Low-momentum π-meson production from evolving quark condensate

A. Filatov

Theoretical and Mathematical Physics Department,
Saratov State University

We'll discuss some features of time dependent fields on the basis of kinetic approach.

The low-momentum problem in the π-meson physics will be considered as an example.

Some kinetics of scalar filed with time dependent mass

The equation of motion

$$[\partial_\mu \partial^\mu + m^2(t)]\varphi(x) = 0.$$

Some kinetics of scalar filed with time dependent mass

The equation of motion

$$[\partial_\mu \partial^\mu + m^2(t)]\varphi(x) = 0.$$

The Lagrange function is

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2(t)\phi^2.$$

Some kinetics of scalar filed with time dependent mass

The equation of motion

$$[\partial_\mu \partial^\mu + m^2(t)]\varphi(x) = 0.$$

The Lagrange function is

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2(t)\phi^2.$$

From the assumption of space homogeneity

$$\varphi(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} e^{i\mathbf{px}} \varphi(\mathbf{p}, t).$$

Some kinetics of scalar filed with time dependent mass

The equation of motion

$$[\partial_\mu \partial^\mu + m^2(t)]\varphi(x) = 0.$$

The Lagrange function is

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2(t)\phi^2.$$

From the assumption of space homogeneity

$$\varphi(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} e^{i\mathbf{px}} \varphi(\mathbf{p}, t).$$

Then the oscillator-type equation of motion is follows

$$\ddot{\varphi}^{(\pm)} + \omega^2(\mathbf{p}, r)\varphi^{(\pm)} = 0,$$

where $\omega(\mathbf{p}, t) = \sqrt{m^2(t) + \mathbf{p}^2}$ is the one-particle energy. The symbols $(\pm)$ correspond to the positive and negative frequency solution, defined by its free asymptotics in the infinite past (future)

$$\varphi^{(\pm)}(\mathbf{p}, t \to \mp\infty) \sim e^{\pm i\omega_\mp t},$$

where $\omega_\mp = \sqrt{m_\mp^2 + \mathbf{p}^2}$ and asymptotic mass is $m_\mp = \lim\limits_{t\to\mp\infty} m(t)$.

Some kinetics of scalar filed with time dependent mass

Quasi particle representation

$$\varphi(x)=\frac{1}{\sqrt{V}}\sum_{\mathbf{p}}\frac{1}{\sqrt{2\omega(\mathbf{p},t)}}e^{i\mathbf{px}}\left\{a^{(-)}(\mathbf{p},t)+a^{(+)}(-\mathbf{p},t)\right\}$$

Some kinetics of scalar filed with time dependent mass

Quasi particle representation

$$\varphi(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} \frac{1}{\sqrt{2\omega(\mathbf{p},t)}} e^{i\mathbf{px}} \left\{ a^{(-)}(\mathbf{p},t) + a^{(+)}(-\mathbf{p},t) \right\}$$

and the generalized momenta

$$\pi(x) = -\frac{i}{\sqrt{V}} \sum_{\mathbf{p}} \sqrt{\frac{\omega(\mathbf{p},t)}{2}} e^{i\mathbf{px}} \left\{ a^{(-)}(\mathbf{p},t) - a^{(+)}(\mathbf{p},t) \right\},$$

where $a^{\pm}(\mathbf{p},t)$ are the positive and negative frequency amplitudes.
This constraints are the basis of nonperturbative dynamics.

Some kinetics of scalar filed with time dependent mass

Quasi particle representation

$$\varphi(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} \frac{1}{\sqrt{2\omega(\mathbf{p},t)}} e^{i\mathbf{px}} \left\{ a^{(-)}(\mathbf{p},t) + a^{(+)}(-\mathbf{p},t) \right\}$$

and the generalized momenta

$$\pi(x) = -\frac{i}{\sqrt{V}} \sum_{\mathbf{p}} \sqrt{\frac{\omega(\mathbf{p},t)}{2}} e^{i\mathbf{px}} \left\{ a^{(-)}(\mathbf{p},t) - a^{(+)}(\mathbf{p},t) \right\},$$

where $a^{\pm}(\mathbf{p},t)$ are the positive and negative frequency amplitudes.
This constraints are the basis of nonperturbative dynamics.

The substitution into the Hamiltonian

$$H(t) = \frac{1}{2} \int d^3x \{ \pi^2(x) + [\Delta\varphi(x)]^2 + m^2(t)\varphi^2(x) \}$$

Some kinetics of scalar filed with time dependent mass

Quasi particle representation

$$\varphi(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} \frac{1}{\sqrt{2\omega(\mathbf{p},t)}} e^{i\mathbf{px}} \left\{ a^{(-)}(\mathbf{p},t) + a^{(+)}(-\mathbf{p},t) \right\}$$

and the generalized momenta

$$\pi(x) = -\frac{i}{\sqrt{V}} \sum_{\mathbf{p}} \sqrt{\frac{\omega(\mathbf{p},t)}{2}} e^{i\mathbf{px}} \left\{ a^{(-)}(\mathbf{p},t) - a^{(+)}(\mathbf{p},t) \right\},$$

where $a^{\pm}(\mathbf{p},t)$ are the positive and negative frequency amplitudes.
This constraints are the basis of nonperturbative dynamics.

The substitution into the Hamiltonian

$$H(t) = \frac{1}{2} \int d^3x \{ \pi^2(x) + [\Delta\varphi(x)]^2 + m^2(t)\varphi^2(x) \}$$

leads to diagonal form, which corresponds to the QPR,

$$H(t) = \frac{1}{2} \sum_{\mathbf{p}} \omega(\mathbf{p},t) \left\{ a^{(+)}(\mathbf{p},t)a^{(-)}(\mathbf{p},t) + a^{(-)}(\mathbf{p},t)a^{(+)}(\mathbf{p},t) \right\}$$

Some kinetics of scalar filed with time dependent mass

The relevant equations of motion

$$\dot{a}^{(\pm)}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)a^{(\mp)}(-\mathbf{p},t) \mp i\omega(\mathbf{p},t)a^{(\pm)}(\mathbf{p},t),$$

where

$$\Delta(\mathbf{p},t) = \frac{\dot{\omega}(\mathbf{p},t)}{\omega(\mathbf{p},t)} = \frac{m(t)\dot{m}(t)}{\omega^2(\mathbf{p},t)}$$

is the transition amplitude between states with positive and negative energies.

Some kinetics of scalar filed with time dependent mass

The relevant equations of motion

$$\dot{a}^{(\pm)}(\mathbf{p}, t) = \frac{1}{2}\Delta(\mathbf{p}, t)a^{(\mp)}(-\mathbf{p}, t) \mp i\omega(\mathbf{p}, t)a^{(\pm)}(\mathbf{p}, t),$$

where

$$\Delta(\mathbf{p}, t) = \frac{\dot{\omega}(\mathbf{p}, t)}{\omega(\mathbf{p}, t)} = \frac{m(t)\dot{m}(t)}{\omega^2(\mathbf{p}, t)}$$

is the transition amplitude between states with positive and negative energies.

The canonical commutation relation

$$[\varphi(x), \pi(x')]_{t=t'} = i\delta(x - x')$$

Some kinetics of scalar filed with time dependent mass

The relevant equations of motion

$$\dot{a}^{(\pm)}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)a^{(\mp)}(-\mathbf{p},t) \mp i\omega(\mathbf{p},t)a^{(\pm)}(\mathbf{p},t),$$

where

$$\Delta(\mathbf{p},t) = \frac{\dot{\omega}(\mathbf{p},t)}{\omega(\mathbf{p},t)} = \frac{m(t)\dot{m}(t)}{\omega^2(\mathbf{p},t)}$$

is the transition amplitude between states with positive and negative energies.

The canonical commutation relation

$$[\varphi(x), \pi(x')]_{t=t'} = i\delta(x - x')$$

forms the standard commutation relation for time dependent creation and annihilation operators

$$[a^{(-)}(\mathbf{p},t), a^{(+)}(\mathbf{p}',t)] = \delta_{\mathbf{p}\mathbf{p}'}$$

Some kinetics of scalar filed with time dependent mass

The corresponding Hamilton operator will be equal

$$H(t) = \sum_{\mathbf{p}} \omega(\mathbf{p}, t) \left\{ a^{(+)}(\mathbf{p}, t) a^{(-)}(\mathbf{p}, t) + \frac{1}{2} \right\}$$

The equation of motion can be rewritten as the Heisenberg-type equation

$$\dot{a}^{(\pm)}(\mathbf{p}, t) = \frac{1}{2} \Delta(\mathbf{p}, t) a^{(\mp)}(-\mathbf{p}, t) + i \left[H(t), a^{(\pm)}(\mathbf{p}, t) \right] .$$

Kinetic equation

The quasiparticle distribution function for the space homogeneous case

$$f(\mathbf{p},t) =< 0 \mid a^{(+)}(\mathbf{p},t)a^{(-)}(\mathbf{p},t) \mid 0 > .$$

Kinetic equation

The quasiparticle distribution function for the space homogeneous case

$$f(\mathbf{p},t) =< 0 \mid a^{(+)}(\mathbf{p},t)a^{(-)}(\mathbf{p},t) \mid 0 > .$$

Using the equation of motion

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\left\{f^{(+)}(\mathbf{p},t) + f^{(-)}(\mathbf{p},t)\right\},$$

where auxiliary correlation functions is introduced

$$f^{(+)}(\mathbf{p},t) =< 0 \mid a^{(+)}(\mathbf{p},t)a^{(+)}(-\mathbf{p},t) \mid 0 >,$$
$$f^{(-)}(\mathbf{p},t) =< 0 \mid a^{(-)}(-\mathbf{p},t)a^{(-)}(\mathbf{p},t) \mid 0 > .$$

Kinetic equation

The quasiparticle distribution function for the space homogeneous case

$$f(\mathbf{p},t) =< 0 \mid a^{(+)}(\mathbf{p},t)a^{(-)}(\mathbf{p},t) \mid 0 > .$$

Using the equation of motion

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\left\{f^{(+)}(\mathbf{p},t) + f^{(-)}(\mathbf{p},t)\right\},$$

where auxiliary correlation functions is introduced

$$f^{(+)}(\mathbf{p},t) =< 0 \mid a^{(+)}(\mathbf{p},t)a^{(+)}(-\mathbf{p},t) \mid 0 >,$$
$$f^{(-)}(\mathbf{p},t) =< 0 \mid a^{(-)}(-\mathbf{p},t)a^{(-)}(\mathbf{p},t) \mid 0 > .$$

Finally, the kinetic equation in integral form is

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\int\limits_{t_0}^{t} dt'\Delta(\mathbf{p},t)[1+2f(\mathbf{p},t)]\cos[2\theta(\mathbf{p};t,t')],$$

where the dynamical phase is equal

$$\theta(\mathbf{p};t,t') = \int\limits_{t'}^{t} d\tau\omega(\mathbf{p},\tau).$$

Assumptions:

The following mechanisms are taken into account:

Assumptions:

The following mechanisms are taken into account:

- σ-mesons decaying into 2π-mesons;

Assumptions:

The following mechanisms are taken into account:

- σ-mesons decaying into 2π-mesons;
- inertial mechanism of σ-meson vacuum production stipulated by fast change of its masses;

Assumptions:

The following mechanisms are taken into account:

- σ-mesons decaying into 2π-mesons;
- inertial mechanism of σ-meson vacuum production stipulated by fast change of its masses;
- and the cooling of the expanding system.

Assumptions:

The following mechanisms are taken into account:

- σ-mesons decaying into 2π-mesons;
- inertial mechanism of σ-meson vacuum production stipulated by fast change of its masses;
- and the cooling of the expanding system.

Limitations

We use the limitations:

Limitations

We use the limitations:

- the masses of σ-mesons are taken in framework of NJL model.

We use the limitations:

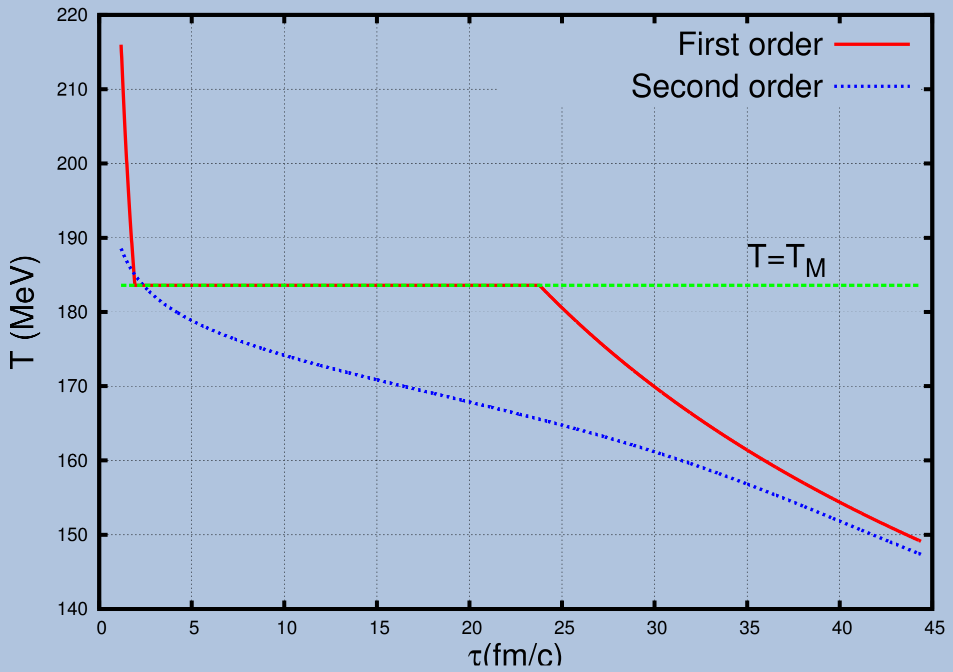
- the masses of σ-mesons are taken in framework of NJL model;

- the decay rate $\Gamma_{\sigma\rightarrow\pi\pi}$ is calculated with zero density approximations in framework of σ-mesons.

Scenario

We use simple hydrodynamic model (based on the Bjorken's scenario) for description the evolution of the system from initial stage to freeze out.

$$\Rightarrow \; T(t)$$



NJL model $\Rightarrow\ M_\sigma(T)$

and hence $M_\sigma(T(t))$

The kinetics of $\sigma - \pi$ subsystems

The kinetic equation for π-mesons:

$$\dot{f}_\pi = I_\pi^{ex} + I_\pi^{\sigma\to\pi\pi},$$

where

$$I_\pi^{ex} = \dot{f}_\pi^{eq}.$$

where f_π^{eq} is an equilibrium Bose-Einstein distribution and

$$I_\pi^{\sigma\to\pi\pi} = \int \frac{d\mathbf{p_1}d\mathbf{p_1}}{\omega_\pi(\mathbf{p_1},t)\omega_\sigma(\mathbf{p_2},t)}\Gamma_{\sigma\to\pi\pi}(\vec{p}_2,\mathbf{p},\mathbf{p}_1;t)\times$$

$$\times f_\sigma(\mathbf{p}_2,t)[1+f_\pi(\mathbf{p}_1,t)][1+f_\pi(\mathbf{p},t)]\times$$

$$\times\delta\{\omega_\sigma(\mathbf{p}_2,t)-\omega_\pi(\mathbf{p},t)-\omega_\pi(\mathbf{p}_1,t)\}\delta(\mathbf{p}_2-\mathbf{p}-\mathbf{p}_1)$$

is the coming term in π-mesons subsystem and

$$\omega_\alpha = \sqrt{M_\alpha^2+\mathbf{p}_\alpha^2}$$

where $\alpha = \sigma$ or π

The kinetic equation for σ-mesons:

$$\dot{f}_\sigma = I_\sigma^{ex} + I_\sigma^{\sigma\to\pi\pi} + I_\sigma^{vac},$$

where

$$I_\sigma^{ex} = \dot{f}_\sigma^{eq}.$$

where f_σ^{eq} is an equilibrium Bose-Einstein distribution and

$$I_\sigma^{\sigma\to\pi\pi} = -\int \frac{d\mathbf{p_1}d\mathbf{p_1}}{\omega_\pi(\mathbf{p_1},t)\omega_\pi(\mathbf{p_2},t)}\Gamma_{\sigma\to\pi\pi}(\mathbf{p},\mathbf{p}_1,\mathbf{p}_2;t)\times$$

$$\times f_\sigma(\mathbf{p},t)[1+f_\pi(\mathbf{p}_1,t)][1+f_\pi(\mathbf{p}_2,t)]\times$$

$$\times\delta\{\omega_\sigma(\mathbf{p},t)-\omega_\pi(\mathbf{p}_1,t)-\omega_\pi(\mathbf{p}_2,t)\}\delta(\mathbf{p}-\mathbf{p}_1-\mathbf{p}_2),$$

is the lost term in σ-mesons subsystem

The last term I_σ^{vac} in kinetic equation is the source term for the σ-meson creation, stipulated by its mass change

$$I_\sigma^{vac}(\mathbf{p},t) = \frac{1}{2}\Delta_\sigma(\mathbf{p},t)\int_{t_0}^{t} dt'\Delta_\sigma(\mathbf{p},t')[1+2f_\sigma(\mathbf{p},t')]\cos[2\theta_\sigma(\mathbf{p};t,t')],$$

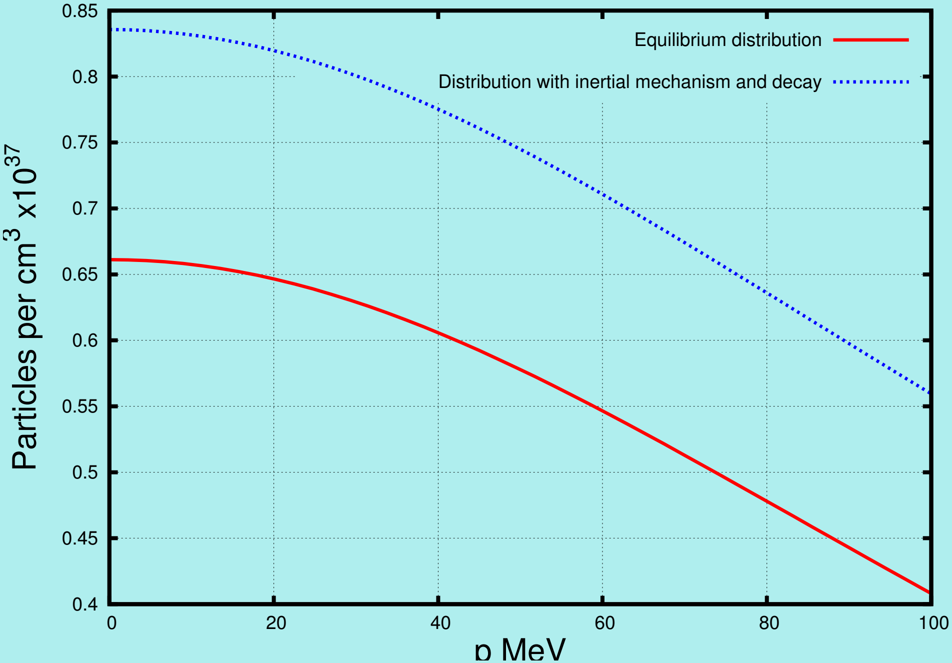
where

$$\Delta_\sigma(\mathbf{p},t) = \frac{\dot{\omega}_\sigma(\mathbf{p},t)}{\omega_\sigma(\mathbf{p},t)} = \frac{M_\sigma(t)\dot{M}_\sigma(t)}{\omega_\sigma^2(\mathbf{p},t)}$$

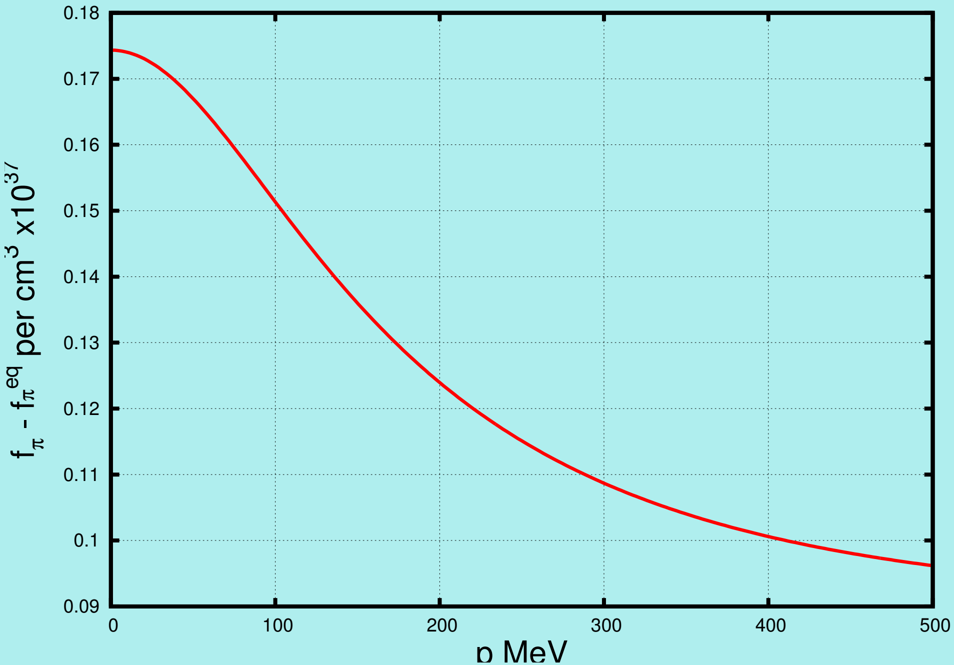
is the transition amplitude between states with positive and negative energies and

$$\theta_\sigma(\mathbf{p};t,t') = \int_{t'}^{t} dt''\omega_\sigma(\mathbf{p},t'').$$

Results



Final distribution of the pions. Initial distribution of the σ-mesons is the Bose-equilibrium.



The difference between final distribution of the pions and Bose-Einstein distribution with Bose-Einstein initial distribution of the σ-mesons.