Pions in the quark matter phase diagram

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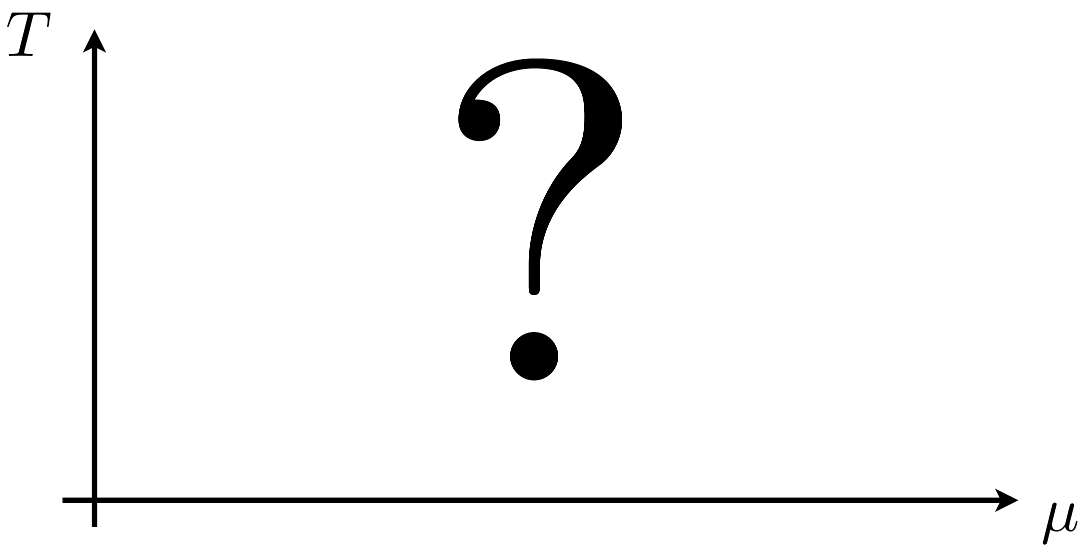
in collaboration with David Blaschke and Roberto Anglani

Aim of the talk

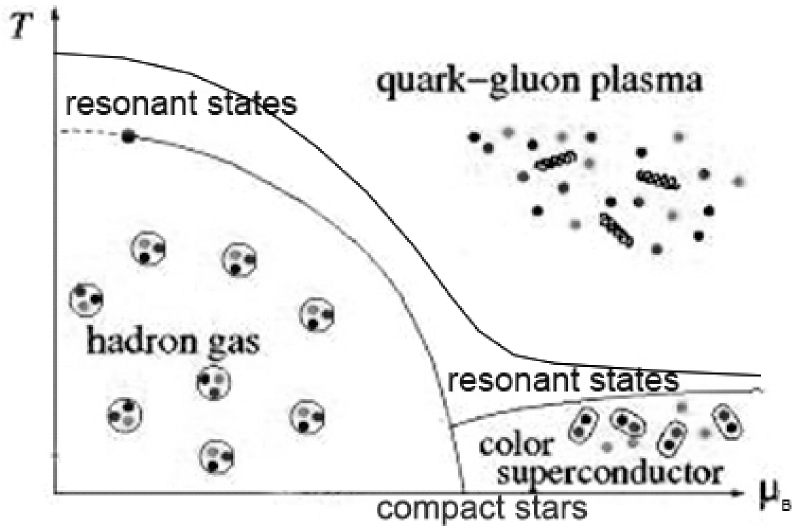
investigation of the
phase diagram of QCD
beyond mean field level
in NJL framework

one challenging problem of quantum chromodynamics is

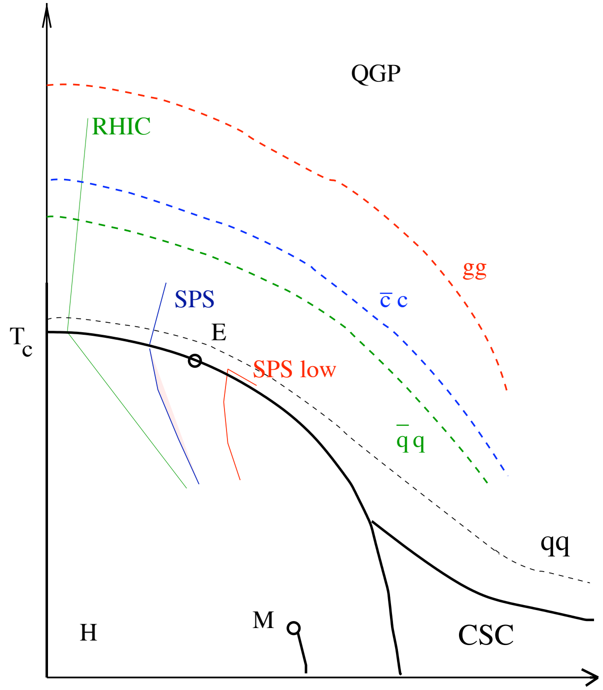
the study of phase diagram



1.1. what we know about phase diagram



Zhuang, P.F. et al. 0710.3634 [hep-ph]



Shuryak and Zahed hep-ph/0403127

2. the effective model

how to give a reliable description in the region around the critical values of chemical potential?

perturbation theory cannot be applied in this region

we have to accept a good compromise.
an effective model:
the Nambu--Jona-Lasinio

The NJL model of QCD mimics the quark–quark interaction mediated by gluons with an effective point–like four fermion interaction

<u>cons</u>

absence of gluon in the Lagrangian; quarks are not confined; etc.

<u>pro</u>

a simple approach to the description of chiral symmetry breaking and phase transitions; analytical calculations possible

2.2 the starting point: the NJL Lagrangian

For the description of hot, dense Fermi-systems,
with strong short-range interactions we consider a Lagrangian
with internal degrees of freedom (2-flavor, 3-color),
with a current -current-type four-Fermion interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{qq} + \mathcal{L}_{q\bar{q}}$$

$$\mathcal{L}_0 = \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q$$

$$\mathcal{L}_{q\bar{q}} = G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]$$

$$\mathcal{L}_{qq} = G_D\sum_{A=2,5,7}\left[\bar{q}i\gamma_5 C\tau_2\lambda_A\bar{q}^T\right]\left[q^T iC\gamma_5\tau_2\lambda_A q\right]$$

$$q = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}\otimes\begin{pmatrix}u\\ d\end{pmatrix}\otimes\begin{pmatrix}r\\ g\\ b\end{pmatrix}$$

$$\boldsymbol{\tau} = (\tau_1,\tau_2,\tau_3) \quad C = i\gamma_2\gamma_0$$

$m_{0,u} = m_{0,d} = m_0$

$\mu_u = \mu_d = \mu$

G_S Scalar and pseudoscalar coupling strength

G_D Scalar diquark coupling strength

the partition function $\mathcal{Z} = \int [dq]\,[d\bar{q}]\,\exp\left[\int_0^\beta d\tau \int d^3x\; \mathcal{L}\right]$ $\qquad \Omega = -T\ln Z$

Hubbard–Stratonovich auxiliary fields

$$\mathcal{Z} \;=\; \int [dq]\,[d\bar{q}]\,[d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\,\exp\left[\int_0^\beta d\tau \int d^3x\,\mathcal{L}\right]$$

$$\mathcal{L}_{\rm eff} = -\frac{\sigma^2+\pi^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D} + \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q - \bar{q}(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})q + i\frac{\Delta_A^*}{2}q^T iC\gamma_5\tau_2\lambda_A q - i\frac{\Delta_A}{2}\bar{q}i\gamma_5 C\tau_2\lambda_A\bar{q}^T$$

Nambu–Gorkov formalism

$\Psi \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ q^c \end{pmatrix} \qquad \bar{\Psi} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{q} & \bar{q}^c \end{pmatrix} \qquad q^c(x) \equiv C\bar{q}^T(x)$

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\,\exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D}\right)\right] \int [d\Psi]\,[d\bar{\Psi}]\,\exp\left[\int_0^\beta d\tau \int d^3x\,\bar{\Psi}\,S^{-1}\Psi\right]$$

$$S^{-1} \equiv \begin{pmatrix} i\partial\!\!\!/ + \mu\gamma_0 - m_0 - \sigma - i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} & \Delta_A\gamma_5\tau_2\lambda_A \\ -\Delta_A^*\gamma_5\tau_2\lambda_A & i\partial\!\!\!/ - \mu\gamma_0 - m_0 - \sigma - i\gamma_5\boldsymbol{\tau}^t\cdot\boldsymbol{\pi} \end{pmatrix}$$

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\left\{\exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D}\right)\right]\exp\left[\mathrm{Tr}\left(\ln S^{-1}\right)\right]\right\}$$

3. the mean field approximation

how to calculate this?

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}] \left\{ \exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2 + \boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^* \Delta_A}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S^{-1} \right) \right] \right\}$$

the mean field approximation (MFA)

decompose bosonic collective fields into a

homogeneous MF part + ~~fluctuation part~~

↓ order parameter: characterization of phase structure

~~↓ correlations~~

$\Delta \rightarrow \Delta_{MF} + \delta \quad \sigma \rightarrow \sigma_{MF} + \sigma$

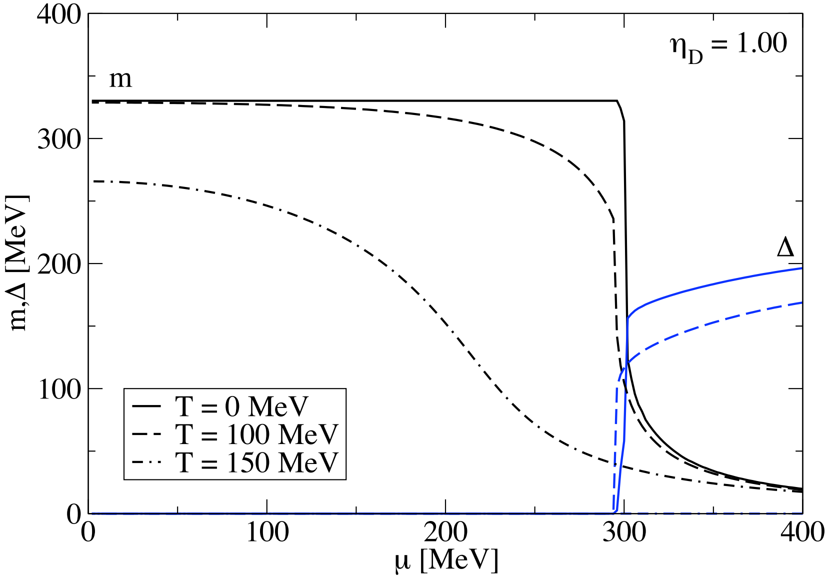
$$\frac{\partial \Omega}{\partial m} = \frac{\partial \Omega}{\partial \Delta} = 0 \qquad \mathcal{Z}_{MF} = \exp\left[\beta V \left(-\frac{\sigma_{MF}^2 + \boldsymbol{\pi}_{MF}^2}{4G_S} - \frac{\Delta_{MF}^* \Delta_{MF}}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S_{MF}^{-1} \right) \right]$$

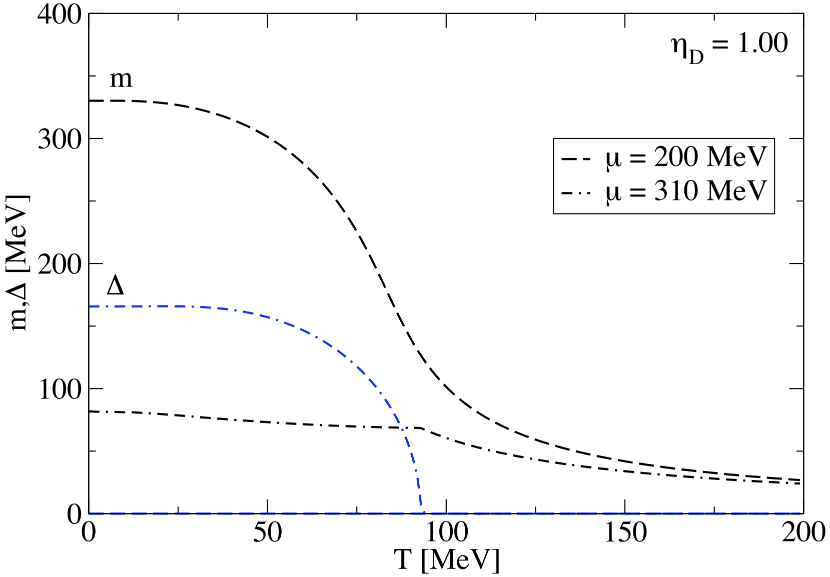
$$m - m_0 = 8G_S\, m \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_{\mathbf{p}}} \left\{ \left[1 - 2n_F(E_{\mathbf{p}}^-)\right] \frac{\xi_{\mathbf{p}}^-}{E_{\mathbf{p}}^-} + \left[1 - 2n_F(E_{\mathbf{p}}^+)\right] \frac{\xi_{\mathbf{p}}^+}{E_{\mathbf{p}}^+} + n_F(-\xi_{\mathbf{p}}^+) - n_F(\xi_{\mathbf{p}}^-) \right\}$$

$$\Delta = 8G_D \int \frac{d^3p}{(2\pi)^3} \left[\frac{1 - 2n_F(E_{\mathbf{p}}^-)}{E_{\mathbf{p}}^-} + \frac{1 - 2n_F(E_{\mathbf{p}}^+)}{E_{\mathbf{p}}^+} \right]$$

$E_{\mathbf{p}}^{\pm} = \sqrt{\left(\xi_{\mathbf{p}}^{\pm}\right)^2 + \Delta^2}$ with $\xi_{\mathbf{p}}^{\pm} = E_{\mathbf{p}} \pm \mu$, $E_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$

3.1 results of MFA





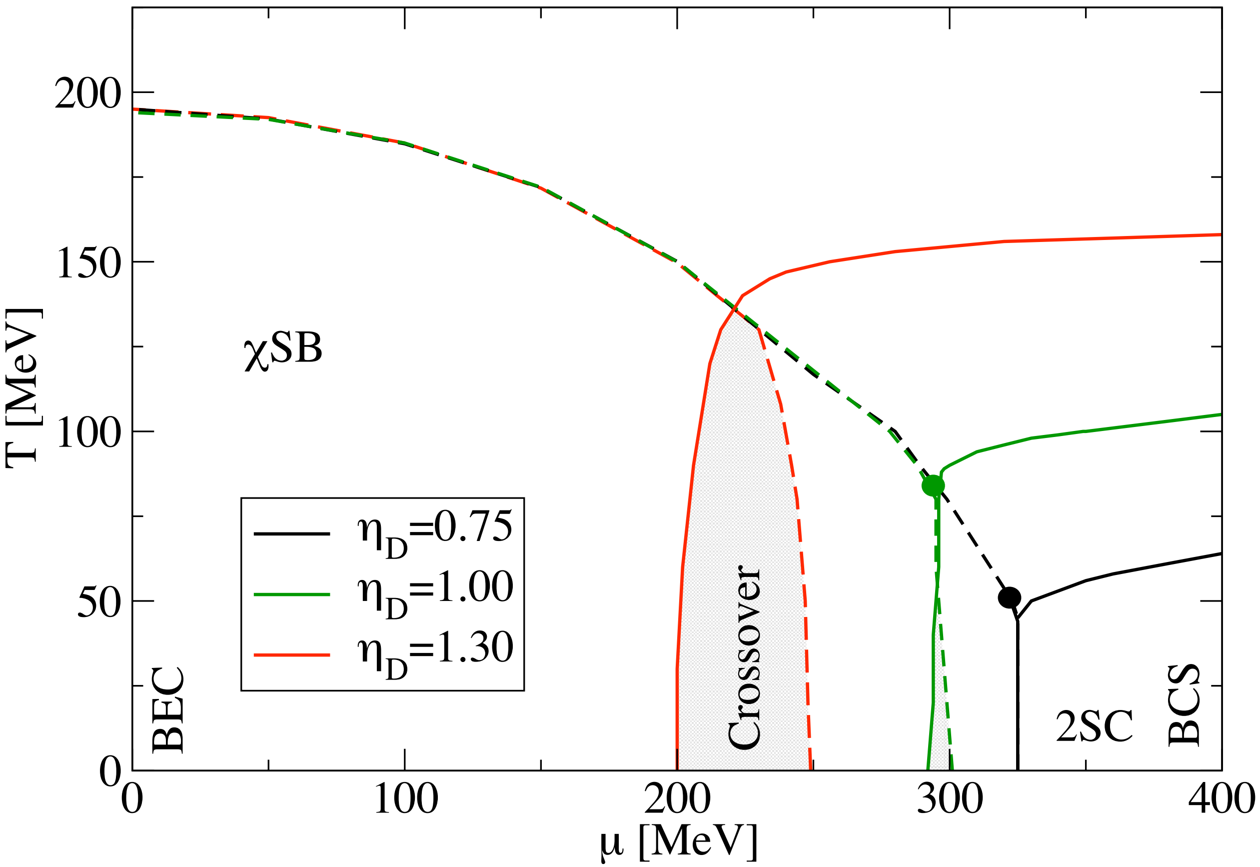
$\Lambda = 629.540$ MeV

$m_0 = 5.27697$ MeV

$G_S \Lambda^2 = 2.17576$

H. Grigorian, Phys. Part. Nucl. Lett. **4**, 223 (2007) [arXiv:hep-ph/0602238].

the phase diagram MF



4 what about fluctuations?

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}] \left\{ \exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S^{-1} \right) \right] \right\}$$

$$S^{-1} = S^{-1}_{MF} + \Sigma$$

ln-expansion around MF values

$$\Sigma \equiv \begin{pmatrix} -\sigma - i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi} & \delta_A \gamma_5 \tau_2 \lambda_A \\ -\delta_A^* \gamma_5 \tau_2 \lambda_A & -\sigma - i\gamma_5 \boldsymbol{\tau}^t \cdot \boldsymbol{\pi} \end{pmatrix}$$

$$\begin{aligned} \mathrm{Tr}[\ln(S^{-1})] &= \mathrm{Tr}[\ln(S^{-1}_{MF} + \Sigma)] \\ &= \mathrm{Tr}\{\ln[S^{-1}_{MF}(1 + S_{MF}\Sigma)]\} \\ &= \mathrm{Tr}\ln S^{-1}_{MF} + \mathrm{Tr}\ln[1 + S_{MF}\Sigma] \\ &= \mathrm{Tr}\ln S^{-1}_{MF} + \mathrm{Tr}[S_{MF}\Sigma - \frac{1}{2} S_{MF}\Sigma S_{MF}\Sigma + \ldots] \end{aligned}$$

$$\mathrm{Tr}\left(S_{MF}\Sigma S_{MF}\Sigma\right) = (\boldsymbol{\pi}, \sigma, \delta_2^*, \delta_2, \delta_5^*, \delta_7^*) \begin{pmatrix} \Pi_{\pi\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & \Pi_{\sigma\sigma} & \Pi_{\sigma\delta_2} & \Pi_{\sigma\delta_2^*} & 0 & 0 \\ 0 & \Pi_{\delta_2^*\sigma} & \Pi_{\delta_2^*\delta_2} & \Pi_{\delta_2^*\delta_2^*} & 0 & 0 \\ 0 & \Pi_{\delta_2\sigma} & \Pi_{\delta_2\delta_2} & \Pi_{\delta_2\delta_2^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{\delta_5^*\delta_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{\delta_7^*\delta_7} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi} \\ \sigma \\ \delta_2 \\ \delta_2^* \\ \delta_5 \\ \delta_7 \end{pmatrix}$$

4.1 meson polarization functions and masses

$$\Pi_{\pi\pi}(q_0,\mathbf{q}) \;=\; 2\int \frac{d^3p}{(2\pi)^3}\sum_{s_p,s_k}\mathcal{T}_-^+(s_p,s_k)\left\{\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0-s_k\xi_{\mathbf{p+q}}^{s_k}+s_p\xi_{\mathbf{p}}^{s_p}}-\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0+s_k\xi_{\mathbf{p+q}}^{s_k}-s_p\xi_{\mathbf{p}}^{s_p}}\right.$$
$$\left.+\sum_{t_p,t_k}\frac{t_pt_k}{E_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}}\frac{n_F(t_pE_{\mathbf{p}}^{s_p})-n_F(t_kE_{\mathbf{p+q}}^{s_k})}{q_0-t_kE_{\mathbf{p+q}}^{s_k}+t_pE_{\mathbf{p}}^{s_p}}\left(t_pt_kE_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}+s_ps_k\xi_{\mathbf{p}}^{s_p}\xi_{\mathbf{p+q}}^{s_k}-|\Delta|^2\right)\right\}$$

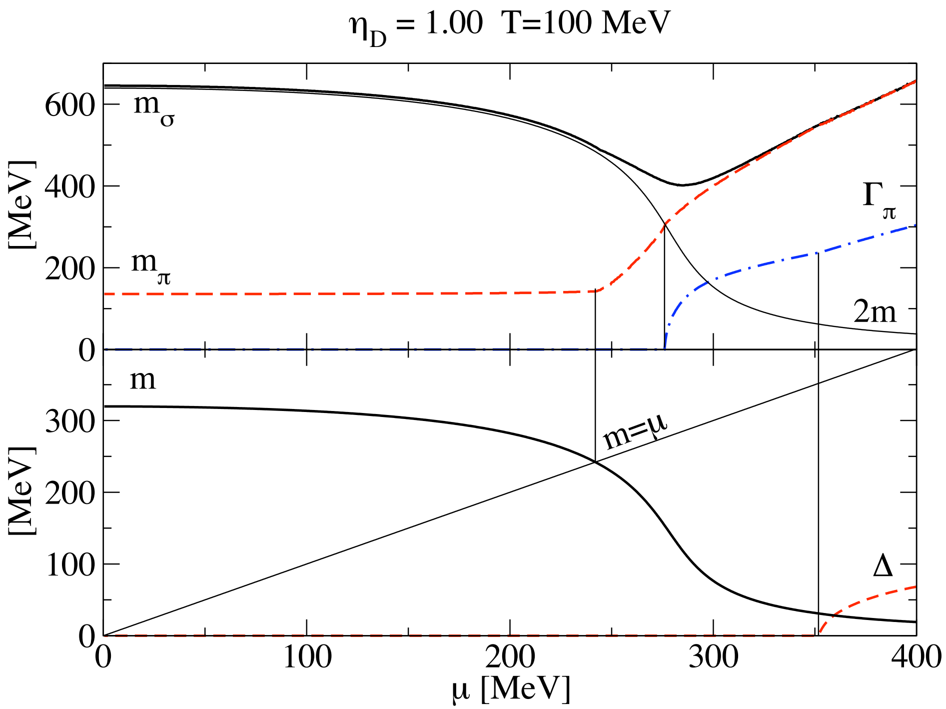
$$\Pi_{\sigma\sigma}(q_0,\mathbf{q}) = 2\int \frac{d^3p}{(2\pi)^3}\sum_{s_p,s_k}\mathcal{T}_-^-(s_p,s_k)\left\{\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0-s_k\xi_{\mathbf{p+q}}^{s_k}+s_p\xi_{\mathbf{p}}^{s_p}}+\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0+s_k\xi_{\mathbf{p+q}}^{s_k}-s_p\xi_{\mathbf{p}}^{s_p}}\right.$$
$$\left.+\sum_{t_p,t_k}\frac{t_pt_k}{E_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}}\frac{n_F(t_pE_{\mathbf{p}}^{s_p})-n_F(t_kE_{\mathbf{p+q}}^{s_k})}{q_0-t_kE_{\mathbf{p+q}}^{s_k}+t_pE_{\mathbf{p}}^{s_p}}\times\left(t_pt_kE_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}+s_ps_k\xi_{\mathbf{p}}^{s_p}\xi_{\mathbf{p+q}}^{s_k}-|\Delta|^2\right)\right\}$$

Similar equations can be derived for the other matrix elements

Sun et al. Phys. Rev. D **75** 096004 (2007)
in the 2-color limit
Ebert et al. Phys. Rev. C **72** 015201 (2005)
in the T=0 limit

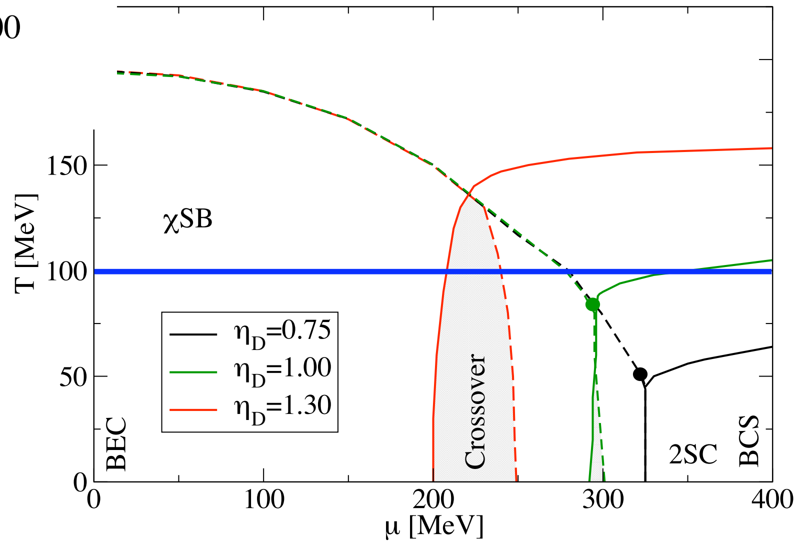
$$\mathcal{T}_\mp^\pm(s_p,s_k) = 1 \pm s_ps_k\frac{\mathbf{p}\cdot\mathbf{k}\mp m^2}{E_{\mathbf{p}}E_{\mathbf{k}}}$$

4.2 the pion mass

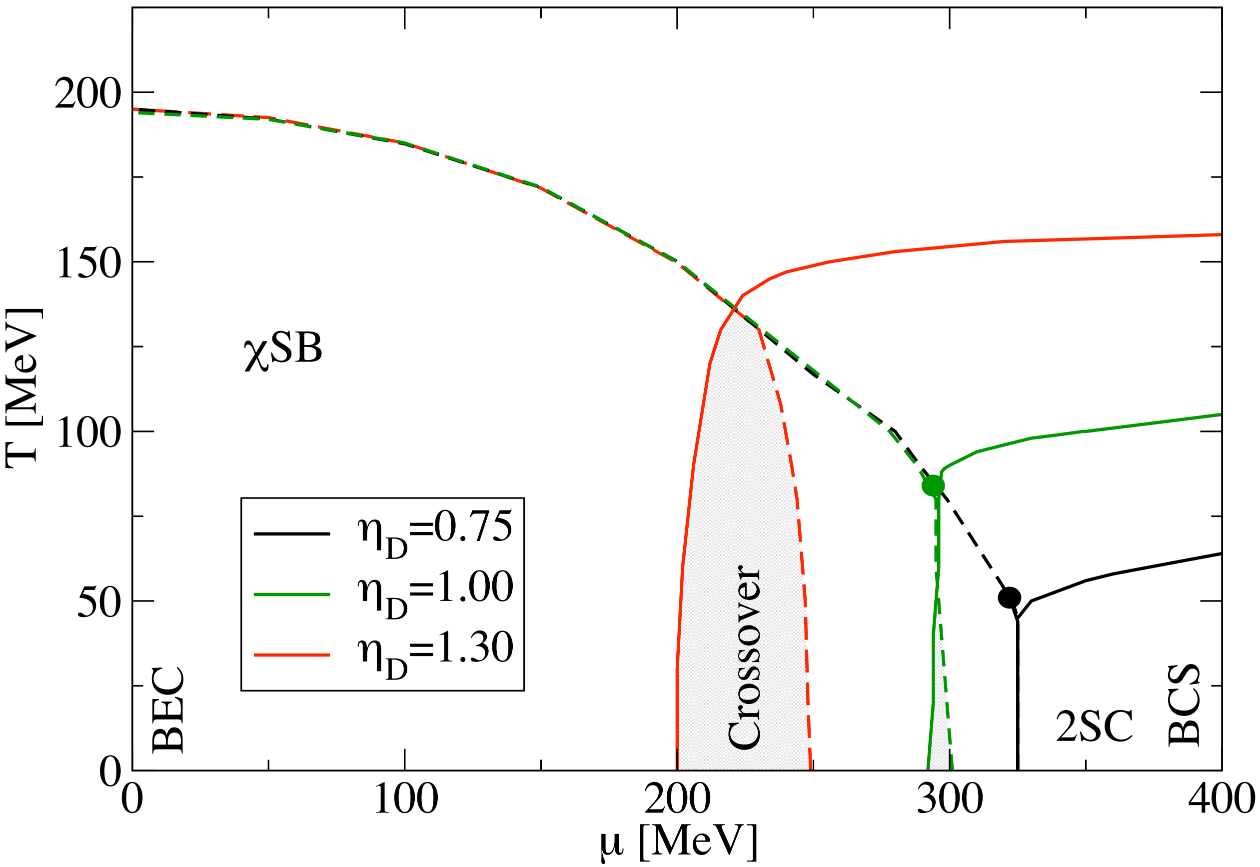


$$\Gamma_\pi = \left(\frac{\partial \mathrm{Re}\Pi_{\pi\pi}}{\partial m_\pi^2}\right)^{-1} \frac{\mathrm{Im}\Pi_{\pi\pi}}{m_\pi}$$

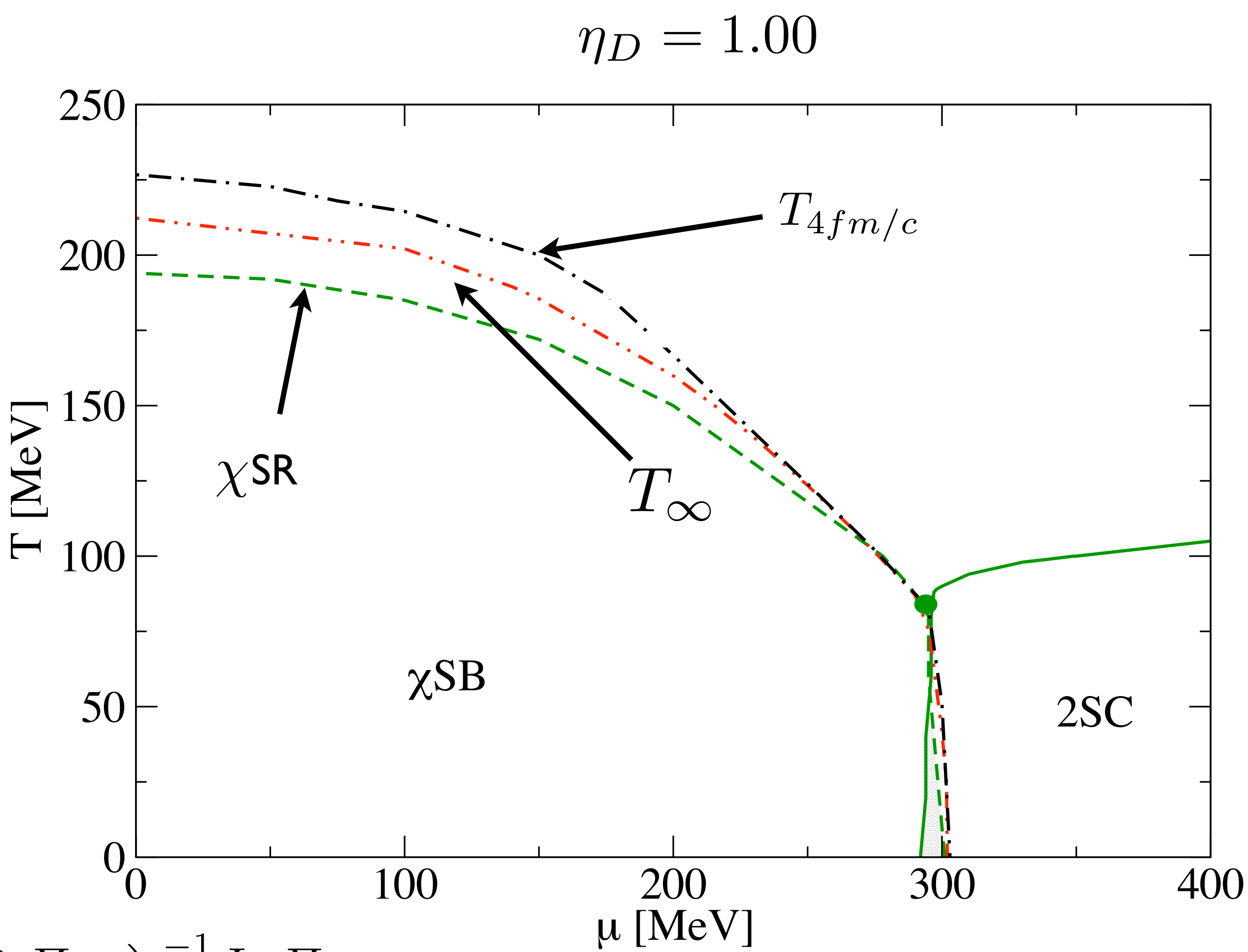
$$1 - 2G_S \Pi(q_0, \boldsymbol{q} = \boldsymbol{0}) = 0$$



the phase diagram MF



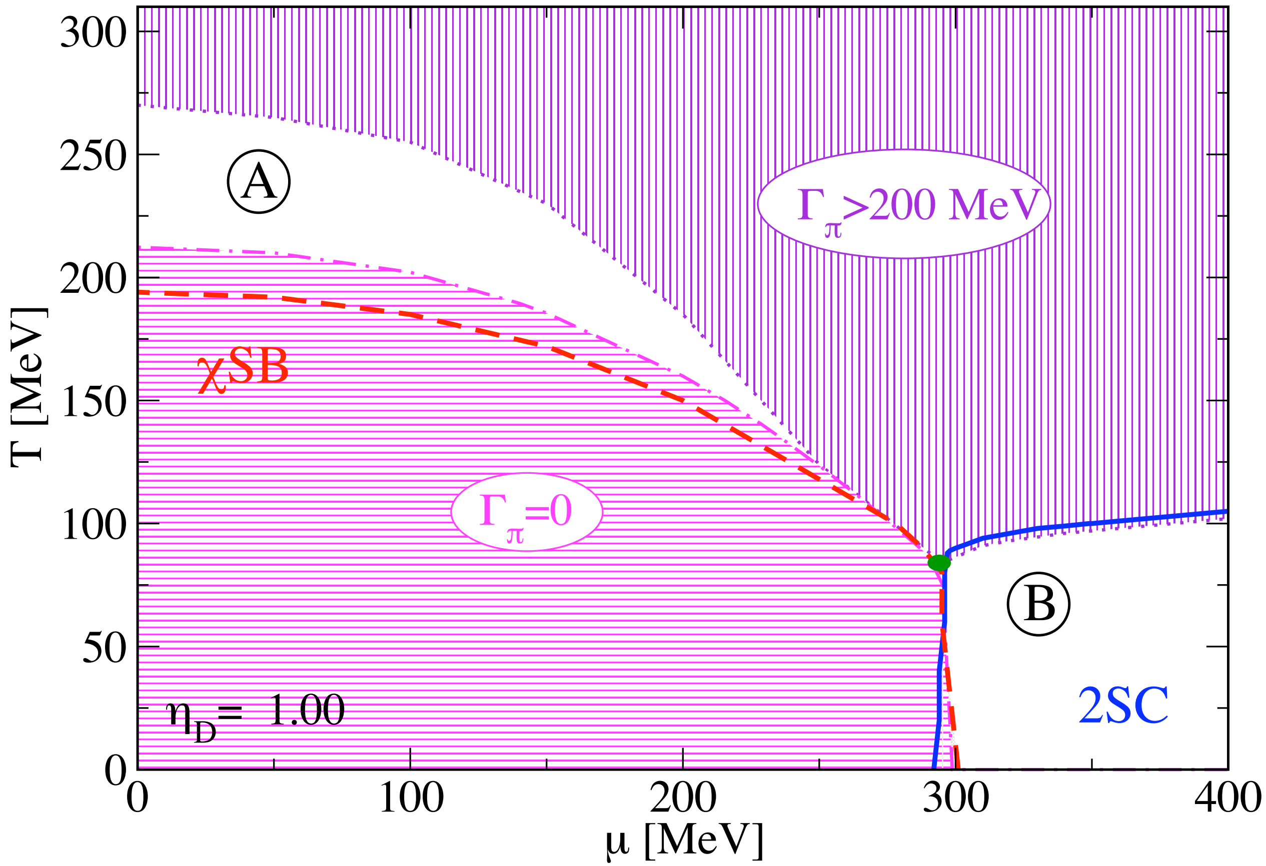
the phase diagram revisited



$$\Gamma_\pi = \left(\frac{\partial \mathrm{Re}\Pi_{\pi\pi}}{\partial m_\pi^2}\right)^{-1} \frac{\mathrm{Im}\Pi_{\pi\pi}}{m_\pi}$$

$$T(\tau = \Gamma_\pi^{-1})$$

RHIC phenomenology



QGP probed in RHIC is far to be a perfect liquid; an explanation: strong correlations in the plasma

Shuryak and Zahed (PRL, 2003)

Region dominated by strong correlated states with a lifetime > 1 fm/c

summary and outlook

fluctuations are included in Gaussian approximation beyond MF;
systematical treatment in the non-perturbative regime possible

some properties of mesons are studied
diquark calculations almost finished
new insight for phase diagram; important for HIC and CSs

investigate $\sigma - \delta$-mixing

constraints of color and electrical neutrality and beta-equilibrium to be implemented (HIC and CSs)
investigation of BEC-BCS crossover (strong coupling);
see lectures of P.F. Zhuang

the same formalism can be applied to Nuclear MF theory
under investigation together with G. Röpke and D. Blaschke

Thanks for your attention