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"Dense Matter In Heavy Ion Collisions and Astrophysics"

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Hydrodynamic Approach to Relativistic Heavy-Ion Collisions

Yu.B. Ivanov

Kurchatov Inst.&GSI

Outline

Why we are not satisfied with kinetics?

- In practice, kinetics $\Rightarrow$ only binary collisions
  mean free path $\lambda \approx 1/(n_B \sigma)$
  if $\sigma \approx 4$ fm^2 and $n_B \approx 5 n_0 \Rightarrow \lambda \approx 0.3$ fm $\sim$ nucleon core
  ($n_0 = 0.15$ fm^{-3} = normal nuclear density)

  Approximation of binary collisions is bad!
- Phase transition into QGP is inaccessible in kinetics as a rule
  The only exception: a multi-phase transport (AMPT) model
  [Lin, Ko and Pal, PRL 89, 152301 (2002)] Simple combinatorics of quarks.

Hydrodynamics

- takes into account any multi-particle interactions
- Phase transition in QGP is accessible through EoS
- However, there are certain **problems**
- directly addresses **Equation of State (EoS)**!

Final Aim: To find a proper EoS, which reproduces all data

Hydrodynamics versus Kinetics

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Hydrodynamics = Conservation laws + assumptions

$\mathbf{J}^{\mu}_{\mathbf{charge}}$ = **Charge Current** (**charge** = baryon charge, electric charge, strangeness, charm, etc.)

$\mathbf{T}^{\mu\nu}$ = **Energy–Momentum Tensor**

Conservation laws:

$$\partial_\mu \mathbf{J}^{\mu}_{\mathbf{charge}} = \mathbf{0} \quad \text{and} \quad \partial_\mu \mathbf{T}^{\mu\nu} = \mathbf{0}$$

Charge conservation: $Q_{charge} = \int d^3x J^0_{charge}$ = total charge of the system

$$\int d^3x \partial_\mu J^{\mu}_{charge} = \partial_t \int d^3x J^0_{charge} = \partial_t Q_{charge} = 0$$

Energy–Momentum conservation: $P^\nu = \int d^3x T^{0\nu}$ = total 4-momentum of the system

$$\int d^3x \partial_\mu T^{\mu\nu} = \partial_t \int d^3x T^{0\nu} = \partial_t P^\nu = 0$$

$\mathbf{J}^{\mu}_{\mathbf{charge}}$ $\Rightarrow$ 4 independent components

$\mathbf{T}^{\mu\nu} = \mathbf{T}^{\nu\mu}$ $\Rightarrow$ symmetric tensor $\Rightarrow$ 10 independent components

If only baryon number conservation is kept:

Number of independent unknown functions **= 14** $>$ Number of equations **= 5**

Assumptions are needed to reduce 14 to 5

Assumption 1: Space–time evolution is smooth

Space–time gradients ∂_μ introduce smallness.

Assumption 2: System is locally characterized by a single 4-vector

$\mathbf{u}^\mu(\mathbf{x})$ = local 4-velocity, normalized as $\mathbf{u}_\mu \mathbf{u}^\mu = 1$

Zero approximation in ∂_μ:

$$\mathbf{J}^\mu_{\mathbf{charge}} = \mathbf{u}^\mu \mathbf{n}_{\mathbf{charge}} \qquad \mathbf{T}^{\mu\nu} = (\varepsilon + \mathbf{P})\, \mathbf{u}^\mu\, \mathbf{u}^\nu - \mathbf{g}^{\mu\nu}\mathbf{P}$$

Here $n_{charge}(x)$, $\varepsilon(x)$ and $P(x)$ are some Lorentz-scalar functions.

Physical meaning of these functions? $\Rightarrow$ in local rest frame $u^\mu = (1, 0, 0, 0)$

$\mathbf{n}_{\mathbf{charge}} = \mathbf{J}^0_{\mathbf{charge}}$ = **proper charge density**

$\varepsilon = \mathbf{T}^{00}$ = **proper energy density**

$\mathbf{P} = \mathbf{T}^{ii}$ = **pressure**

Number of independent unknown functions **= 6** $>$ **5 =** Number of equations

Assumption 3: Local thermal equilibrium

$$\mathbf{EoS} \Rightarrow \mathbf{P}(\mathbf{x}) = \mathbf{P}(\mathbf{n}_{\mathbf{charge}}(\mathbf{x}), \varepsilon(\mathbf{x}))$$

Number of independent unknown functions = Number of equations = 5

This is perfect hydrodynamics: $\partial_\mu \mathbf{S}^\mu = 0$ Entropy is conserved.

Exercise

Please, derive entropy conservation

$$\partial_\mu \mathbf{S}^\mu = \mathbf{0}$$

proceeding from:

Continuity eq. $\partial_\mu J^\mu = 0$

Euler eq. $\partial_\mu T^{\mu\nu} = 0$

Thermodynamic eq. $dE = T\, dS - P\, dV$

where

$$V = 1/n = \text{volume containing one particle}$$

$$E = \varepsilon V, \quad S = Vs, \quad S^\mu = su^\mu$$

Applications of perfect hydrodynamics

First application: L.D. Landau, Izv.Akad.Nauk Ser.Fiz.17,51,**1953**
"On the multiparticle production in high-energy collisions"

Numerous applications at energies $E_{lab} = 0.2 - 2{\cdot}10^4$ GeV/nucl.
(from Bevalac, Synchrophasotron and SIS to AGS, SPS and RHIC)
R.B. Clare, D. Strottman, Phys.Rept.141:177-280,1986
H. Stoecker, W. Greiner, Phys.Rept.137:277-392,1986
I.N. Mishustin, V.N. Russkikh, L.M. Satarov, Sov.J.Nucl.Phys.54:260-314,1991
D.H. Rischke, nucl-th/9809044

Below 0.2 GeV/nucl. Pauli blocking makes hydro inapplicable

Hydro is perfect at RHIC. [P. Huovinen, P.V. Ruuskanen, nucl-th/0605008]

At 0.2–2 GeV/nucl. direct description proceeding from 2 cold nuclei
(shock-wave generation of entropy)

Problem 1: initial stage is too nonequilibrium at higher energies

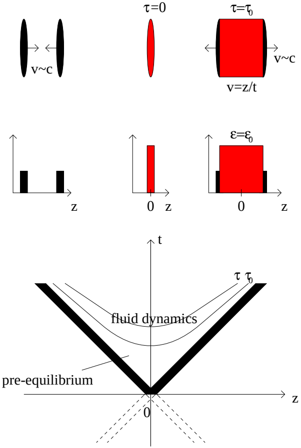
- either parametrization of initial fireball [P. Huovinen, P.V. Ruuskanen, nucl-th/0605008]
- or kinetic calculation of it [Y. Hama *et al.*, hep-ph/0510101; V.V. Skokov and V.D. Toneev, Phys.Rev. C73, 021902(R) (2006), nucl-th/0601160.]
- or multi-fluid dynamics (described below)

Problem 2: Termination of hydro $\Rightarrow$ freeze-out

When mean-free-path $\sim$ system size, hydro is inapplicable!

Popular solution to Problem 1: Bjorken model

Bjorken model is applicable only at very high incident energies



Assume no conserved charges: $n \equiv 0$

Assume scaling solution: $\mathbf{v} = \mathbf{z}/\mathbf{t}$ at $\tau > \tau_0$

New variables:
Proper time: $\tau = t\,(1 - v^2)^{1/2} = (t^2 - z^2)^{1/2}$
Rapidity: $\eta = \mathrm{Arctanh}(v) = \mathrm{Arctanh}(z/t)$

Then, $\partial_t T^{00} + \partial_z T^{z0} = 0$ and $\partial_t T^{0z} + \partial_z T^{zz} = 0$ are reduced to

$$\frac{\partial \varepsilon}{\partial \tau} + \frac{\varepsilon + P}{\tau} = 0 \quad \text{and} \quad \frac{\partial P}{\partial \eta} = 0$$

EoS independent consequences:

$\mathbf{P} = \mathbf{P}(\tau)$

$\partial P/\partial \eta = s\partial T/\partial \eta + n\partial \mu/\partial \eta = 0 \quad \Rightarrow \quad \mathbf{T} = \mathbf{T}(\tau)$

Entropy density $\quad \mathbf{s} = \mathbf{s_0}\tau$

For ultrarelativistic EoS $P = \frac{1}{3}\varepsilon$ **exact solution:** $\varepsilon = \varepsilon_0 \tau^{-4/3}$

Outline

Freeze-out is still unsolved problem!

Standard now, Cooper–Frye prescription:

- "decoupling" from hydro happens on **a continues hypersurface Σ**, defined by some criterion: e.g. $\varepsilon = \varepsilon_{frz}$ or $T = T_{frz}$ or $n_{bar} = n_{bar}^{frz}$.
- after this "decoupling" **particles stream freely to detectors**

$$f(k,x) = \frac{1}{\exp\{[k_\nu u^\nu(x) - \mu(x)]/T(x)\} \pm 1} = \text{equilibrium distribution function}$$

Number of particles contained in hypersurface element $d\Sigma$ ($n^\mu =$ unit vector normal to this element):

$$N_\Sigma \equiv d\Sigma_\mu N^\mu = d\Sigma\, n_\mu\, N^\mu = \int \frac{d^3k}{E} d\Sigma\, (n_\mu k^\mu)\, f(k,x)$$

Invariant momentum spectrum of particles contained in hypersurface element $d\Sigma$:

$$E\frac{dN_\Sigma}{d^3k} = d\Sigma\,(n_\mu k^\mu)\, f(k,x)$$

Invariant momentum spectrum of particles contained in hypersurface Σ:

$$E\frac{dN}{d^3k} = \int_\Sigma E\frac{dN_\Sigma}{d^3k} = \int_\Sigma d\Sigma\,(n_\mu k^\mu)\, f(k,x)$$

This is observable spectrum.

Problems of Cooper–Frye Freeze-out

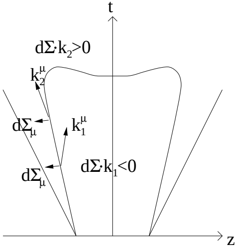
Observable momentum spectrum of particles:

$$E\frac{dN}{d^3k} = \int_\Sigma d\Sigma\ (\mathbf{n}_\mu \mathbf{k}^\mu)\ f(k,x)$$

Problem: $E\frac{dN}{d^3k} < 0$ when $n_\mu k^\mu < 0$
These are particles returning to hydro phase.

$E\frac{dN}{d^3k} < 0$ **is unacceptable for observables!**

There is no problem, e.g. when $n^\mu = u^\mu$:
$u_\mu k^\mu > 0$ always.



Remedy for the Problem:

- Cut-off (Sinyukov, Bugaev, Csernai *et al.*)

$$E\frac{dN}{d^3k} = \int_\Sigma d\Sigma\ (n_\mu k^\mu)\ f(k,x)\ \Theta(n_\mu k^\mu)$$

  Energy, momentum and charge are not conserved.
  To conserve them, hydro should contain sources for returning particles.
- Cut-off with renormalization to fulfill conservations (Bugaev, Csernai *et al.*)
  Too artificial construction?
- More sophisticated Cut-off's with renormalization (Csernai *et al.*)
  How physical these are?

Further modifications of Cooper–Frye assumptions:

- **Freeze-out hypersurface Σ can be discontinues** rather than continues.
  $n^\mu = u^\mu$ solves the problem of negative spectrum. (Milekhin)
  Problem of "shadowing" (i.e. particles returning to hydro phase) still persists.
- After hydro freeze-out **particles still exercise collisions** rather than stream freely to detectors. (Shuryak *et al.*)
  This helps to resolve points of "chemical" and "thermal" freeze-out.

Models, using hypersurface, can be called "geometrical".

Dynamical models of freeze-out:

- **Freeze-out occurs in 4-dimensional region** rather than at hypersurface.
  "Continuous emission" of particles occurs in the system surface layer of the mean-free-path width. (Grassi, Hama, Kodama)

It is the most prominent among others but very difficult for numeric realization.

- **Simplified "continuous emission"** with the mean-free path srunk to zero.
  (Russkikh, Ivanov, Toneev)
  This can be described by hydro equations with sinks at the system surface.
  Problem of "shadowing" (i.e. particles returning to hydro phase) still persists.

Outline

Assumptions revisited

Perfect hydrodynamics is based on

Assumption 1: Space–time evolution is smooth
Zero approximation in ∂_μ.

Assumption 2: System is locally characterized by a single 4-vector

$$\mathbf{u}^\mu(\mathbf{x}) = \text{local 4-velocity, normalized as } \mathbf{u}_\mu \mathbf{u}^\mu = 1$$

Assumption 3: Local thermal equilibrium

Viscous Hydrodynamics

Assumption 1: Space–time evolution is smooth

Expansion in ∂_μ up to 1st order:

$$\mathbf{J}^{\mu}_{\mathbf{charge}} = \mathbf{u}^{\mu}\mathbf{n}_{\mathbf{charge}} + \nu^{\mu} \qquad \mathbf{T}^{\mu\nu} = (\varepsilon + \mathbf{P})\,\mathbf{u}^{\mu}\,\mathbf{u}^{\nu} - \mathbf{g}^{\mu\nu}\mathbf{P} + \pi^{\mu\nu}$$

Requiments:

- ν^{μ} and $\pi^{\mu\nu}$ contain only 1st-order ∂_μ terms
- entropy grows: $\partial_\mu \mathbf{S}^{\mu} > \mathbf{0}$
- Landau choice: u^{μ} = velocity of energy transfer $\Rightarrow u^{\mu}\pi^{\mu\nu} = 0$.

$$\nu^{\mu} = \kappa \left(\frac{nT}{\varepsilon + P}\right)^2 (\partial^{\mu} - u^{\mu}\,u^{\nu}\partial_{\nu}) \left(\frac{\mu}{T}\right)$$

$$\pi^{\mu\nu} = \eta\left(\partial^{\mu}u^{\nu} + \partial^{\nu}u^{\mu} - u^{\mu}u_{\lambda}\partial^{\lambda}u^{\nu} - u^{\nu}u_{\lambda}\partial^{\lambda}u^{\mu}\right) + \left(\zeta - \frac{2}{3}\eta\right)(g^{\mu\nu} - u^{\mu}u^{\nu})\,\partial_{\lambda}u^{\lambda}$$

μ = chemical potential T = temperature

η = **shear viscosity** ζ = **bulk viscosity** κ = **thermal conductivity**

viscosity, etc. = corrections due to weak nonequilibrium

Nonrelativistic viscous hydro: H. Stoecker, W. Greiner, Phys.Rept.137:277-392,1986

Problem: violation of relativistic causality

Parabolic equations for heat conduction and shear diffusion

$\Downarrow$

infinite speed of propagation for thermal and viscous signals

Causal Viscous Hydrodynamics

Causal theory of dissipative fluids:
Grad (1949), Müller (1967), Israel and Stewart (1976–1979)

Expansion in ∂_μ up to 2d order:

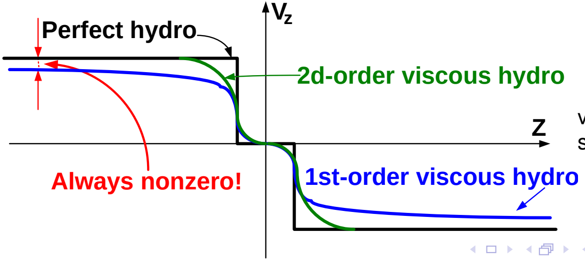
2d-order equations are hyperbolic $\Rightarrow$ causal propagation

Thermodynamics is modified:

Space of thermodynamic variables is extended to include dissipative flows.
Heat flows and viscous pressures are considered as independent variables.

Application to heavy-ion collisions [(1+1)-dimensional simulations]
A. Muronga, Phys.Rev.C69:034903,2004

2d-order results are closer to perfect hydro than 1st-order results.



velocity (v_z) across shock-wave front

Outline

Assumption 2: no 4-vectors

Assumption 2.0: System is not characterized by any 4-vector

Zero approximation in ∂_μ:

$$\mathbf{J}^{\mu}_{\mathbf{charge}} \equiv \mathbf{0} \qquad \mathbf{T}^{\mu\nu} = -\mathbf{g}^{\mu\nu}\mathbf{P}$$

This the case of **cosmological dark energy**

$$-P \equiv \Lambda = \textbf{cosmological constant}$$

$$\Lambda = \langle T^{00} \rangle_{vacuum} > 0$$

Properties of vacuum are indeed independent of reference frame.

Therefore

$$\mathbf{P}_{\mathbf{dark\ energy}} < \mathbf{0}$$

Cosmological dark energy accelerates the Universe expansion.

A good review: V. Sahni, "Dark Matter and Dark Energy", Lect.Notes Phys. 653 (2004) 141, astro-ph/0403324

Assumption 2: two 4-vectors

Assumption 2.2: System is locally characterized by two 4-vectors

Let the first vector be $u^\mu(x)$ = local 4-velocity, normalized as $u_\mu u^\mu = 1$

Physical nature of the other vector is important

- Other vector is **ω^μ-field** as in Walecka $\sigma\omega$ model

$$J^\mu_{bar} = u^\mu n_{bar}, \quad T^{\mu\nu} = (\varepsilon + P)\, u^\mu\, u^\nu - g^{\mu\nu} P + (1/2) g_\omega (J^\mu_{bar}\omega^\nu + J^\nu_{bar}\omega^\mu) + m^2_\omega \omega^\mu\omega^\nu$$

This is **$\sigma\omega$-mean-field hydrodynamics**. Additional field equations are required

$$(\Box + m^2_\sigma)\sigma + dU/d\sigma = g_\sigma \rho, \qquad (\Box + m^2_\omega)\omega^\mu = g_\omega J^\mu.$$

$$\textbf{EoS} \Rightarrow P(x) = P[n_{bar}(x), \varepsilon(x), \sigma(x), \omega^\mu(x)]$$

This is a good model for heavy-ion collisions at $E_{lab} \sim 1$ GeV/nucleon.

Ivanov, Nucl.Phys.A474:669,1987; Russkikh, *et al.*, Nucl.Phys.A572:749,1994

- Other vector is **electromagnetic field A^μ**

$$J^\mu_{bar} = u^\mu n_{bar} \quad J^\mu_{em} = u^\mu n_{em}$$

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu\, u^\nu - g^{\mu\nu} P + (1/2)(J^\mu_{em} A^\nu + J^\nu_{em} A^\mu)$$

Relativistic magnetohydrodynamics ($m^2_{photon} = 0$)

Additional field equation: $\Box A^\mu = J^\mu_{em}$

$$\textbf{EoS} \Rightarrow P(x) = P[n_{bar}(x), n_{em}(x), \varepsilon(x), A^\mu(x)]$$

It is highly desirable to develop such a model in view of predicted strong isotopic dependence of EoS.

Assumption 2.3: System is locally characterized by two or more 4-vectors

- **two vectors are 4-velocities $u_p^\mu(x)$ and $u_t^\mu(x)$: 2-fluid hydrodynamics**

  In plasma physics: electron and ion fluids.

  In nuclear physics: **p**rojectile and **t**arget fluids.
  It takes into account **finite stopping power of nuclear matter**.

  Los Alamos 1978–1986 (Amsden, Harlow, Nix, Clare, Strottman)
  Kurchatov Inst. 1988–1991 **"2-fluid hydro with free-streaming pions"** (Mishustin, Russkikh, Satarov)

- **two 4-velocities and one field vector: 2-fluid mean-field hydrodynamics**
  GSI 1991–1997 (Russkikh, Ivanov, *et al.*)

- **three 4-velocities:**

  3-fluid hydro with instant formation of fireball: Frankfurt University 1993–2000 (Brachmann, Katscher, Dumitru, Rischke, Maruhn, Stöcker, Greiner, Mishustin, Satarov, *et al.*)

  3-fluid hydrodynamics with delayed formation of fireball
  GSI 2003–now (Ivanov, Russkikh, Toneev, Phys. Rev. C 73, 044904 (2006))

This is not the end of the list!
E.g., "chiral hydrodynamics" = hydrodynamics with chiral mean fields for dynamics near a chiral critical point (Paech, Stoecker, A. Dumitru, Phys.Rev.C68:044907,2003)

Target laboratory: **Equation of State**

Targets are interchangeable!

Accelerator department, providing the Heavy-Ion Beam:

Hydro Code

There is a great variety of hydrodynamic models

Particle detection and identification:

Freeze-out and
transforming of Hydro results into Observables

Local thermodynamic equilibrium???

It is absent, when mean-free-path $\sim$ system size:

- **at low incident energies** because of Pauli blocking
  $\Longrightarrow$ **we do not consider them**
- **in small systems** (collisions of light nuclei)
  $\Longrightarrow$ **we do not consider them**
- **at the final stage of the reaction,** $\Longrightarrow$ **freeze-out**
  (there are recipes rather than theory)
- **at the initial stage of the reaction,**
  when nuclei interpenetrate each other
  - Parametrization of initial state ("fireball")
  - Calculation of initial "fireball" within some kinetic model
  - **Multi-fluid description**

Outline

Aims: 3-Fluid Model

- Simulations of A+A collisions at $1 < E_{lab} < 160$ A·GeV (from SIS to SPS)
  proceeding from 2 cold nuclei till particles in the detector

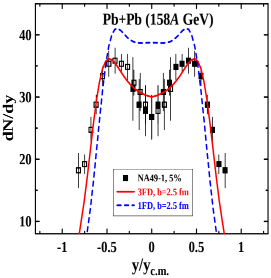
- Tool: **3-Fluid Hydrodynamics**

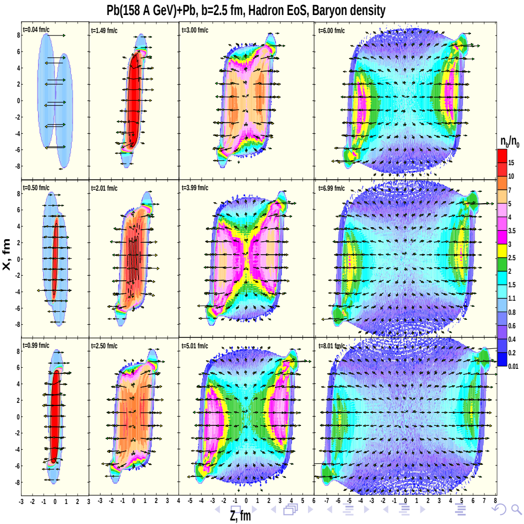
  Ivanov, Russkikh, Toneev, Phys. Rev. C 73, 044904 (2006)

Finite nuclear stopping power

Niclei do not instantly stop each other (like in 1FD).
They interpenetrate each other ⇒ **Finite nuclear stopping power**

Baryon density evolution in reaction plane (3-fluid calculation)





$v_{\parallel} = c \tanh y$
y = rapidity

in nonrelativism $v_{\parallel} = cy$

- Why this tool?
  - Finite stopping power $\Longrightarrow$ **Nonequilibrium**
    $\Rightarrow$ No Conventional Hydrodynamics
  - Weak nonequilibrium $\Rightarrow$ viscosity, thermal conductivity
  - Strong nonequilibrium $\Rightarrow$ **Multi-Fluid Approximation**
  - For **MFA** we need **friction** $\Rightarrow$ **we start from kinetics**

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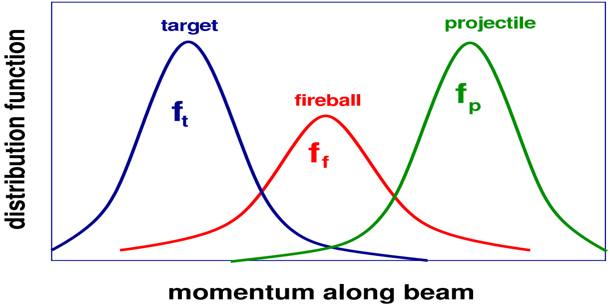
From Kinetics to Multi-Fluids



- Distributions are separated in momentum space
  $\Rightarrow$ **can be associated with different fluids**
- Leading particles carry baryon charge
  $\Rightarrow$ **2 baryon-rich fluids: projectile-like and target-like**
- Produced particles populate mid-rapidity
  $\Rightarrow$ **fireball fluid**
- Intra-fluid equilibration is faster than inter-fluid stopping
  $\Rightarrow$ **local equilibrium in each fluid**

From Kinetics to Multi-Fluids

From single kinetic Eq.

$$p_\mu \partial^\mu f = C(f,f)$$

to set of kinetic Eqs: $f = f_{projectile} + f_{target} + f_{fireball}$

$$\begin{aligned} p_\mu \partial^\mu f_p &= C_p(f_p,f_p) + C_p(f_p,f_t) + C_p(f_p,f_f) \\ p_\mu \partial^\mu f_t &= C_t(f_t,f_t) + C_t(f_p,f_t) + C_t(f_t,f_f) \\ p_\mu \partial^\mu f_f &= C_f(f_f,f_f) + C_f(f_p,f_t) + C_f(f_p,f_f) + C_f(f_t,f_f) \end{aligned}$$

C_α = collision terms
$C_p(f_p,f_p)$, etc. = intra-fluid collision terms $= 0 \Rightarrow f^{(equilib.)}$
Nondiagonal $C_\alpha(f_\beta,f_\gamma)$ (i.e. with $\beta \neq \gamma$) $\Longrightarrow$ **inter-fluid friction**.

and then to 3-fluid Eqs:

Baryon number conservation

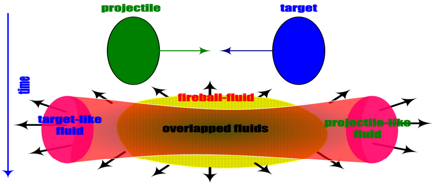
$$\sum_{\text{``baryons''}} \int \frac{d^3p}{p^0} p_\mu \partial^\mu f_\alpha = \partial_\mu J^\mu_\alpha = 0$$

Energy–momentum exchange

$$\sum_{all\ species} \int \frac{d^3p}{p^0} p_\nu p_\mu \partial^\mu f_\alpha = \partial_\mu T^{\mu\nu}_\alpha = \text{Friction} \equiv \sum_{\beta\gamma} \int \frac{d^3p}{p^0} p_\nu C_\alpha(f_\beta,f_\gamma)$$

α = projectile-like or target-like or fireball

3-Fluid Equations



Target-like fluid: $\partial_\mu J_t^\mu = 0$ $\quad \partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$

Leading particles carry bar. charge exchange/emission

Projectile-like fluid: $\partial_\mu J_p^\mu = 0,$ $\quad \partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$

Fireball fluid: $J_f^\mu = 0,$ $\quad \partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$

Baryon-free fluid Source term Exchange

The source term is delayed due to a formation time $\tau \sim 1$ fm/c

Total energy-momentum conservation: $\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$

Each α-fluid is perfect fluid

FOR BARYON-RICH FLUIDS ($\alpha =$ P AND T):

$$\mathbf{J}^{\mu}_{\alpha} = \mathbf{u}^{\mu}_{\alpha}\mathbf{n}_{\alpha} \qquad \mathbf{T}^{\mu\nu}_{\alpha} = (\varepsilon_{\alpha} + \mathbf{P}_{\alpha})\, \mathbf{u}^{\mu}_{\alpha}\, \mathbf{u}^{\nu}_{\alpha} - \mathbf{g}^{\mu\nu}\mathbf{P}_{\alpha}$$

n_{α} = proper baryon density u_{α} = hydro 4-velocity
ε_{α} = proper energy density P_{α} = pressure

FOR FIREBALL FLUID, only thermalized part is of hydrodynamic form:

$$\mathbf{n}_{\mathrm{f}} = 0 \qquad \mathbf{T}^{(\mathbf{eq})\mu\nu}_{\mathbf{f}} = (\varepsilon_{\mathrm{f}} + \mathbf{P}_{\mathrm{f}})\, \mathbf{u}^{\mu}_{\mathrm{f}}\, \mathbf{u}^{\nu}_{\mathrm{f}} - \mathbf{g}^{\mu\nu}\mathbf{P}_{\mathrm{f}}$$

Its evolution is defined by a retarded source term

$$\partial_{\mu} T^{(eq)\mu\nu}_{f}(x) = \int d^4x'\,\delta^4\left(x - x' - U_F(x')\tau\right)\left[F^{\nu}_{pt}(x') + F^{\nu}_{tp}(x')\right] - F^{\nu}_{fp}(x) - F^{\nu}_{ft}(x)$$

where $\tau =$ **formation time**, and

$$U^{\nu}_{F}(x') = \left\{u^{\nu}_{p}(x') + u^{\nu}_{t}(x')\right\} / |u_p(x') + u_t(x')|$$

is a free-streaming 4-velocity of the produced fireball matter.

The residual, free-streaming part of fireball matter

$$T^{(fs)\mu\nu}_{f} = T^{\mu\nu}_{f} - T^{(eq)\mu\nu}_{f}$$

Each α-fluid is perfect fluid

FOR BARYON-RICH FLUIDS ($\alpha =$ P AND T):

$$\mathbf{J}^{\mu}_{\alpha} = \mathbf{u}^{\mu}_{\alpha}\mathbf{n}_{\alpha} \qquad \mathbf{T}^{\mu\nu}_{\alpha} = (\varepsilon_\alpha + \mathbf{P}_\alpha)\,\mathbf{u}^{\mu}_{\alpha}\,\mathbf{u}^{\nu}_{\alpha} - \mathbf{g}^{\mu\nu}\mathbf{P}_\alpha$$

n_α = proper baryon density u_α = hydro 4-velocity
ε_α = proper energy density P_α = pressure

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The residual, free-streaming part of fireball matter

$$T^{(fs)\mu\nu}_f = T^{\mu\nu}_f - T^{(eq)\mu\nu}_f$$

Delayed Formation of Fireball

Each α-fluid is perfect fluid

FOR BARYON-RICH FLUIDS ($\alpha =$P AND T):

$$\mathbf{J}^{\mu}_{\alpha} = \mathbf{u}^{\mu}_{\alpha}\mathbf{n}_{\alpha} \qquad \mathbf{T}^{\mu\nu}_{\alpha} = (\varepsilon_{\alpha} + \mathbf{P}_{\alpha})\, \mathbf{u}^{\mu}_{\alpha}\, \mathbf{u}^{\nu}_{\alpha} - \mathbf{g}^{\mu\nu}\mathbf{P}_{\alpha}$$

n_{α} = proper baryon density u_{α} = hydro 4-velocity
ε_{α} = proper energy density P_{α} = pressure

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$$\mathbf{n}_{\mathrm{f}} = 0 \qquad \mathbf{T}^{(\mathrm{eq})\mu\nu}_{\mathrm{f}} = (\varepsilon_{\mathrm{f}} + \mathbf{P}_{\mathrm{f}})\, \mathbf{u}^{\mu}_{\mathrm{f}}\, \mathbf{u}^{\nu}_{\mathrm{f}} - \mathbf{g}^{\mu\nu}\mathbf{P}_{\mathrm{f}}$$

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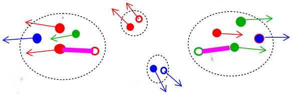
The residual, free-streaming part of fireball matter

$$T^{(fs)\mu\nu}_{f} = T^{\mu\nu}_{f} - T^{(eq)\mu\nu}_{f}$$

Formation Time

In hadronic phase

delay due to string formation, $\tau \sim 1$ fm/c



In quark-gluon phase

Two-stage production of quark–gluon plasma:

(1) First a coherent color field is produced.

(2) Then it decays into incoherent color-field flactuations – quarks and gluons.

$\tau =$ time between field production and its decay.

Outline

Friction

Projective-Target Friction:

$$\mathbf{F}^{\nu}_{\mathbf{pt}} = \rho_{\mathbf{p}} \rho_{\mathbf{t}} \left[\left(\mathbf{u}^{\nu}_{\mathbf{p}} - \mathbf{u}^{\nu}_{\mathbf{t}} \right) \mathbf{D}_{\mathbf{P}} + \left(\mathbf{u}^{\nu}_{\mathbf{p}} + \mathbf{u}^{\nu}_{\mathbf{t}} \right) \mathbf{D}_{\mathbf{E}} \right]$$

⇑ heating

⇑ fireball production

ρ_α = scalar density of α fluid

$D_{P/E}$ = transport coeff. in terms of cross sections

$D_E \neq 0$ only when $\sigma(NN \rightarrow NN + \textit{secondary particles}) \neq 0$

Projective(Target)-Fireball Friction:

Absorption of a fireball matter by baryon-rich fluids:

$\mathbf{N_p}\pi_\mathbf{f} \rightarrow \mathbf{(Baryon\ Resonance)_p} \rightarrow \mathbf{N_p}\pi_\mathbf{p}$

$$\mathbf{F}^{\nu}_{\mathbf{fp}} = \mathbf{D}_{\mathbf{fp}} \frac{\mathbf{T}^{(\mathbf{eq})0\nu}_{\mathbf{f}}}{\mathbf{u}^{0}_{\mathbf{f}}} \rho_{\mathbf{p}}$$

D_{fp} = transport coeff. in terms of cross sections

PROJECTIVE-TARGET FRICTION:

$$\mathbf{F}^{\nu}_{\mathbf{pt}} = \rho_{\mathbf{p}}\rho_{\mathbf{t}}\left[\left(\mathbf{u}^{\nu}_{\mathbf{p}} - \mathbf{u}^{\nu}_{\mathbf{t}}\right)\mathbf{D}_{\mathbf{P}} + \left(\mathbf{u}^{\nu}_{\mathbf{p}} + \mathbf{u}^{\nu}_{\mathbf{t}}\right)\mathbf{D}_{\mathbf{E}}\right]$$

⇑ heating ⇑ fireball production

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PROJECTIVE(TARGET)-FIREBALL FRICTION:

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D_{fp} = transport coeff. in terms of cross sections

In fact:

$D_{P/E}$ = in terms of only proton-proton cross sections (L.M. Satarov, (1990))

$D_{fireball-projectile(target)}$ = in terms of only pion-nucleon resonance cross sections (V.N. Russkikh, et al., (2004))

Uncertainties in Friction:

- poor usage of various cross sections
- medium effects
- multiparticle collisions

Therefore:

PROJECTIVE-TARGET FRICTION:

$$F^{\nu}_{pt} = \xi^2(s_{pt})\rho_p\rho_t\left[\left(u^{\nu}_p - u^{\nu}_t\right)D_P + \left(u^{\nu}_p + u^{\nu}_t\right)D_E\right]$$

$\xi^2(s_{pt})$ = tuning factor for Friction

$s_{pt} = m_N^2\left(u^{\nu}_p + u^{\nu}_t\right)^2$ = mean invariant energy squared (rather than E_{lab}!)

Friction Fit

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Outline

Important Physical Input

- Equation of State (EoS)
- Friction: $\xi^2(s_{pt})$ = tuning factor for Friction
- Formation Time (τ)
- Freeze-out energy-density (ε_{frz})

  It can be different for different species
- Coalescence coefficients for fragments

(!) Protons and neutrons are not distinguished!

Hadronic EoS (Galitsky&Mishustin 1979)

Energy density: $$\varepsilon(n_B,T) = \underbrace{\varepsilon_{gas}(n_B,T)}_{\text{hadron gas in mean field}} + \underbrace{W(n_B)}_{\text{mean field}}$$

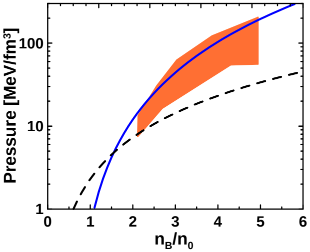
Pressure: $$P(n_B,T) = \underbrace{P_{gas}(n_B,T)}_{\text{hadron gas in mean field}} \underbrace{+ n_B \frac{dW(n_B)}{dn_B} - W}_{\text{mean field}}$$

$T = 0$

$$\frac{\varepsilon(n_B,0)}{m_N n_0} = a\left(\frac{n_B}{n_0}\right)^{5/3} - b\left(\frac{n_B}{n_0}\right)^{2} + c\left(\frac{n_B}{n_0}\right)^{7/3}$$

$\varepsilon(n_B,0))$ **saturates the cold matter at**
$n_0 =$ **0.15 fm^{-3} and**
$\varepsilon(n_0, T=0)/n_0 - m_N =$ **-16 MeV,**
and provides incompressibility $K =$ 210 MeV.

Astrophysical constraints on EoS
T. Klahn et al., nucl-th/0602038:
hard EoS's are preferable at $T = 0$.



Danielewicz, Lacey, Lynch Science 298, 1592 (2002)

Freeze-out

- Criterion: $\underbrace{Local}_{(at\ x\ position)}$ $\underbrace{proper}_{(in\ local\ rest\ frame)}$ $\underbrace{energy\ \ density\ \ of\ \ matter}_{(summed\ over\ all\ fluids)}$

  is less than ε_{frz}

- Freeze-out shock: T_{hydro} and μ_{hydro} are mapped to T_{gas} and μ_{gas} proceeding from baryon, energy and momentum conservations.

Energy accumulated in “mean fields” is released.

- Freeze-out *a là* Milekhin

$$E\frac{dN}{d^3p} = \int f_{gas}(x,p)p^\mu d\sigma_\mu, \qquad d\sigma_\mu = u_\mu (d^3x)_{proper}$$

u_μ =hydro 4-velocity proper = in the frame, where u_μ = (1,0,0,0)

- No problem with Cooper-Frye’s negative contributions into particle numbers
- Baryon number, energy and momentum are exactly conserved!
- Problem of shadowing still persists
- **Further study of Freeze-out is needed!**

Freeze-out

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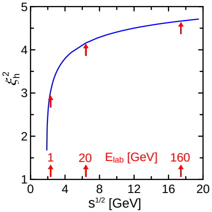
- ○ Baryon number, energy and momentum are exactly conserved!
- ○ Problem of shadowing still persists
- ○ **Further study of Freeze-out is needed!**

Physical Input for Hadronic Scenario

- Galitsky–Mishustin Hadronic EoS
- **Enhanced Friction** (fitted to observed stopping power)



$$F_{pt}^{\nu} = \xi_h^2(s)\rho_p\rho_t \left[\left(u_p^{\nu} - u_t^{\nu}\right) D_P + \left(u_p^{\nu} + u_t^{\nu}\right) D_E\right]$$

$$s = m_N^2 \left(u_p^{\nu} + u_t^{\nu}\right)^2 = \text{mean inv. energy squared}$$

$$\xi_h^2(s) = 1 + 3 \left[\ln\left(s/(2m_N)^2\right)^{1/2}\right]^{1/4}$$

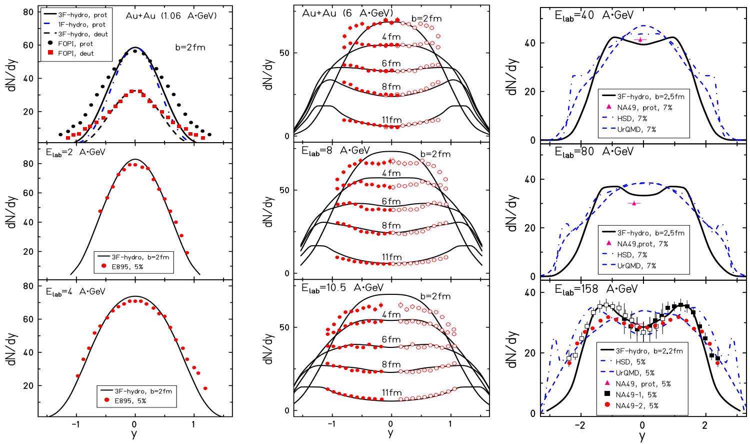
- Formation Time $\tau = \mathbf{2\ fm/c}$
  (affects pion production only at $E_{lab} > 30$ A·GeV)
  - $\tau_{particle} < \tau = \int d^3p\, \tau_{particle}\gamma_{particle} f(p) \Big/ \int d^3p\, f(p)$

  If $T \approx 100$ MeV, $\tau_{particle} \approx 1$ fm/c.
- Freeze-out: $\varepsilon_{frz} \approx \mathbf{0.4\ GeV/fm^3}$ (mainly affects m_T spectra)
  (the same for all species, for chemical and thermal freeze-out)

Outline

Proton and $(p - \bar{p})$ Rapidity Distributions

SIS AGS SPS



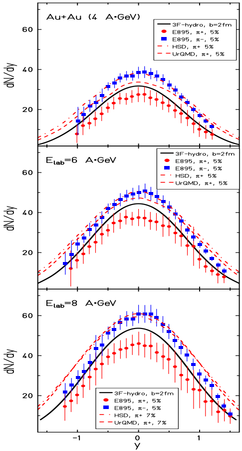
$$y = \frac{1}{2} \ln \frac{E + p_{\parallel}}{E - p_{\parallel}} = \text{rapidity}$$

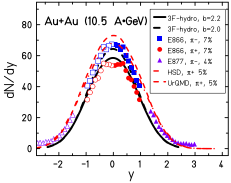
For comrarison

Blue: Kinetic Models (HSD and UrQMD)

H. Weber, et al., PR **C67** (2003) 014904

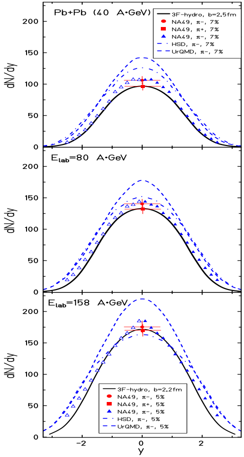
Pion Rapidity Spectra





Red: Kinetic models

Weber, et al., (2003)



3FD: $\pi = (\pi_+ + \pi_0 + \pi_-)/3$

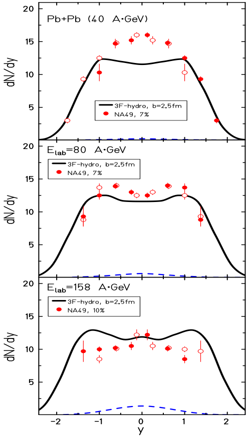
Blue: Kinetic models

Weber, et al., (2003)

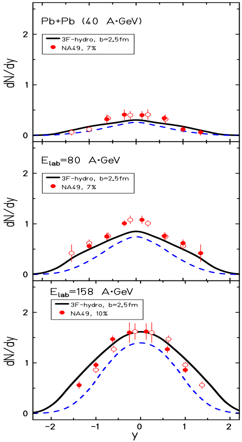
3-fluid hydro can compete with kinetics!

Rare-Particle Rapidity Distributions

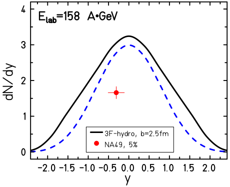
$\Lambda + \Sigma^0$



$\bar{\Lambda} + \bar{\Sigma}^0$



$\bar{p}$



NA49: NP **A661** (1999) 45c

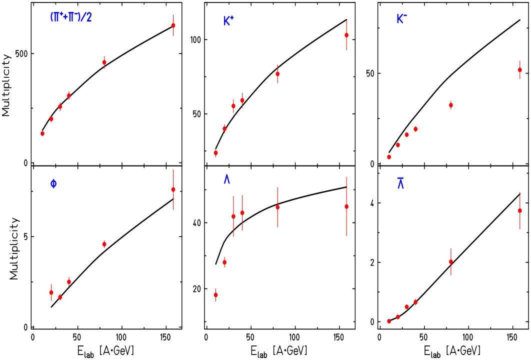
NA49: nucl-ex/0311024

dashed line = contribution from the fireball fluid

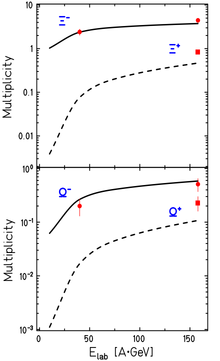
Fireball fluid really works!

Multiplicities

non-strange strange multi-strange



as summarized by F.Becattini, M.Gazdzicki et al., hep-ph/0310049



NA49: nucl-ex/0409004

K^- are essentially absorbed after freeze-out: $K^- N \to \pi \Sigma^\pm \to N\pi\pi$

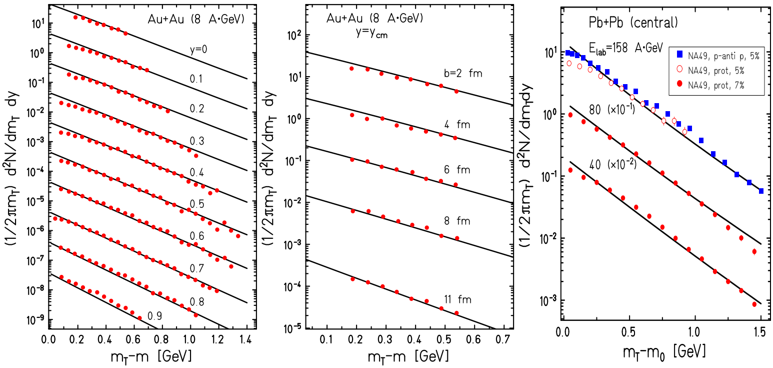
Post-freeze-out kinetics is needed!

Outline

Transverse-Mass Spectra

$$m_T = (m^2 + p_T^2)^{1/2} = \text{transverse mass}$$

AGS SPS



E917: PRL **86** (2001) 1970

NA49: PRL **82** (1999) 2471
NA49: NP **A715** (2003) 166c

For better reproduction **Post-freeze-out kinetics is needed!**

Teaney, Lauret, and E.V. Shuryak, nucl-th/0110037

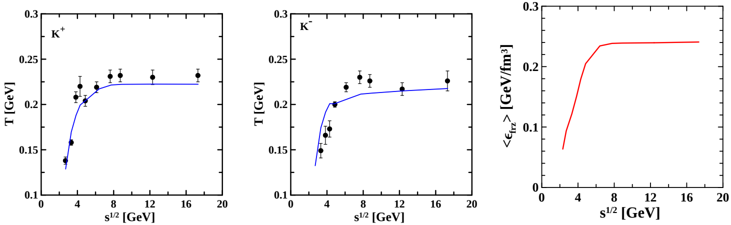
Transverse temperatures of kaons

Ivanov, Russkikh, nucl-th/0607070

$s^{1/2} = [2m_N(m_N + E_{lab})]^{1/2}$ = incident energy in c.m. frame

$$\frac{d^2N}{m_T\, dm_T\, dy} \propto \exp\left(-m_T/\mathbf{T}\right),$$

$\langle \epsilon_{\mathbf{frz}} \rangle$ = **actual** freeze-out energy density



Freeze-out criterion: $\varepsilon < \varepsilon_{frz} = 0.4$ GeV/fm^3

Freeze-out is similar to liquid–gas transition:
When matter is dense $\max(\varepsilon) > \varepsilon_{frz}$, it **evaporates** from the surface.
When it is over-rarefied $\max(\varepsilon) < \varepsilon_{frz}$, it **explodes** transforming into frozen-out gas.

Natural value of actual ϵ_{frz} is $\epsilon_{frz} \approx \varepsilon_{frz}/2$, if ε_{frz} is achieved at the surface.
However, at **low** E_{lab} (i.e. $s^{1/2}$), ε_{frz} is not achieved $\Rightarrow \epsilon_{frz} < \varepsilon_{frz}/2$
At **high** E_{lab} (i.e. $s^{1/2}$), ε_{frz} is achieved at the surface $\Rightarrow \epsilon_{frz} \approx \varepsilon_{frz}/2$

Heavy systems really reveal hydrodynamic behavior.

Outline

Flow

p_x = in-plane transverse momentum
p_y = out-of-plane transv. momentum
$p_T = (p_x^2 + p_y^2)^{1/2}$ = total tr. mom.

Transverse momentum per hadron

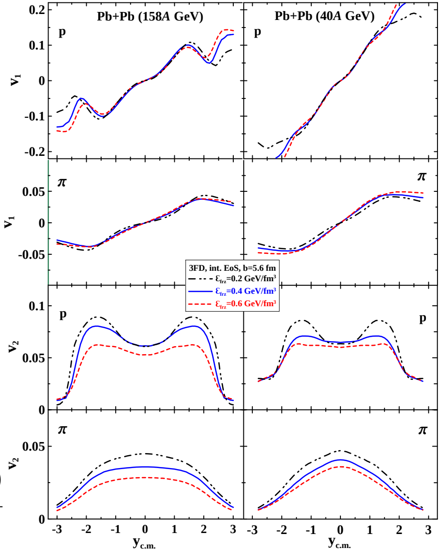
$$\langle p_x \rangle (y) = \frac{\int d^2p_T \; p_x \; (dN/dy \; d^2p_T)}{\int d^2p_T \; (dN/dy \; d^2p_T)}$$

Directed flow:

$$v_1(y) = \frac{\int d^2p_T \; \frac{p_x}{p_T} \; (dN/dy \; d^2p_T)}{\int d^2p_T \; (dN/dy \; d^2p_T)}$$

Elliptic flow:

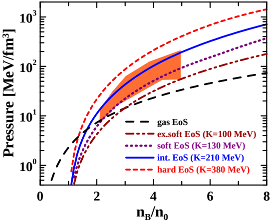
$$v_2(y) = \frac{\int d^2p_T \; \frac{p_x^2 - p_y^2}{p_T^2} \; (dN/dy \; d^2p_T)}{\int d^2p_T \; (dN/dy \; d^2p_T)}$$

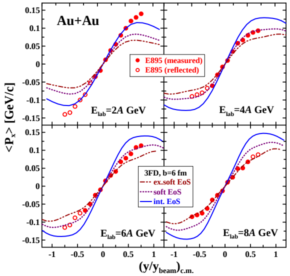


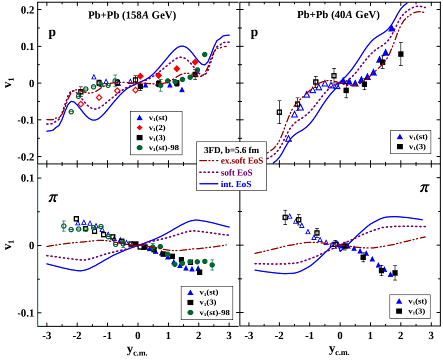
$\langle p_x \rangle$ and v_1 reflect early-stage dynamics.
v_2 is only low sensitive to freeze-out.

Directed Flow: softening EoS

Russkikh, Ivanov, nucl-th/0606007







Problem of shadowing at freeze-out

⇓

Poor reproduction of pion v_1

Directed flow requires softer EoS at high incident energies!

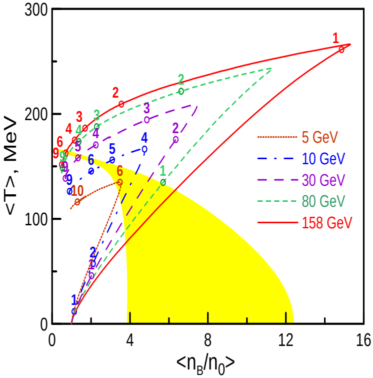
Phase transition to quark–gluon matter???

Outline

Phase Trajectories

What baryon densities (n_B) and temperatures (T) are achieved in central Pb+Pb collisions?

$\langle n_B \rangle$ and $\langle T \rangle$ are values averaged over system.



Phase-separation region only to guide an eye: No phase transition in GM EoS!

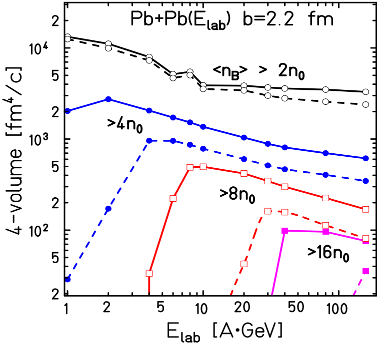
Phase-separation region:
MIT-bag model + hadronic gas
(Toneev, et al., Eur. Phys. J. **C32**, 399 (2004))

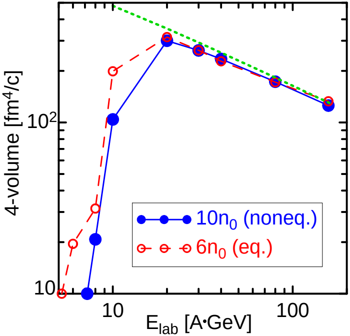
High $\langle n_B \rangle$ and $\langle T \rangle$ values hint at possibility of phase transition.

Invariant 4-Volume

How long and in which volume a quantity Q exceeds a Q_0 value?

$$V_4(Q) = \int d^4x\, \Theta(q - Q)$$





Solid lines: $Q = n_B =$ baryonic density

Dashed lines: $Q = n_B^{(eq)} =$ baryonic density of thermalized matter

$E_{lab} \sim 20$ **A·GeV is preferable for production of thermalized matter with** $n_B^{eq} > 6n_0$

This is the energy of future GSI accelerator.

What have we learned about nature?

- **Hadronic EoS** reasonably reproduces observables (except flow)

However

- **Directed flow** requires softer EoS at high incident energies: **hint at possibility of phase transition to quark-gluon phase?**
- High $\langle n_B \rangle$ and $\langle T \rangle$ values: **this transition is possible**

Other conclusions:

- Transverse-Mass Spectra: **Heavy systems really reveal hydrodynamic behavior.**
- $E_{lab} \sim 20$ A·GeV is preferable for production of dense thermalized matter with $n_B^{eq} > 6 n_0$

Outline

Estimate of Projective-Fireball Friction

Let p-fluid consist only of nucleons, and fireball fluid, of only pions.

Collision term $\mathbf{N_p}\pi_\mathbf{f} \longrightarrow$ **(Baryon Resonance)**

$$C_f(f_p, f_f) = -\int \frac{d^3q}{q_0} W^{N\pi\to R}(s)\, f_f^{(eq)}(p) f_p(q), \qquad s = (p+q)^2$$

$W^{N\pi\to R} = \frac{1}{2}\sqrt{(s - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2}\; \sigma_{tot}^{N\pi\to R} = R$ production rate

$\sigma_{tot}^{N\pi\to R}(s) =$ pion–nucleon cross sections.

Multiplying C_f by p^ν and integrating,

$$F_{fp}^\nu(x) = \int \frac{d^3q}{q_0}\frac{d^3p}{p_0} p^\nu W^{N\pi\to R}(s)\, f_f^{(eq)}(p)\, f_p(q)$$
$$\simeq \frac{W^{N\pi\to R}(s_{fp})}{m_\pi u_f^0}\left(\int \frac{d^3q}{q_0} f_p(q)\right)\left(\int \frac{d^3p}{p_0} p^0 p^\nu f_f^{(eq)}(p)\right) = D_{fp}\frac{T_f^{(eq)0\nu}}{u_f^0}\rho_p,$$

we substituted $p^0 \approx \langle p^0\rangle = m_\pi u_f^0$ and $s \approx s_{fp} = (m_\pi u_f + m_N u_p)^2$.

$$\textbf{Transport coefficient} = \mathbf{D_{fp}} = \frac{\mathbf{W^{N\pi\to R}(s_{fp})}}{\mathbf{m_N m_\pi}} = \mathbf{V_{rel}^{fp}\, \sigma_{tot}^{N\pi\to R}(s_{fp})}.$$

$V_{rel}^{fp} = [(s_{fp} - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2]^{1/2}/(2m_N m_\pi) =$ p-fireball relative velocity

Outline

Numerics

- **Particle-in-Cell Method for Hydro**
  - Roshal'&Russkikh (1982)
  - Euler stage: transfer due to pressure gradients (on a grid)
  - Lagrange stage: transfer due to drift of the matter ($\partial_\mu J_t^\mu$, $\partial_\mu T_t^{\mu\nu}$, etc.) is simulated by test-particle motion
  - Computation in the c.m. frame
- **Careful choice of the grid** to avoid numerical diffusion:
  - Number of cells per Lorentz contracted diameter > 30
  - $\Delta x : \Delta y : \Delta z = 1 : 1 : 1$ is best of all (Waldhauser, et al., 1992)
  - $\Delta x : \Delta t = 3.5$ is best of all (1D simulations)
  - Number of test-particles per cell > 3
    $\approx 10^7$ test-particles for Pb(158 GeV/nucl.) + Pb
- **Required Resources:**
  - Hydro: 30 h of CPU time at AMD Opteron 64 bit 2.0 GH and 7.5 GB for central collision Pb(158 GeV/nucl.) + Pb
  - Hydro: 10 h of CPU time at P4 2.6 GH and 1.5 GB memory for central collision Au(10 GeV/nucl.) + Au
  - Converting the hydro data into observables: few hours of CPU time