodynamic limit, the operators $a_0/\sqrt{V}$ and $a_0^{\dagger}/\sqrt{V}$ commute

$$\lim_{V\to\infty}\left[a_0/\sqrt{V}, a_0^{\dagger}/\sqrt{V}\right] = \frac{1}{V} \to 0 \tag{17}$$

and are c-numbers. Hence, the operator of the number of particles $N_0 = a_0^{\dagger}a_0$ is a c-number, too. The concept of quasiaverages was introduced by Bogoliubov on the basis of an analysis of many-particle systems with a degenerate statistical equilibrium state. Such states are inherent to various physical many-particle systems. Those are liquid helium in the superfluid phase, metals in the superconducting state, magnets in the ferromagnetically ordered state, liquid crystal states, the states of superfluid nuclear matter, etc.
From the other hand, it is clear that only a thorough experimental and theoretical investigation of quasiparticle many-body dynamics of the many-particle systems can provide the answer on the relevant microscopic picture [20]. As is well known, Bogoliubov was first to emphasize the importance of the time scales in the many-particle systems thus anticipating the concept of emergence of macroscopic irreversible behavior starting from the reversible dynamic equations.
More recently it has been possible to go step further. This step leads to a deeper understanding of the relations between microscopic dynamics and macroscopic behavior on the basis of emergence concept [21, 22, 23]. There has been renewed interest in emergence within discussions of the behavior of complex systems and debates over the reconcilability of mental causation, intentionality, or consciousness with physicalism. This concept is also at the heart of the numerous discussions on the interrelation of the reductionism and functionalism.
A vast amount of current researches focuses on the search for the organizing principles responsible for emergent behavior in matter [23, 24], with particular attention to correlated matter, the study of materials in which unexpectedly new classes of behavior emerge in response to the strong and competing interactions among their elementary constituents.

As it was formulated by D.Pines [24], "we call *emergent behavior* . . . the phenomena that owe their existence to interactions between many subunits, but whose existence cannot be deduced from a detailed knowledge of those subunits alone".
Emergence - macro-level effect from micro-level causes - is an important and profound interdisciplinary notion of modern science. There has been renewed interest in emergence within discussions of the behavior of complex systems. In the search for a "theory of everything," scientists scrutinize ever-smaller components of the universe. String theory postulates units so minuscule that researchers would not have the technology to detect them for decades. R.B. Laughlin [21, 22], argued that smaller is not necessarily better. He proposes turning our attention instead to emerging properties of large agglomerations of matter. For instance, chaos theory has been all the rage of late with its speculations about the "butterfly effect," but understanding how individual streams of air combine to form a turbulent flow is almost impossible. It may be easier and more efficient, says Laughlin, to study the turbulent flow. Laws and theories follow from collective behavior, not the other way around, and if one will try to analyze things too closely, he may not understand how they work on a macro level. In many cases, the whole exhibits properties that can not be explained by the behavior of its parts. As Laughlin points out, mankind use computers and internal combustion engines every day, but scientists do not totally understand why all of their parts work the way they do. It is well known that there are many branches of physics and chemistry where phenomena occur which cannot be described in the framework of interactions amongst a few particles. As a rule, these phenomena arise essentially from the cooperative behavior of a large number of particles. Such many-body problems are of great interest not only because of the nature of phenomena themselves, but also because of the intrinsic difficulty in solving problems which involve interactions of many particles in terms of known Anderson statement that "more is different". It is often difficult to formulate a fully consistent and adequate microscopic theory of complex cooperative phenomena. R. Laughlin and D. Pines invented an idea of a quantum protectorate [21, 23], "a stable state of matter, whose generic low-energy properties are determined by a higher-organizing principle and nothing else" [23]. This idea brings into physics the concept that emphasize the crucial role of low-energy and high-energy scales for treating the propertied of the substance. It is known that a many-particle system (e.g. electron gas) in the low-energy limit can be characterized by a small set of *collective* (or hydrodynamic) variables and equations of motion corresponding to these variables. Going beyond the framework of the low-energy region would require the consideration of plasmon excitations, effects of electron shell reconstructing, etc. The existence of two scales, low-energy and high-energy, in the description of physical phenomena is used in physics, explicitly or implicitly.
According to R. Laughlin and D. Pines, "The emergent physical phenomena regulated by higher organizing principles have a property, namely their insensitivity to microscopics, that is directly relevant to the broad question of what is knowable in the deepest sense of the term. The low energy excitation spectrum of a conventional superconductor, for example, is completely generic and is characterized by a handful of parameters that may

be determined experimentally but cannot, in general, be computed from first principles. An even more trivial example is the low-energy excitation spectrum of a conventional crystalline insulator, which consists of transverse and longitudinal sound and nothing else, regardless of details. It is rather obvious that one does not need to prove the existence of sound in a solid, for it follows from the existence of elastic moduli at long length scales, which in turn follows from the spontaneous breaking of translational and rotational symmetry characteristic of the crystalline state. Conversely, one therefore learns little about the atomic structure of a crystalline solid by measuring its acoustics. The crystalline state is the simplest known example of a quantum protectorate, a *stable state of matter whose generic low-energy properties are determined by a higher organizing principle and nothing else* ... Other important quantum protectorates include superfluidity in Bose liquids such as ^4He and the newly discovered atomic condensates, superconductivity, band insulation, ferromagnetism, antiferromagnetism, and the quantum Hall states. The low-energy excited quantum states of these systems are particles in exactly the same sense that the electron in the vacuum of quantum electrodynamics is a particle ... Yet they are not elementary, and, as in the case of sound, simply do not exist outside the context of the stable state of matter in which they live. These quantum protectorates, with their associated emergent behavior, provide us with explicit demonstrations that the underlying microscopic theory can easily have no measurable consequences whatsoever at low energies. The nature of the underlying theory is unknowable until one raises the energy scale sufficiently to escape protection". The notion of *quantum protectorate* was introduced to unify some generic features of complex physical systems on different energy scales, and is a complimentary unifying idea resembling the symmetry breaking concept in a certain sense.

The sources of quantum protection in high-T_c superconductivity and low-dimensional systems were discussed as well. According to Anderson and Pines, the source of quantum protection is likely to be a collective state of the quantum field, in which the individual particles are sufficiently tightly coupled that elementary excitations no longer involve just a few particles, but are collective excitations of the whole system. As a result, macroscopic behavior is mostly determined by overall conservation laws.

It is worth also noticing that the notion of quantum protectorate [21, 23] complements the concepts of broken symmetry and quasiaverages by making emphasis on the hierarchy of the energy scales of many-particle systems. In an indirect way these aspects arose already when considering the scale invariance and spontaneous symmetry breaking.

D.N. Zubarev showed [18] that the concepts of symmetry breaking perturbations and quasiaverages play an important role in the theory of irreversible processes as well. The method of the construction of the nonequilibrium statistical operator becomes especially deep and transparent when it is applied in the framework of the quasiaverage concept. For detailed discussion of the Bogoliubov's ideas and methods in the fields of nonlinear oscillations and nonequilibrium statistical mechanics see Refs. [1, 25, 26]. It was demonstrated in Ref. [4] that the connection and interrelation of the conceptual advances of the many-body physics discussed above show that those concepts, though different in

details, have complementary character. Many problems in the field of statistical physics of complex materials and systems (e.g. the chirality of molecules) and the foundations of the microscopic theory of magnetism and superconductivity were discussed in relation to these ideas.
To summarize, it was demonstrated that the Bogoliubov's method of quasiaverages plays a fundamental role in equilibrium and nonequilibrium statistical mechanics and quantum field theory and is one of the pillars of modern physics. It will serve for the future development of physics as invaluable tool. All the methods developed by N. N. Bogoliubov are and will remain the important core of a theoretician's toolbox, and of the ideological basis behind this development. Additional material and discussion of these problems can be found in recent publications [27, 28, 29, 30].

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