Transport and Optical Properties in Dense Plasmas

UNIVERSITÄT ROSTOCK

Heidi Reinholz

in collaboration with J. Adams, C. Fortmann, S. Glenzer, I. Morozov, T. Raitza, R. Redmer, G. Röpke, A. Sengebusch, R. Thiele

Dense Matter In Heavy Ion Collisions and Astrophysics

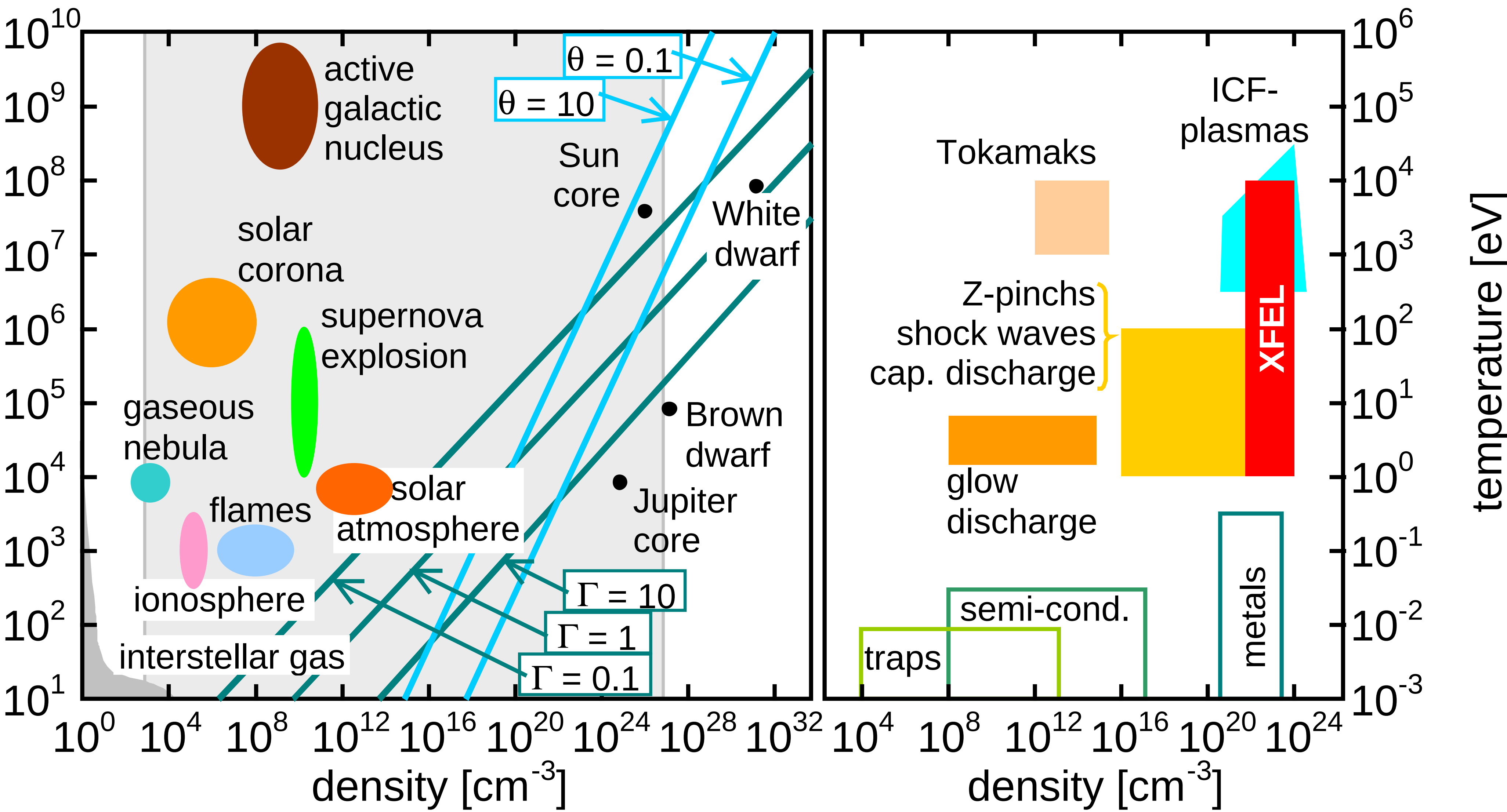
21.8.-1.9.2006, JINR, Dubna/Russia

Outline

- Plasma in electric and magnetic fields, interaction with radiation
  $\Rightarrow$ production, excitation, diagnostic tool
- Many–particle theory
  - kinetic equations, linear response, molecular dynamics simulations

  $\Rightarrow$ dielectric function $\epsilon(k, \omega)$, dynamical collision frequency $\nu(\omega)$
- Applications
  $\Rightarrow$ transport properties
  (dc–conductivity, thermopower, Hall effect)
  $\Rightarrow$ optical properties
  (dynamical conductivity, reflectivity, absorption,
  Thomson scattering, bremsstrahlung, spectral lines)

Density-temperature regions



A. Höll, PhD thesis (Rostock, 2002)

Electrical Conductivity $\sigma(\omega)$

induced charge current density $\vec{J}$ in many-particle system under the influence of electric field $\vec{E}$

$$\vec{J} = \sigma \vec{E}$$

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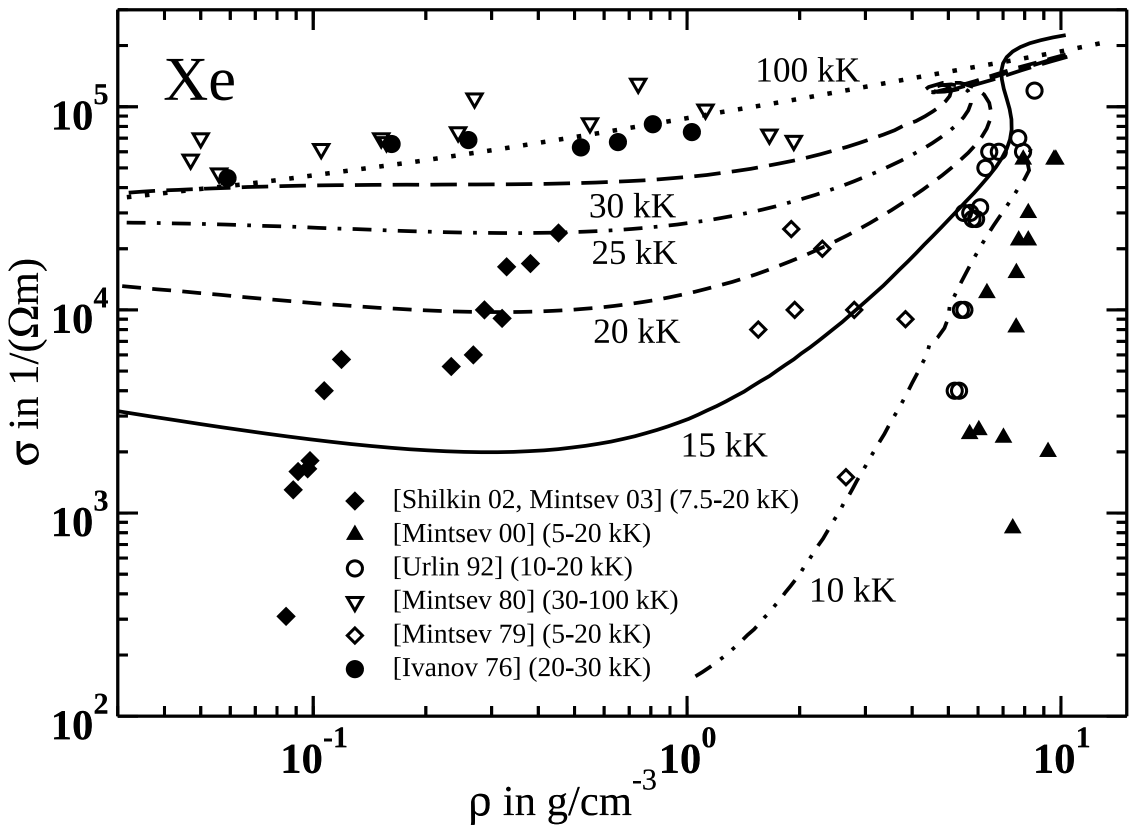
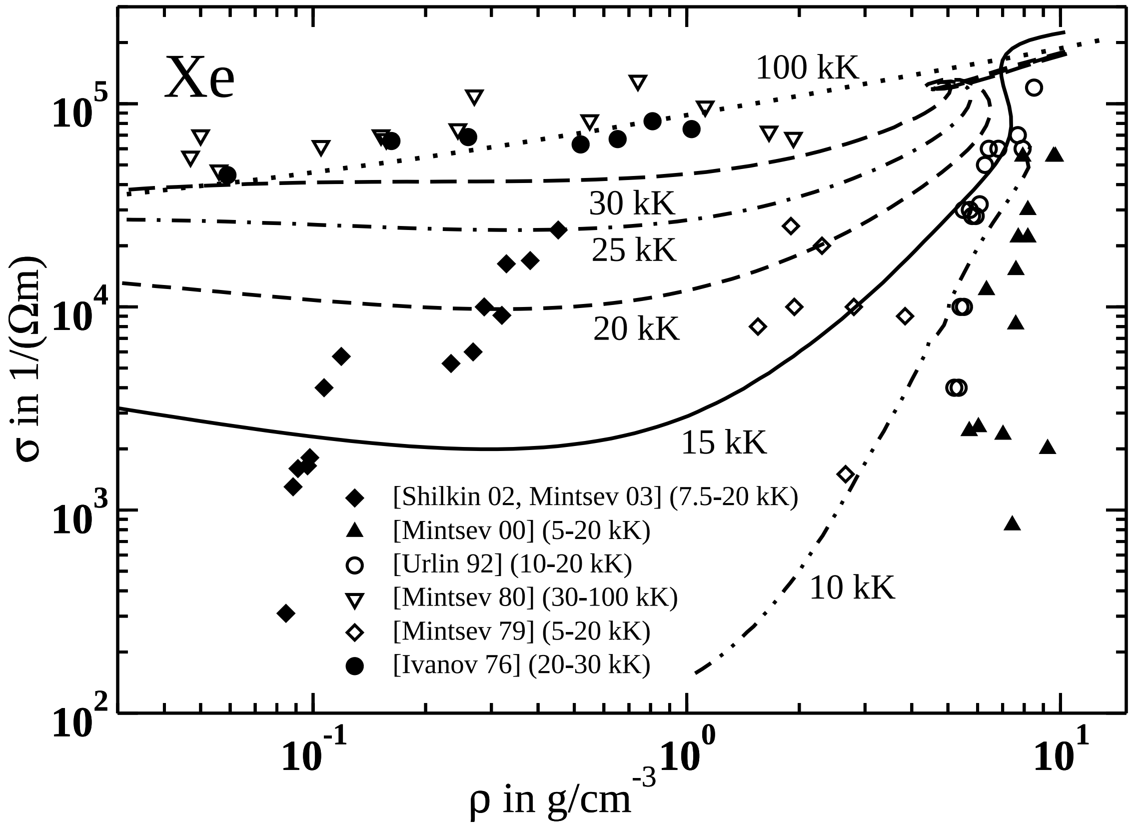


S. Kuhlbrodt et al., Contr Plasma Phys. 45 (2005) 61

Electrical Conductivity $\sigma(\omega)$

induced charge current density $\vec{J}$ in many-particle system under the influence of electric field $\vec{E}$

$$\vec{J} \;=\; \sigma\,\vec{E} \;=\; \boxed{\mathrm{Tr}\left\{\hat{\vec{j}}\,\hat{\rho}\right\}}$$



statistical operator $\hat{\rho}$

currrent operator

$$\hat{\vec{j}} \;=\; \frac{1}{\Omega}\sum_c \frac{e_c}{m_c}\hat{\vec{P}}_c$$

S. Kuhlbrodt et al., Contr Plasma Phys. 45 (2005) 61

Linear response theory

- statistical operator for generalized grand canonical ensemble by introducing set of relevant observables $\{B_n\}$

$$\rho_{\rm rel} = \frac{1}{Z_{\rm rel}} {\rm e}^{-[\hat{\mathcal{H}} - \mu N + \sum_n \Phi_n B_n]} \qquad \hat{\mathcal{H}} = \hat{\mathcal{H}}_{\rm eq} - \sum_c e_c \vec{R}_c \vec{E}$$

- self-consistency condition for response parameter Φ_n

$\boxed{{\rm Tr}\,(B_n \rho_{rel}) = {\rm Tr}\,(B_n \rho)}$ with statistical operator $\rho = \rho_{rel} + \rho_{irrel}$

- solution in linear response:
  response equation containing equilibrium correlation functions/ generalized BOLTZMANN equation

$$\langle B_m; \dot{\vec{R}}_c \rangle e_c \vec{E} = \sum_n \langle B_m; \dot{B}_n \rangle \Phi_n$$

$${\rm Tr}\,\{B_n \rho\} = \sum_m (B_n; B_m) \Phi_n$$

Zubarev, Morozov, Röpke, Stat. Mech. of Non-equilibrium Processes, Berlin 1996

Linear response theory

$$\langle B_m;\dot{\vec{R}}_c\rangle e_c\vec{E} = \sum_n \langle B_m;\dot{B}_n\rangle \Phi_n$$

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equilibrium correlation functions

$$\langle A;B\rangle_z = \int_0^\infty dt\, e^{izt}\,(A(t);B) = -\frac{i}{\beta}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\;\frac{1}{z-\omega}\;\frac{1}{\omega}\;\text{Im}\,G_{AB^+}(\omega-i0)$$

$$(A(t);B) = \frac{1}{\beta}\int_0^\infty d\tau\;\text{Tr}\,[A(t-i\hbar\tau)\;B^+\,\rho_0]$$

- application to electrical current density using set $\{B_n\}=\vec{P}=\dot{\vec{R}}$

$$\vec{J}=\left\langle\vec{j}\right\rangle=\text{Tr}\left\{\rho\vec{j}\right\}=\frac{e}{\Omega}\text{Tr}\left\{\rho\dot{\vec{R}}\right\}=\frac{e}{\Omega}\text{Tr}\left\{\rho_{\text{rel}}\vec{P}\right\}=\sigma\vec{E}$$

*Röpke, Meister, Ann. Phys. **36** (1979) 377; Röpke, PRA **38** (1988) 3001; Reinholz et al. , PRE 52 (1995) 6368*

$$\begin{aligned} \langle B_m; \dot{\vec{R}}_c \rangle e_c \vec{E} &= \sum_n \langle B_m; \dot{B}_n \rangle \Phi_n \\ \text{Tr}\,\{B_n \rho\} &= \sum_m (B_n; B_m) \Phi_n \end{aligned}$$

- application to electrical current density using set $\{B_n\} = \vec{P} = \dot{\vec{R}}$

$$\vec{J} = \left\langle \vec{j} \right\rangle = \mathrm{Tr}\left\{\rho \vec{j}\right\} = \frac{e}{\Omega} \mathrm{Tr}\left\{\rho \dot{\vec{R}}\right\} = \frac{e}{\Omega} \mathrm{Tr}\left\{\rho_{\mathrm{rel}} \vec{P}\right\} = \sigma \vec{E}$$

- solution for electrical conductivity

$$\sigma = \beta \Omega \langle j; j \rangle = \frac{\beta e^2}{\Omega\, m_e} \frac{(P;P)^2}{\langle \dot{P}; \dot{P} \rangle}$$

Kubo-Greenwood formula $\Longleftrightarrow$ force force correlation functions

$(\dot{P} = F_{ei} + F_{ee} + F_{ea})$

Röpke, Meister, Ann. Phys. ***36*** *(1979) 377; Röpke, PRA* ***38*** *(1988) 3001; Reinholz et al. , PRE 52 (1995) 6368*

Dynamical conductivity

time dependent external field

$$\boxed{\sigma(\omega) = \frac{\epsilon_0 \omega_{\mathrm{pl}}^2}{-i\omega + \nu(\omega)}} \qquad \omega_{\mathrm{pl}}^2 = \frac{e^2 n_e}{\epsilon_0 m_e}$$

generalized Drude formula

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generalized Drude formula

- kinetic theory
  collision integrals, moment expansion for distribution function, relaxation time ansatz
- linear response theory
  generalized statistical operator, equilibrium correlation functions

$$\nu(\omega) = \frac{\beta}{n_e \Omega} \left\langle \dot{\vec{P}}; \dot{\vec{P}} \right\rangle_{\omega + i\eta}$$

Reinholz, Redmer, Röpke, Wierling, PRE 62 (2000) 5648

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- molecular dynamic simulations
  normalized current auto–correlation function (ACF)

$$\langle J; J \rangle_{\omega + i\eta} = \frac{\epsilon_0 \omega_{\rm pl}^2}{\beta \Omega} \lim_{\epsilon \to 0} \int_0^\infty {\rm e}^{i(\omega + i\epsilon)t} K(t)\, {\rm d}t \qquad K(t) = \frac{1}{\langle J^2 \rangle} \frac{1}{\delta} \int_0^\delta d\tau\, J(t+\tau) J(\tau)$$

MD simulations

normalized current auto-correlation function (ACF)

$$K(t)^{L/T} = \frac{1}{\langle J^2 \rangle} \frac{1}{\delta} \int_0^{\delta} d\tau \, J^{z/x}(t+\tau) J^{z/x}(\tau)$$

with

$$\vec{J}(t) = \frac{1}{\Omega_0} \sum_c \sum_{\alpha=1}^{N} e_c \vec{v}^{z}_{c,\alpha}(t)$$

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equation of motion with mean field E^z due to

average current density $\vec{j}(t) = j_z \vec{e}_z$
has only z-component due to $\vec{k} = k \vec{e}_z$

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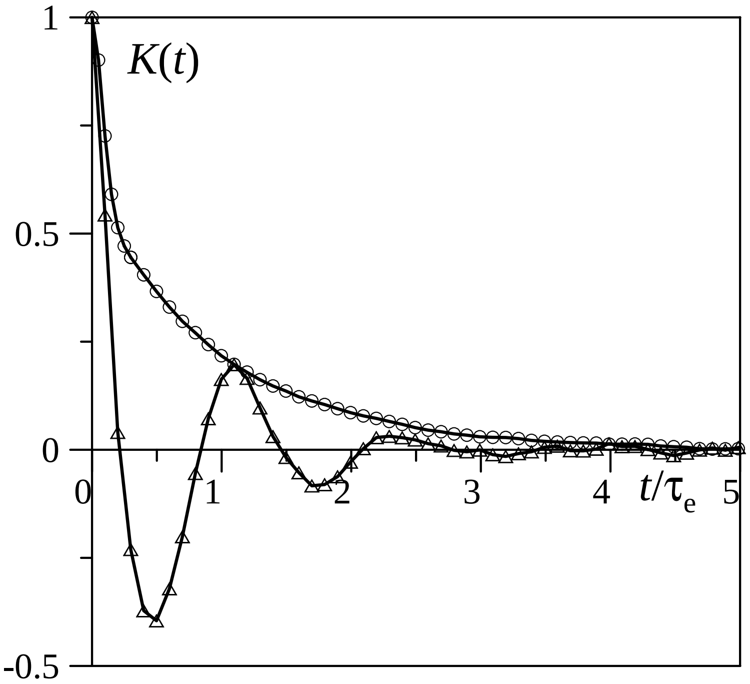


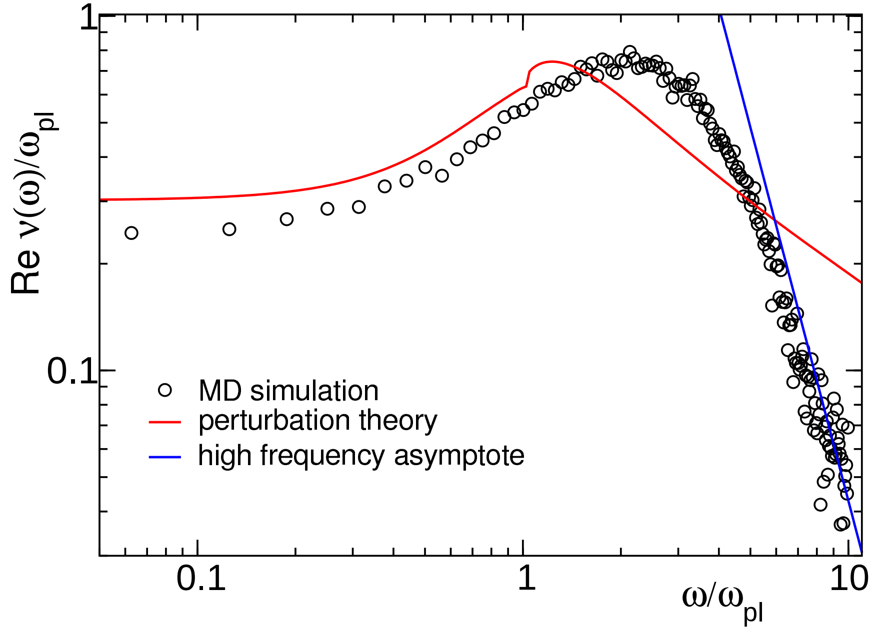
Kelbg pseudopotential

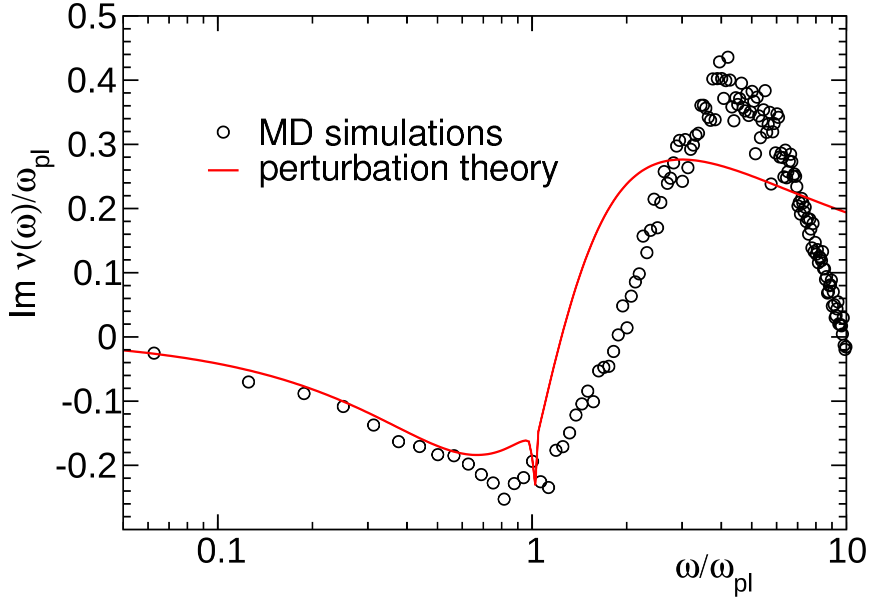
Figure: normalized current ACF for $\Gamma = 1.28$, $\tau_e = 2\pi/\omega_{pl}$ – period of electron plasma oscillations: MD simulations without (o) and including (△) an additional mean-field term

Reinholz et al., PRE 69 (2004) 066412; Morozov et al. PRE 71 (2005) 066408

Dynamical collision frequency

fully ionized plasma at $n = 3.8 \times 10^{21}$ cm^{-3}, $T = 33\,000$ K, $\Gamma = 1.28$, $\Theta = 3.2$





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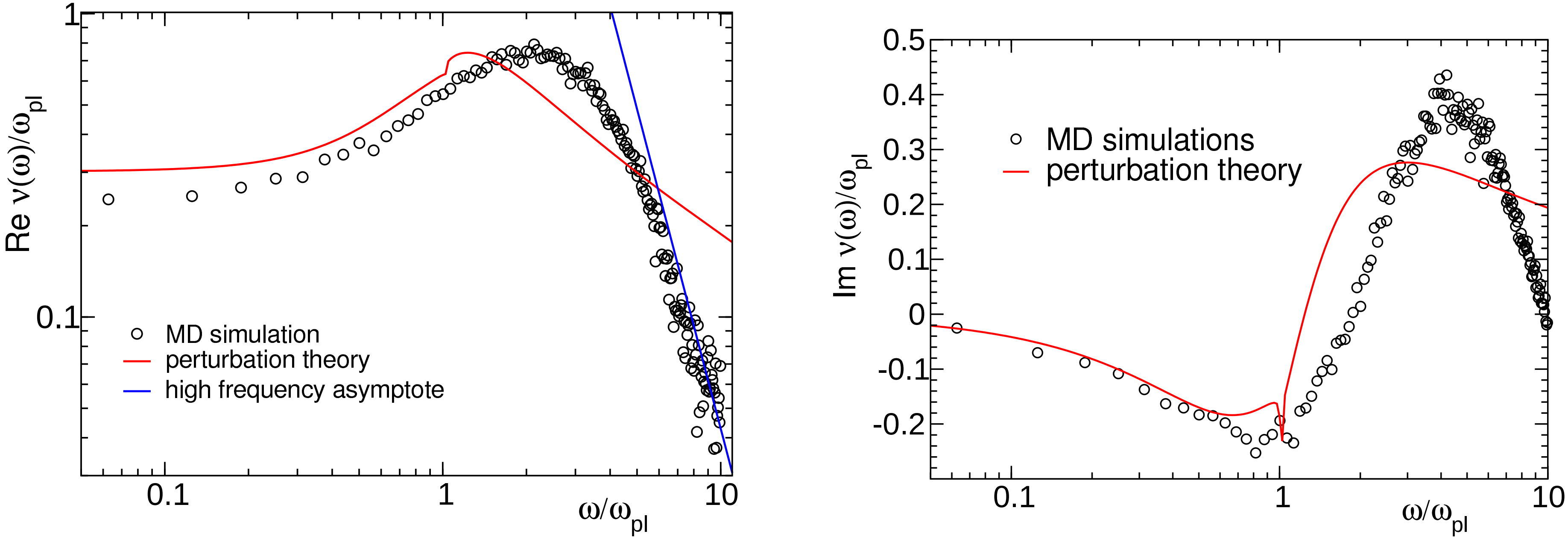

perturbation theory: *Reinholz et al., PRE* ***62*** *(2000) 5648; PRE* ***69*** *(2004) 066412*

$$\nu(\omega) \approx r(\omega)\nu^{(P_0)}(\omega) = r(\omega)\nu^{\mathrm{GD}}(\omega) = r(\omega)(\nu^{\mathrm{ladder}}\left[(\omega) - \nu^{\mathrm{Born}}(\omega) + \nu^{\mathrm{LB}}(\omega)\right]$$

dynamically screened binary collision approximation using Gould-deWitt ansatz (dynamical screening and strong collisions); higher moments of single–particle distribution function via renormalization factor (electron-electron interaction)

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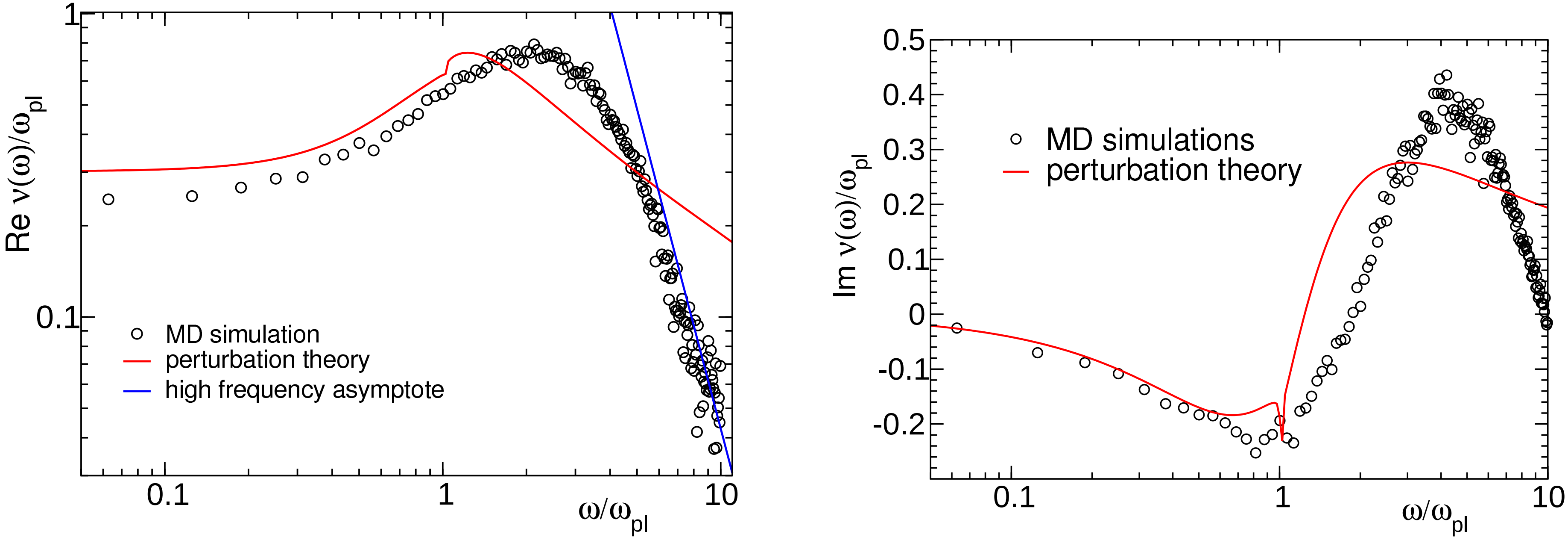

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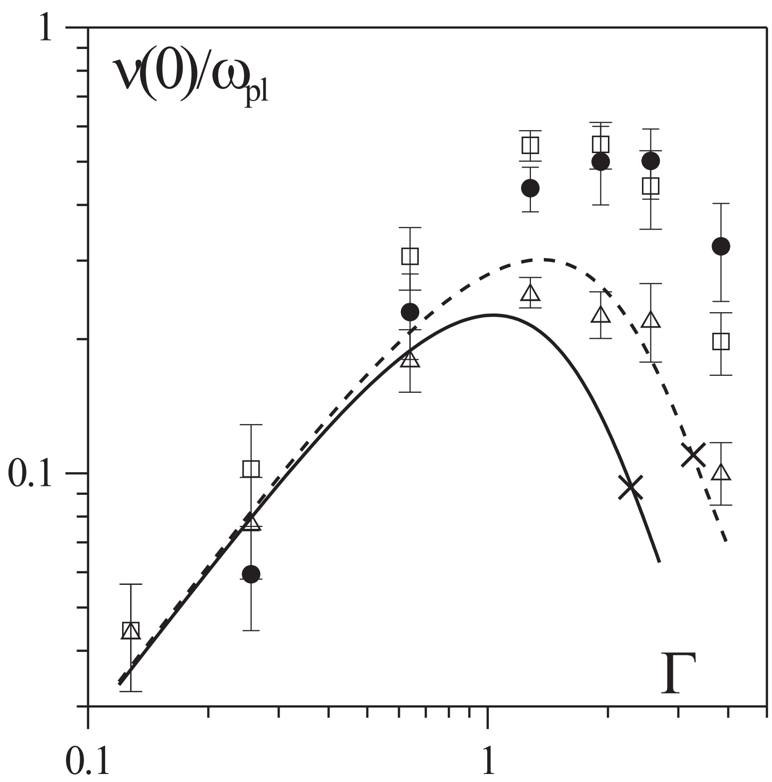
high frequency asymptotes in Born approximation

$$\text{Re } \nu^{\rm Kelbg}(\omega) \propto \omega^{-7/2} \qquad \nu^{\rm Coulomb}(\omega) \propto \omega^{-3/2}$$

DC Conductivity comparison



collision frequency

- □ from MD simulations at $\omega_{\rm pl}$
- • collisional damping of Langmuir waves, $\nu = 2\delta_c$

(*Morozov, Norman, JETP 100 (2005)*)

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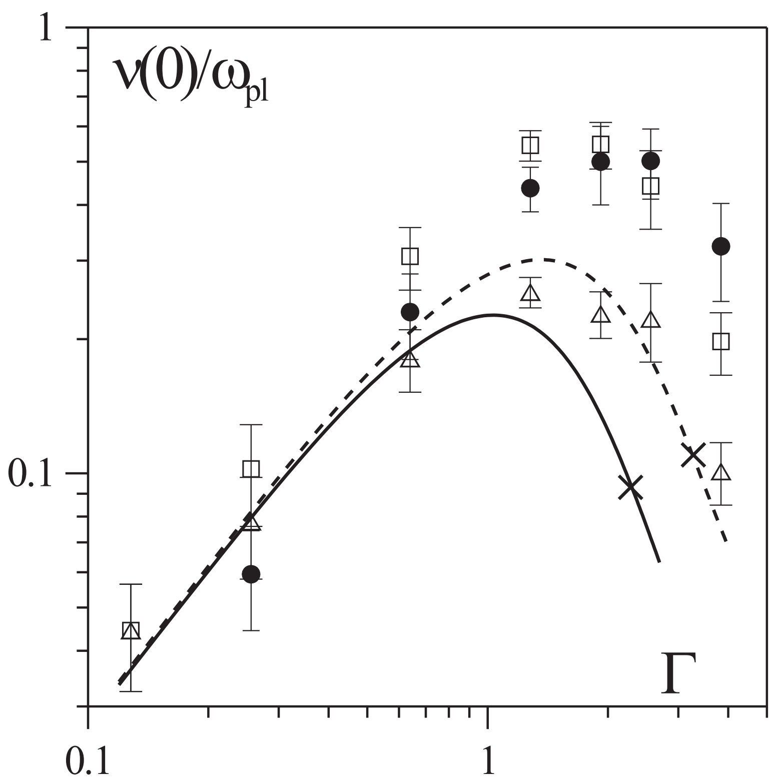

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△ — MD simulations $T = 33000$K

full line — interpolation formula $T = 33000$K

filled symbols — experimental data (*Mintsev et al.*)

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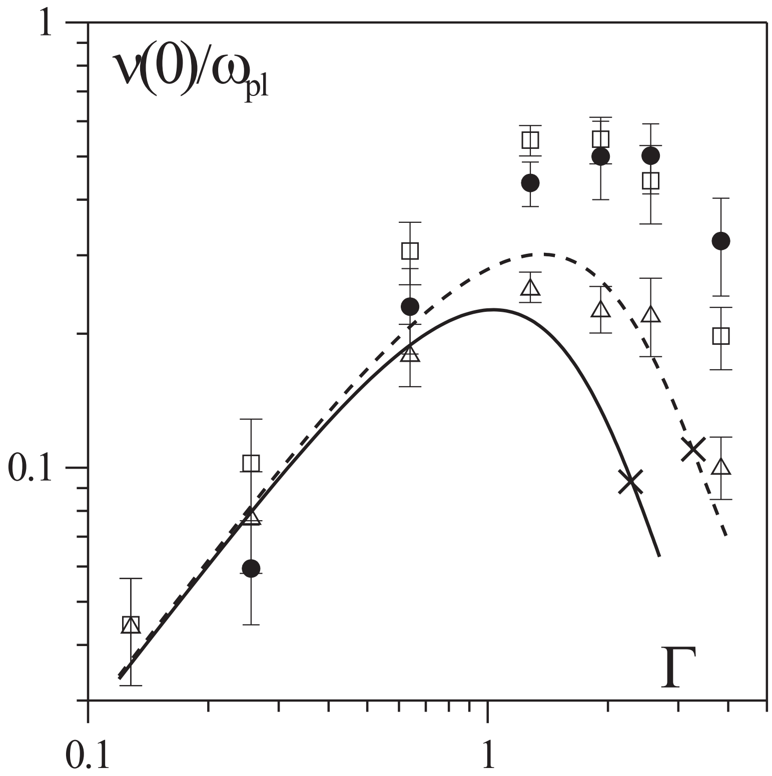
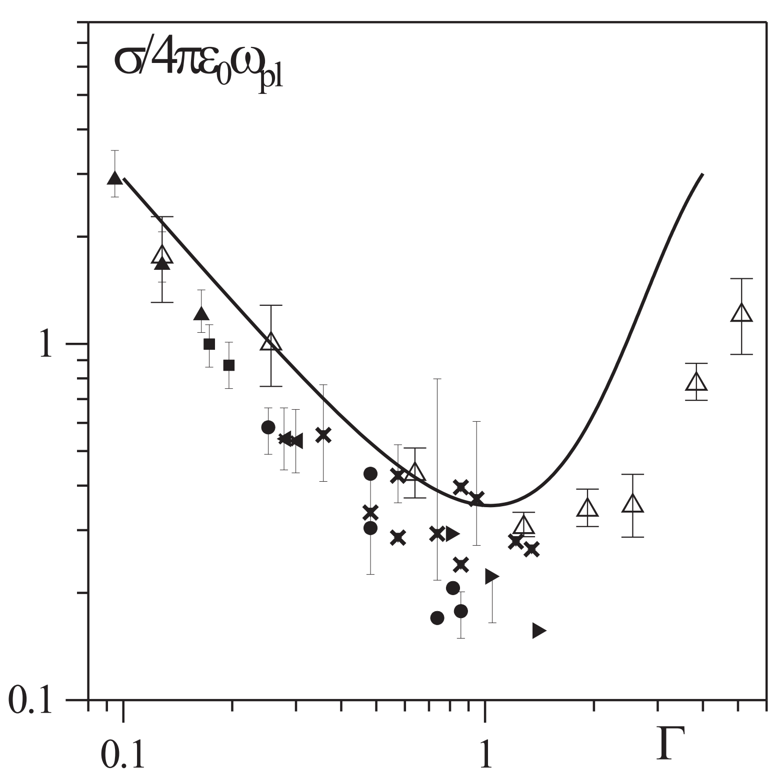


collision frequency



Static conductivity

- □ from MD simulations at $\omega_{\rm pl}$
- • collisional damping of Langmuir waves, $\nu = 2\delta_c$

(*Morozov, Norman, JETP 100 (2005)*)

$$\sigma_{dc} = \lim_{\omega \to 0} \sigma(\omega) = \frac{\epsilon_0\, \omega_{pl}^2}{\nu_{\rm dc}} = \frac{{\rm e}^2 n_e}{m_e}\, \tau_{\rm dc}$$

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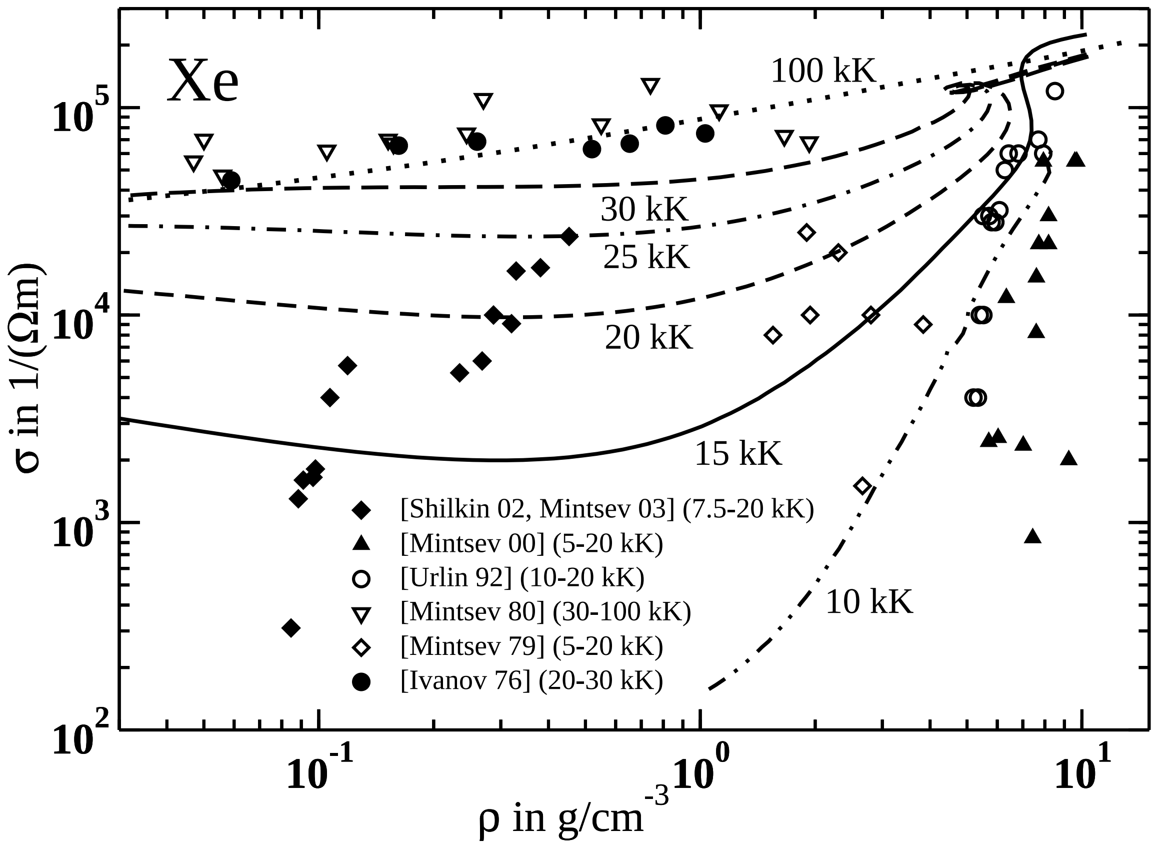
DC conductivity in xenon

electrical conductivity of xenon in comparison to experimental data



partially ionized plasma ⇒ bound states, depletion of free charge carriers, additional scattering mechanisms (COMPTRA04)

S. Kuhlbrodt et al., Contr Plasma Phys. 45 (2005) 61

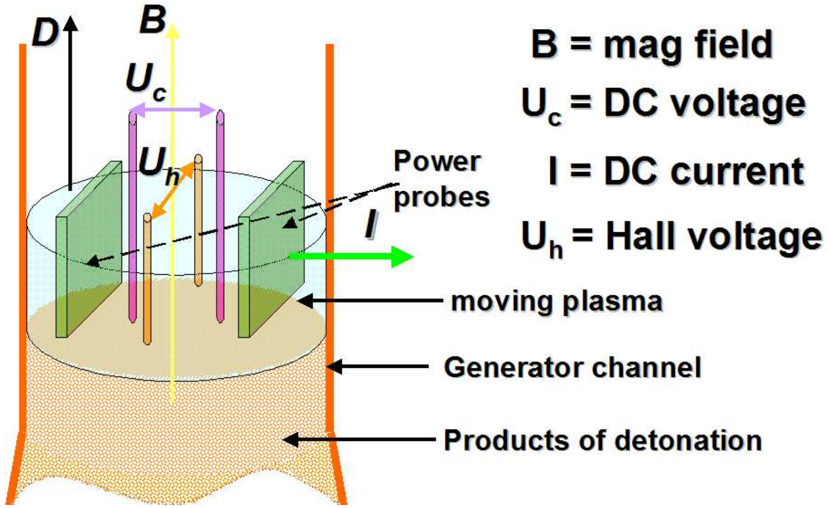
Conclusions I

- linear response theory (Zubarev approach) to derive expressions for static and dynamical conductivity
- Kubo formula or force-force correlation functions
- systematic and consistent inclusion of strong collisions, dynamical screening, e-e interaction and effects in partially ionized systems
- results from perturbation theory, molecular dynamics simulation and experiments are consistent in weakly coupled plasmas
- high frequency behaviour of collision frequency
- quantum statistical simulations: QMD, WPMD, PIMC

Applications

- Hall effect
- optical properties
- reflectivity
- Thomson scattering

Hall measurement



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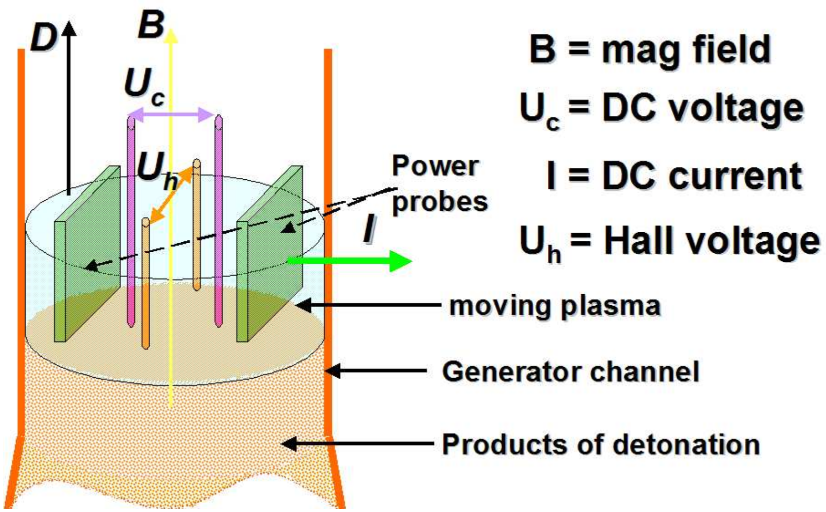

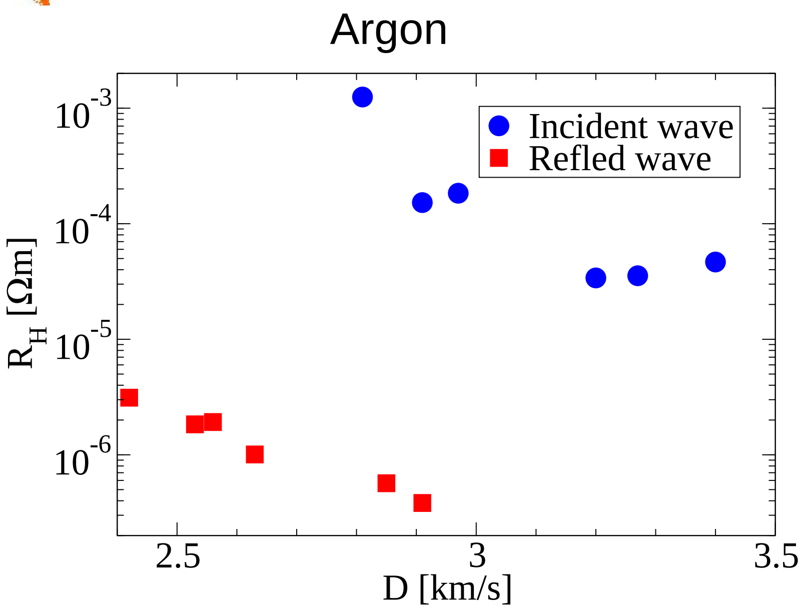

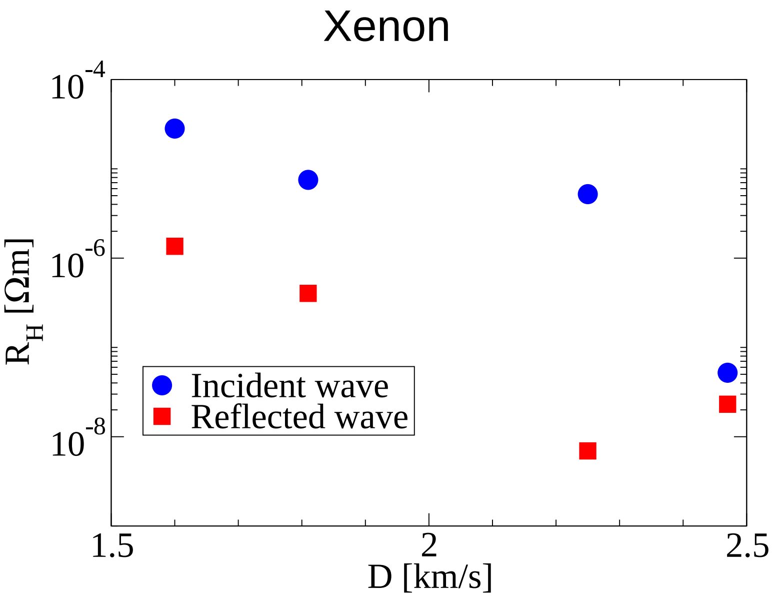

*Shilkin et al. , JETP **77** 486 (2003)*

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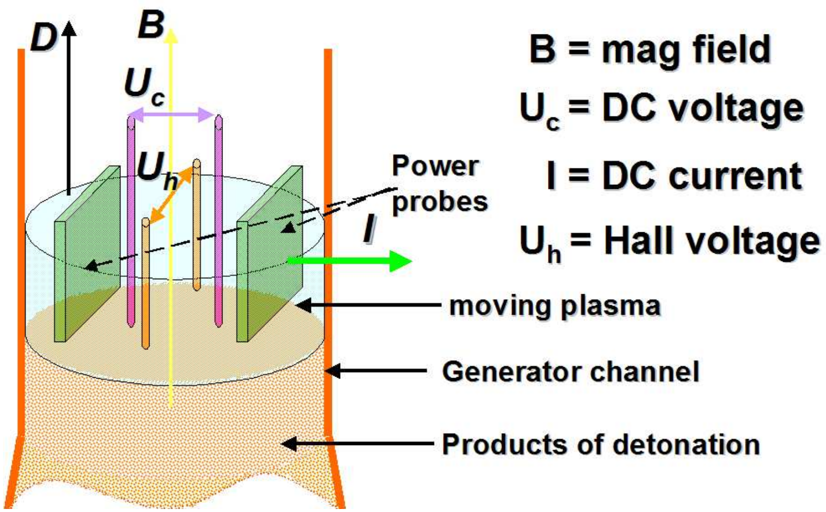
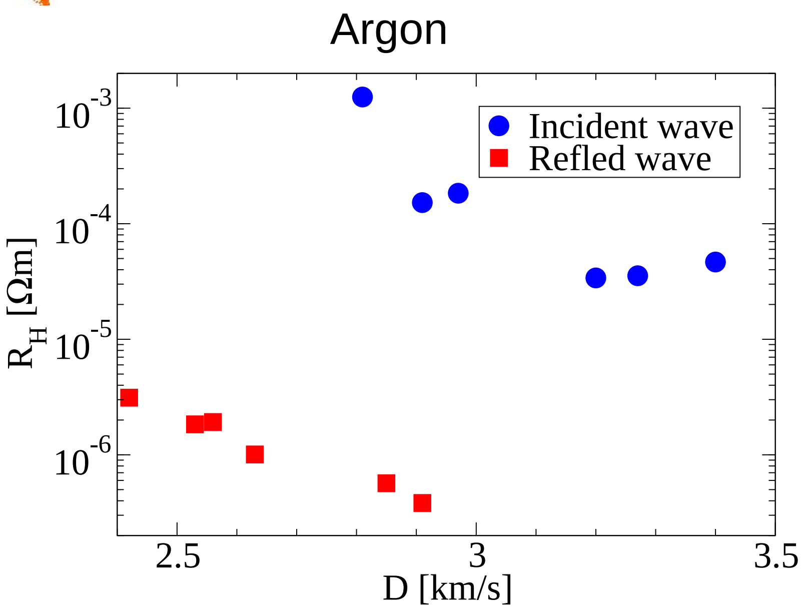
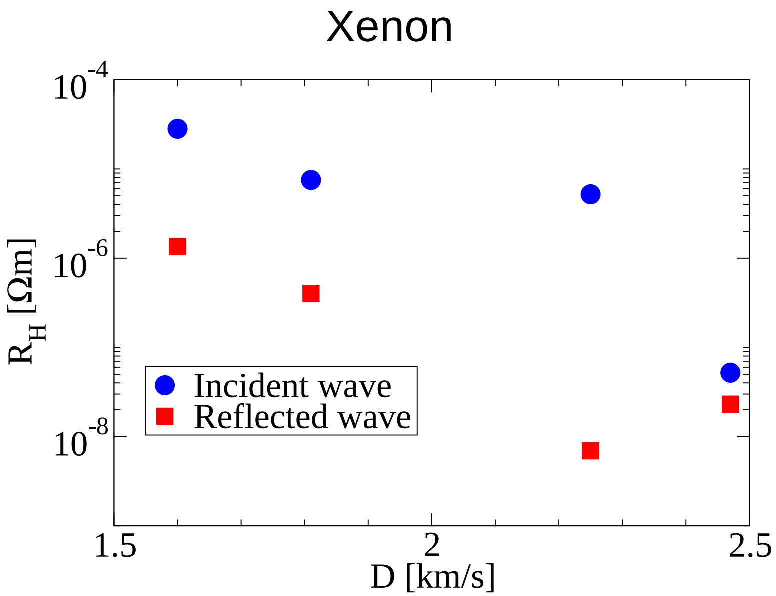


- Hall voltage U_h

$$\vec{E}_h = R_h \left(\vec{J} \times \vec{B}\right) = \frac{U_h}{d}$$

- free electron densities n_e calculated from a thermodynamic model
$\Rightarrow R_h = r_h/(e\, n_e)$





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Electromagnetic transport properties

many-particle system under the influence electric field $\vec{E}$ and magnetic field $\vec{B}$

electric current densities: $\vec{J} = \sigma \vec{E} + \sigma R_h \left(\vec{J}_{\rm el} \times \vec{B} \right) \; = \; -\frac{e}{\Omega} \; \left\langle \dot{\hat{R}} \right\rangle$

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time evolution:

$$\frac{\mathrm{d}\hat{R}_n}{\mathrm{d}t} \; = \; \dot{\hat{R}}_n \; = \; \frac{i}{\hbar}[\hat{\mathcal{H}}, \hat{R}_n], \qquad \hat{\mathcal{H}} = \hat{\mathcal{H}}_0 - e\vec{E}\hat{\vec{R}} + \frac{e}{2m_{\rm e}}\vec{B}(\hat{\vec{R}} \times \hat{\vec{P}})$$

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$$\begin{aligned} m_e \dot{\hat{R}} &= \hat{P} + \frac{e}{2}\left(\vec{B} \times \hat{R}\right) \\ m_e \ddot{\hat{R}} &= \dot{\hat{P}} - e\left(\vec{E} + \frac{1}{m_e}\left(\dot{\hat{R}} \times \vec{B}\right)\right) \end{aligned}$$

Lorentz force

relevant observables

$$\begin{aligned} \{B_n\} &= \dot{\hat{\vec{R}}}_n \quad \text{with} \\ \dot{\vec{R}}_0 &= \dot{\hat{R}} \end{aligned}$$

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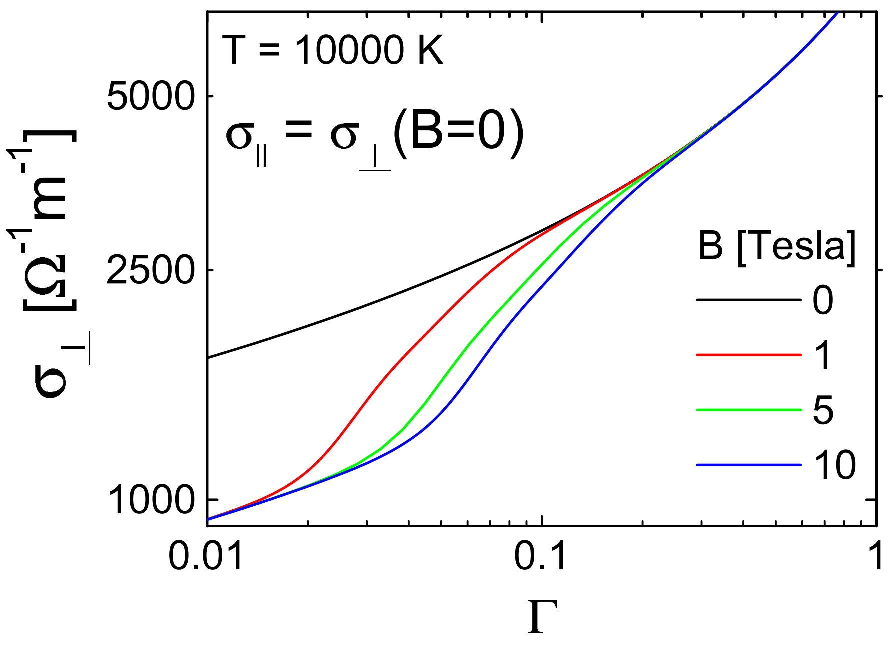
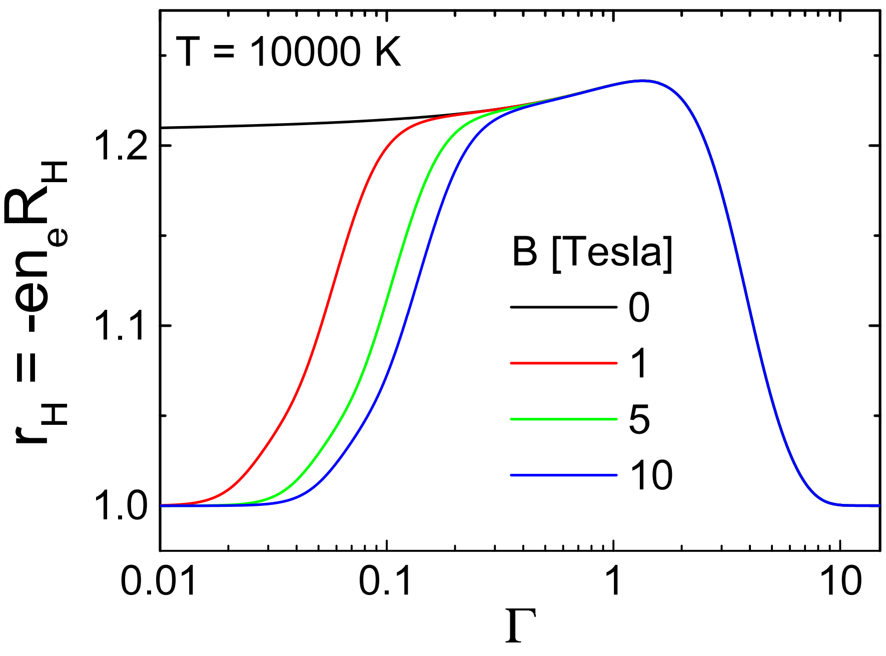
equilibrium correlation functions $\boxed{\langle \dot{\hat{P}}_m \,;\, \dot{\hat{P}}_n \rangle + {\omega_e}^2 \langle \hat{P}_n \,;\, \hat{P}_m \rangle}$

with electron cyclotron frequency $\omega_{\rm e} = \frac{eB}{m_e}$

fully ionized plasma in magnetic field

- LRT within five moment approximation
- transport cross section in Born approximation for statically screened e-i and e-e potential (Debye potential)
- note that $r_H = 1.93$ for fully ionized Lorentz plasma





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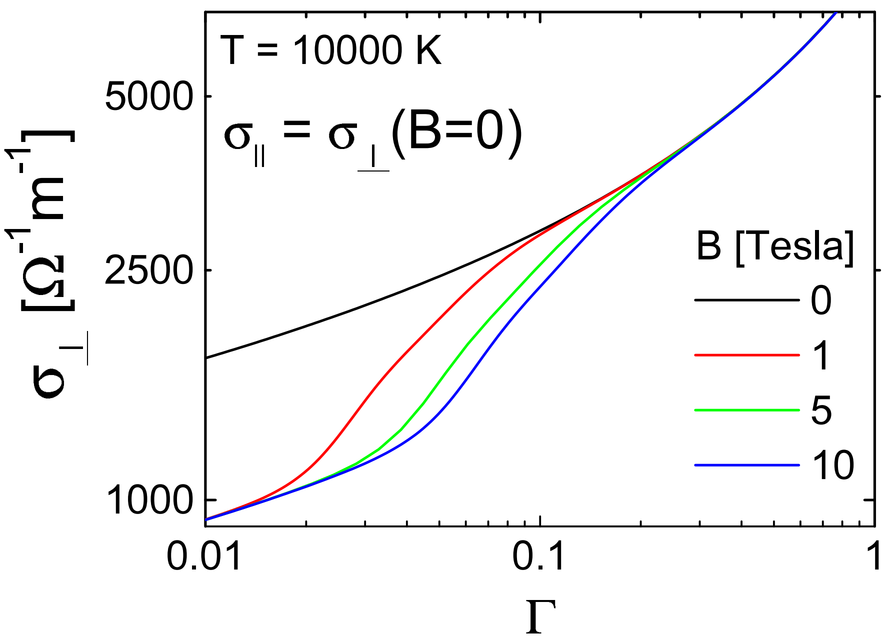
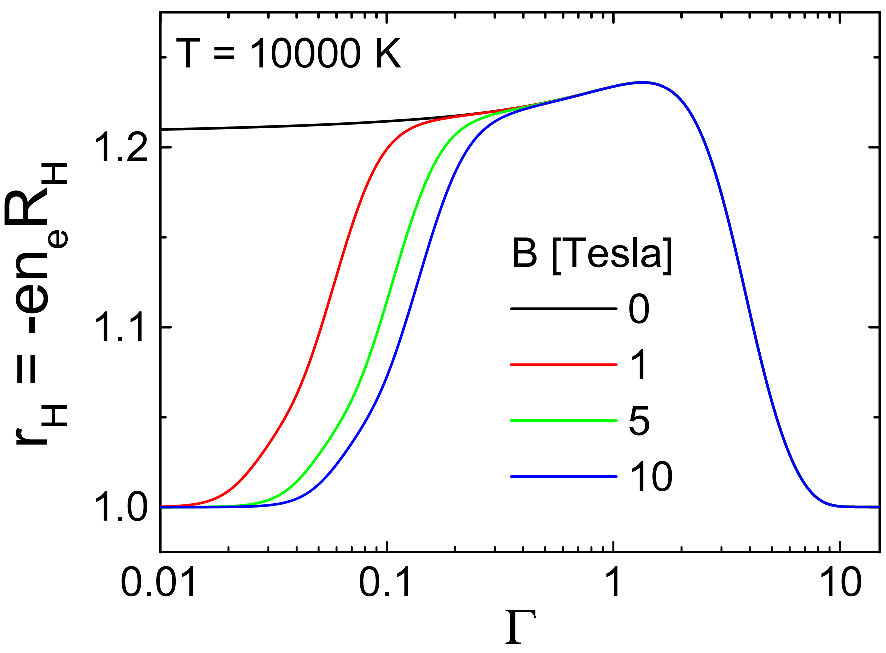


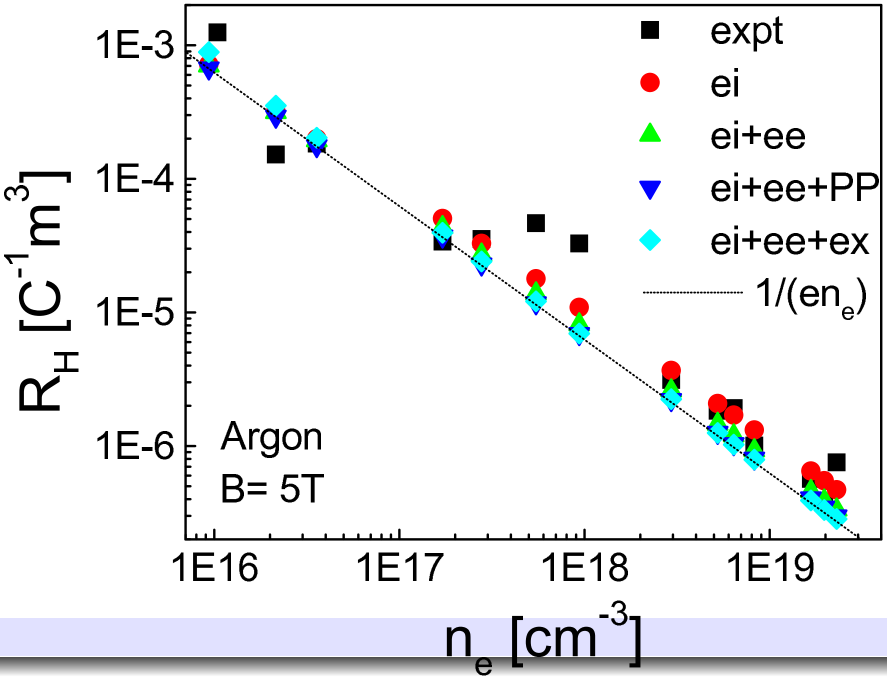


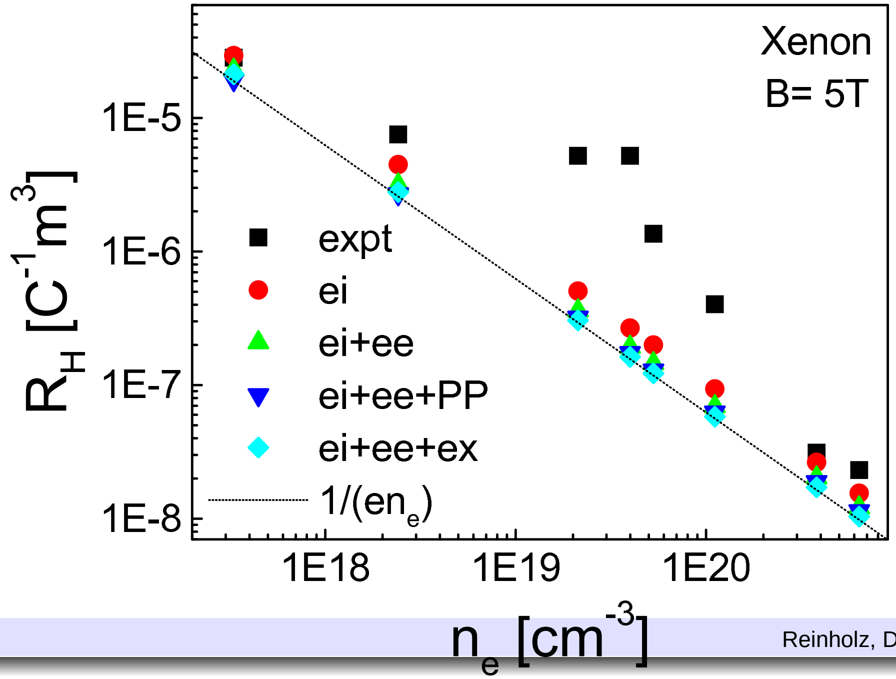
- conductivity is reduced in low density limit (higher moments are not relevant)
- maximum in Hall factor shifts to higher densities with decreasing temperature

Hall coefficient in partially ionized plasma

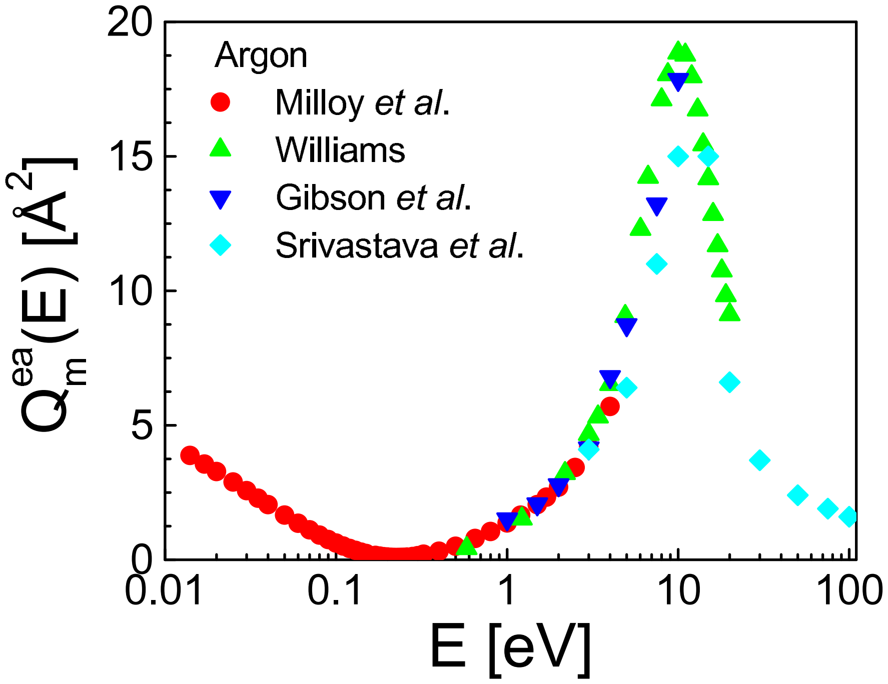
scattering mechanisms:

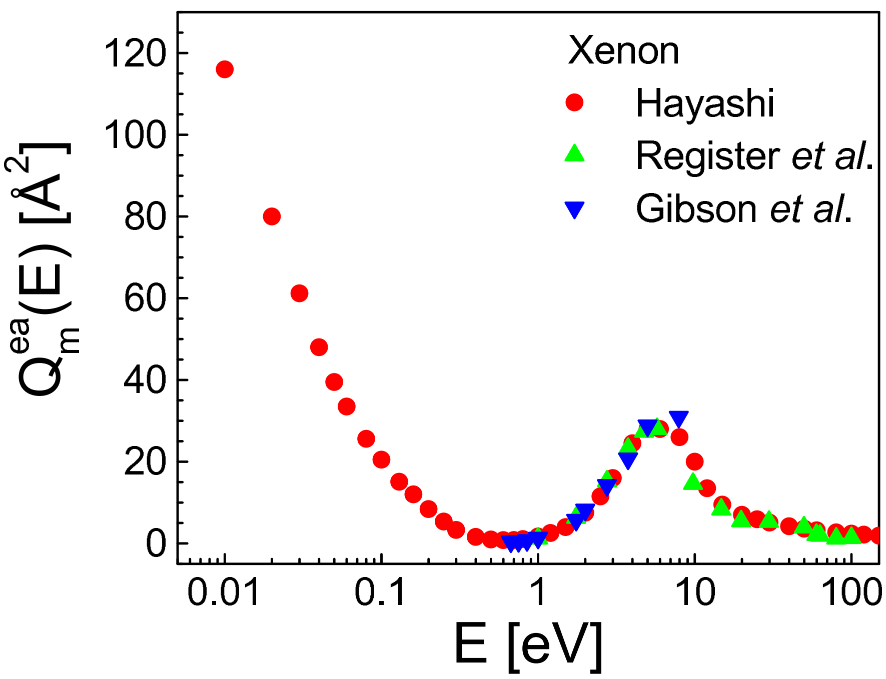
$$F = F_{ei} + F_{ee} + F_{ea}$$



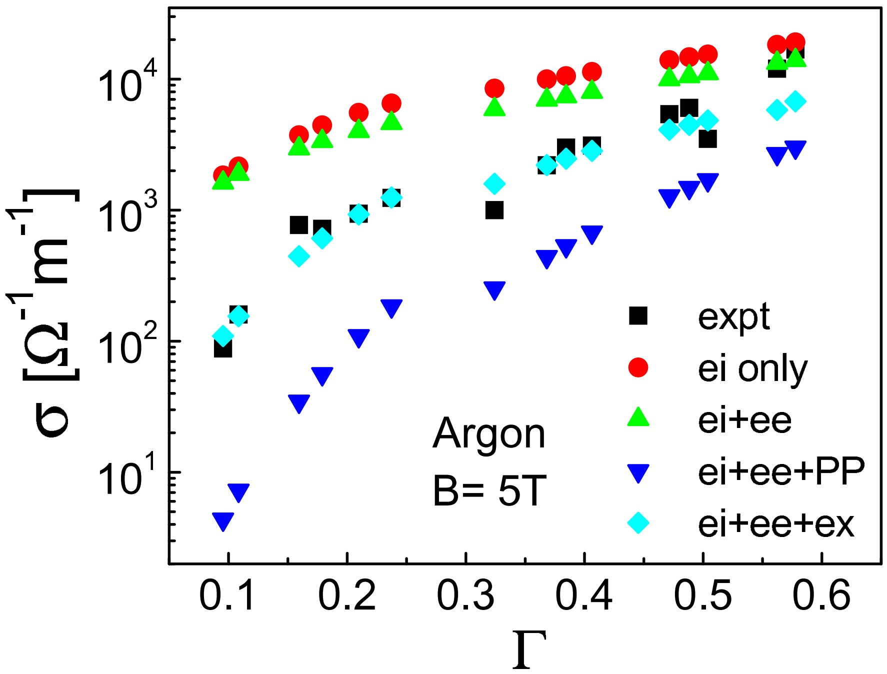


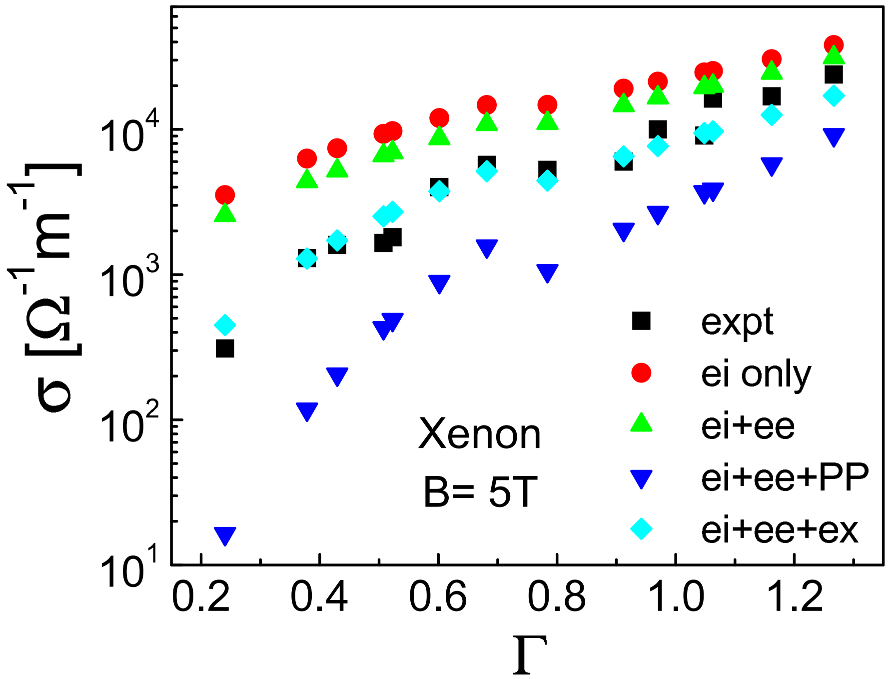
transport cross section





conductivity in magnetic field





Conclusions II

- extension of linear response theory to include magnetic field effects in order to describe Hall effect
- reproduction of results from relaxation time approach
- systematic and consistent inclusion of e-e interaction and effects in partially ionized systems

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- correlations are not relevant for high magnetic fields (r_H=1) except in a small parameter region of intermediate correlation strength
- outlook
  - more detailed calculations necessary for comparison with experiment (ionization degree, transport cross section)
  - Hall factor as diagnostic tool for determination of system parameters

Dielectric and optical response

$$\epsilon(\mathbf{k},\omega) = 1 + \frac{i}{\epsilon_0\omega}\sigma(\mathbf{k},\omega) = 1 - \frac{\omega_{\rm pl}^2}{\omega(\omega - i\nu(\omega))}$$

- dynamical conductivity - generalized Drude formula

Reinholz, Redmer, Röpke, Wierling, PRE 62 (2000) 5648

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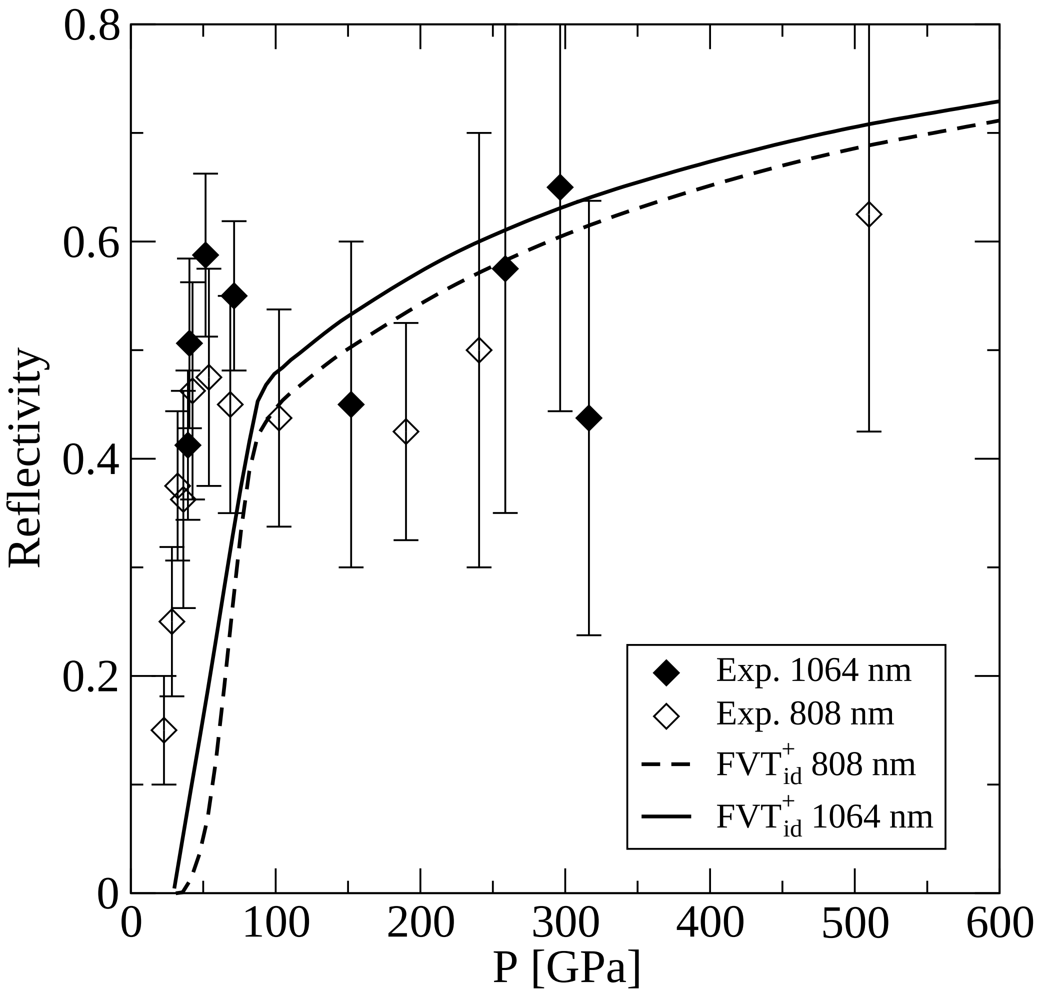
- reflectivity at normal incidence

$$R(\omega) = \left|\frac{\sqrt{\epsilon(\omega)} - 1}{\sqrt{\epsilon(\omega)} + 1}\right|^2$$

Reinholz, Redmer, Röpke, Wierling, PRE 62 (2000) 5648

Reflectivity of Deuterium



- shock wave experiments along Hugeniot
- assuming step like shock wave front

$$R(\omega) = \left| \frac{\sqrt{\epsilon(\omega)} - 1}{\sqrt{\epsilon(\omega)} + 1} \right|^2$$

with

$$\text{RPA} \quad \epsilon(\omega) = 1 - \frac{\omega_{\text{pl}}^2}{\omega^2}$$

$$\text{Drude} \quad \epsilon(\omega) = 1 - \frac{\omega_{\text{pl}}^2}{\omega(\omega + i\nu_{dc}}$$

- insulator-plasma transition

Redmer et al. , AIP Conf. Proc. 845, 127 (2006

Reflectivity of xenon

shock compressed dense plasma:

$\Rightarrow$ pressure 1.6 - 20 GPa, T$\approx 33\,000$ K, density 0.5 - 4 g cm^{-3}

laser: $\lambda = 1.06\ \mu\text{m}, 0.694\,\mu\text{m}, 0.532\ \mu\text{m}$

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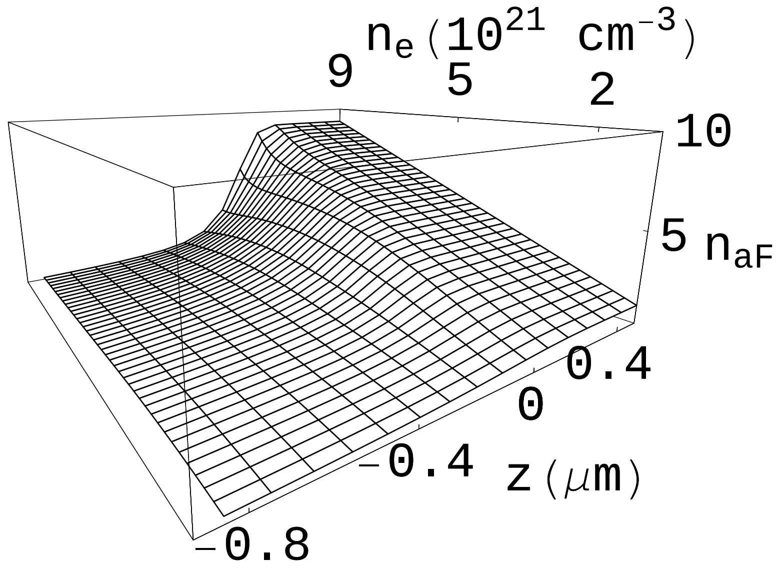
distribution of density of free electrons $n_e(z)$ in shock wave front in dependence on electron density n_e



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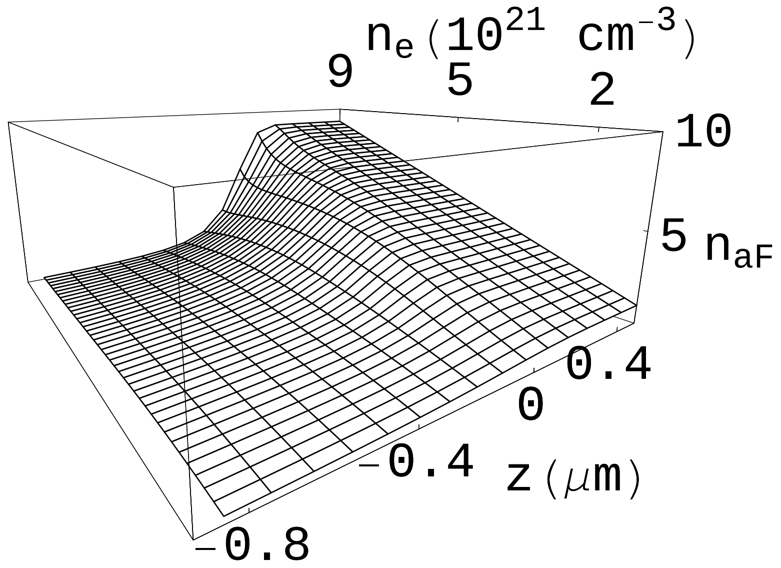
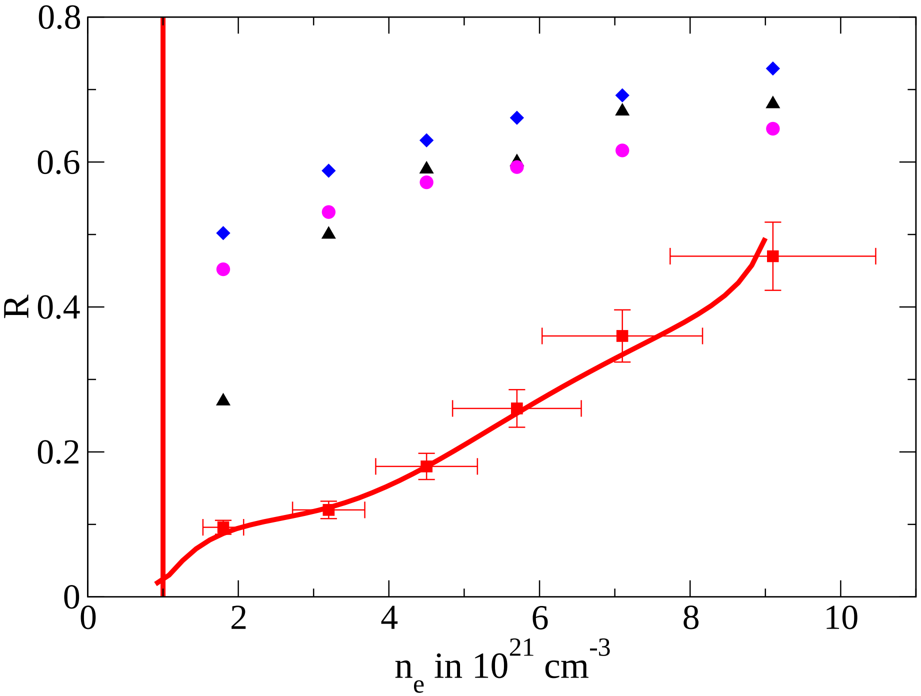
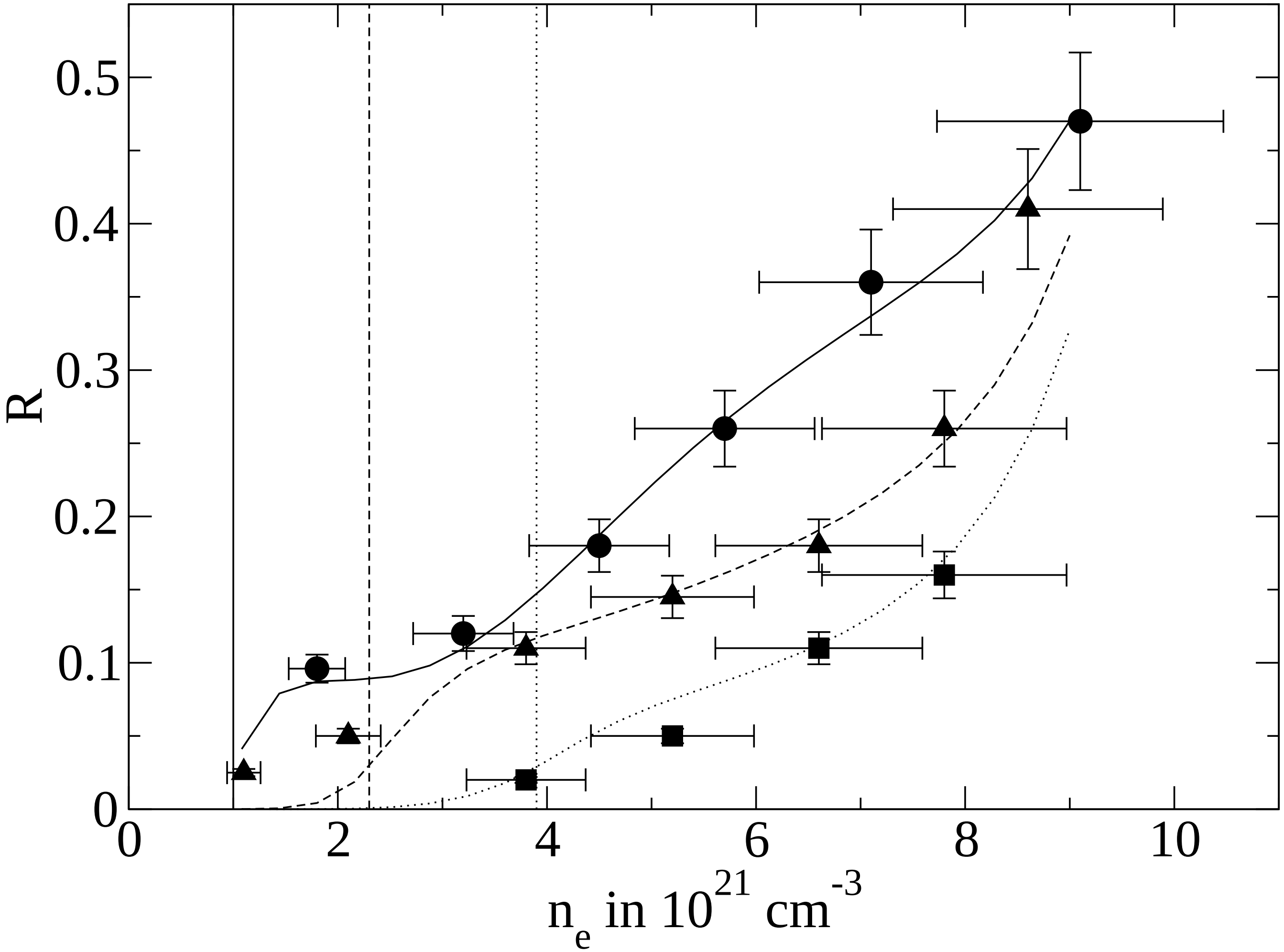




comparison of experiment with theoretical calculations for $1.06\ \mu\text{m}$: molecular dynamic simulation (▲); ERR_{dc} Padé formula for $\nu_{\text{ei,ee}}$ (◊); ERR_{dc} incl. ν_{ea} (●); full line - shock front profile

Reflectivity of xenon



Reflectivity coefficient R for Xenon calculated with asymmetric Fermi profile in comparison with measurements (symbols with error bars) for laser wavelengths $1.06\ \mu$m (solid line, ●), $0.694\ \mu$m (dashed line, ▲), and $0.532\ \mu$m (dotted line, ■), the corresponding critical densities n_e^{cr} are indicted with vertical lines

Raitza et al. , J. Phys. A 39 (2006) 4393

Thomson Scattering

scattering cross section:

$$\frac{d^2\sigma}{d\Omega\, d\omega} = \sigma_{\mathrm{T}} \frac{k_1}{k_0} S(k,\omega)$$

$\mathbf{k} = \mathbf{k_0} - \mathbf{k_1}$, $\omega = \omega_0 - \omega_1$
$k_0(k_1)$: incident (scattered) wavevector
σ_{T}: Thomson cross section

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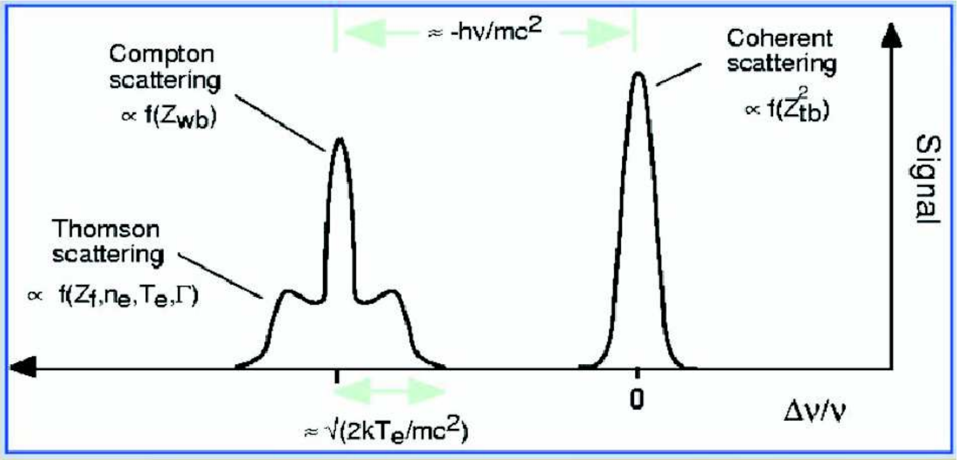

O.L. Landen et al., JQSRT 71 (2001) 465

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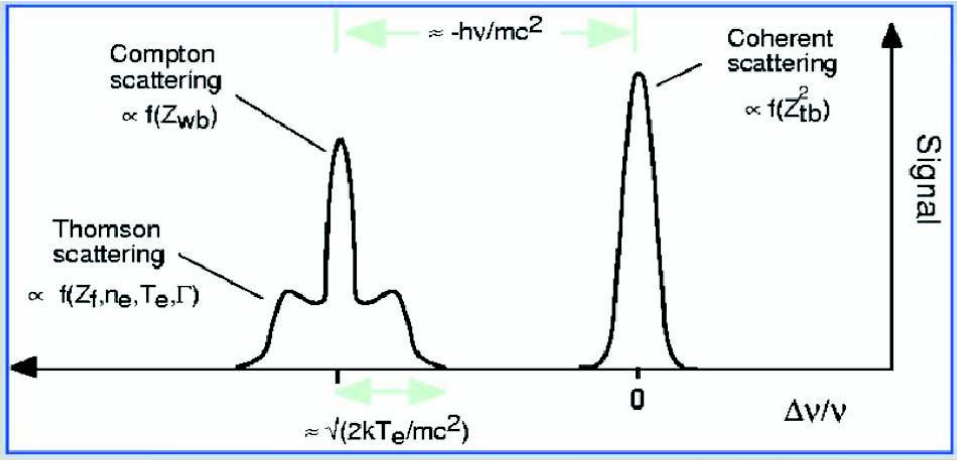

O.L. Landen et al., JQSRT 71 (2001) 465

$$S(k,\omega) = |f_{\mathrm{I}}(k) + q(k)|^2\, S_{\mathrm{ii}}(k,\omega) + Z_{\mathrm{f}} S_{\mathrm{ee}}(k,\omega) + Z_C \int d\omega'\, \widetilde{S}(k,\omega-\omega') S_{\mathrm{s}}(k,\omega')$$

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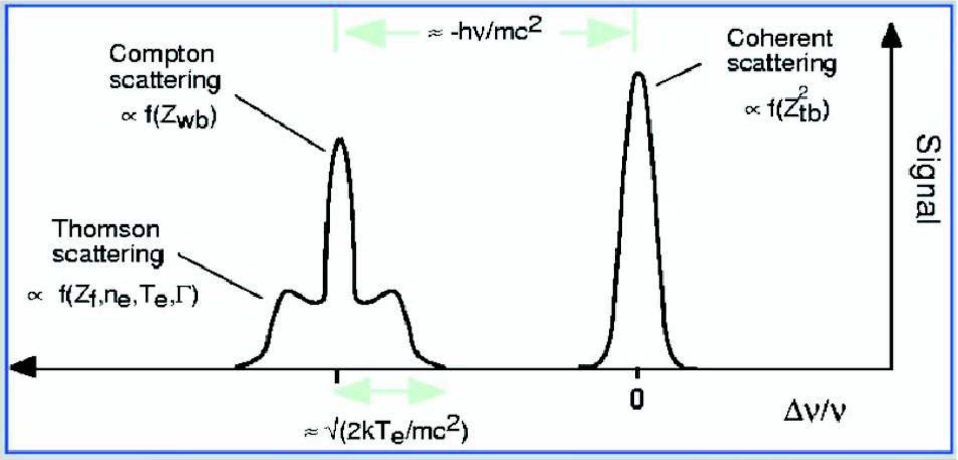

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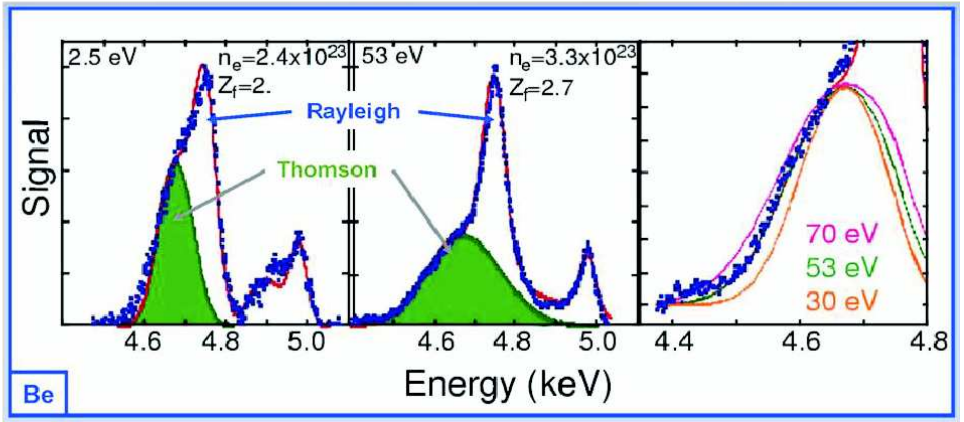
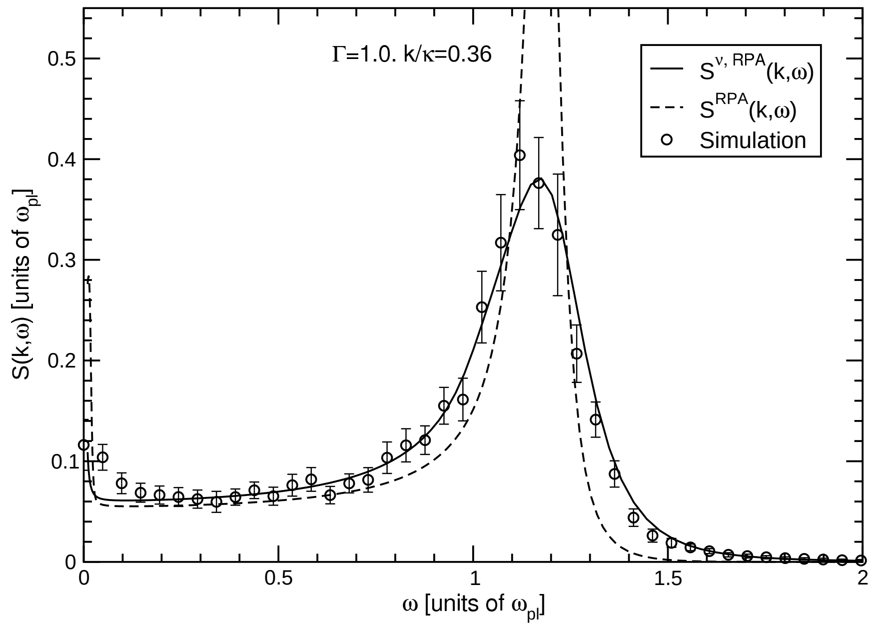


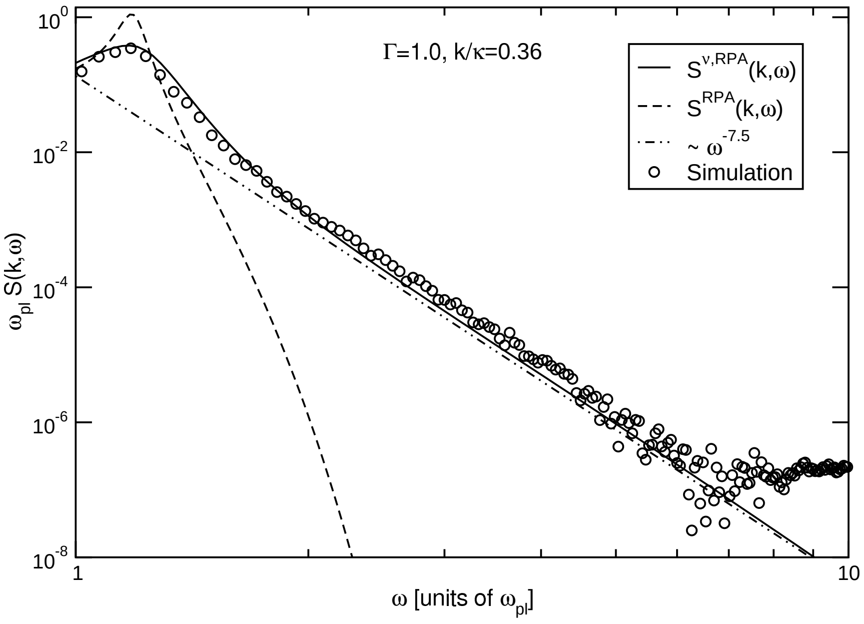
Glenzer et al., PRL 90 (2003) 175002

Experiment on Beryllium at 30 kJ Omega laser facility in Rochester
heating: Rh X-ray (2.7 keV - 3.4 keV)
scattering source: He-like Ti α-line (4.75 keV)
scattering angle: $\Theta_S = 125^\circ$

Dynamic structure factor

$$S_{\rm ee}(\mathbf{k},\omega) = \frac{1}{\pi\, V(k)}\,\frac{1}{{\rm e}^{-\beta\hbar\omega}-1}\,{\rm Im}\epsilon^{-1}_{\rm long}(\mathbf{k},\omega)$$





dynamic structure factor for an electron-proton model plasma with Deutsch-like effective interaction in RPA and in Mermin-like approximation which utilizes a dynamically screened collision frequency $\nu(\omega)$

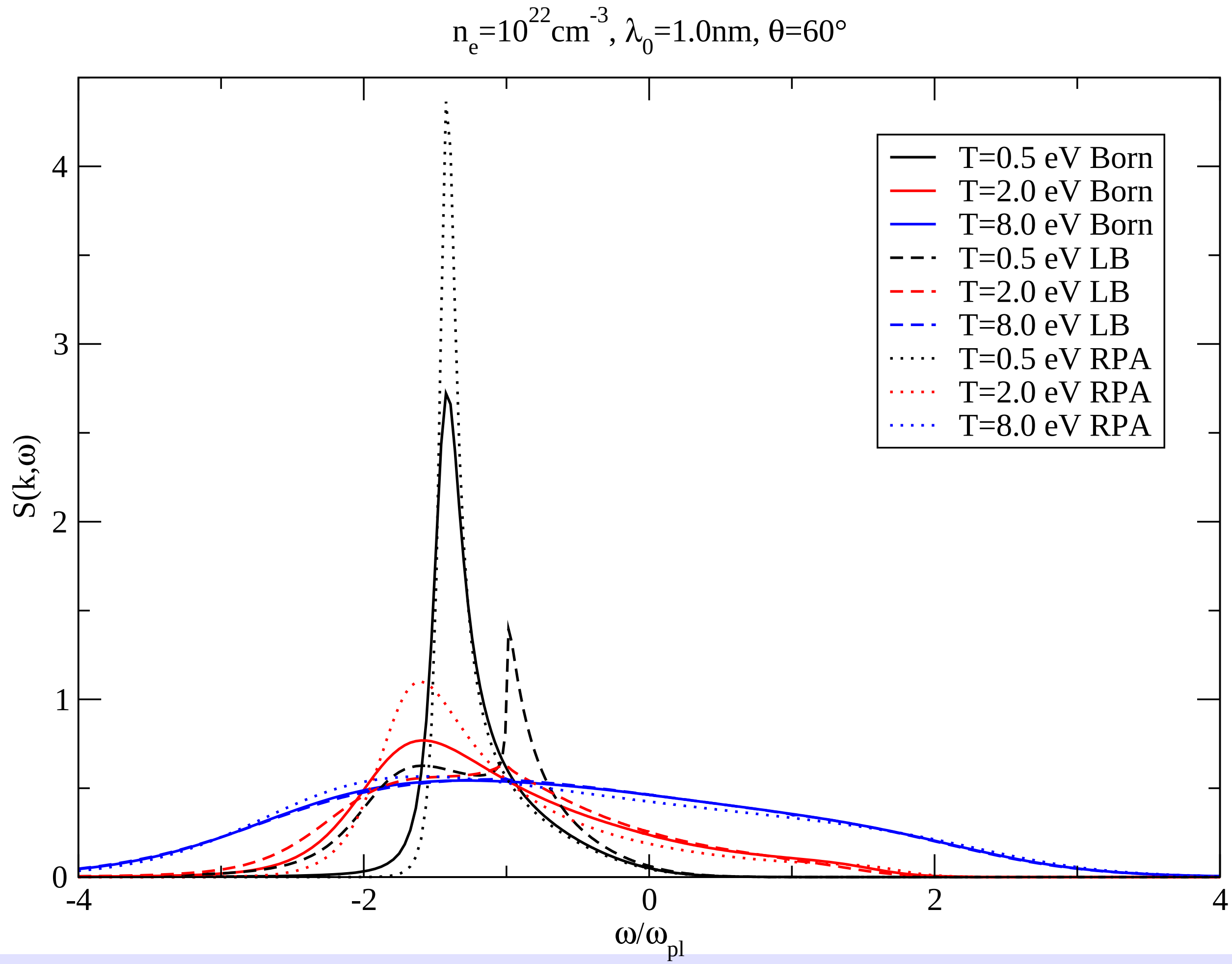
A. Selchow et al. PRE 64 (2001) 056410

Thomson scattering

diagnostic tool for warm dense matter:

determination of plasma parameters using VUV and X-ray

relevance of collision



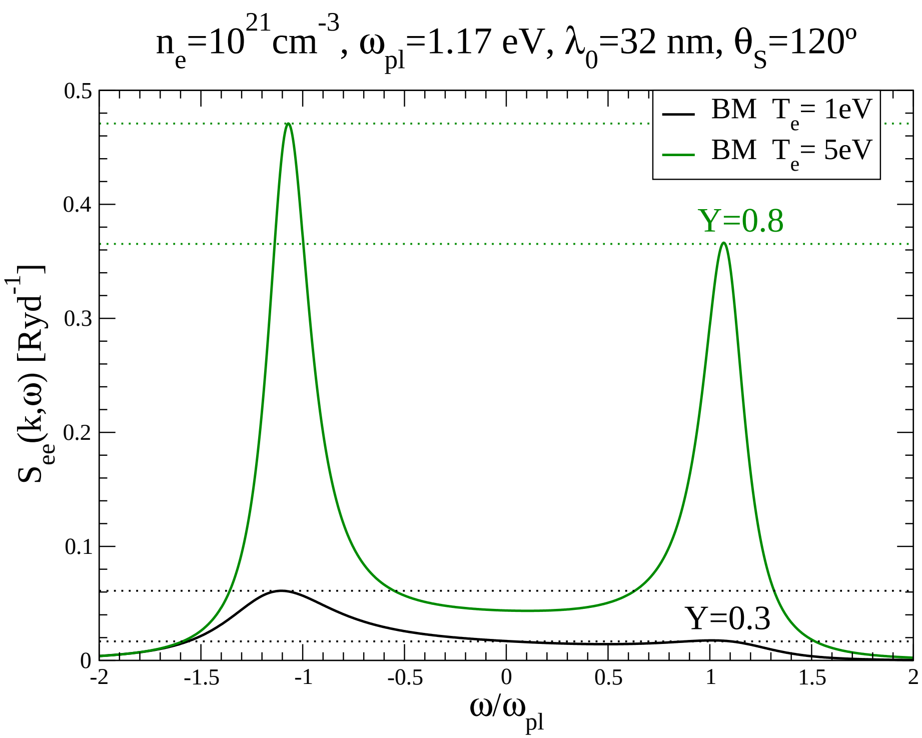
Höll et al. , EPJD 29 (2004) 159, Redmer et al. , IEEE Transact. on Plasma Sc. 33 (2005)

Determination of temperature and density

- Electron temperature T_e can be determined for all frequencies via detailed balance:

$$Y = \frac{S_{ee}(k,\omega)}{S_{ee}(-k,-\omega)} = e^{-\frac{\hbar\omega}{k_B T_e}}$$

- electron density n_e is given by position of plasmon peak, related to dispersion relation Re $\epsilon(k,\omega) = 0$
  $\rightsquigarrow$ in RPA: Gross-Bohm relation [1]:
  $$\omega_R^2 \approx \omega_{pl}^2 + 3k^2 v_{th}^2$$



$\rightsquigarrow$ with $\omega_{pl} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$ and the thermal velocity $v_{th}^2 = k_B T_e / m_e$

- Note: for warm dense matter resonance position ω_R can be shifted by collisions

[1] Bohm, Gross, PR 75 (1949) 1851

Conclusions III

- The dielectric function governs different physical properties such as the dc and optical conductivity.
- Central quantity in describing transport and optical properties is the dynamical collision frequency.
- Results obtained from linear response theory and MD simulations are in good agreement for dynamic structure factor as well as in long-wavelength limit.
- In order to describe experimental results
  - inclusion of bound states (partial ionization, polarization potential, spectral function, Mott effect)
  - density and temperature profiles
- Thomson scattering in warm dense matter and bremsstrahlung are of particular interest due to recent developments of experimental opportunities.

Transport properties

many-particle system under the influence of external perturbations X_i:
electric field $\vec{E}$, temperature gradient ∇T, magnetic field $\vec{B}$

consider electric charge and energy current densities:

$$\vec{J}_{\mathrm{el}} = \sum_i \hat{\mathcal{L}}_{0i}\,\vec{X}_i = \sigma\left(\vec{E} - S\nabla T\right) + \sigma R_H\left(\vec{J}_{\mathrm{el}} \times \vec{B}\right) + \sigma N\left(\nabla T \times \vec{B}\right)$$

$$\vec{J}_q = \sum_i \hat{\mathcal{L}}_{1i}\,\vec{X}_i = TS\vec{J}_{\mathrm{el}} - K\nabla T - NT\left(\vec{J}_{\mathrm{el}} \times \vec{B}\right) - L\left(\nabla T \times \vec{B}\right)$$

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σ – electrical conductivity
K – thermal conductivity
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Hall voltage U_H

$$\vec{E}_H = R_H\left(\vec{J}_{\rm el} \times \vec{B}\right) = \frac{U_H}{d}$$

Transport and Onsager coefficients

assume geometrical configuration: $\nabla T \parallel \vec{E} \perp \vec{B}$

components parallel to $\vec{E}$ and $\vec{B}$ or $\vec{B} = 0$

$$\sigma_{\parallel} = e^2 \mathcal{L}_{01}$$

$$S_{\parallel} = \frac{1}{eT}\left(h - \frac{\mathcal{L}_{11}}{\mathcal{L}_{01}}\right)$$

components perpendicular to $\vec{B}$

$$\sigma_{\perp} = \frac{e^2 \widetilde{\mathcal{L}}_{01}}{\eta}$$

$$S_{\perp} = \frac{1}{eT}\left(h - \frac{\tilde{\mathcal{L}}_{11}}{\tilde{\mathcal{L}}_{01}}\left(1 + \omega_{\mathrm{e}}^2 \frac{\widetilde{\mathcal{L}}_{02}\widetilde{\mathcal{L}}_{12}}{\widetilde{\mathcal{L}}_{01}\widetilde{\mathcal{L}}_{11}}\right)\eta\right)$$

$$R_H = -\frac{\widetilde{\mathcal{L}}_{02}}{e m_{\mathrm{e}} \widetilde{\mathcal{L}}_{01}{}^2}\,\eta$$

$$\eta = \left(1 + \omega_{\mathrm{e}}^2 \frac{\widetilde{\mathcal{L}}_{02}^2}{\widetilde{\mathcal{L}}_{01}^2}\right)^{-1}$$

h - enthalpy

electron cyclotron frequency $\omega_{\mathrm{e}} = \frac{eB}{m_e}$

$$\boxed{\vec{J} = \mathrm{Tr}\left\{\hat{\vec{j}}\,\hat{\rho}\right\}}$$

Relaxation time approach

kinetic approach: solving linearized Boltzmann equation with relaxation time approach

$$\tilde{\mathcal{L}}_{ij} = \frac{n_e}{m_e} \left\langle \frac{\epsilon^i \, \tau^j}{1 + \omega_e^2 \tau^2} \right\rangle$$

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$$\mathcal{L}_{ij} = \frac{n_e}{m_e} \left\langle \epsilon^i \; \tau^j \right\rangle \qquad \langle \ldots \rangle = -\frac{2 \, m_e}{3 \, n_e \, \hbar^2} \int d^3p \; \ldots \; v^2 \; \frac{df_0}{d\epsilon}$$

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$$\sigma_\perp = \frac{e^2 n_e}{m_e}\left\langle \frac{\tau}{1+\omega_e^2\tau^2}\right\rangle \left(1+\frac{\left\langle \frac{\omega_e^2\tau^2}{1+\omega_e^2\tau^2}\right\rangle^2}{\left\langle \frac{\tau}{1+\omega_e^2\tau^2}\right\rangle^2}\right) \Rightarrow \qquad \sigma_{\parallel} = \frac{e^2 n_e}{m_e}\langle\tau\rangle$$

$$r_H = -e\ n_e\ R_H = \frac{\left\langle \frac{\tau^2}{1+\omega_e^2\tau^2}\right\rangle^2}{\left\langle \frac{\tau}{1+\omega_e^2\tau^2}\right\rangle^2 + \left\langle \frac{\omega^2\tau^2}{1+\omega_e^2\tau^2}\right\rangle^2} \Rightarrow \qquad r_H = \frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}$$

Lee, More Phys. Fluids ***27*** *(1984)*

Low density limit

$\{P_n\}$	$\sigma^*\ ln\Lambda$		$-eS/k_B$		r_H	
	ei	$ei+ee$	ei	$ei+ee$	ei	$ei+ee$
0	0.2992	0.2992	0	0	1	1
$0,1$	0.9724	0.5781	1.1538	0.8040	1.5325	1.2586
$0,1,2$	1.0145	0.5834	1.5207	0.7110	1.9786	1.2068
$0,1,2,3$	1.0157	0.5875	1.5017	0.7139	1.9343	1.2077
$0,1,2,3,4$	1.0158	0.5892	1.5004	0.7039	1.9333	1.2036
$0,1,\ldots,10$	1.0159	$\ldots$	1.5000	$\ldots$	1.9328	$\ldots$
Relax app	1.0159	–	1.5000	–	1.9328	–
Spitzer[2]	–	0.591	–	–	–	–
Arbitrary B	0.2992	0.2992	0	0	1	1

[1] Reinholz, Redmer and Tamme, Contrib. Plasma Phys. ***29*** *(1989)*

[2] Spitzer and Härm, Phys. Rev. ***89*** *(1953)*