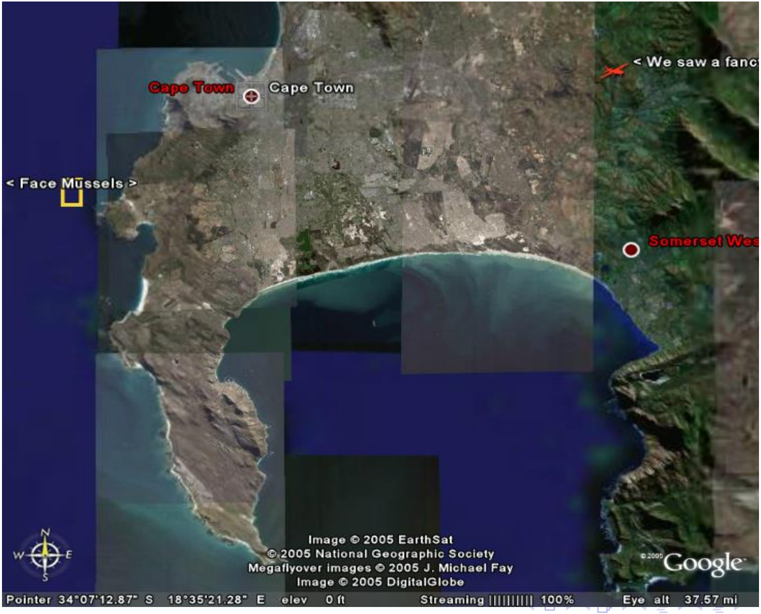
The Thermal-Statistical Model for Particle Production I.

J. Cleymans

23 - 25 July 2008 / JINR, Dubna

Outline

Cape Peninsula



South Africa

UCT

ALICE Collaboration

The ALICE Collaboration
27 countries, 83 Institutes and 1000 members

A Large Ion Collider Experiment at CERN LHC

Armenia
Yerevan, Yerevan Physics Institute

China
Beijing, Institute of Atomic Energy
Wuhan, Hua-Zhong Normal University, Institute of Particle Physics

Croatia
Zagreb, University of Zagreb, Prirodoslovno-Matematicki Fakultet
Zagreb, Ruder Boskovic Institute

Czech Republic
Praha, Academy of Sciences of the Czech Republic, Institute of Physics
Rez, Academy of Sciences of the Czech Republic, Nuclear Physics Institute

Denmark
Copenhagen, Niels Bohr Institute, University of Copenhagen

inland
Jyväskylä, University of Jyväskylä, Department of Physics and Helsinki Institute of Physics

rance
Aubière, Université Blaise Pascal (Clermont-Ferrand II), Institut National de Physique Nucléaire et de Physique des Particules (IN2P3-CNRS), Laboratoire de Physique Corpusculaire (LPC)
Gif-sur-Yvette, DAPNIA CEA Saclay
Nantes, Ecole des Mines, Institut National de Physique Nucléaire et de Physique des Particules (IN2P3-CNRS), Université de Nantes, Laboratoire de Physique Subatomique et des Technologies Associées (SUBATECH)
Orsay, Université de Paris XI, Institut National de Physique Nucléaire et de Physique des Particules (IN2P3-CNRS), Institut de Physique Nucléaire (IPN)
Strasbourg, Université Louis Pasteur, Institut National de Physique Nucléaire et de Physique des Particules (IN2P3-CNRS), Institut de Recherches Subatomiques (IReS)
Villeurbanne, Université Claude Bernard Lyon-I, Institut National de Physique Nucléaire et de Physique des Particules (IN2P3-CNRS, Institut de Physique Nucléaire de Lyon (IPNL)

Germany
Darmstadt, Gesellschaft für Schwerionenforschung (GSI)
Darmstadt, Technische Universität Darmstadt, Institut für Kernphysik
Frankfurt, Johann-Wolfgang-Goethe Universität, Institut für Kernphysik
Heidelberg, Ruprecht-Karls-Universität Heidelberg, Physikalisches Institut
Heidelberg, Ruprecht-Karls-Universität Heidelberg, Kirchhoff Institute for Physics
Köln, Fachhochschule
Münster, Westfälische Wilhelms-Universität Münster, Institut für Kernphysik

Greece

Hungary
- Budapest, Hungarian Academy of Sciences, KFKI Research Institute for Particle and Nuclear Physics

India
- Aligarh, Department of Physics, Aligarh Muslim University
- Bhubaneswar, Institute of Physics
- Calcutta, Variable Energy Cyclotron Centre (VECC)
- Calcutta, Saha Institute of Nuclear Physics
- Chandigarh, Panjab University, High Energy Physics group
- Jaipur, University of Rajasthan, Physics Department
- Jammu, University of Jammu, Physics Department

Italy
- Alessandria Università del Piemonte Orientale, Istituto Nazionale di Fisica Nucleare (INFN)
- Bari, Politecnico di Bari
- Bari, Università degli Studi di Bari and INFN
- Bologna, Università degli Studi di Bologna and INFN
- Cagliari Università degli Studi di Cagliari and INFN
- Catania, Università di Catania and INFN
- Legnaro, Laboratori Nazionali di Legnaro, INFN
- Padova, Università degli Studi di Padova and INFN
- Roma, Università di Roma I 'La Sapienza' and INFN
- Salerno, Università degli Studi di Salerno and INFN
- Torino, Università degli Studi di Torino and INFN
- Trieste, Università degli Studi di Trieste and INFN

Mexico
- Mexico, D.F., Centro de Investigación y de Estudios Avanzados del IPN (CINVESTAV), Departamento de Fisica and Departamento de Fisica Aplicada
- Mexico, D.F., Universidad Nacional Autonoma de Mexico, Instituto de Fisica and Instituto de Ciencias Nucleares
- Morelia, Michoacan, Universidad Michoacana de San Nicolas de Hidalgo, Inst de Fisica y Matematicas
- Puebla, Universidad Autonoma de Puebla

Netherlands
- Utrecht, Rijksuniversiteit Utrecht (NIKHEF) and Amsterdam, Nuclear Physics Section (NIKHEF)

Norway
- Bergen, University of Bergen, Department of Physics
- Bergen, Bergen University College Faculty of Engineering
- Oslo, University of Oslo, Department of Physics

Poland
- Cracow, H. Niewodniczanski Institute of Nuclear Physics, High Energy Physics Department
- Warszawa, Soltan Institute for Nuclear Studies
- Warszawa, Warsaw University of Technology

Portugal

Rep. of Korea
- Kangnung, Kangnung National University
- Pohang, Pohang Accelerator Laboratory

Romania
- Bucharest, National Institute for Physics and Nuclear Engineering

Russia
- Dubna, Moscow Region, Joint Institute for Nuclear Research (JINR)
- Gatchina, St. Petersburg Nuclear Physics Institute (PNPI)
- Moskva, Academy of Sciences, Institute for Nuclear Research (INR)
- Moskva, Institute for Theoretical and Experimental Physics (ITEP)
- Moskva, Russian Research Centre Kurchatov Institute
- Moskva, Moscow Engineering Physics Institute (MePhI)
- Novosibirsk, Budker Institute for Nuclear Physics (INP)
- Protvino, Institute for High Energy Physics (IHEP)
- Sarov, Russian Federal Nuclear Center (VNIIEF)
- St. Petersburg, State University, Institute for Physics, Mendeleev Institute for Metrology and Meson Scientific Association

Slovakia
- Bratislava, Comenius University, Faculty of Mathematics, Physics and Informatics
- Kosice, Institute of Experimental Physics, Slovak Academy of Sciences
- Kosice, Faculty of Sciences, P.J. Safarik University

South Africa
- Rondebosch, University of Cape Town (UCT)

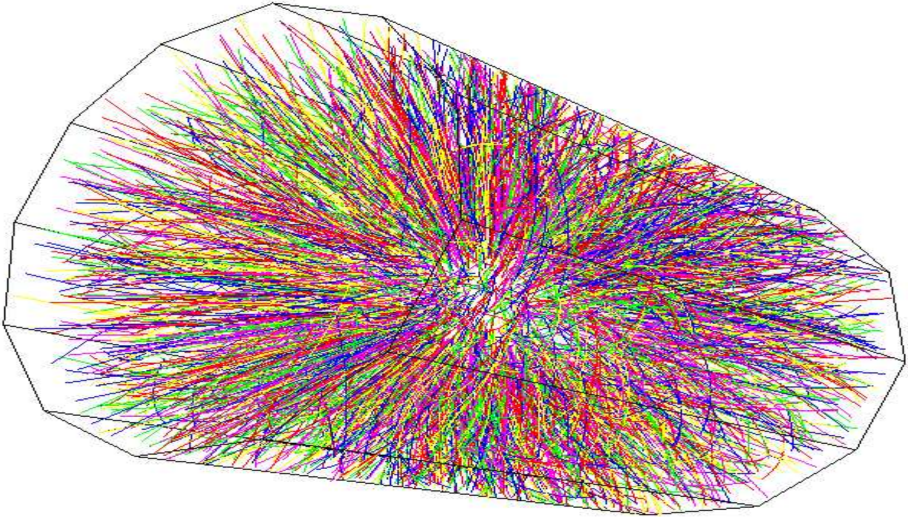
Sweden
- Lund, University of Lund, Division of Cosmic and Subatomic Physics

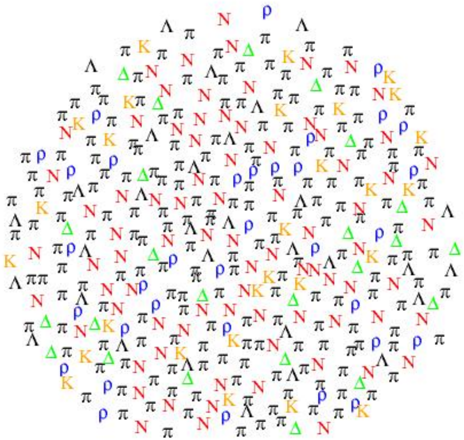
Switzerland
- Geneva, European Organization for Nuclear Research (CERN)
- Lausanne, Ecole Polytechnique Fédérale Lausanne (EPFL)

Ukraine
- Kharkov, Ukrainian Academy of Sciences, National Scientific Centre 'Kharkov Institute of Physics and Technology'
- Kharkov, Scientific and Technological Research Institute of Instrument Engineering (SRTIIE)
- Kiev, Ukrainian Academy of Sciences, Bogolyubov Institute for Theoretical Physics, Department of High Energy Density Physics

United Kingdom
- Birmingham, University of Birmingham, School of Physics and Space Research

United States of America
- Columbus OH, Ohio State University, Department of Physics
- Columbus OH, Ohio Supercomputer Centre
- Oak Ridge TN, Oak Ridge National Laboratory (ORNL), Instrumentation and Controls Division





J.C. and H. Satz, Zeitschrift fuer Physik C57, 135 (1993)

Thermal Equilibrium

In thermal equilibrium

$$\boxed{Z = \mathrm{Tr}\ e^{-\dfrac{H}{T} + \dfrac{\mu N}{T}}}$$

$$\boxed{\langle N \rangle = \frac{\mathrm{Tr}\ N e^{-\dfrac{H}{T} + \dfrac{\mu N}{T}}}{\mathrm{Tr}\ e^{-\dfrac{H}{T} + \dfrac{\mu N}{T}}}}$$

$$\boxed{\langle E \rangle = \frac{\mathrm{Tr}\ E e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\ e^{\frac{-H}{T} + \frac{\mu N}{T}}}}$$

Full Hydrodynamic Flow

Bjorken scaling + Transverse expansion

After integration over m_T

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0}$$

where N_i^0 is the particle yield
as calculated in a fireball **AT REST!**

Effects of hydrodynamic flow cancel out in ratios.

Thermal Equilibrium

Particle Number

$$\begin{aligned}
\langle N\rangle &= \frac{\mathrm{Tr}\, N e^{-\frac{H}{T}+\frac{\mu N}{T}}}{\mathrm{Tr}\, e^{-\frac{H}{T}+\frac{\mu N}{T}}} \\
&= \frac{T}{Z}\frac{\partial}{\partial\mu}\mathrm{Tr}\, e^{\frac{-H}{T}+\frac{\mu N}{T}} \\
&= T\frac{1}{Z}\frac{\partial Z}{\partial\mu} \\
&= T\frac{\partial}{\partial\mu}\ln Z
\end{aligned}$$

Thermal Equilibrium

Average Energy

$$
\begin{aligned}
\langle E \rangle &= \frac{\mathrm{Tr}\, H\, e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{\frac{-H}{T} + \frac{\mu N}{T}}} \\
&= -\frac{1}{Z}\frac{\partial Z}{\partial \beta} + \mu \langle N \rangle \\
&= T^2 \frac{\partial}{\partial T} \ln Z + \mu \langle N \rangle
\end{aligned}
$$

Thermal Equilibrium

$$\begin{aligned}
N_i &= g_i\, V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T}\right) e^{\frac{\mu_i}{T}} \\
&= g_i\, V \frac{4\pi}{(2\pi)^3} \int p^2\, dp \exp\left(-\frac{\sqrt{p^2+m_i^2}}{T}\right) e^{\frac{\mu_i}{T}} \\
&= g_i\, V \frac{4\pi}{(2\pi)^3} T^3 \int x^2\, dx \exp\left(-\sqrt{x^2+m_i^2/T^2}\right) e^{\frac{\mu_i}{T}} \\
&= g_i\, V \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
\end{aligned}$$

Thermal Equilibrium

$$
\begin{aligned}
n_i &= g_i \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} \\
\epsilon_i &= g_i \frac{1}{2\pi^2} T m_i^3 \left[K_1\left(\frac{m_i}{T}\right) + 3\frac{T}{m} K_2\left(\frac{m_i}{T}\right)\right] e^{\frac{\mu_i}{T}} \\
s_i &= g_i \frac{1}{2\pi^2} m_i^3 \left[K_1\left(\frac{m_i}{T}\right) + \frac{4T}{m} K_2\left(\frac{m_i}{T}\right) - \frac{\mu_i}{m} K_2\left(\frac{m_i}{T}\right)\right] e^{\frac{\mu_i}{T}} \\
P_i &= g_i \frac{1}{2\pi^2} T^2 m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
\end{aligned}
$$

Chemical Equilibrium

In equilibrium

$$E_1 + E_2 + \cdots = E_3 + E_4 + E_5 + \cdots \tag{1}$$

for the chemical potentials

$$\mu_1 + \mu_2 + \cdots = \mu_3 + \mu_4 + \mu_5 + \cdots \tag{2}$$

As an example

$$\pi^0 + p \leftrightarrow \pi_0 + p + \pi^0 \tag{3}$$

leads to

$$\mu_{\pi^0} + \mu_p = \mu_{\pi^0} + \mu_p + \mu_{\pi^0} \tag{4}$$

which leads to

$$\mu_{\pi^0} = 0 \tag{5}$$

Chemical Equilibrium

In equilibrium

$$B + B \leftrightarrow B + B + B + \bar{B} \tag{6}$$

$$dE = -pdV + TdS + \mu_B dN_B + \mu_{\bar{B}} dN_{\bar{B}}$$

Due to baryon number conservation one has

$$N_B - N_{\bar{B}} = \mathrm{constant}$$

and

$$dN_B = dN_{\bar{B}}$$

The energy is a minimum for

$$dE = (\mu_B + \mu_{\bar{B}}) dN_B = 0 \tag{7}$$

$$\mu_B = -\mu_{\bar{B}} \tag{8}$$

Chemical Equilibrium

In equilibrium

$$N_B = g\ V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T} + \frac{\mu_B}{T}\right)$$

$$N_{\bar{B}} = g\ V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T} - \frac{\mu_B}{T}\right)$$

$$\begin{array}{rcl} N_B & = & N_{\bar{B}} \rightarrow \mu_B = 0 \\ N_B & \geq & N_{\bar{B}} \rightarrow \mu_B \geq 0 \\ N_B & \leq & N_{\bar{B}} \rightarrow \mu_B \leq 0 \end{array}$$

	Chemical Equilibrium	No Chem. Equil.
π	$\exp\left[-\frac{E_\pi}{T}\right]$	$\exp\left[-\frac{E_\pi}{T}+\frac{\mu_\pi}{T}\right]$
N	$\exp\left[-\frac{E_N}{T}+\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_N}{T}\right]$
$\overline{N}$	$\exp\left[-\frac{E_N}{T}-\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_{\overline{N}}}{T}\right]$
Λ	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_B}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_\Lambda}{T}\right]$
$\overline{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T}-\frac{\mu_B}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_{\overline{\Lambda}}}{T}\right]$
K	$\exp\left[-\frac{E_K}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_K}{T}\right]$
$\overline{K}$	$\exp\left[-\frac{E_K}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_{\overline{K}}}{T}\right]$

The number of particles of type *i* is determined by:

$$\boxed{N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)}$$

For bosons:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) - 1}$$

For fermions:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) + 1}$$

Chemical Equilibrium

Only conserved quantum numbers matter for chemical equilibrium: In equilibrium

$$\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S + C_i\mu_C + .. \quad (9)$$

g_i	m_i	stat	S_i	B_i	Q_i	Particle i
1	0.140	-1	0	0	1.	π^+
1	0.135	-1	0	0	0.	π^0
1	0.140	-1	0	0	-1.	π^-
1	0.547	-1	0	0	0.	η
3	0.770	-1	0	0	1.	ρ^+
3	0.770	-1	0	0	0.	ρ^0
3	0.770	-1	0	0	-1.	ρ^-
3	0.782	-1	0	0	0.	ω
1	0.958	-1	0	0	0.	η'
1	0.980	-1	0	0	0.	f_0
1	0.982	-1	0	0	1.	a_0^+
1	0.982	-1	0	0	0.	a_0^0
1	0.982	-1	0	0	-1.	a_0^-
3	1.019	-1	0	0	0.	ϕ
3	1.170	-1	0	0	0.	
3	1.230	-1	0	0	1.	
3	1.230	-1	0	0	0.	
3	1.230	-1	0	0	-1.	
3	1.229	-1	0	0	1.	
3	1.229	-1	0	0	0.	
3	1.229	-1	0	0	-1.	
5	1.275	-1	0	0	0.	
3	1.282	-1	0	0	0.	
1	1.297	-1	0	0	0.	
1	1.300	-1	0	0	1.	
1	1.300	-1	0	0	0.	

The Role of Resonances

Example: ρ's

$$\rho \rightarrow \pi^+\pi^-$$

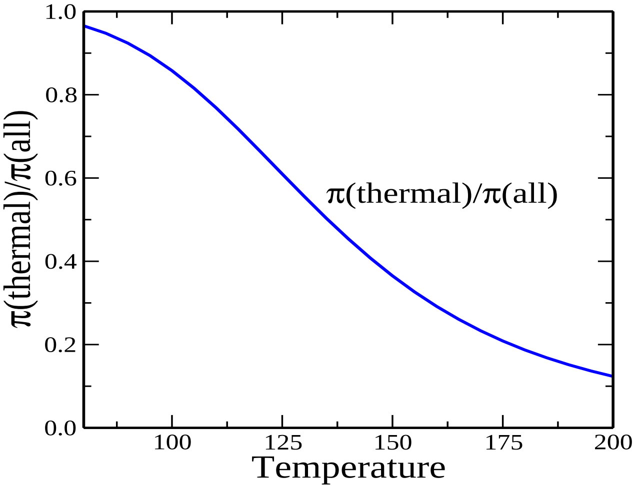
Final, observed, number of π^+ is given by

$$N_{\pi^+} = N_{\pi^+}(\text{thermal}) + N_{\pi^+}(\text{resonance decays})$$

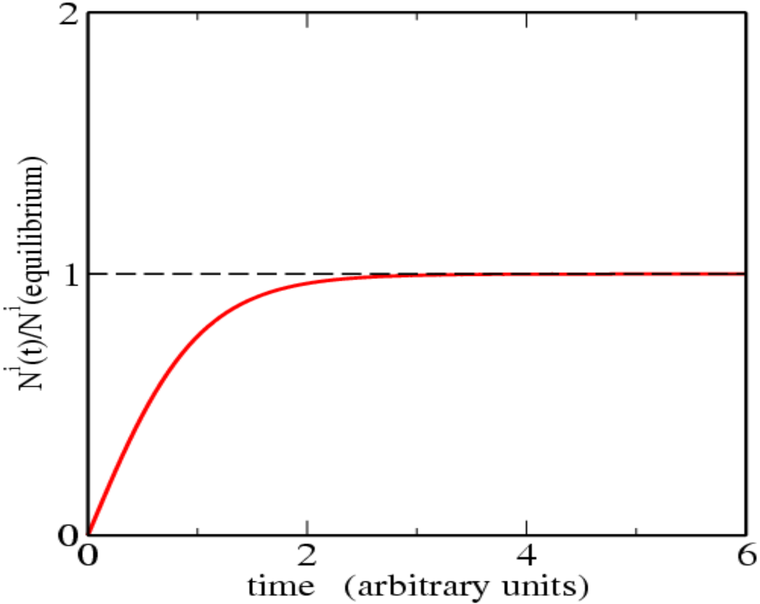
depending on the temperature, over 80% of observed pions are due to resonance decays

g_i	m_i	stat	S_i	B_i	Q_i	BR$\longrightarrow \pi^+$	Particle i
1	0.140	-1	0	0	1.	1.000	π^+
1	0.135	-1	0	0	0.	0.000	π^0
1	0.140	-1	0	0	-1.	0.000	π^-
1	0.547	-1	0	0	0.	0.285	η
3	0.770	-1	0	0	1.	1.000	ρ^+
3	0.770	-1	0	0	0.	1.000	ρ^0
3	0.770	-1	0	0	-1.	0.000	ρ^-
3	0.782	-1	0	0	0.	0.910	ω
1	0.958	-1	0	0	0.	0.965	η'
1	0.980	-1	0	0	0.	0.521	f_0
1	0.982	-1	0	0	1.	1.285	a_0^+
1	0.982	-1	0	0	0.	0.285	a_0^0
1	0.982	-1	0	0	-1.	0.285	a_0^-
3	1.019	-1	0	0	0.	0.155	ϕ
3	1.170	-1	0	0	0.	1.000	h_1
3	1.230	-1	0	0	1.	1.500	
3	1.230	-1	0	0	0.	0.50	
3	1.230	-1	0	0	-1.	0.50	
3	1.229	-1	0	0	1.	1.91	
3	1.229	-1	0	0	0.	0.91	
3	1.229	-1	0	0	-1.	0.91	
5	1.275	-1	0	0	0.	0.69	
3	1.282	-1	0	0	0.	1.00	
1	1.297	-1	0	0	0.	1.11	
1	1.300	-1	0	0	1.	2.00	
1	1.300	-1	0	0	0.	1.50	

Importance of Resonances.



Strangeness saturation?

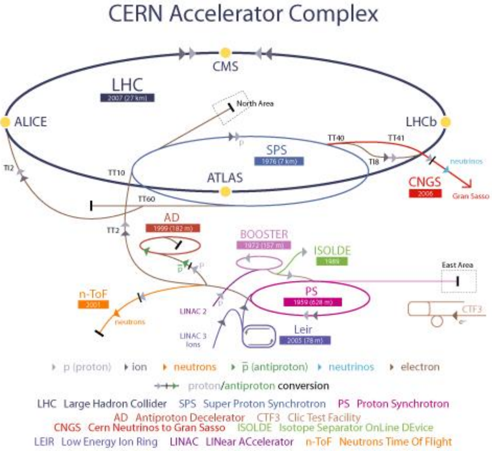


Strangeness saturation?

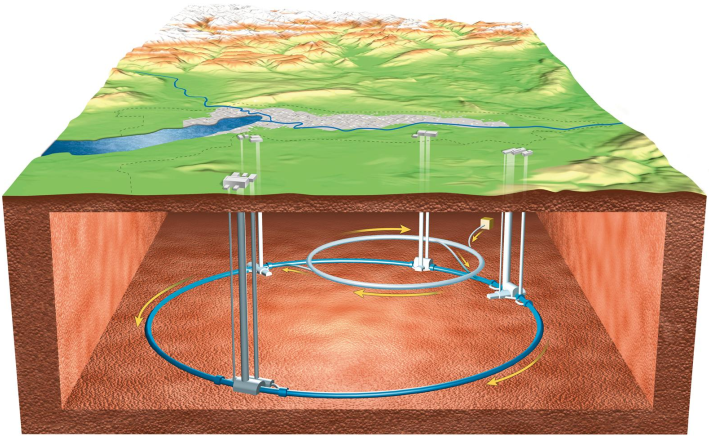
$$N_i = \boxed{\gamma_s^{|S|}} V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)$$

with
$\gamma_s < 1$ strangeness under-saturation
$\gamma_s = 1$ strangeness in chemical equilibrium
$\gamma_s > 1$ strangeness over-saturation

SPS


CERN Accelerator Complex
CMS
LHC
2007 (27 km)
North Area
ALICE
LHCb
TT40
TT41
SPS
1976 (7 km)
TI8
neutrinos
TI2
TT10
ATLAS
CNGS
2006
Gran Sasso
TT60
AD
1999 (182 m)
TT2
BOOSTER
1972 (157 m)
ISOLDE
1989
East Area
p̄
p
n-ToF
2001
PS
1959 (628 m)
CTF3
e⁻
LINAC 2
neutrons
LINAC 3
Ions
Leir
2005 (78 m)
p (proton)
ion
neutrons
p̄ (antiproton)
neutrinos
electron
proton/antiproton conversion
LHC Large Hadron Collider
SPS Super Proton Synchrotron
PS Proton Synchrotron
AD Antiproton Decelerator
CTF3 Clic Test Facility
CNGS Cern Neutrinos to Gran Sasso
ISOLDE Isotope Separator OnLine DEvice
LEIR Low Energy Ion Ring
LINAC LINear ACcelerator
n-ToF Neutrons Time Of Flight

SPS



SPS data.

	Measurement
Pb–Pb 158A GeV	
$(\pi^+ + \pi^-)/2.$	600±30
K^+	95 ±10
K^-	50 ±5
K_S^0	60 ±12
p	140±12
$\bar{p}$	10 ±1.7
ϕ	7.6±1.1
Ξ^-	4.42±0.31
$\overline{\Xi^-}$	0.74±0.04
$\overline{\Lambda}/\Lambda$	0.2±0.04

SPS data.

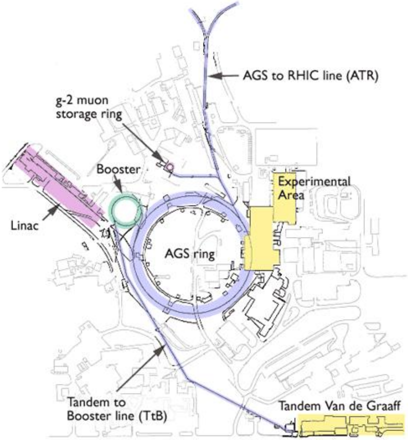
SPS: Chemical Freeze-Out Parameters:

$$\begin{aligned} T &= 156.0 \pm 2.4\mathrm{MeV} \\ \mu_B &= 239 \pm 12\mathrm{MeV} \\ \gamma_s &= 0.862 \pm 0.036 \end{aligned}$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

AGS

AGS


AGS to RHIC line (ATR)
g-2 muon storage ring
Booster
Experimental Area
Linac
AGS ring
Tandem to Booster line (TtB)
Tandem Van de Graaff

AGS data.

	Measurement
Au–Au 11.6A GeV	
Participants	363±10
K^+	23.7±2.9
K^-	3.76±0.47
π^+	133.7±9.9
Λ	20.34±2.74
p/π^+	1.234±0.126
$\bar{p}$	$>$0.0185±0.0018

AGS data.

AGS: Chemical Freeze-Out Parameters:

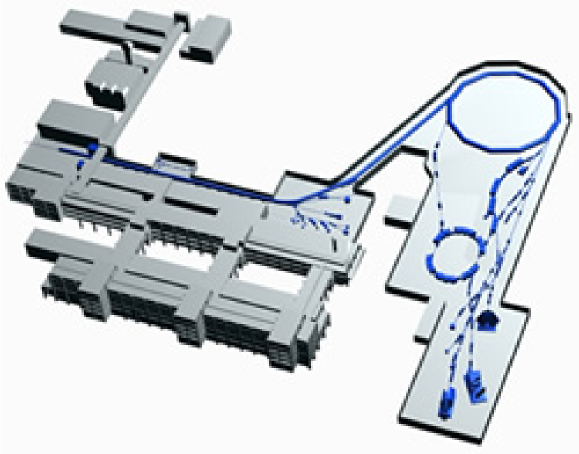
$$\begin{aligned} T &= 130.6 \pm 5.5\mathrm{MeV} \\ \mu_B &= 594 \pm 26\mathrm{MeV} \\ \gamma_s &= 0.883 \pm 0.124 \end{aligned}$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

SIS data.

	Measurement
Au–Au 1.7A GeV	
π^+/p	0.052±0.013
K$^+/\pi^+$	0.003±0.00075
π^-/π^+	2.05±0.51
η/π^0	0.018±0.007

GSI



SIS data.

SIS: Chemical Freeze-Out Parameters:

$$\begin{aligned} T &= 49.7 \pm 1.1\mathrm{MeV} \\ \mu_B &= 818 \pm 15\mathrm{MeV} \\ \gamma_s &= 1\ \mathrm{(fixed)} \end{aligned}$$

J. C., H. Oeschler and K. Redlich)
Physical Review C59, (1999) 1663.

RHIC data.

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu, Phys. Rev. C71, 0409071 (2005)

Ratio	Experiment	Central	Mid-Central	Peripheral
$\pi^-_{(2)}/\pi^+_{(2)}$	BRAHMS	0.990±0.100		
	PHENIX	0.960±0.177	0.920±0.170	0.933±0.172
	PHOBOS	1.000±0.022		
	STAR	1.000±0.073	1.000±0.073	1.000 ± 0.073
$K^+_{(2)}/K^-_{(2)}$	PHENIX	1.152±0.240	1.292±0.268	1.322±0.284
	PHOBOS	1.099±0.111		
	STAR	1.109±0.022	1.105±0.036	1.120±0.040
$\bar{p}_{(1)}/p_{(1)}$	PHENIX	0.680±0.149	0.671±0.142	0.717±0.157
$\bar{p}_{(2)}/p_{(2)}$	BRAHMS	0.650±0.092		
	PHOBOS	0.600±0.072		
	STAR	0.714±0.050	0.724±0.050	0.764±0.053
$\bar{\Lambda}_{(1)}/\Lambda_{(1)}$	PHENIX	0.750±0.180	0.798±0.197	0.795±0.197
$\bar{\Lambda}_{(2)}/\Lambda_{(2)}$	STAR	0.719±0.090	0.739±0.092	0.744±0.100
$\bar{\Xi}^+_{(2)}/\Xi^-_{(2)}$	STAR	0.840±0.053	0.822±0.114	0.815±0.096
$\bar{\Omega}^+/\Omega^-$	STAR	1.062±0.410		
$K^-_{(2)}/\pi^-_{(2)}$	PHENIX	0.151±0.030	0.134±0.027	0.116±0.023
	STAR	0.151±0.022	0.147±0.022	0.130±0.019
$K^0_S/\pi^-_{(2)}$	STAR	0.134±0.022	0.131±0.022	0.108±0.018
$\bar{p}_{(1)}/\pi^-_{(2)}$	PHENIX	0.049±0.010	0.047±0.010	0.045±0.009
$\bar{p}_{(2)}/\pi^-_{(2)}$	STAR	0.069±0.019	0.067±0.019	0.067±0.019
$\Lambda_{(1)}/\pi^-_{(2)}$	STAR	0.043±0.008	0.043±0.008	0.039±0.007
$\Lambda_{(2)}/\pi^-_{(2)}$	PHENIX	0.072±0.017	0.068±0.016	0.074±0.017
$< K^{*0} > /\pi^-_{(2)}$	STAR	0.039±0.011		
$\phi/\pi^-_{(2)}$	STAR	0.022±0.003	0.021±0.004	0.022±0.004
$\Xi^-_{(2)}/\pi^-_{(2)}$	STAR	0.0093±0.0012	0.0072±0.0011	0.0060±0.0008

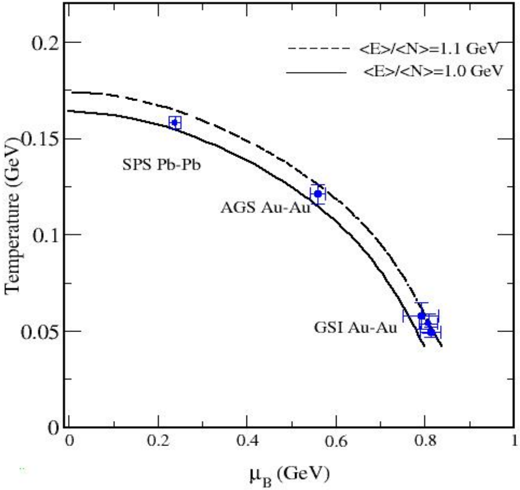
RHIC data.

RHIC: Chemical Freeze-Out Parameters:

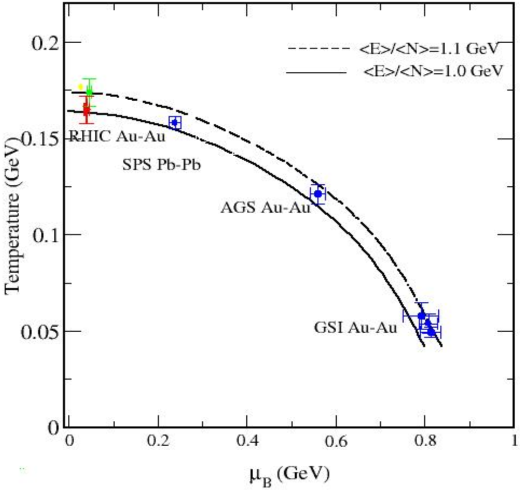
$$\begin{aligned} T &= 169 \pm 4.2\mathrm{MeV} \\ \mu_B &= 39.6 \pm 6\mathrm{MeV} \\ \gamma_s &= 0.9 \pm 0.1 \end{aligned}$$

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu
Phys. Rev. C71, 0409071 (2005)

E/N in 1999

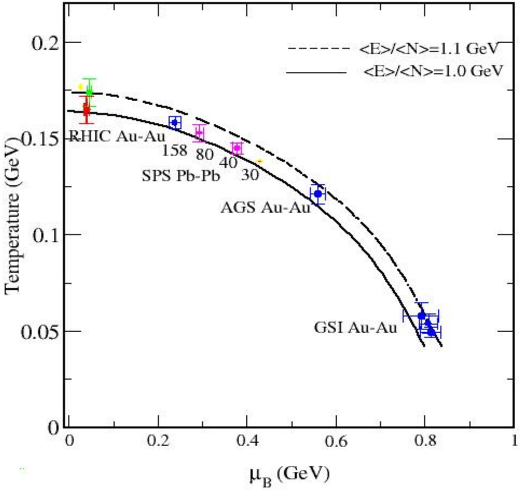

0.2
0.15
0.1
0.05
0
Temperature (GeV)
0
0.2
0.4
0.6
0.8
1
μ_B (GeV)
<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
SPS Pb-Pb
AGS Au-Au
GSI Au-Au

E/N in 2000



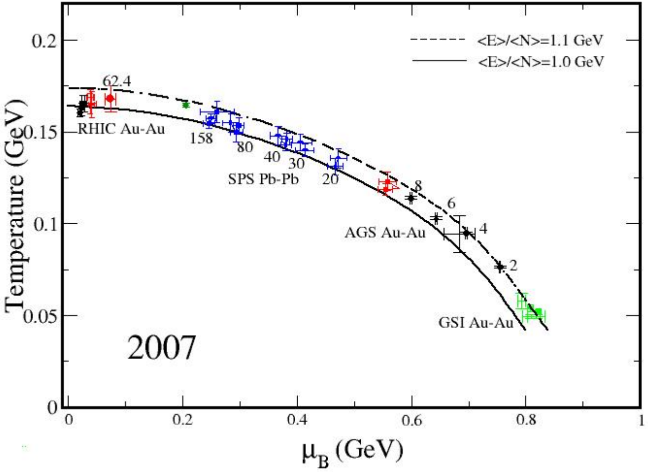
<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
RHIC Au-Au
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
0
0.05
0.1
0.15
0.2
μ_B (GeV)
0
0.2
0.4
0.6
0.8
1

E/N in 2005

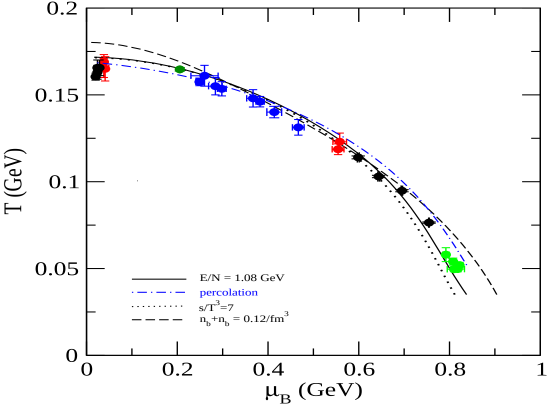


<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
RHIC Au-Au
158
80
40
30
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
μ_B (GeV)
0
0.05
0.1
0.15
0.2
0
0.2
0.4
0.6
0.8
1

E/N in 2007


<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
62.4
RHIC Au-Au
158
80
40
30
20
SPS Pb-Pb
8
6
4
2
AGS Au-Au
GSI Au-Au
2007
Temperature (GeV)
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
μ_B (GeV)

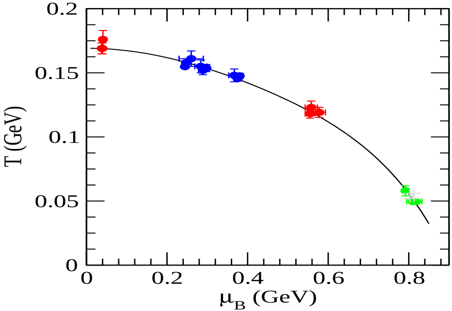
Chemical Freeze-Out: Status in 2005



V. Magas and H. Satz, Eur. Phys. J. **C32** 115 (2003).

P. Braun-Munzinger and J. Stachel, J. Phys. G:Nucl. Part. Phys. **28** 1971 (2002).

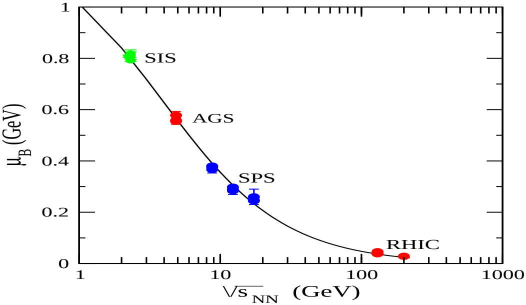
Chemical Freeze-Out: Status in 2005



$$T(\mu_B) = 0.169 - 0.189\mu_B^2 + 0.165\mu_B^4 - 0.229\mu_B^6.$$

J. C., H. Oeschler, K. Redlich, S. Wheaton

μ_B as a function of $\sqrt{s_{NN}}$

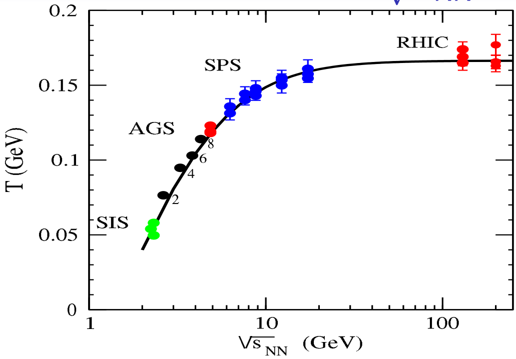


$$\mu_B(\sqrt{s}) = \frac{1.273 \text{ GeV}}{1 + 0.258 \text{ GeV}^{-1}\sqrt{s}}.$$

This predicts at LHC $\mu_B \approx 1$ MeV.

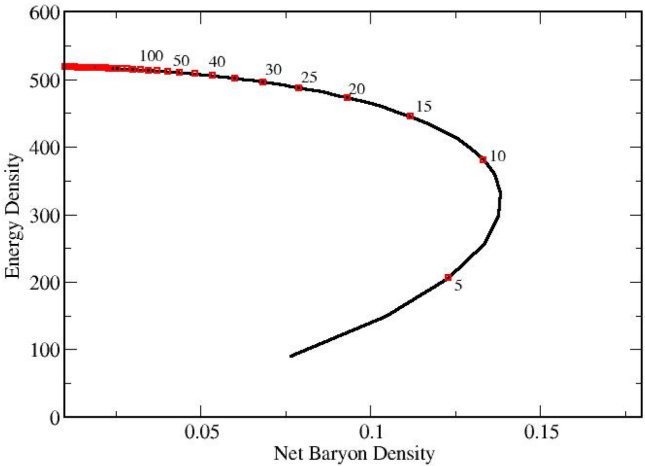
J. C., H. Oeschler, K. Redlich, S. Wheaton

T as a function of $\sqrt{s_{NN}}$

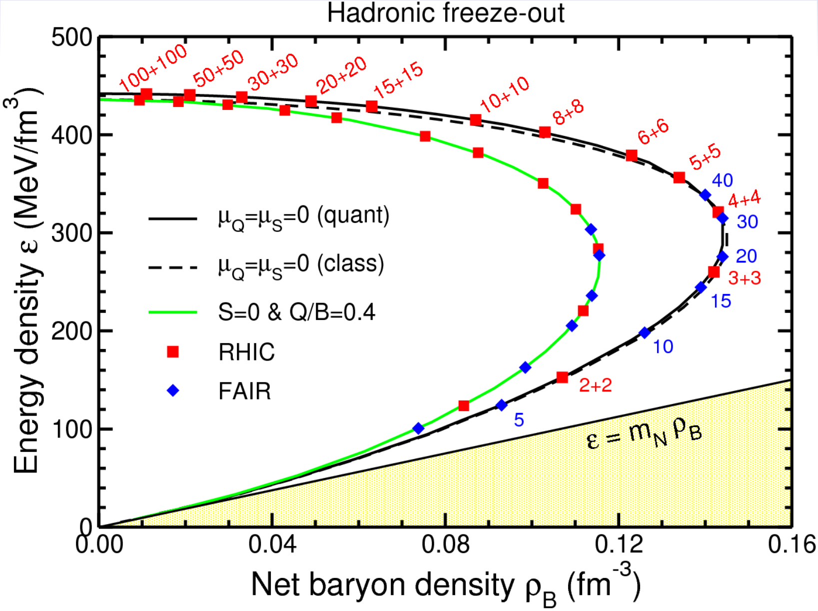


This predicts at LHC $T \approx 170$ MeV.

J. C., H. Oeschler, K. Redlich, S. Wheaton



J. Randrup and J.C., Phys. Rev. C74 (2006) 047901



J. Randrup and J.C., Phys. Rev. C74 (2006) 047901

Will it be possible to determine directly E/N ?

E: energy of primordial hadrons
N: number of primordial hadrons

$$\begin{aligned}\langle E_T\rangle &= \langle E\sin\theta\rangle \\ &= \frac{\pi}{4}\langle E\rangle\end{aligned}$$

Low energy limit

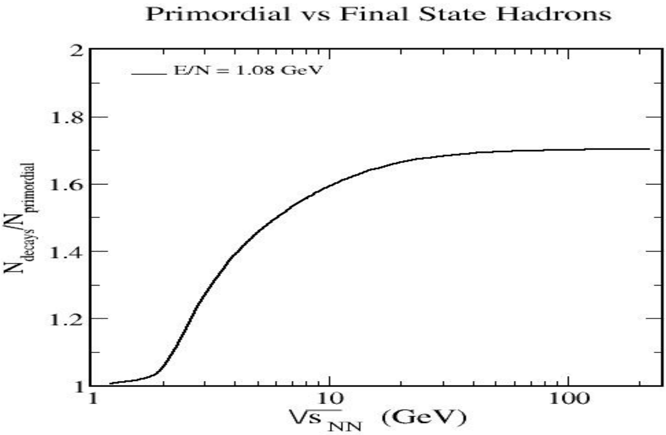
$$\lim \frac{E_T}{N_{ch}} = \frac{\frac{\pi}{4} m_N}{0.4} \approx 1.8 \,\mathrm{GeV}$$

High energy limit

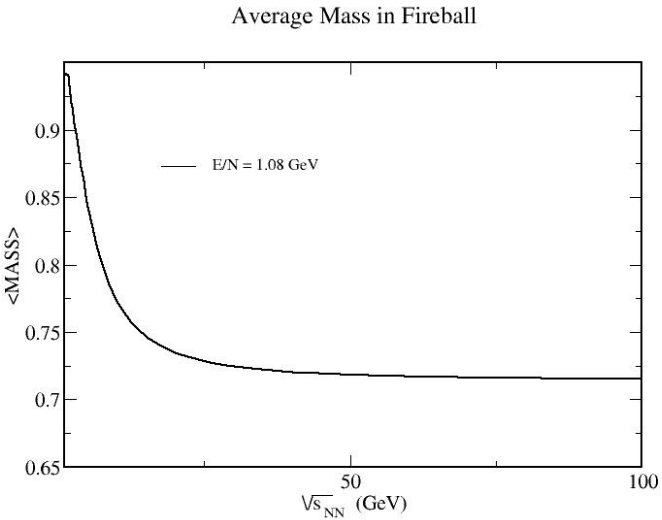
$$\lim \frac{E_T}{N_{ch}} = \frac{\frac{\pi}{4} \langle M \rangle}{2/3} \approx 0.9 \;\; \mathrm{GeV}$$

However

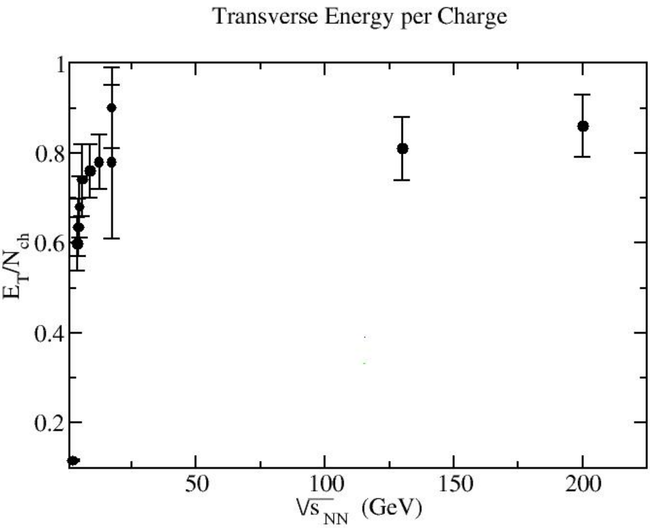
E_T : subtract m_N for baryons
add m_N for antibaryons.

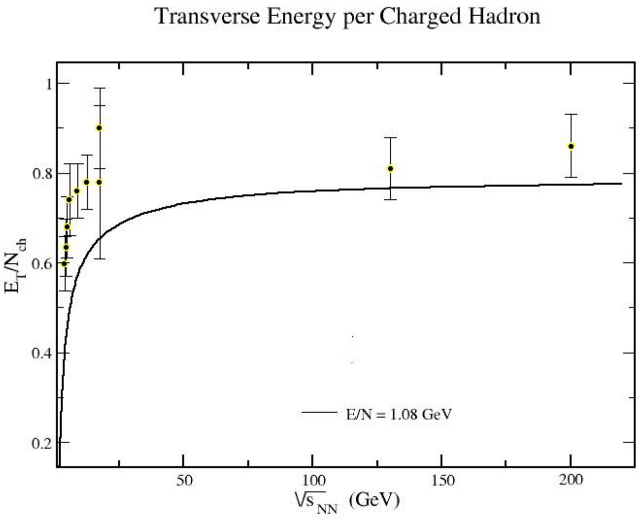


J.C., R. Sahoo, D.K. Srivastava, S. Wheaton

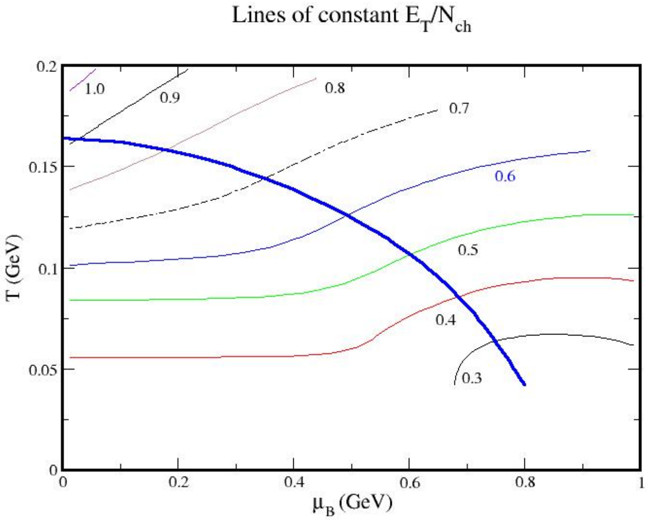


J.C., R. Sahoo, D.K. Srivastava, S. Wheaton


Transverse Energy per Charge
E_T/N_{ch}
1
0.8
0.6
0.4
0.2
50
100
150
200
$\sqrt{s}_{NN}$ (GeV)

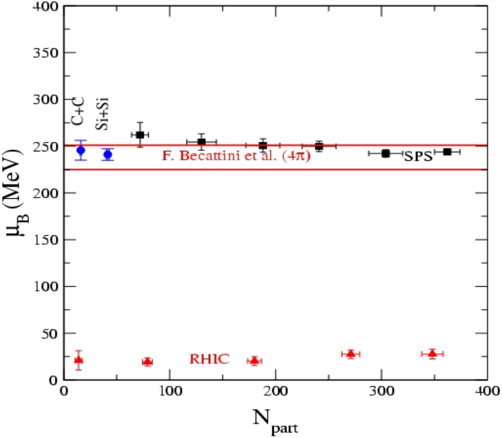


J.C., R. Sahoo, D.K. Srivastava, S. Wheaton



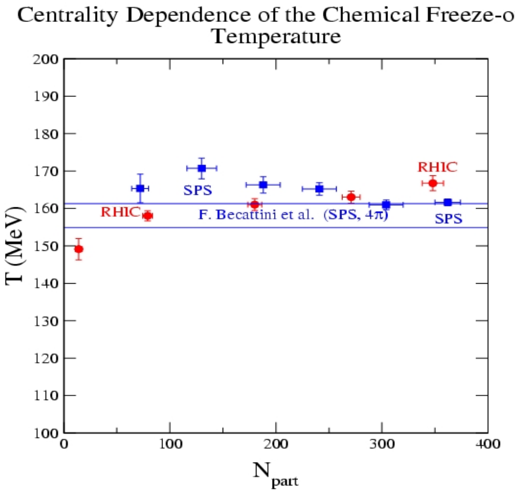
J.C., R. Sahoo, D.K. Srivastava, S. Wheaton

E_T/N_{ch} mainly follows T and is determined by E/N,

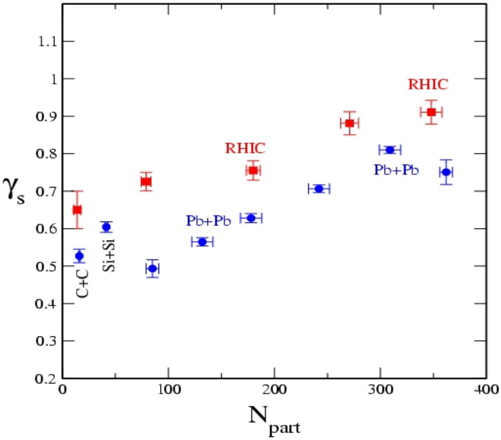


Centrality Dependence of the Baryon Chemical Potential

J. C., B. Kämpfer, P. Steinberg and S. Wheaton, Journal of Physics G30 S595-S598 (2004).



J. C., B. Kämpfer, P. Steinberg and S. Wheaton, Journal of Physics G30 S595-S598 (2004).

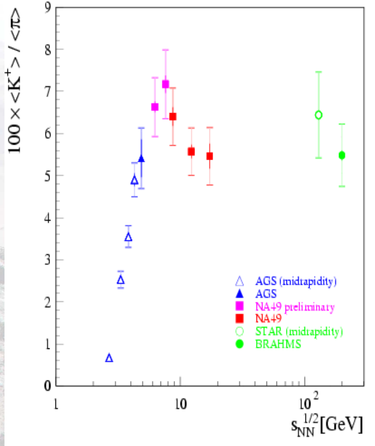


J. C., B. Kämpfer, P. Steinberg and S. Wheaton, Journal of Physics G30 S595-S598 (2004).

The NA49 Collaboration has recently performed a series of measurements of Pb-Pb collisions at 20, 30, 40, 80 and 158 AGeV beam energies . When these results are combined with measurements at lower beam energies from the AGS they reveal an unusually sharp variation with beam energy in the $\Lambda/\langle\pi\rangle$, with $\langle\pi\rangle \equiv 3/2(\pi^+ + \pi^-)$, and K^+/π^+ ratios. Such a strong variation with energy does not occur in pp collisions and therefore indicates a major difference in heavy-ion collisions. This transition has been referred as the “horn”.

The Elephant in the Room

Friese
Dinkelaker
Blume
Speltz



Difficult to avoid, Hard to Model
→ But no unambiguous corroborating evidence

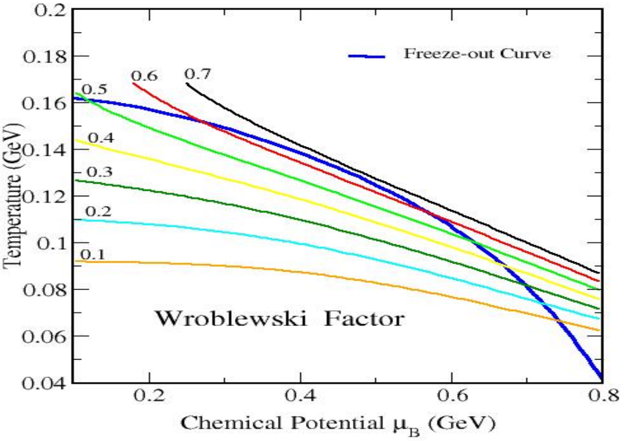
Strangeness in Heavy Ion Collisions
vs
Strangeness in pp - collisions

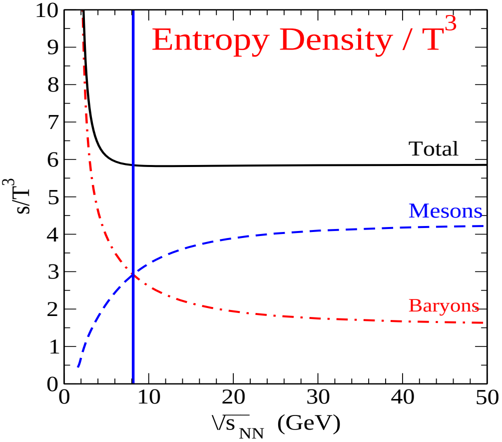
Use the Wroblewski factor

$$\lambda_s = \frac{2\left\langle s\bar{s}\right\rangle}{\left\langle u\bar{u}\right\rangle + \left\langle d\bar{d}\right\rangle}$$

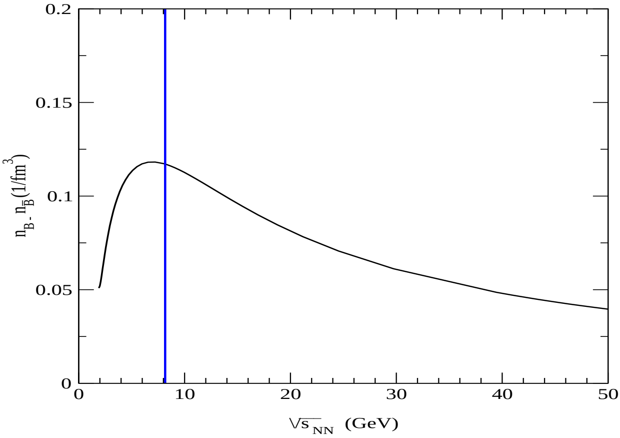
This is determined by the number of **newly** created quark – anti-quark pairs and **before** strong decays, i.e. before ρ's and Δ's decay.

Limiting values :
$\lambda_s = 1$ all quark pairs are equally abundant, SU(3) symmetry.
$\lambda_s = 0$ no strange quark pairs.

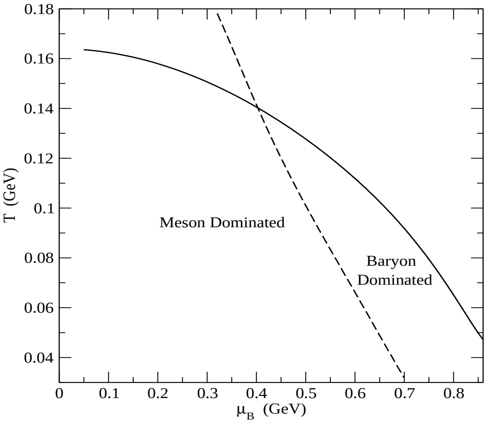

Freeze-out Curve
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Wroblewski Factor
Temperature (GeV)
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Chemical Potential μ_B (GeV)
0.2
0.4
0.6
0.8



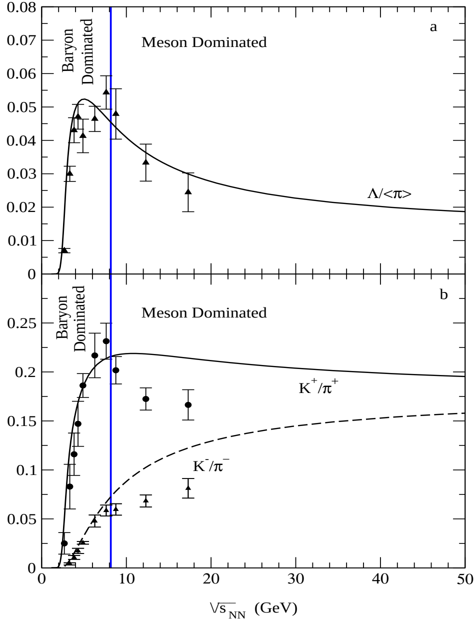
J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.



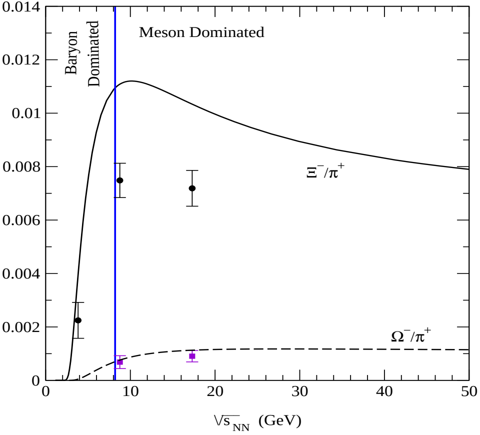
J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.



J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.



J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.



J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.

Maxima in Particle Ratios predicted by the Thermal Model.

Ratio	Maximum at $\sqrt{s_{NN}}$ (GeV)	Maximum Value
$\Lambda/\langle\pi\rangle$	5.1	0.052
Ξ^-/π^+	10.2	0.011
K^+/π^+	10.8	0.22
Ω^-/π^+	27	0.0012

J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.