Kinetics of Dense Matter: Correlations and Memory

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Kinetic description

- The model (for illustration)

$$H = \sum_{11'} h(1',1)\, a^\dagger_{1'} a_1 + \frac{1}{2} \sum_{121'2'} V_2(1'2',12)\, a^\dagger_{2'} a^\dagger_{1'} a_1\, a_2$$

- Kinetic description in terms of reduced density matrices:

$$f_s(1\ldots s, 1'\ldots s'; t) = \langle a^\dagger_{s'} \ldots a^\dagger_{1'} a_1 \ldots a_s \rangle^t, \quad s = 1, 2, \ldots$$

- The quantum Liouville equation with a boundary condition (Zubarev's method)

$$\frac{\partial \varrho(t)}{\partial t} + \frac{1}{i\hbar}\left[\varrho(t), H\right] = -\varepsilon \left\{\varrho(t) - \varrho_{\rm rel}(t)\right\}, \quad \varepsilon \to +0$$

- The general form of the relevant statistical operator (summation over repeated indices)

$$\varrho_{\rm rel}(t) = Z^{-1}(t) \exp\left\{ -\sum_{s\geq 1} \frac{1}{s!}\, \lambda_s(1'\ldots s', 1\ldots s; t)\, a^\dagger_{s'} \ldots a^\dagger_{1'} a_1 \ldots a_s \right\}$$

Derivation of kinetic equations

- Quantum hierarchy for reduced density matrices

$$\frac{\partial}{\partial t} f_s(1\dots s, 1'\dots s'; t) - \frac{1}{i\hbar} \langle [\, a^{\dagger}_{s'} \dots a^{\dagger}_{1'} a_1 \dots a_s, H \,] \rangle^t$$
$$= -\varepsilon \{ f_s(1\dots s, 1'\dots s'; t) - \overline{f}_s(1\dots s, 1'\dots s'; t) \},$$

Notation: $\overline{f}_s(1\dots s, 1'\dots s'; t) = \mathrm{Tr}\left(\varrho_{\mathrm{rel}}(t)\, a^{\dagger}_{s'} \dots a^{\dagger}_{1'} a_1 \dots a_s\right)$

- Comments:

1) For macroscopic systems, it is expected that all boundary conditions are equivalent if one deals with exact solutions of the hierarchy.

2) Similar approximations in the hierarchy lead to different kinetic equations for different boundary conditions.

3) For instance, Markovian approximation is adequate only if $\varrho_{\mathrm{rel}}(t)$ describes all relevant long-lived correlations.

Special boundary conditions

- Complete weakening of correlations (Bogoliubov's boundary condition)

$$\varrho_{\rm rel}(t) = Z^{-1}(t)\exp\left\{-\lambda_1(1',1;t)\,a^\dagger_{1'}a_1\right\}, \quad f_1(t) = \overline{f}_1(t)$$

Relevant correlations: $\overline{g}_2(t) = \overline{f}_2 - \overline{f}_1\overline{f}_1 = 0.$

- "Cluster" correlations [Röpke(1988)] (e.g., binary correlations)

$$\varrho_{\rm rel}(t) = Z^{-1}(t)\exp\left\{-\lambda_1(1',1;t)a^\dagger_{1'}a_1 - \frac{1}{2}\lambda_2(1'2',12;t)\,a^\dagger_{2'}a^\dagger_{1'}a_1a_2\right\}$$

Relevant correlations: $\overline{g}_2(t) = g_2(t)$.

- "Hydrodynamic" correlations [Morozov,Röpke(1995)]:

$$\varrho_{\rm rel}(t) = Z^{-1}(t)\exp\left\{-\lambda_1(1',1;t)a^\dagger_{1'}a_1 - \int d\boldsymbol{r}\,\beta(\boldsymbol{r},t)H(\boldsymbol{r})\right\}$$

Relevant correlations: $\overline{g}_2(t) = \overline{g}_2[\beta(t),\lambda_1(t)]$.
T is the "quasitemperature".

Features of kinetic equations

- Bogoliubov's boundary condition (weak interaction, low density)
  Markovian Boltzmann-type kinetic equations for $f_1(t)$. Correlations are included through memory effects. Problems with the equilibrium solution.
- "Cluster" correlations (dense systems with bound states)
  A set of coupled equations for f_1 and the "cluster" correlation functions, e.g., for $g_2(t)$. Correct conservation laws and equilibrium solutions.
- "Hydrodynamic" correlations
  Markovian Enskog-type kinetic equations for $f_1(t)$ coupled with hydrodynamic equations. Cross-sections in kinetic equations depend on $\overline{g}_2$. Correct conservation laws and equilibrium solutions. Unification of kinetics and hydrodynamics. Inclusion of "hydrodynamic" correlations improves the properties of non-Markovian Boltzmann-type kinetic equations [Morozov,Röpke,(2001)].

Some challenges

- Inclusion of nonequilibrium “cluster” and/or “hydrodynamic” correlations in the Green’s function method.
  The “Mixed” Green’s function approach to quantum kinetics with initial correlations [Morozov,Röpke,(1999)].
- Evolution of nonequilibrium correlations in relativistic kinetics.
  At the moment the relativistic kinetic theory does not go far beyond the quasiparticle picture.
- Application of the Enskog-type quantum kinetic equations to heavy-ion collisions.
  An attractive feature of the Enskog-type equations: an interpolation approach applicable to the transition
  (Fermi liquid) $\rightarrow$ (semi-quantum dense hot matter) $\rightarrow$
  (a low density gas).