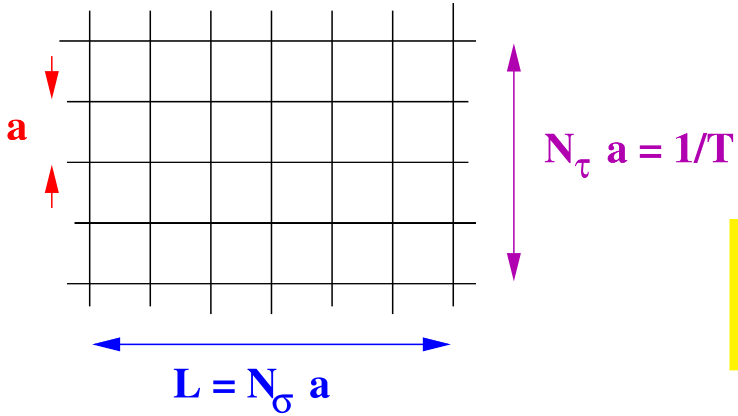
Bulk QCD Thermodynamics on the lattice

numerical treatment of QCD $\Rightarrow$ discretize (Euclidean) space-time

$\Rightarrow$ **lattice**

$N_\sigma^3 \times N_\tau$



$$Z(T,V) = \int \prod_{i=1}^{N_\tau N_\sigma^3} d\phi(x_i) \exp\{-S[\phi(x_i)]\}$$

finite yet high-dimensional path integral

$\longrightarrow$ **Monte Carlo**

- thermodynamic limit, IR - cut-off effects $\quad LT = \frac{N_\sigma}{N_\tau} \rightarrow \infty$
- continuum limit, UV - cut-off effects $\quad aT = \frac{1}{N_\tau} \rightarrow 0$
- chiral limit $\quad m \rightarrow m_{\text{phys}} \simeq 0$

  numerical effort $\sim (1/m)^p$

I Phase diagram at $\mu = 0$

Localisation of the phase transition

- **order parameter**

1.) **Polyakov loop** $L(\vec{x}) = \frac{1}{3}\mathrm{tr}\prod_\tau U_\tau(\vec{x},\tau)$

- sensitive on $Z(3)$ symmetry (pure gauge theory only)
- measures free energy of an isolated quark $\langle L \rangle \sim e^{-F_{\mathrm{quark}}/T}$

hadron phase	$F_{\mathrm{quark}} \to \infty$ $\langle L \rangle = 0$
plasma phase	F_{quark} endlich $\langle L \rangle \neq 0$

2.) **chiral condensate**

- sensitive on chiral symmetry ($m \to 0$)

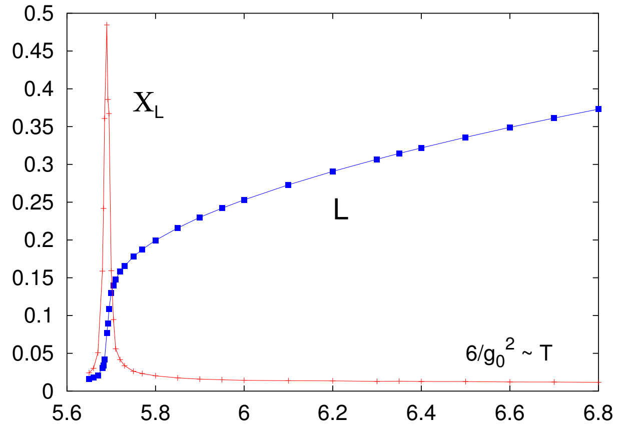
hadron phase	$\langle \bar{q}q \rangle \neq 0$
plasma phase	$\langle \bar{q}q \rangle = 0$

- **susceptibilities**

e.g. chiral susceptibility
- measures fluctuations

$$\chi_m \sim \frac{\partial^2 \ln Z}{\partial m^2} \sim \langle\, (\overline{q}q)^2 \,\rangle - \langle\, \overline{q}q \,\rangle^2$$

quenched QCD

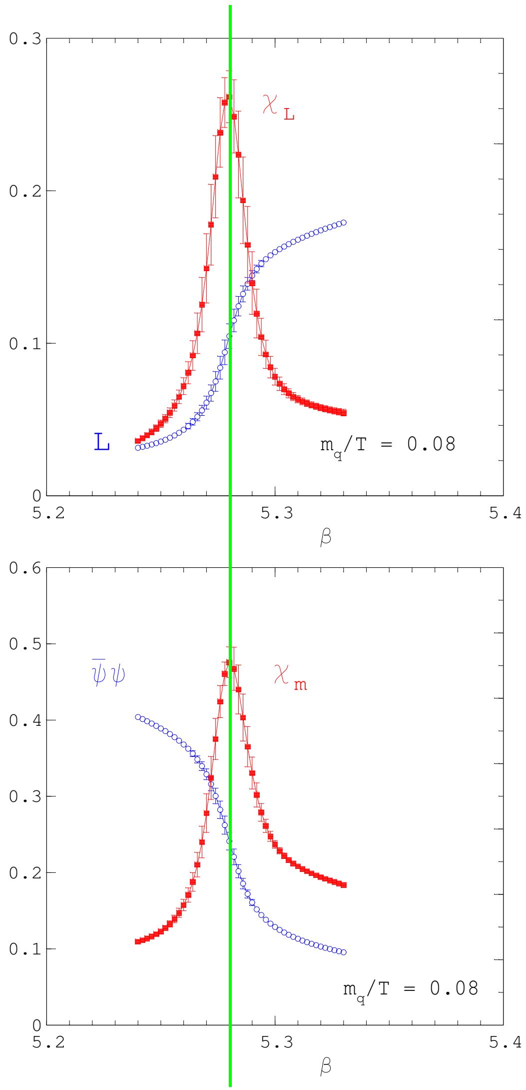


$$T = \frac{1}{N_\tau a(g)}$$

in perturbation theory

$$a = \frac{1}{\Lambda} \exp\left\{ -\frac{24\pi^2}{33 - 2N_F} \frac{1}{g^2} \right\}$$

full QCD ($N_F = 2$)



Polyakov loop

chiral condensate

$$g_{crit}^{Polyakov} = g_{crit}^{chiral}$$

$\beta = 6/g^2$

critical temperature T_c

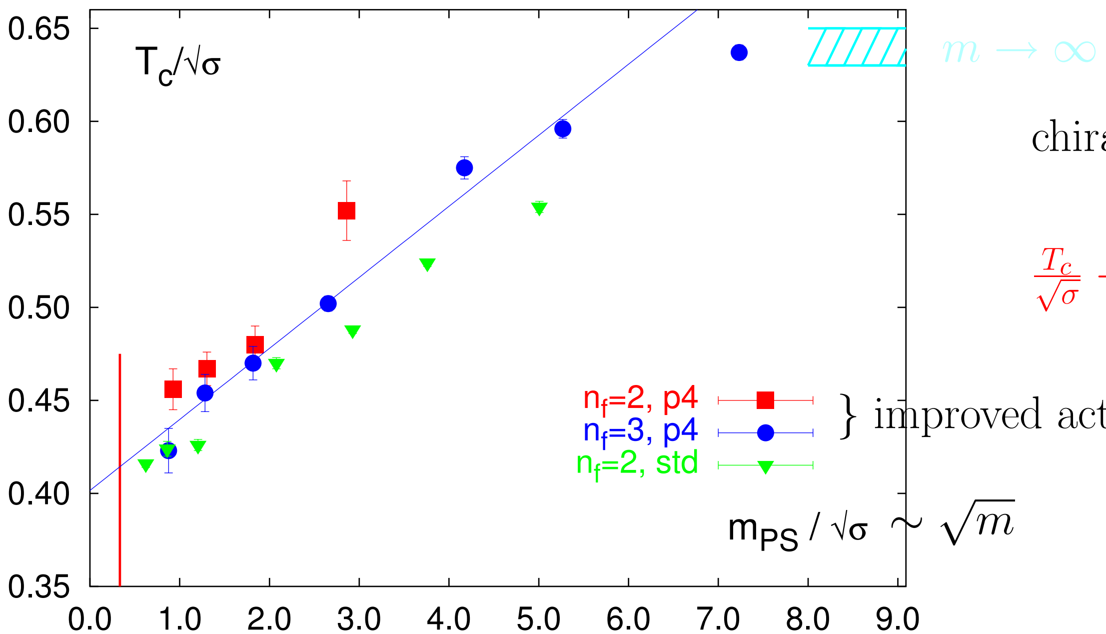
$$T_c = \frac{1}{N_\tau\, a(g_c)}$$

at $T = 0$, same (bare) coupling g_c, measure e.g. string tension σ $\Rightarrow$: $\sqrt{\sigma}\, a(g_c) = number$

$\Rightarrow$ dimension less ratio

$$\frac{T_c}{\sqrt{\sigma}} = \frac{1}{N_\tau\, a(g_c)} \star \frac{a(g_c)}{number} = \frac{1}{N_\tau \star number}$$

$\sqrt{\sigma}$ only weakly affected by quark mass



chiral extrapolation $m \to 0$

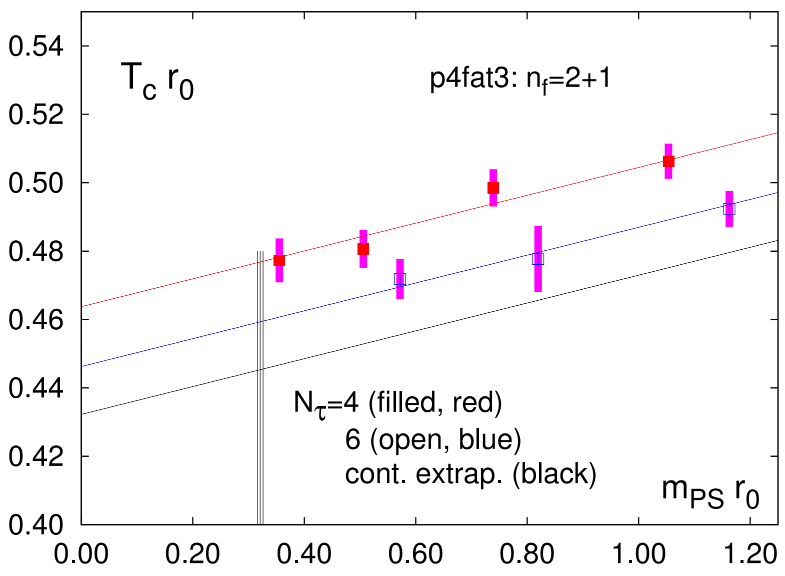
$$\frac{T_c}{\sqrt{\sigma}} \to \begin{cases} 0.42\,(\pm 0.01)\,(\text{syst.})\ \text{MeV} & N_F = 2\ \text{Wilson}^{\star} \\ 0.42\,(\pm 0.01)\,(\text{syst.})\ \text{MeV} & N_F = 2\ \text{staggered} \\ 0.40\,(\pm 0.01)\,(\text{syst.})\ \text{MeV} & N_F = 3\ \text{staggered} \end{cases}$$

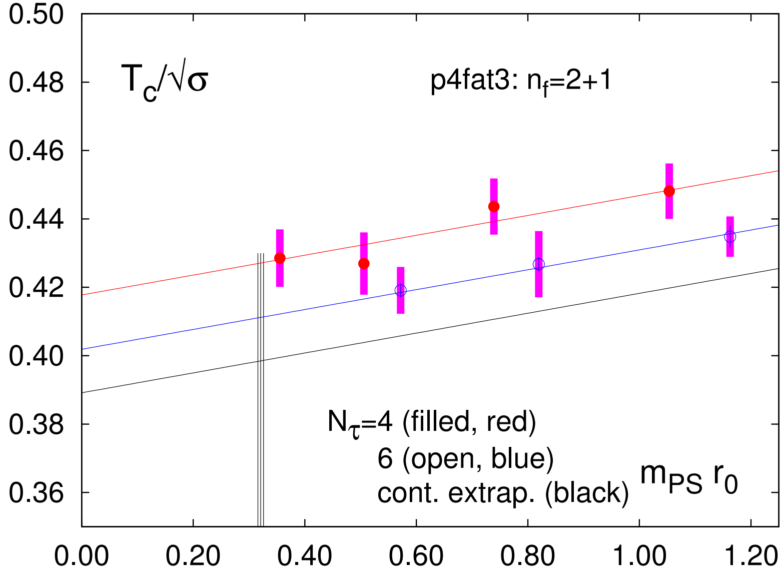
[cp-pacs collaboration]

Caveat: not yet continuum limit

but: improved actions

new results: $N_F = 2+1$, $N_\tau = 4, 6$, exact algorithm (RHMC)





combined continuum/chiral extrapolation ($d = 1.08$ for $O(4)$, $d = 2$ for first order)

$$(T_c r_0)_{m_l,N_\tau} = T_c r_0 + A(m_{PS} r_0)^d + B/N_\tau^2$$

chiral limit $T_c r_0 = 0.444(6)^{+12}_{-2}$ $T_c/\sqrt{\sigma} = 0.399(5)^{+10}_{-1}$

phys. point $T_c r_0 = 0.456(7)^{+3}_{-1}$ $T_c/\sqrt{\sigma} = 0.408(7)^{+3}_{-1}$

with new $T = 0$ MILC (lattice) results for $r_0 = 0.469(7)$fm obtain: $T_c = 192(5)(4)$MeV

nature of the phase transition (at $\mu = 0$)

expected

- $\Phi(\tau) = \Sigma_n \exp\{i\omega_n\tau\}\ \Phi(\omega_n)$ with Matsubara frequencies $\omega_n = \begin{cases} 2\pi T n & \text{bosons} \\ \pi T(2n+1) & \text{fermions} \end{cases}$
  $\longrightarrow$ for high temperatures static boson-modes only
  $\longrightarrow$ three-dimensional effective theory
- long range correlations
  $\longrightarrow$ global symmetries count, microscopic details don't (universality)
  $\longrightarrow$ here: chiral symmetries, σ models

$\Rightarrow$ $N_F = 2$ [Wilczek,Pisarski]

- if phase transition continuous (2nd order), then $SU_R(2) \otimes SU_L(2) \simeq O(4)$
- if $U_A(1)$ effectively restored (no non-trivial topological configurations at T_c^{chiral}), then phase transition discontinuous (1st order).

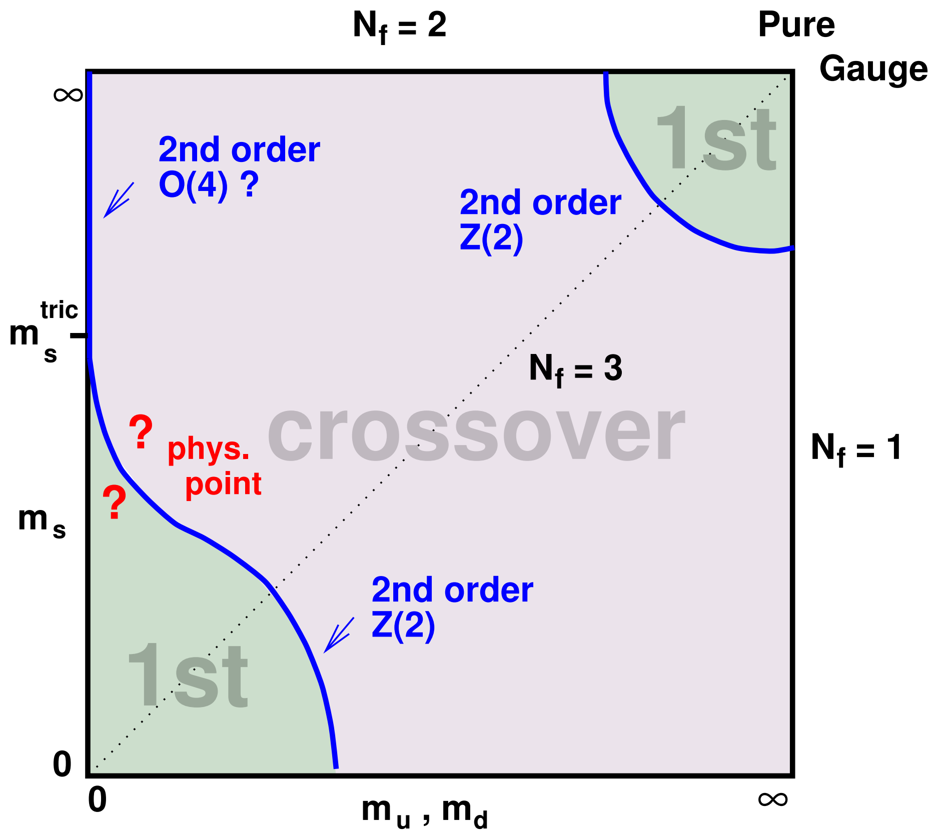
$\Rightarrow$ $N_F = 3$ [Wilczek,Pisarski]

- phase transition discontinuous $\longrightarrow$ even at $m \leq m_c \neq 0$
- at critical end point m_c: $Z(2)$ Ising universality [Gavin,Gocksch,Pisarski]

$\Rightarrow$ $N_F = 2 + 1$

- depending on quark masses $m_{u,d}, m_s$

expected phase diagram in the $m_{u,d} - m_s$ plane ($\mu = 0$)



critical behavior

in the vicinity of a phase transition: correlation length $\longrightarrow \infty$

$\Rightarrow$ scaling behavior of the free energy density

$$f(t, m, L) = b^{-d} f(b^{y_t} t, b^{y_h} m, L/b) \qquad \text{with reduced temperature } t = \frac{|T - T_c|}{T_c}$$

$\Rightarrow$ scaling laws, e.g.

$$\langle M \rangle \sim m^{1/\delta}$$

$$\chi_m \sim L^{\gamma/\nu}$$

$$B_4 = \frac{\langle (\delta M)^4 \rangle}{\langle (\delta M)^2 \rangle^2}$$

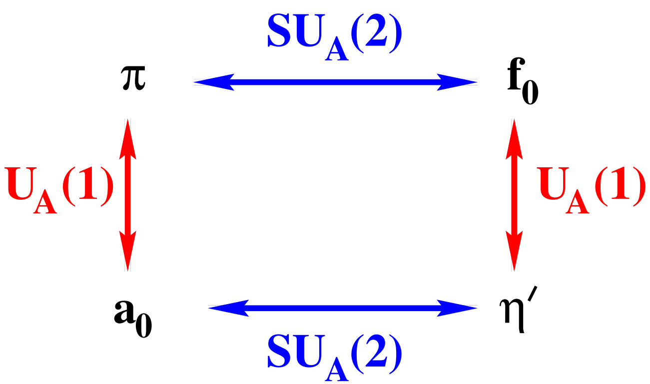
(here: $\delta M = M - \langle M \rangle, M \simeq \overline{q}q$)

with critical exponents $\delta, \gamma, \nu, \ldots$ und Binder-cumulant B_4 universal

	Z(2)	O(2)	O(4)
γ/ν	1.963(3)	1.962(5)	1.975(4)
B_4	1.604(1)	1.242(2)	1.092(3)

$N_F = 2$

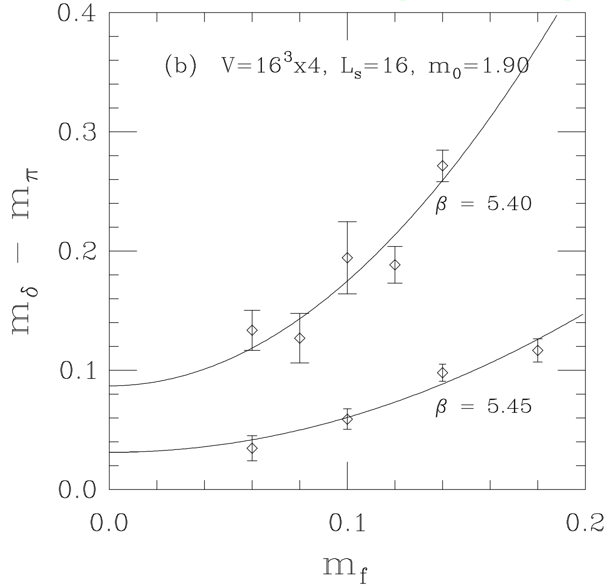
- conflicting results for critical behavior
  - Wilson $\langle M \rangle$ scales $\sim O(4)$ in m [Iwasaki et al.]
  - staggered χ_m, χ_t does not scale $\sim O(4)$ in m [Karsch,EL; JLQCD; MILC]
  - staggered $\langle M \rangle$ scales $\sim O(4)$ in L [Engels et al.]
  - staggered c_V scales as 1st order [Di Giacomo et al.]
  - staggered $\langle M \rangle$ is as in $O(2)$ at finite L [Kogut, Sinclair]
- $U_A(1)$:
  - if effectively restored, then mass spectrum degeneracy [Shuryak; Cohen et al;, ...]



screening masses
in accord with
theoretical expectation

$m_{a_0} - m_\pi = c_0 + c_2\, m^2 + \ldots$

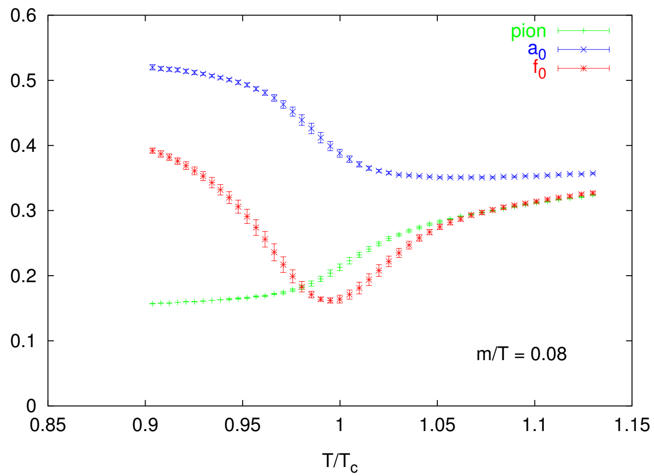
[Columbia]



at $\beta = 5.40 (T \simeq 1.2 T_c)$: c_0 small $\neq 0$
$\Rightarrow U_A(1)$ weakly broken

similarly:
generalized susceptibilities

$$\chi_H \sim 1/M_H^2$$



at $T \simeq T_c$:

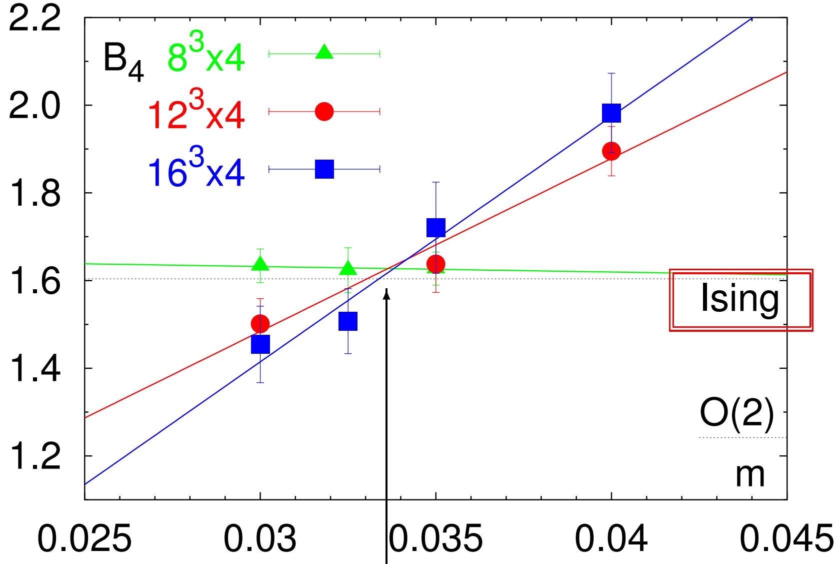
$\chi_{f_0} \simeq \chi_\pi$ $SU_A(2)$ restored
$\chi_{a_0} \neq \chi_\pi$ $U_A(1)$ not restored

$N_F = 3$

Binder cumulant B_4

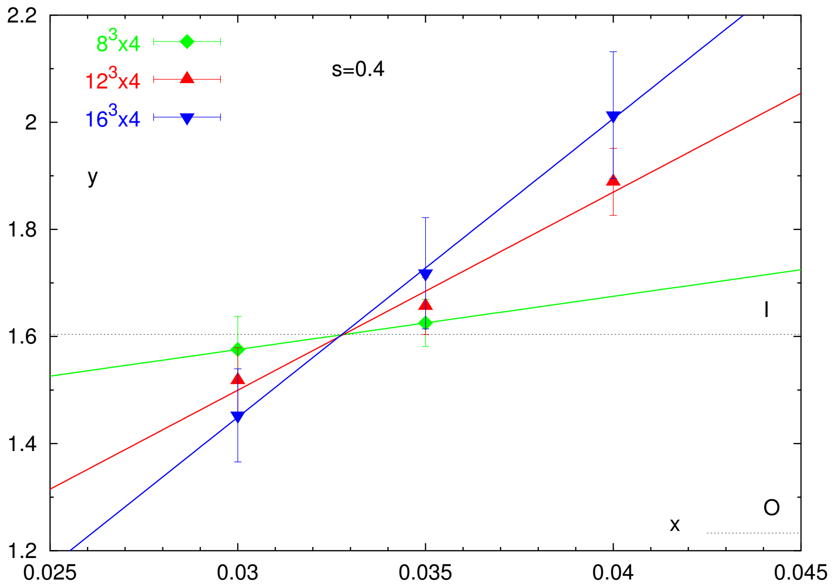
- intersection for various V yields critical value of m
- value of B_4 is universal
- corrections from V finite and 'order parameter not matched correctly'

standard action



$m_c \simeq 4\, m_u^{phys} \Leftrightarrow m_{PS} \simeq 290$ MeV

[Bielefeld; deForcrand,Philipsen]



magnetization-like order parameter $\mathcal{M}$

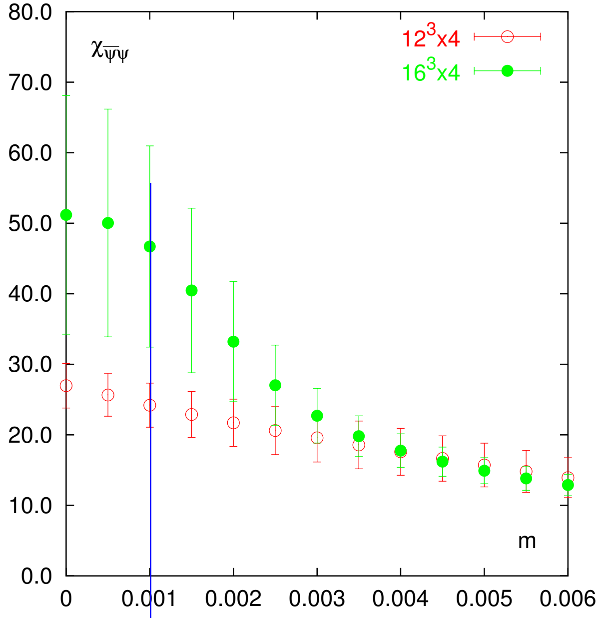
not identical with chiral condensate $\langle \bar{q} q \rangle$

(chiral symmetry broken by $m_q \neq 0$ anyway)

$$\mathcal{M} = \langle \bar{q} q \rangle + s\, S$$

improved action: ‘reweighting’ in quark mass (see later)

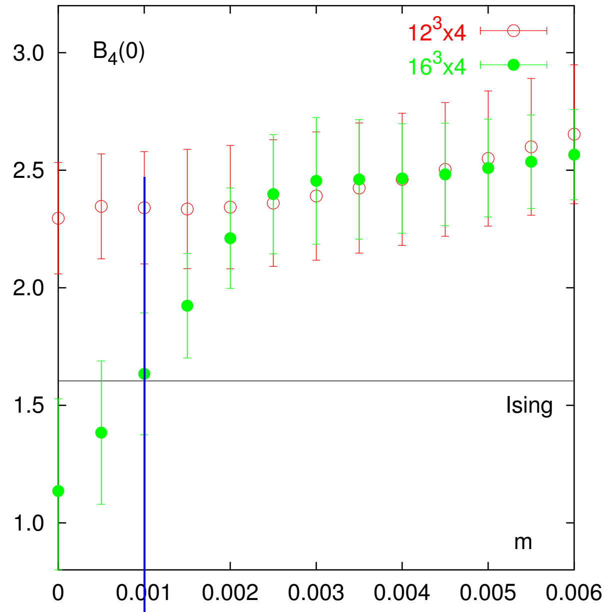
chiral susceptibility



1^{st} crossover

$\chi_m \sim V$

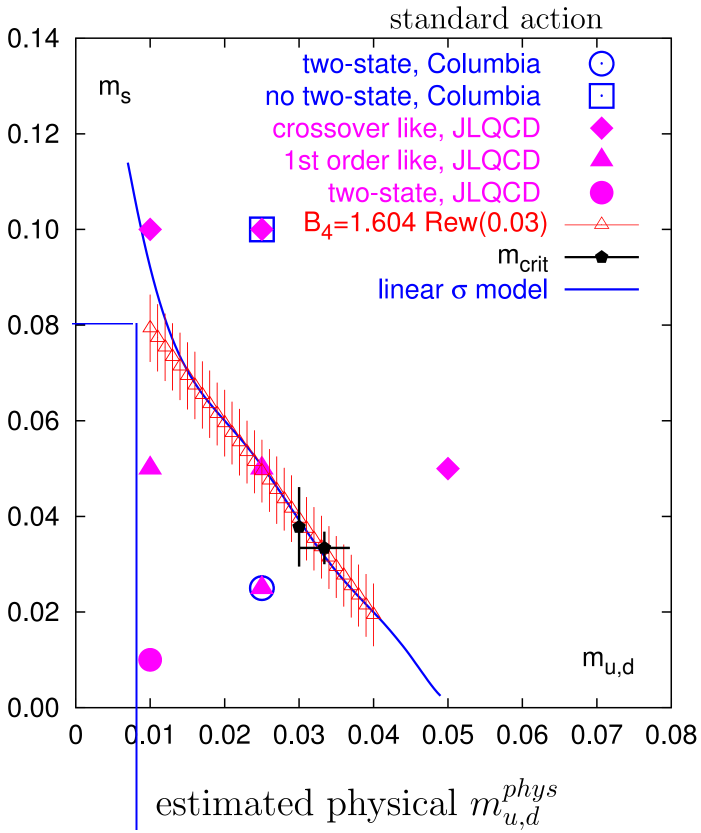
Binder cumulant



$m_c \simeq 0.001 \Leftrightarrow m_\pi \simeq 70\text{MeV}$

i.e. large cut-off effects

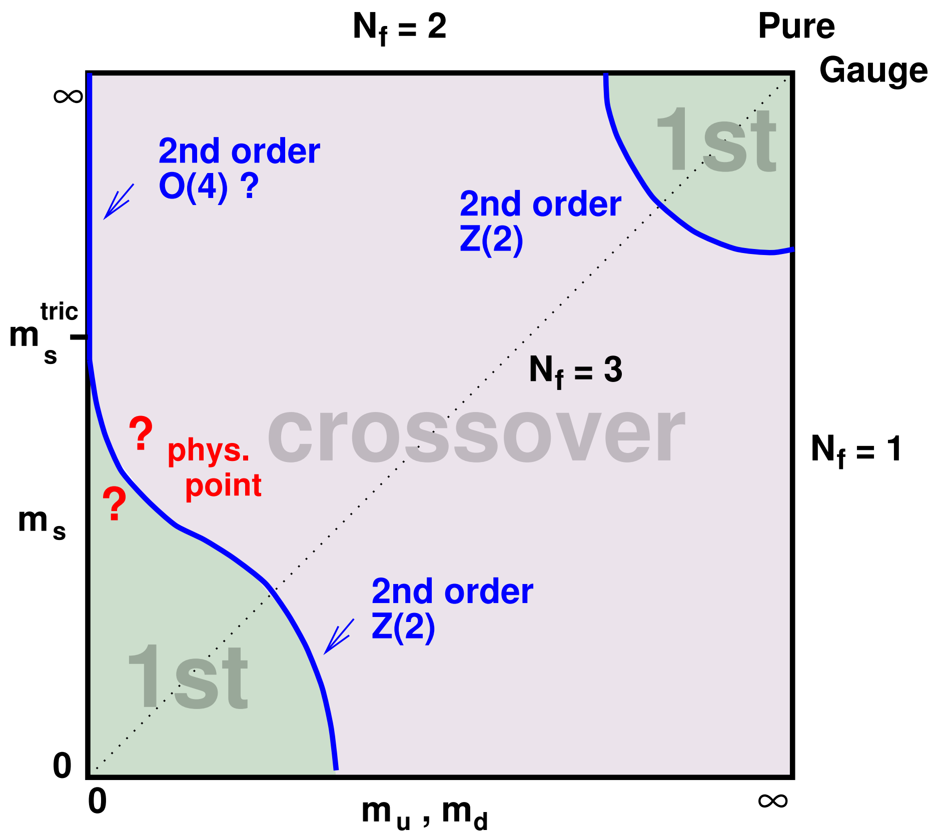
$N_F = 2 + 1$



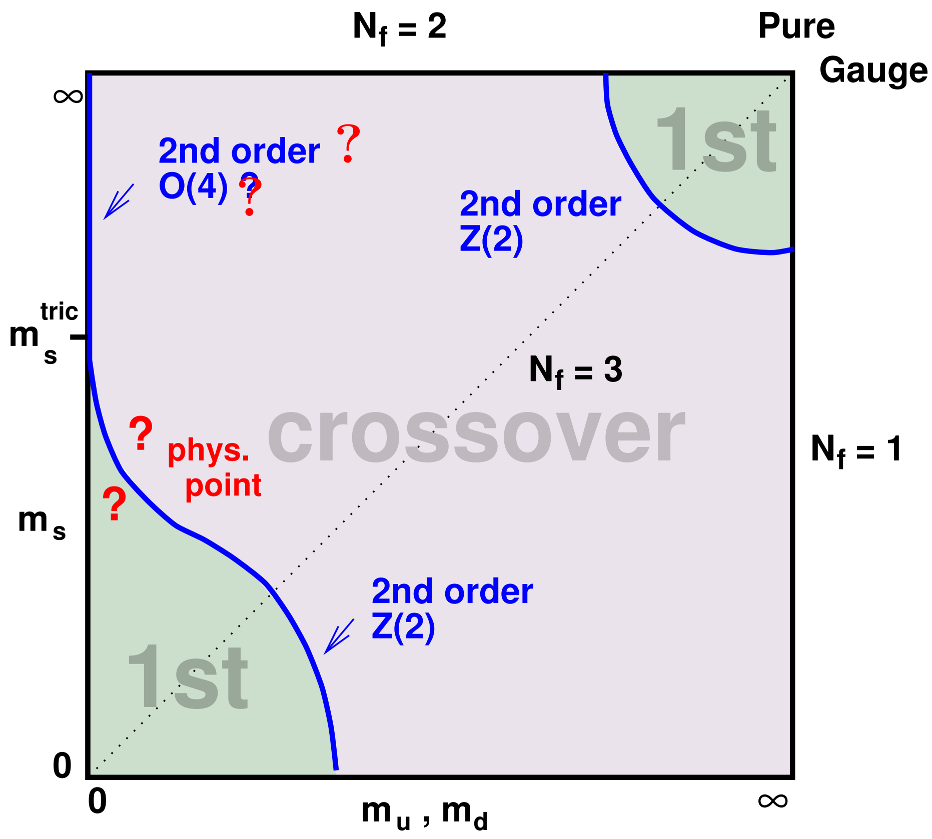
estimated physical $m_{u,d}^{phys}$

$$\rightarrow \quad \frac{m_s^{crit}}{m_{u,d}^{phys}} \simeq 10 \qquad \text{compare} \quad \frac{m_s^{phys}}{m_{u,d}^{phys}} \simeq 20$$

1st order at physical point unlikely



N_f = 2
Pure Gauge
∞
2nd order O(4) ?
1st
2nd order Z(2)
m_s^tric
N_f = 3
? phys. point
crossover
N_f = 1
?
m_s
2nd order Z(2)
1st
0
0
m_u , m_d
∞



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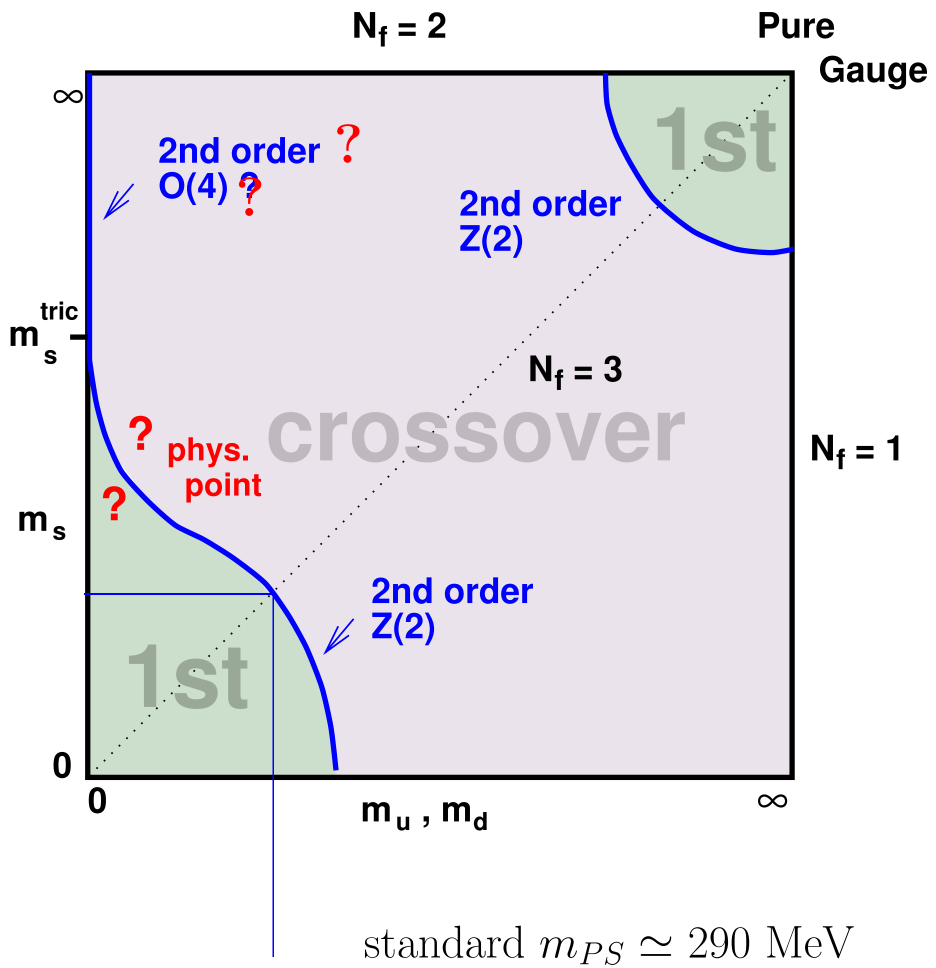
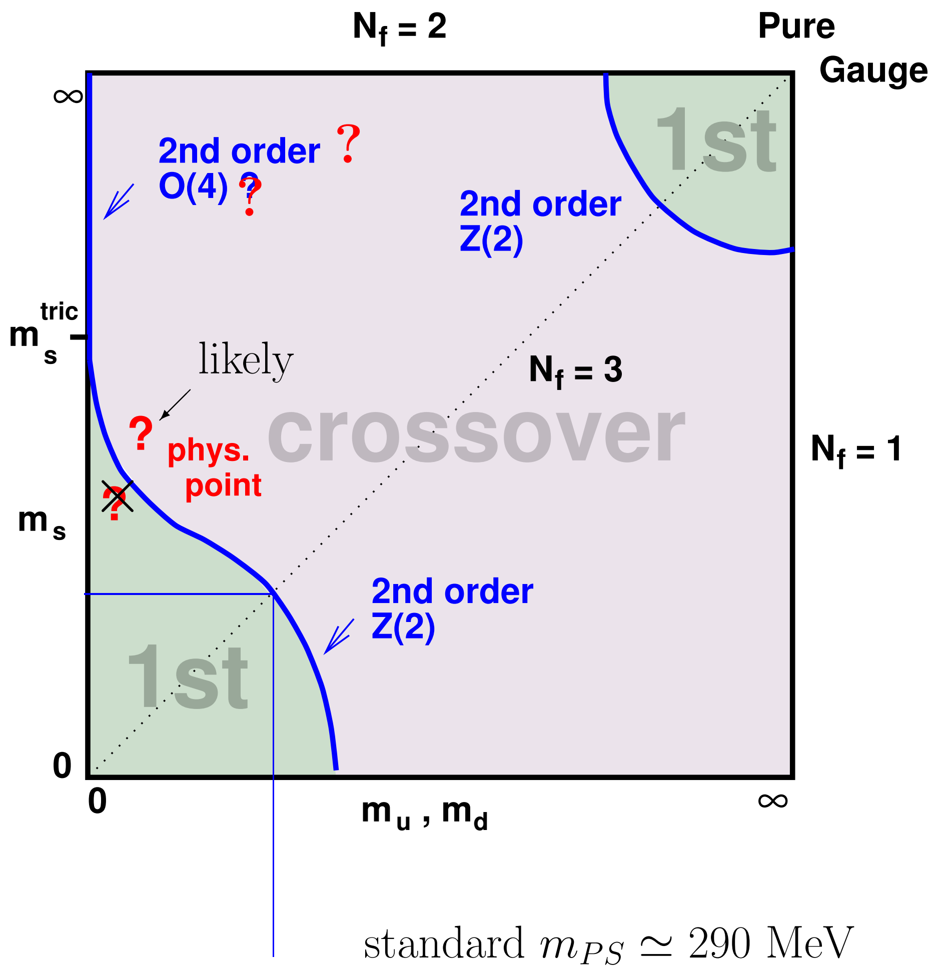


standard $m_{PS} \simeq 290$ MeV
$\longrightarrow m_{PS} \simeq 70$ MeV improved

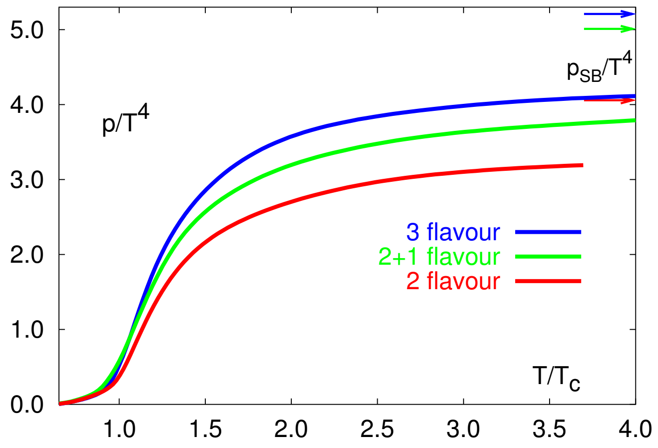


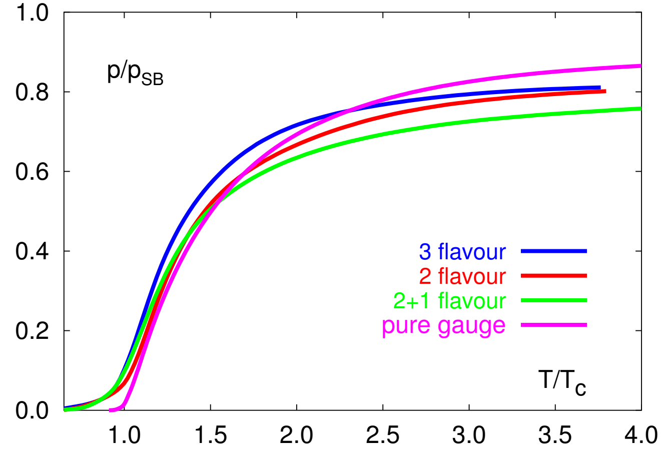
standard $m_{PS} \simeq 290$ MeV

$\longrightarrow m_{PS} \simeq 70$ MeV improved

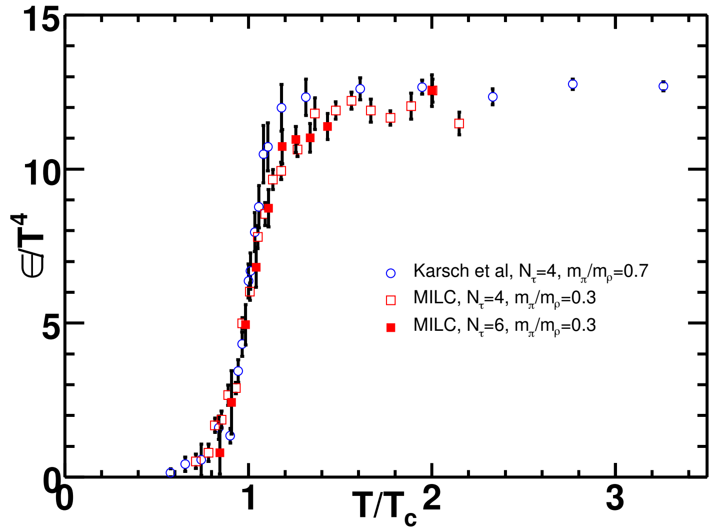
II. Equation of state at $\mu = 0$

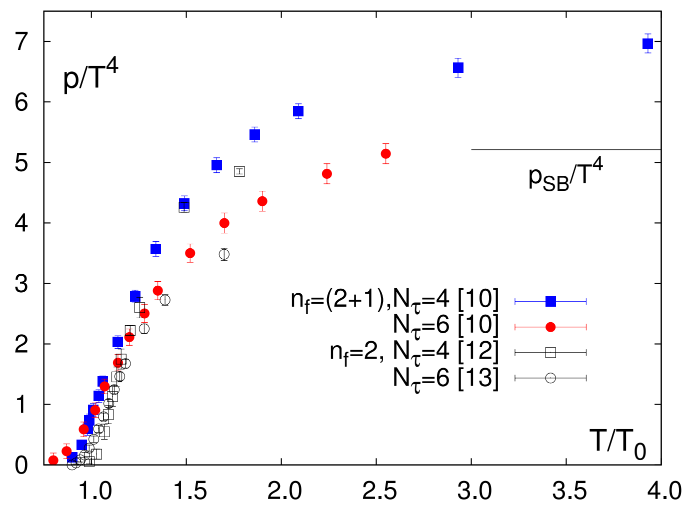
- old results: $16^3 \times 4, \quad m_\pi/m_\rho \simeq 0.7$ [Karsch, EL, Peikert]





- new results: MILC, LAT2005 Y.Aoki et al., JHEP 2006

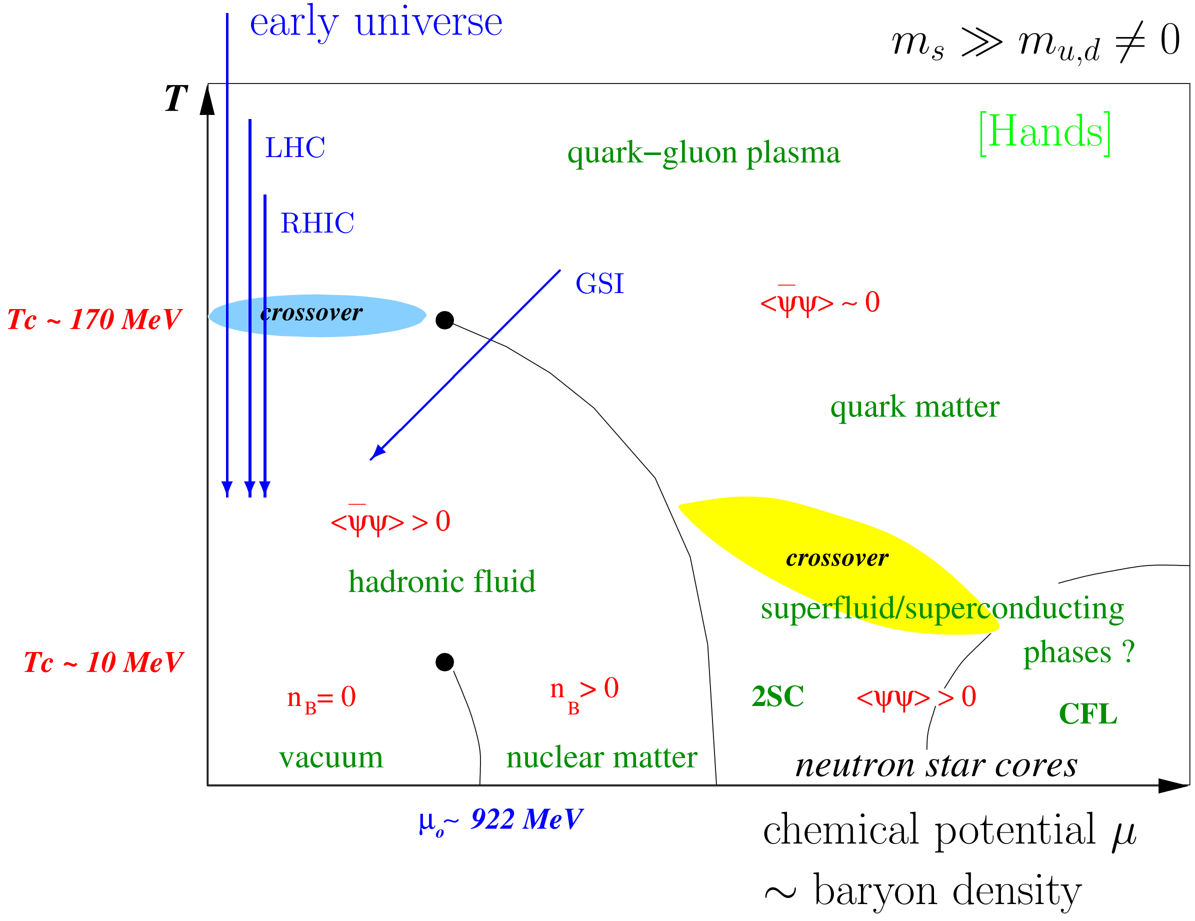




quark masses seem to not matter too much – controlling/reducing UV effects important

III. Phase diagram at $\mu \neq 0$

expected properties :



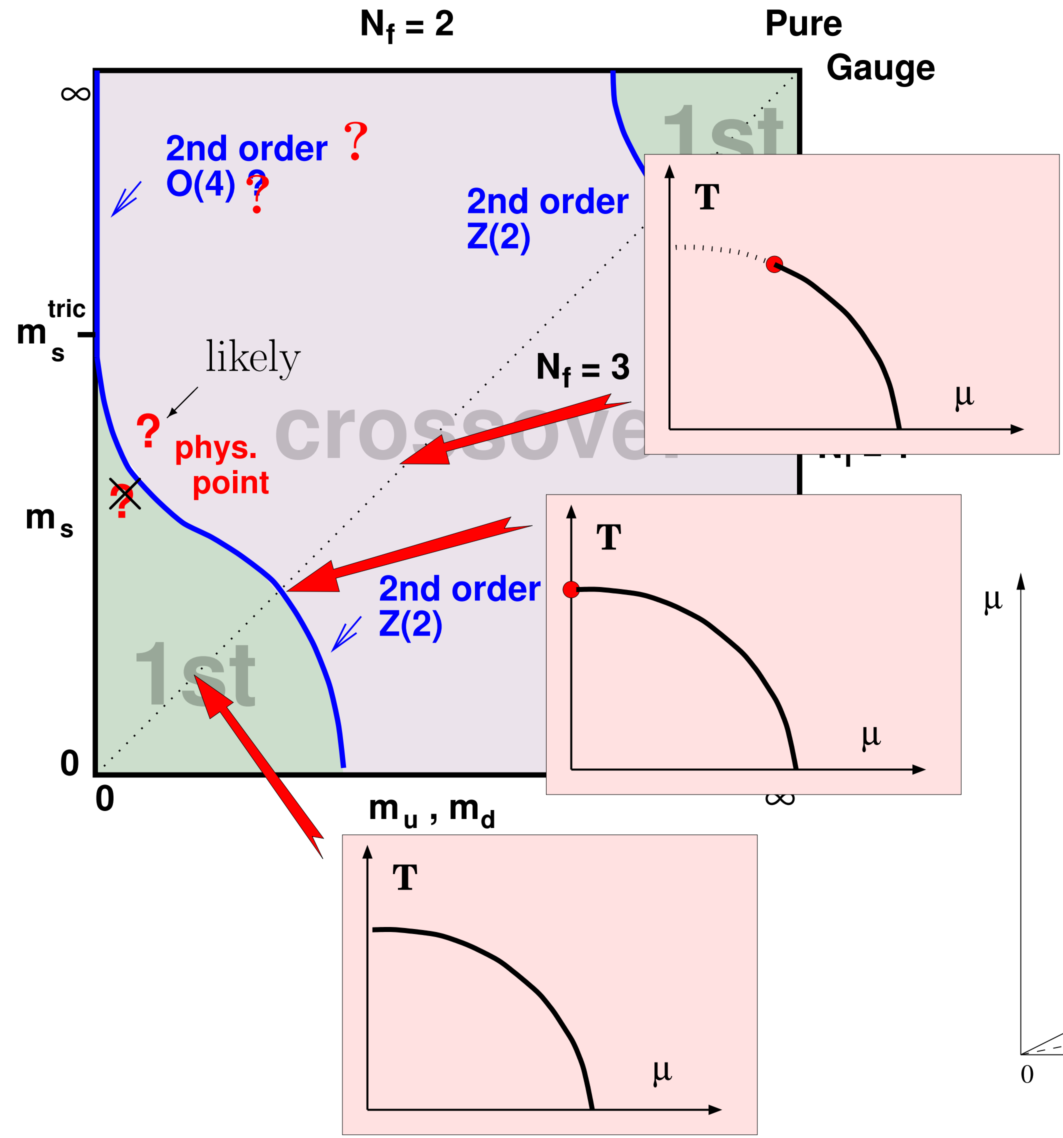
in detail dependent on masses of light flavors

$m_{u,d} \ll m_s \quad N_F = 2$

$m_{u,d} < m_s \quad N_F = 2+1$

$m_{u,d} \simeq m_s \quad N_F = 3$

see e.g. Rajagopal, Wilczek, hep-ph/0011333


N_f = 2
Pure
Gauge
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1st
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N_f = 3
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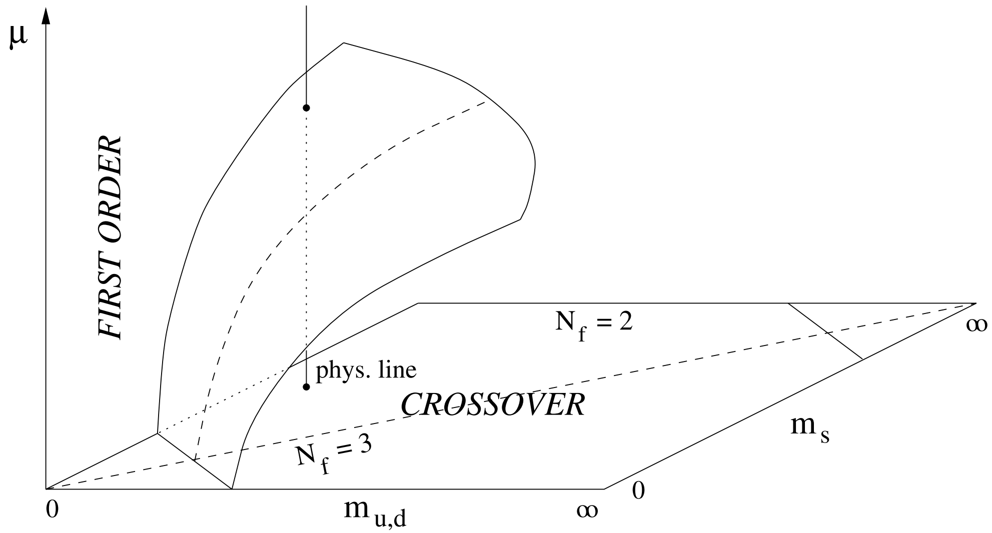
1st
0
0
m_u , m_d
∞
T
μ
T
μ
T
μ

summarized:


μ
FIRST ORDER
N_f = 2
∞
phys. line
CROSSOVER
N_f = 3
m_s
0
0
m_u,d
∞

the problem :

$$Z_{GC}(T,V,\mu) = \int \mathcal{D}U_\mu \, \mathcal{D}q \, \mathcal{D}\overline{q} \, \exp\{-S_G(U) + \overline{q}M(\mu)q\}$$

integrate over quark fields

$$Z_{GC}(T,V,\mu) = \int \mathcal{D}U_\mu \, \det M(\mu) \, \exp\{-S_G(U)\}$$

- for $\mu \neq 0$: $\det M(\mu)$ complex $\Rightarrow$ can not be used as statistical weight in Monte Carlo
- reformulate: $\det M(\mu) = |\det M(\mu)|\, e^{i\Theta}$ and use phase Θ as (part of the) observable:

$$\langle\, \mathcal{O} \,\rangle_{\det M} = \langle\, \mathcal{O}\, e^{i\Theta} \,\rangle_{|\det M|} \,/\, \langle\, e^{i\Theta} \,\rangle_{|\det M|}$$

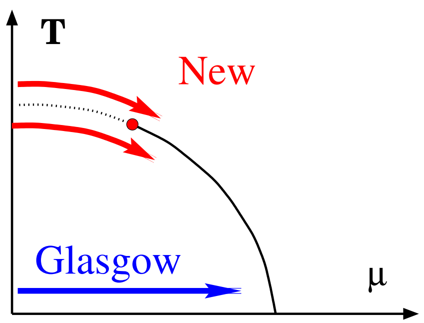
- but : $\langle\, e^{i\Theta} \,\rangle_{|\det M|} \sim e^{-V}$ 'sign problem'

* ‘Reweighting’

[Glasgow; Fodor,Katz]

simulate at parameters $p_0 = (g, m, \mu)_0$ and reweight to $p = (g, m, \mu)$

$$\mathcal{D}U e^{-S_G(p)} \det M(p) = \underbrace{\mathcal{D}U e^{-S_G(p_0)} \det M(p_0)}_{\text{simulation}} \star \underbrace{e^{-[S_G(p)-S_G(p_0)]} \frac{\det M(p)}{\det M(p_0)}}_{\text{correction-factor}}$$



- limited by overlap

* ‘Taylor-expansion’

[Bielefeld-Swansea; Gavai,Gupta]

$$\langle \mathcal{O} \rangle \left(\frac{\mu}{T}\right) = \langle \mathcal{O} \rangle_{\mu=0} + \langle \tilde{\mathcal{O}}_2 \rangle_{\mu=0} \star \left(\frac{\mu}{T}\right)^2 + \langle \tilde{\mathcal{O}}_4 \rangle_{\mu=0} \star \left(\frac{\mu}{T}\right)^4 + \ldots \qquad \text{with} \quad \tilde{\mathcal{O}}_k = \frac{1}{k!} \frac{\partial^k \mathcal{O} \det M}{\partial \mu^k}$$

- limited by convergence radius

* ‘imaginary μ’

[Forcrand,Philipsen; D’Elia,Lombardo]

- $\mu = i\mu_I \Rightarrow \det M$ real and positive
- analytic continuation to real μ

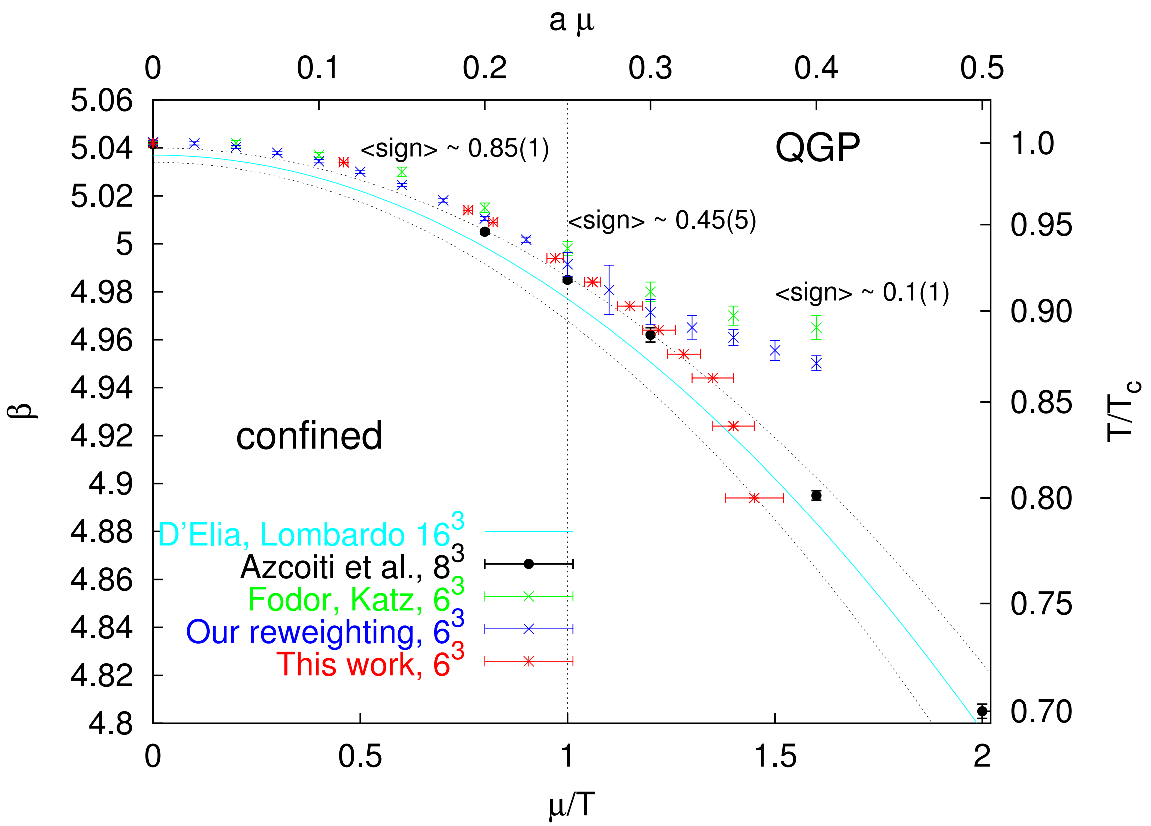
- limited by $Z(\mu_I/T) = Z(\mu_I/T + 2\pi/3)$

* **'canonical'** [Forcrand,Kratochvila]

$$Z_C(B) = \frac{1}{2\pi} \int d\left(\frac{\mu_I}{T}\right) \exp\left\{-i3B\frac{\mu_I}{T}\right\} Z_{GC}(\mu = \mu_I)$$

- sample at fixed μ_I
- Fourier transform each determinant $\longrightarrow$ work $\sim N_\sigma^9 \times N_\tau$
- combine with reweighting in μ_I
- back to $Z_{GC}(\mu)$ by $\Sigma_B \exp\left\{+3B\frac{\mu}{T}\right\} Z_C(B)$

Phase Diagram $T - \mu$: comparing apples with apples



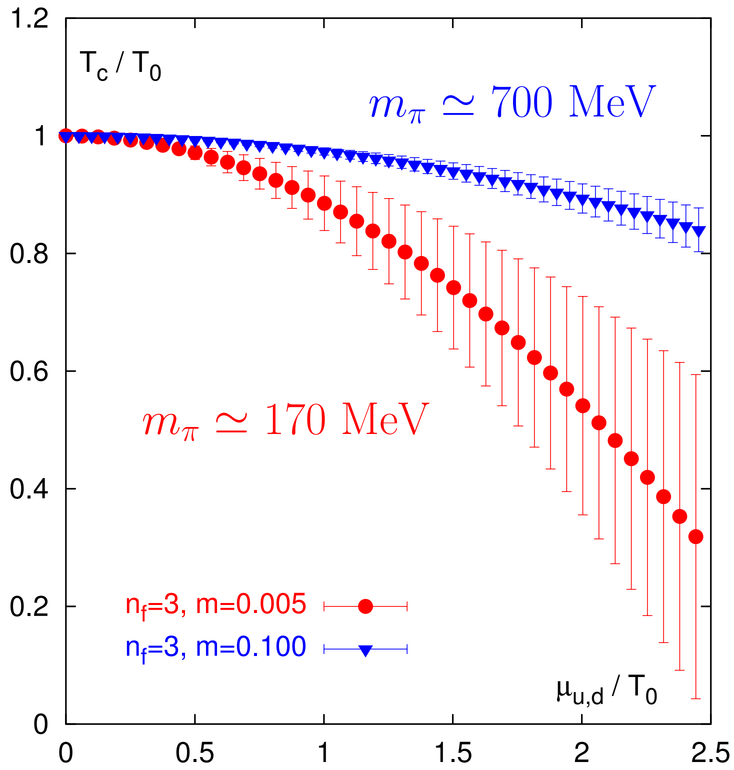
taken from Forcrand
see hep-lat/0602024

canonical partition fct.
$N_F = 4$
small lattices

agreement at small μ/T
seems to hold also at
- $N_F < 4$
- bigger lattices

i) reweighting becomes unreliable

- applicable at small values for μ in the phenomenologically relevant range for RHIC, LHC
- first, exploratory results in qualitative agreement, further systematic investigations required
- in particular at smaller quark masses :



$N_F = 3$, improved action

$$\frac{T_c(\mu)}{T_c(0)} = 1 - 0.025(6)\left(\frac{\mu}{T_c(0)}\right)^2$$

$$\frac{T_c(\mu)}{T_c(0)} = 1 - 0.114(46)\left(\frac{\mu}{T_c(0)}\right)^2$$

(perturbative β-function $d\beta_c/d\ln a$)

- considerable quark mass dependence

IV. Equation of state at $\mu \neq 0$

Observables

pressure ($\mu = (\mu_u, \mu_d, \ldots)$)

$$\frac{p}{T^4} = \Omega(T,\mu) = \frac{1}{VT^3} \ln Z(T,\mu) = \sum_{n=0}^{\infty} c_n(T,m_q) \left(\frac{\mu}{T}\right)^n$$

with

$$c_n(T,m_q) = \frac{1}{n!}\frac{1}{VT^3}\frac{\partial^n \ln Z(T,\mu)}{\partial(\mu/T)^n}\Big|_{\mu=0}$$

number density

$$\frac{n_q}{T^3} = \frac{1}{VT^3}\frac{\partial \ln Z(T,\mu)}{\partial(\mu/T)} = \sum_{n=2}^{\infty} n c_n(T,m_q) \left(\frac{\mu}{T}\right)^{n-1}$$

interaction measure

$$\frac{\epsilon - 3p}{T^4} = \sum_{n=0}^{\infty} c_n'(T,m_q) \left(\frac{\mu}{T}\right)^n$$

with

$$c_n'(T,m_q) = T\frac{dc_n(T,m_q)}{dT}$$

from those, energy density

$$\frac{\epsilon}{T^4} = \sum_{n=0}^{\infty} (3c_n(T,m_q) + c_n'(T,m_q)) \left(\frac{\mu}{T}\right)^n$$

and entropy density

$$\frac{s}{T^3} = \frac{\epsilon + p - \mu n_q}{T^4} = \sum_{n=0}^{\infty} ((4-n)c_n(T,m_q) + c_n'(T,m_q)) \left(\frac{\mu}{T}\right)^n$$

diagonal and off-diagonal susceptibilities

$$\frac{\chi_{ff}(T,\mu)}{T^2} = \frac{\partial^2 \Omega(T,\mu)}{\partial(\mu_f/T)^2}$$

$$\frac{\chi_{fk}(T,\mu)}{T^2} = \frac{\partial^2 \Omega(T,\mu)}{\partial(\mu_f/T)\partial(\mu_k/T)}$$

with $\mu_q = \frac{1}{2}(\mu_u + \mu_d)$ and $\mu_I = \frac{1}{2}(\mu_u - \mu_d)$

quark number susceptibility

$$\frac{\chi_q(T,\mu)}{T^2} = \frac{\partial^2 \Omega(T,\mu)}{\partial(\mu_q/T)^2} = 2\,(\chi_{uu} + \chi_{ud})$$

isovector susceptibility

$$\frac{\chi_I(T,\mu)}{T^2} = \frac{\partial^2 \Omega(T,\mu)}{\partial(\mu_I/T)^2} = 2\,(\chi_{uu} - \chi_{ud})$$

charge susceptibility

$$\frac{\chi_Q(T,\mu)}{T^2} = \frac{\partial^2 \Omega(T,\mu)}{\partial(\mu_Q/T)^2} = \frac{1}{9}\,(5\chi_{uu} - 4\chi_{ud})$$

and higher moments/derivatives

What is known analytically

(A) **high temperature** : **perturbation theory** [Vuorinen]

$$\Omega(T,\mu) = \Omega^{(0)}(T,\mu) + g^2\,\Omega^{(2)}(T,\mu) + g^3\,\Omega^{(3)}(T,\mu) + \mathcal{O}(g^4)$$

with Stefan-Boltzmann (free gas) limit

$$\frac{p_{SB}}{T^4} = \Omega^{(0)}(T,\mu) = \frac{8\pi^2}{45} + \sum_{f=u,d,..}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$$

diagonal suscept.

$$\frac{\chi_{ff}(T,\mu)}{T^2} = 1 + \frac{3}{\pi^2}\left(\frac{\mu_f}{T}\right)^2 + \mathcal{O}(g^2)$$

off-diagonal suscept.

$$\frac{\chi_{fk}(T,\mu)}{T^2} = g^3\,\kappa\,\frac{\mu_f}{T}\frac{\mu_k}{T} + \mathcal{O}(g^4)$$

$$\frac{\chi_{fk}(T,0)}{T^2} = -\frac{5}{144\pi^6}\,g^6 \ln 1/g$$

(B) **low temperature** : **hadron resonance gas model**

$$\Omega_{HRG}(T,\mu_q,\mu_I) = \sum_{i\in\ mesons} \Omega^M_{m_i}(T,\mu_q,\mu_I) + \sum_{i\in\ baryons} \Omega^B_{m_i}(T,\mu_q,\mu_I)$$

where

$$\Omega^{M/B}_{m_i} = \frac{1}{2\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{\ell=1}^{\infty} \left\{ \frac{1}{(-1)^{\ell+1}} \right\} \ell^{-2} K_2\left(\frac{\ell m_i}{T}\right) z_i^\ell \quad \text{with} \quad z_i = \exp((3B_i\mu_q + 2I_{3i}\mu_I)/T)$$

fugacities

- for baryons, $\ell \geq 2$ terms can safely be neglected[1] $\Rightarrow$ at $\mu_I = 0$:

$$\frac{p(T,\mu_q,\mu_I=0)}{T^4} \simeq G(T) + F(T)\cosh\left(\frac{3\mu_q}{T}\right) \qquad \Rightarrow \qquad \frac{\chi_q}{T^2} = 9F(T)\cosh\left(\frac{3\mu_q}{T}\right)$$

likewise, $\displaystyle \frac{\chi_I(T,\mu_q,\mu_I=0)}{T^2} \simeq G^I(T) + F^I(T)\cosh\left(\frac{3\mu_q}{T}\right)$

- For all quantities X of the form $X = G^X(T) + F^X(T)\cosh(3\mu_q/T)$:

$$X = \sum_{n=0}^{\infty} c_n^X(T)\,(\mu_q/T)^n \qquad \text{with} \qquad \frac{c^X_{2n+2}}{c^X_{2n}} = \frac{9}{(2n+2)(2n+1)} \quad \text{for } n \geq 1$$

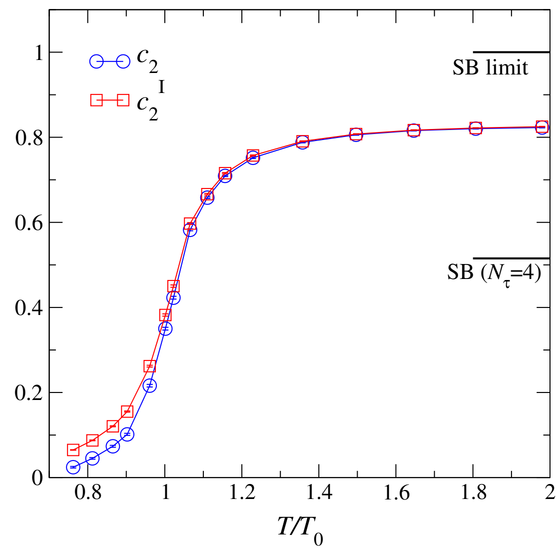
[1] $K_2(x) \sim e^{-x}(1+P(1/x))/\sqrt{x}$

pressure $$\frac{\Delta p}{T^4} = \frac{p(\mu_q)}{T^4} - \frac{p(\mu_q = 0)}{T^4} = c_2 \left(\frac{\mu_q}{T}\right)^2 + c_4 \left(\frac{\mu_q}{T}\right)^4 + c_6 \left(\frac{\mu_q}{T}\right)^6 + \cdots$$

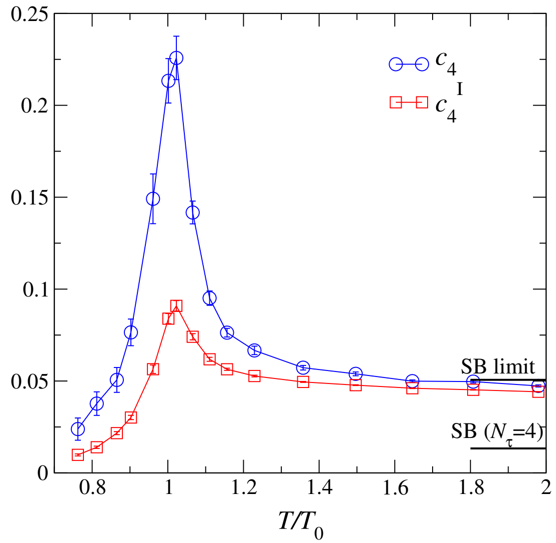
quark number $$\frac{n_q(T,\mu_q)}{T^3} = 2c_2 \left(\frac{\mu_q}{T}\right) + 4c_4 \left(\frac{\mu_q}{T}\right)^3 + 6c_6 \left(\frac{\mu_q}{T}\right)^5 + \cdots$$

q number suscept. $$\frac{\chi_q(T,\mu_q)}{T^2} = 2c_2 + 12c_4 \left(\frac{\mu_q}{T}\right)^2 + 30c_6 \left(\frac{\mu_q}{T}\right)^4 + \cdots$$

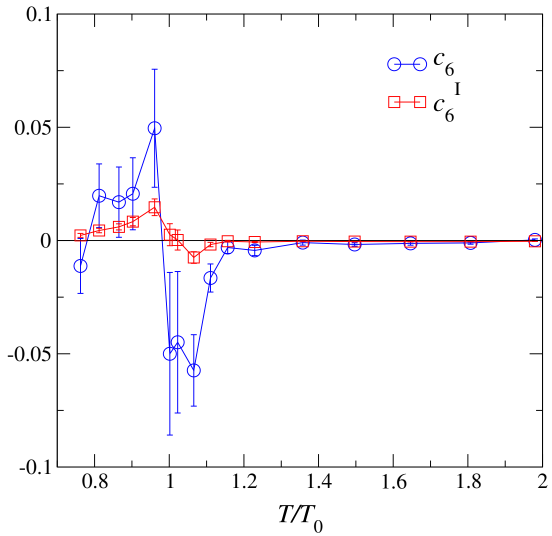
Isovector suscept. $$\frac{\chi_I(T,\mu_q)}{T^2} = 2c_2^I + 12c_4^I \left(\frac{\mu_q}{T}\right)^2 + 30c_6^I \left(\frac{\mu_q}{T}\right)^4 + \cdots$$



- approaching SB
- discretisation effects !

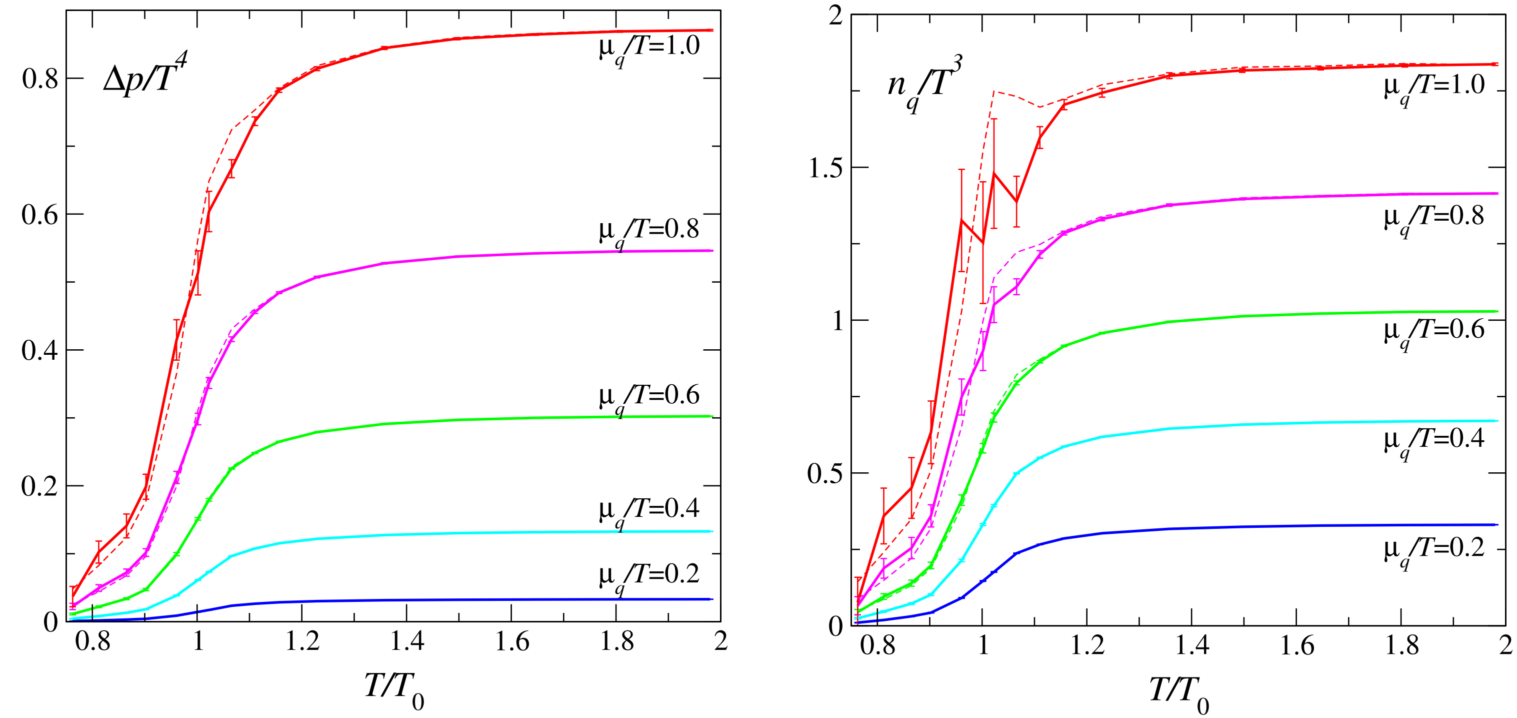


- peak around T_c



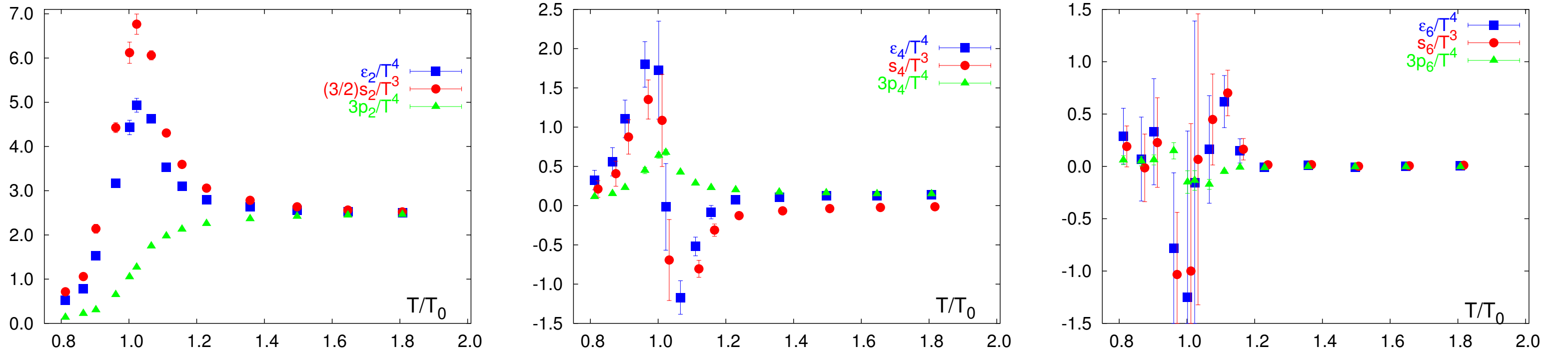
- sign change around $T \lesssim T_c$
- small

pressure **quark number density**

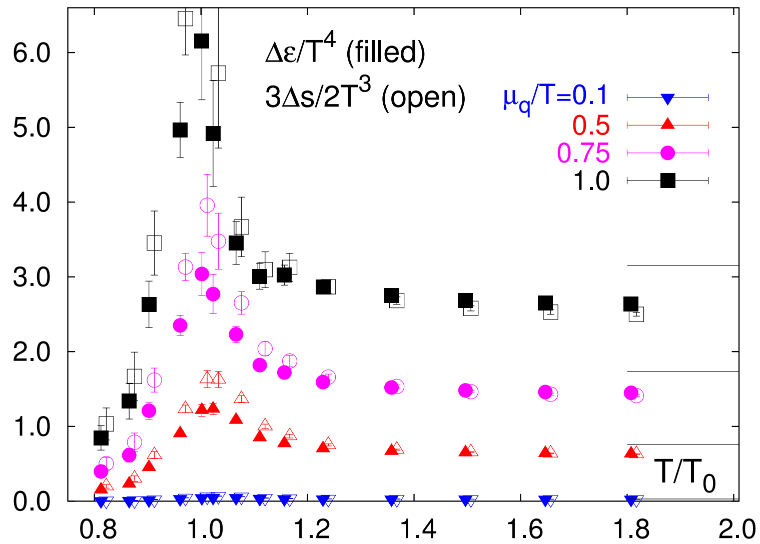


- comparison: up to $\mathcal{O}(\mu^4)$ (dashed) with up to $\mathcal{O}(\mu^6)$ (full) suggests rapid convergence
- contribution to total p is small: $p(\mu=0)/T^4 \simeq \mathcal{O}(4)$

together with the c'_n coefficients ...

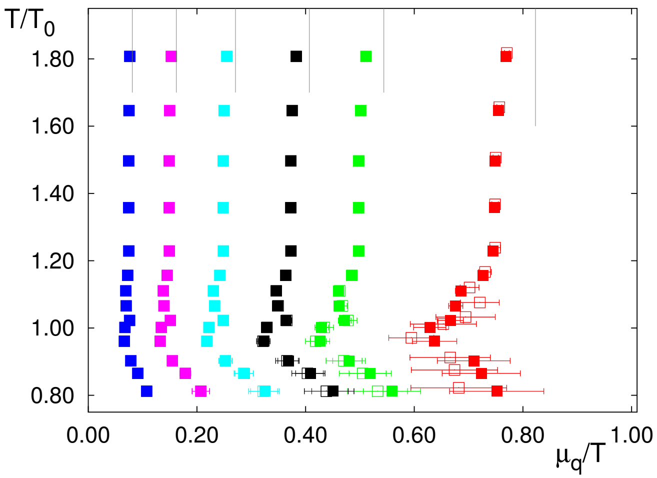


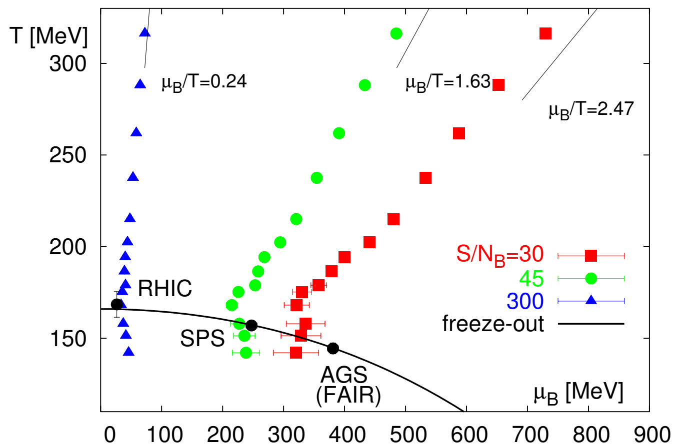
obtain **energy** and **entropy**



Lines of fixed entropy over baryon number

it is generally believed that the fireball expansion follows a line of fixed S/N_B





in the ideal gas limit
$$\frac{S}{N_B} = 3\frac{\frac{37\pi^2}{45} + \left(\frac{\mu_q}{T}\right)^2}{\frac{\mu_q}{T} + \frac{1}{\pi^2}\left(\frac{\mu_q}{T}\right)^3} \qquad \Rightarrow \frac{\mu_q}{T} = \text{const (vertical lines)}$$

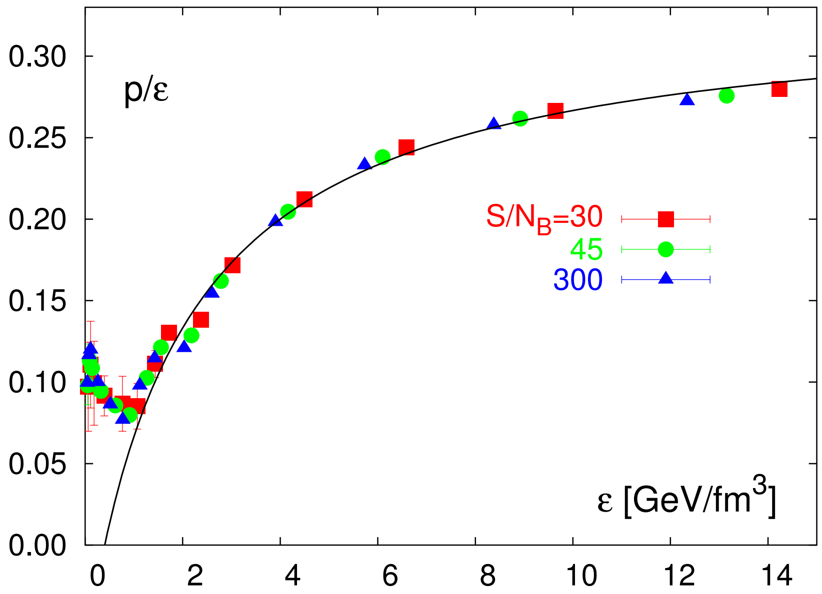
isentropic expansion lines for SPS: $S/N_B \simeq 45$

RHIC: $S/N_B \simeq 300$

FAIR: $S/N_B \simeq 30$

keep in mind: feasibility study of what one can do with lattice data

Isentropic EoS

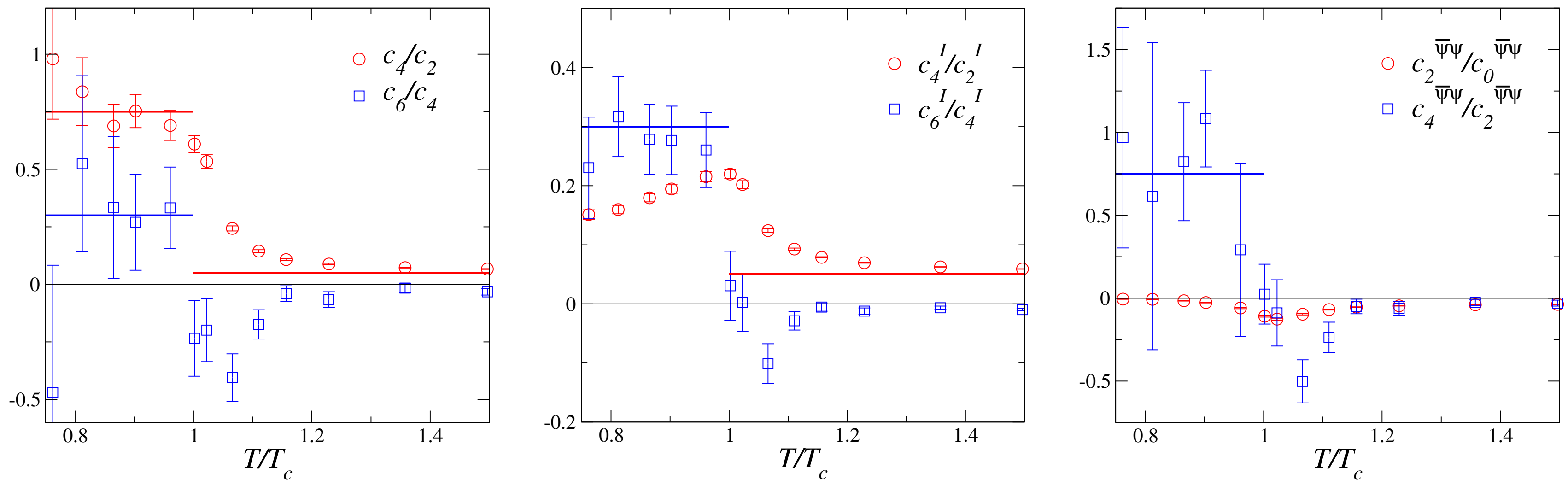


- $p(\epsilon)$ to a good approximation independent of S/N_B
- $p(\epsilon)$ well parametrized by

$$\frac{p}{\epsilon} \simeq \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\,\epsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

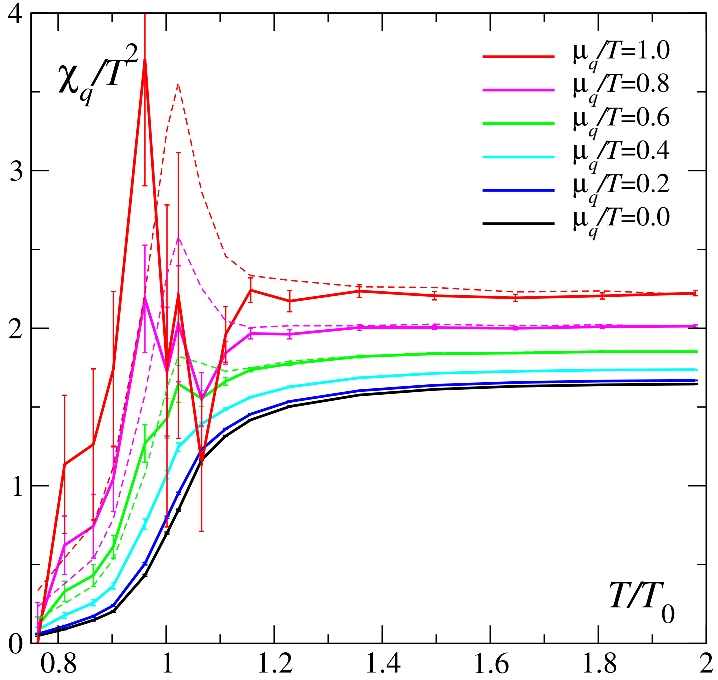
Comparison with analytic results

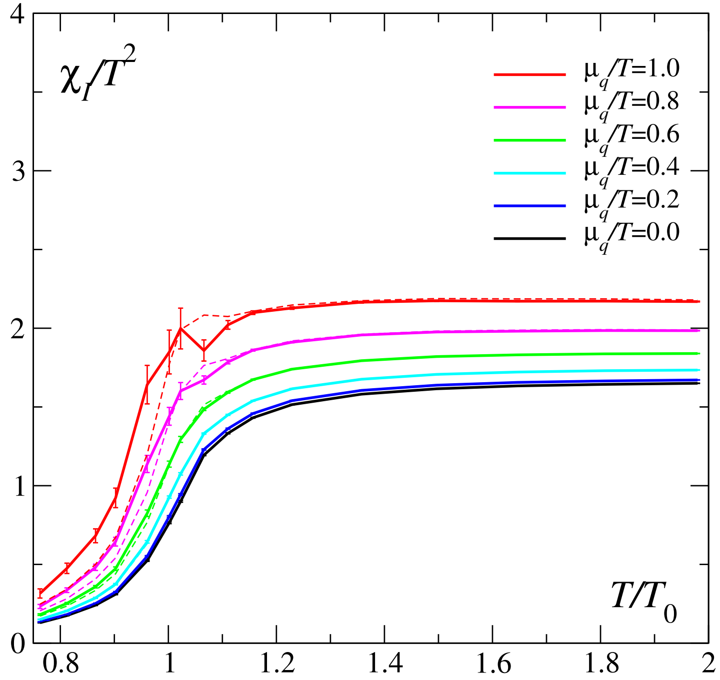
recall: ratios $\frac{c_{2n}}{c_{2n+2}}$ allow comparison with the hadron resonance gas model fairly detail independent



- above T_0: ratios approaching SB values
- below T_0: ratios except those involving $c_0, c_2^I, c_0^{\bar{\psi}\psi}$ (depend on $G^X(T)$) are
  - temperature independent
  - taking hadron resonance gas values $\longrightarrow$ do not indicate critical behavior

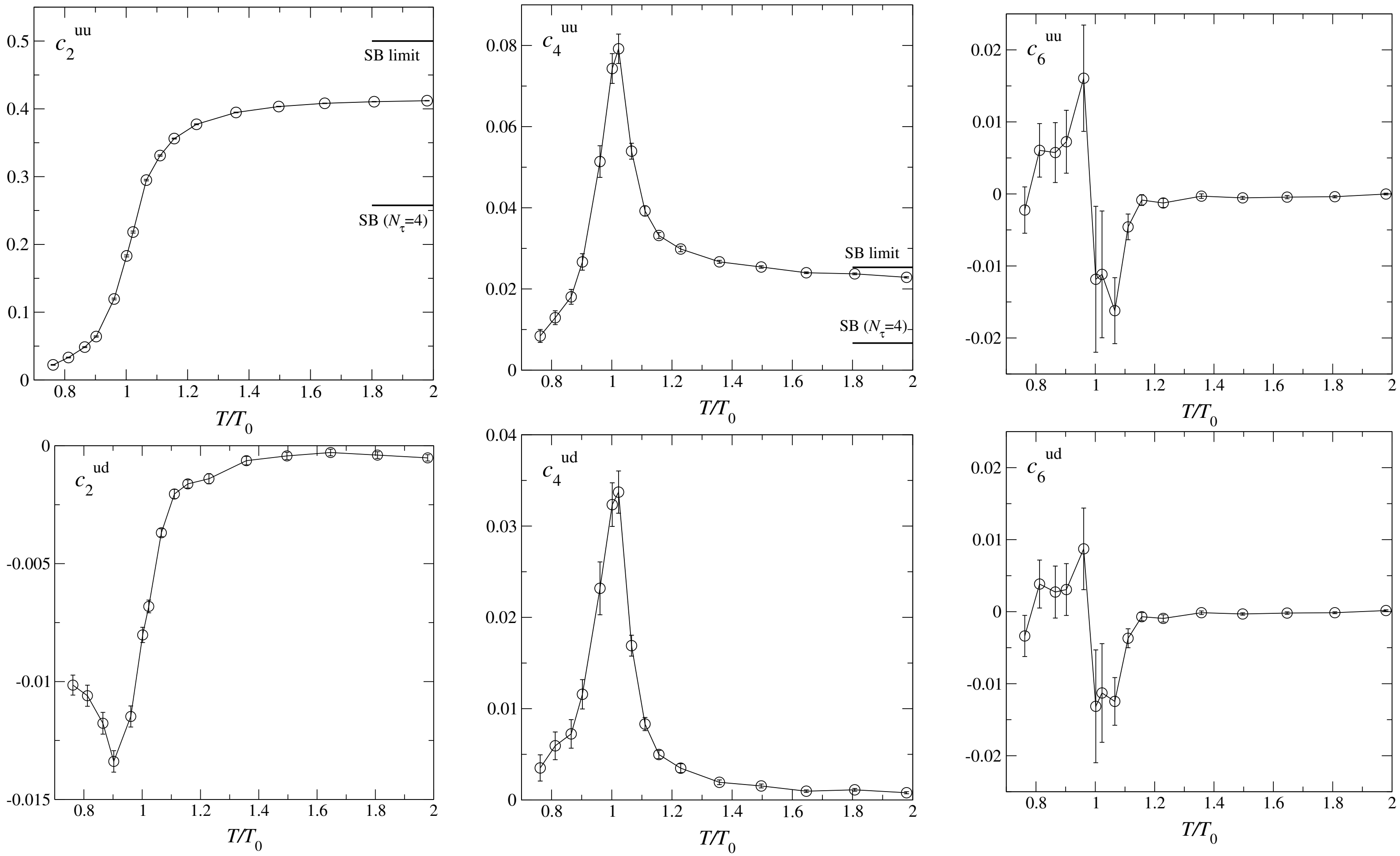
quark number susceptibility **isovector susceptibility**





- again comparison: up to $\mathcal{O}(\mu^4)$ (dashed) with up to $\mathcal{O}(\mu^6)$
- peak in χ_q developing with increasing μ, coming from c_4
- c_6 shifts peak in χ_q to smaller T
- peak less convincing because of error bars and dip $\longrightarrow$ more statistics needed here
- no peak in χ_I $\longrightarrow$ strong correlations between χ_{uu} and χ_{ud}

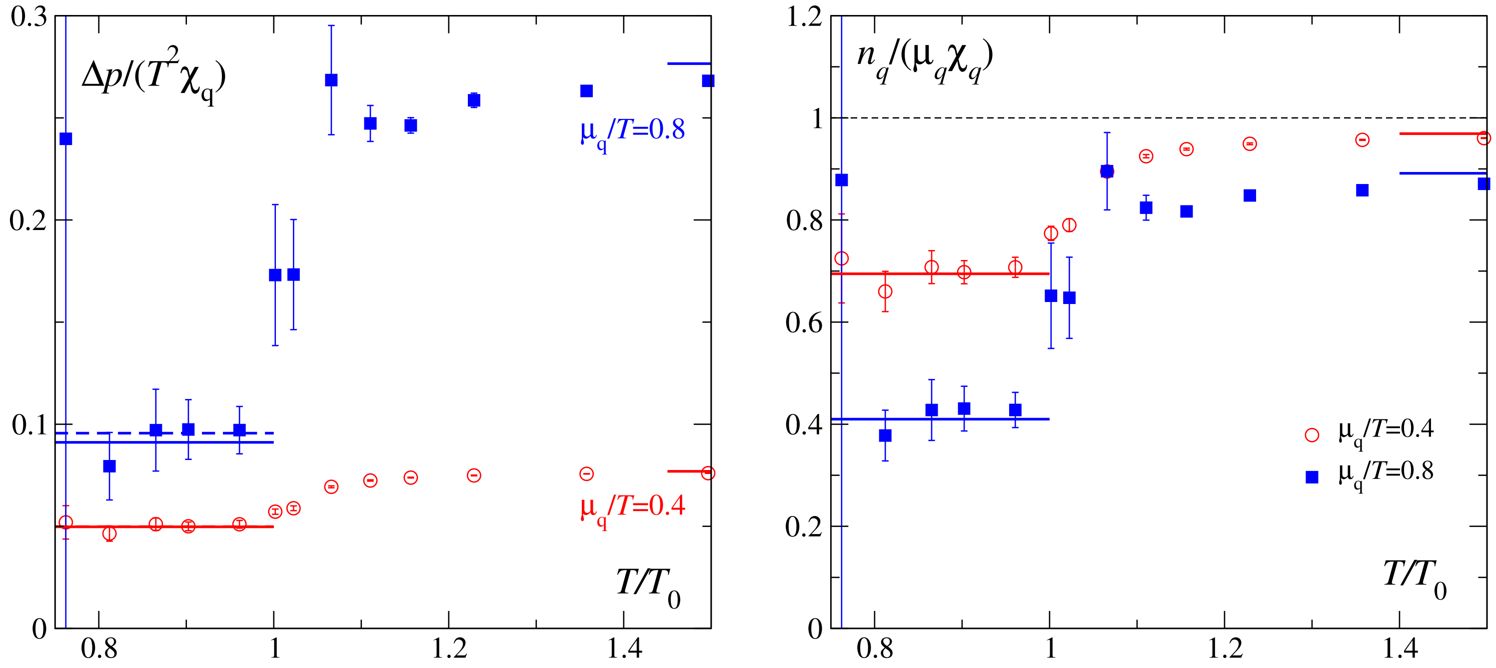
diagonal and off-diagonal susceptibilities



- at $T > T_0$: χ_{uu} and χ_{ud} approach SB limit, i.e. $\chi_{ud} \to 0$
- at $T > T_0$ signs in agreement with perturbation theory [Blaizot, Iancu, Rebhan]
- at $T \lesssim T_0$: $\chi_{ud} \neq 0$
- around T_0: $c_n^{ud} \simeq c_n^{uu}$ for $n > 2 \to$ at $\mu_q = \mu_c$, peaks in both, χ_{uu} and χ_{ud}

  $\to$ at $\mu_q > 0$, fluctuations in different flavor channels are correlated

- χ_q rises rapidly with increasing μ_q, but rise to a large extent due to rise in pressure



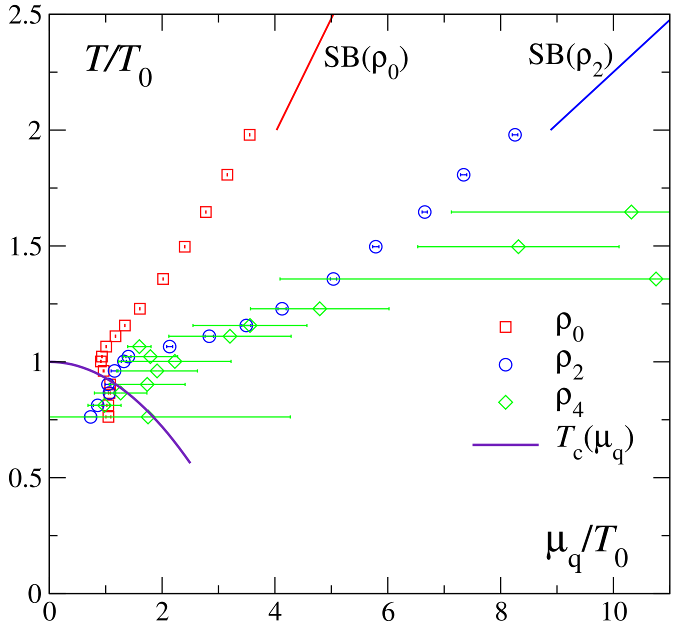
- $$\frac{\partial p}{\partial n_q} = \frac{\partial p/\partial \mu_q}{\partial n_q/\partial \mu_q} = \frac{n_q}{\chi_q} = \frac{1}{\kappa_T\, n_q} \longrightarrow 0$$ at $2^{\rm nd}$ order phase transition (isothermal compressibility $\kappa_T \longrightarrow \infty$)

- no indication of criticality

- but, for $T \leq 0.96 T_c$, consistency with hadron resonance gas model (at $\mu_I = 0$)

$$\frac{n_q^{HRG}}{\mu_q \chi_q^{HRG}} = \frac{T}{3\mu_q} \tanh\left(\frac{3\mu_q}{T}\right)$$

convergence radius



critical point limits convergence radius

$$\rho = \lim_{k\to\infty} \rho_k = \lim_{k\to\infty} \sqrt{\frac{c_k}{c_{k+2}}}$$

- SB limit: $\rho_k = \infty$ for $k \geq 4$
- for T big: approaching SB limit

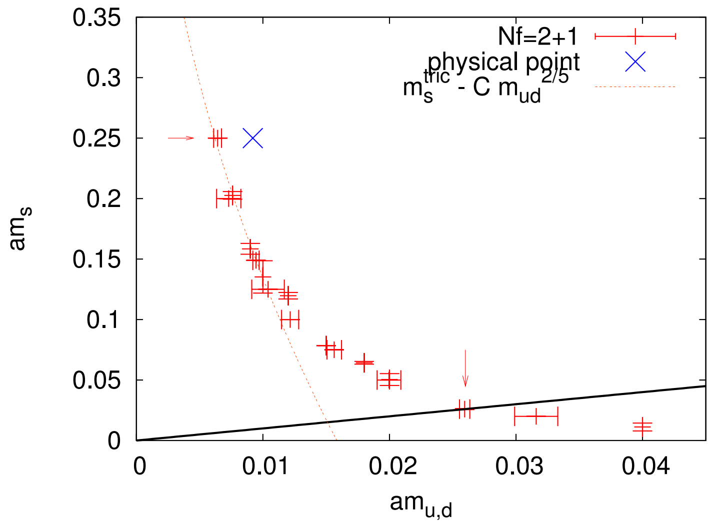
- at $T_c(\mu)$: $\rho_k \simeq 1$

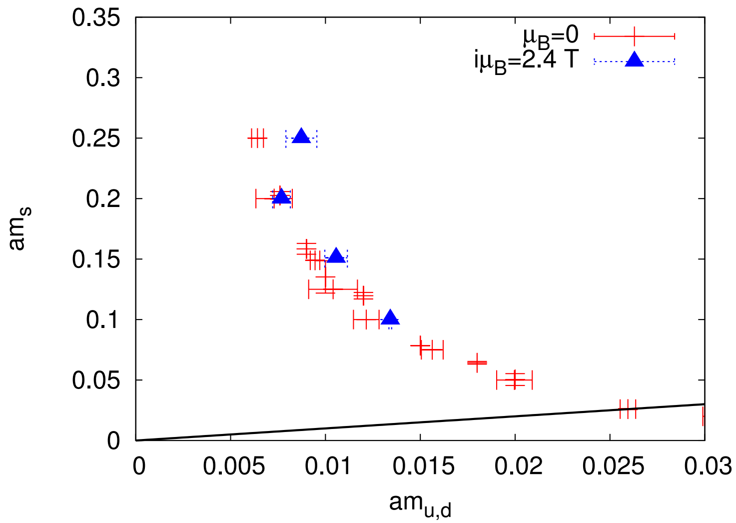
- $c_k > 0 \Rightarrow$ convergence radius indicates critical point

Results:

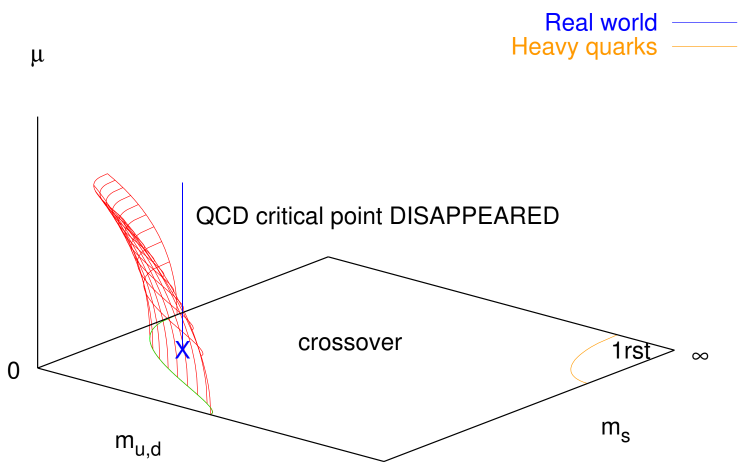
- Gavai, Gupta: $\mu_B \simeq 180\text{MeV}$ (Taylor expansion)
- Fodor, Katz: $\mu_B \simeq 360\text{MeV}$ (Lee-Yang zeroes)
- Bi-Swansea: LGT consistent with HRG, HRG analytic (Taylor expansion)

existence of a critical endpoint ? [deForcrand, Philipsen]





critical region has the tendency to grow with $\mu_I \Rightarrow$ shrink with real μ



but: finer lattices/improved actions needed

$N_\tau = 4$ here