Statistical Model for the QCD Phase Diagram - II -

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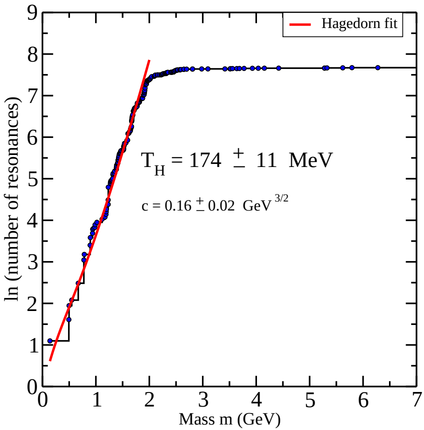
Outline

THE HAGEDORN TEMPERATURE.



Keep on adding the number of hadronic resonances.

J.C. and Dawit Worku, Mod. Phys. Lett. A26 (2011) 1197; arXiv: 1103.1463

HADRONS DO NOT EXIST ABOVE THE HAGEDORN TEMPERATURE.

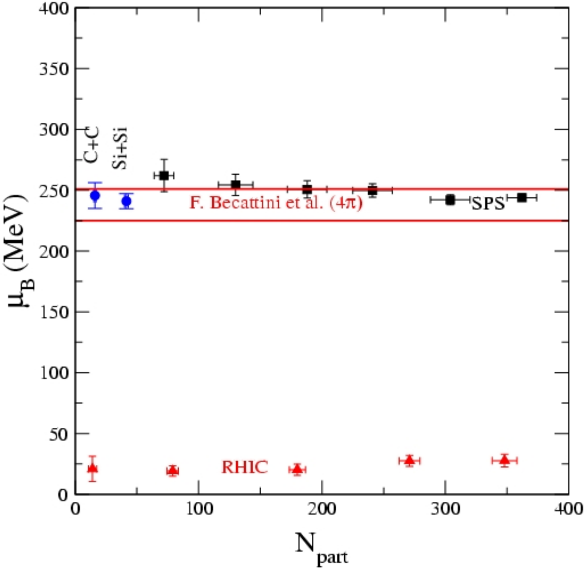
Thermodynamic quantities like particle density, energy density, pressure ,... all involve a summation over hadron species:

$$\sum_i \exp - \frac{E_i}{T}$$

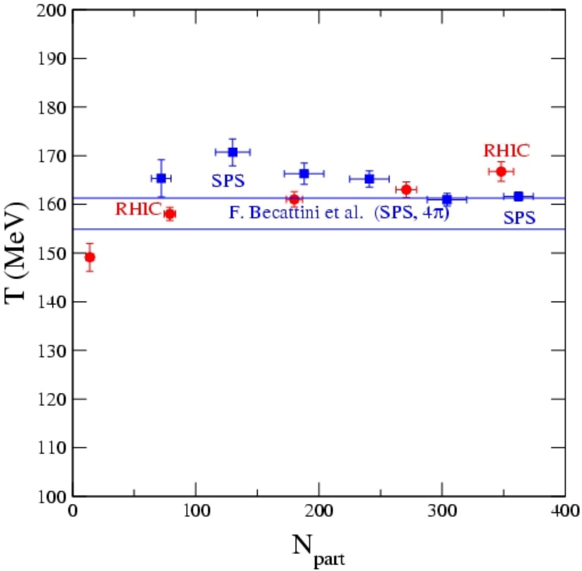
and the sum becomes (too) large due to the number of resonances.

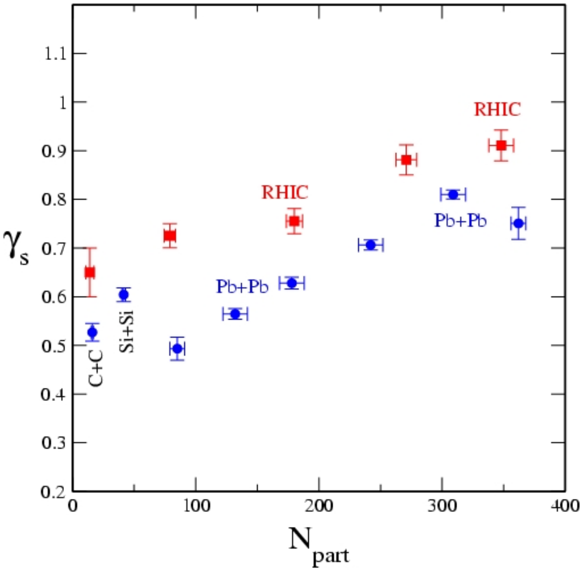
Size of System.

Centrality Dependence of the Baryon Chemical Potential



Centrality Dependence of the Chemical Freeze-out Temperature





RHIC
RHIC
Pb+Pb
Pb+Pb
Si+Si
C+C
γs
Npart
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
100
200
300
400

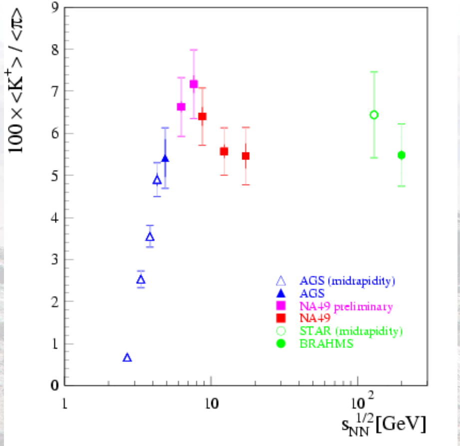
Strangeness and Heavy Ion Collisions Implications for NICA/FAIR.

The NA49 Collaboration has performed a series of measurements of Pb-Pb collisions at 20, 30, 40, 80 and 158 AGeV beam energies . When these results are combined with measurements at lower beam energies from the AGS they reveal an unusually sharp variation with beam energy in the $\Lambda/\langle\pi\rangle$, with $\langle\pi\rangle \equiv 3/2(\pi^+ + \pi^-)$, and K^+/π^+ ratios.

Such a strong variation with energy does not occur in pp collisions and therefore indicates a major difference in heavy-ion collisions. This transition has been referred as the "horn".

The Elephant in the Room

Friese
Dinkelaker
Blume
Speltz



Difficult to avoid, Hard to Model

→ But no unambiguous corroborating evidence

Strangeness in Heavy Ion Collisions
vs
Strangeness in pp - collisions

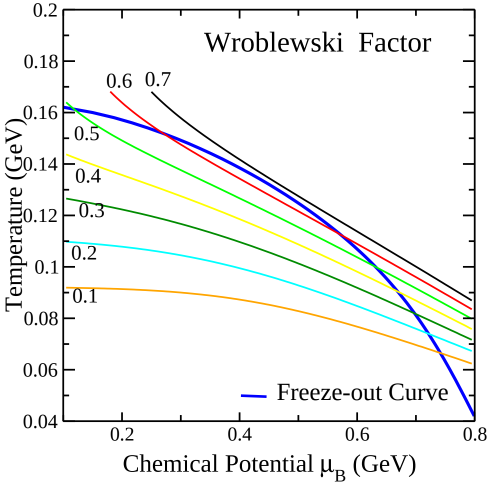
Use the Wroblewski factor

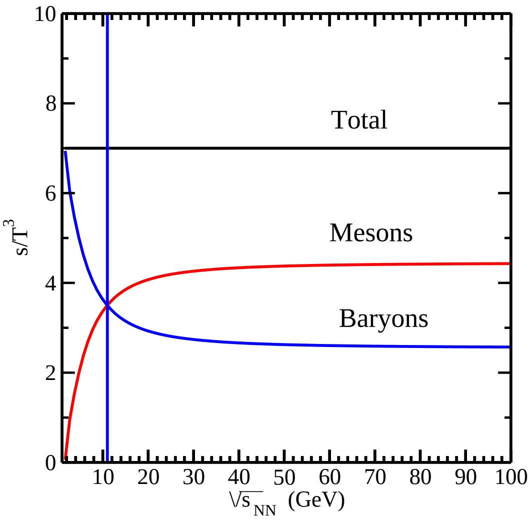
$$\lambda_s = \frac{2\,\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

This is determined by the number of **newly** created quark – anti-quark pairs and **before** strong decays, i.e. before ρ's and Δ's decay.

Limiting values :
$\lambda_s = 1$ all quark pairs are equally abundant, SU(3) symmetry.
$\lambda_s = 0$ no strange quark pairs.

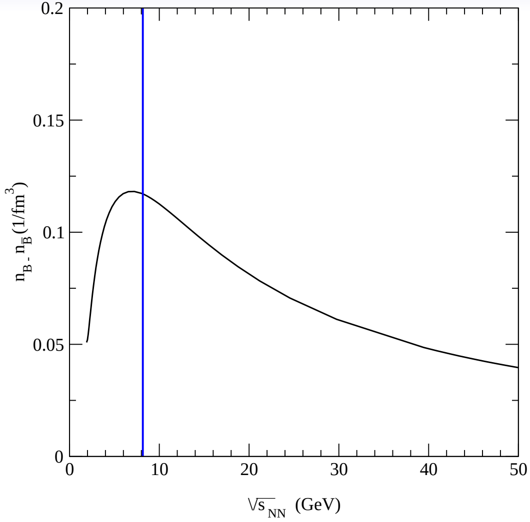
Maxima in particle ratios : K^+/π^+

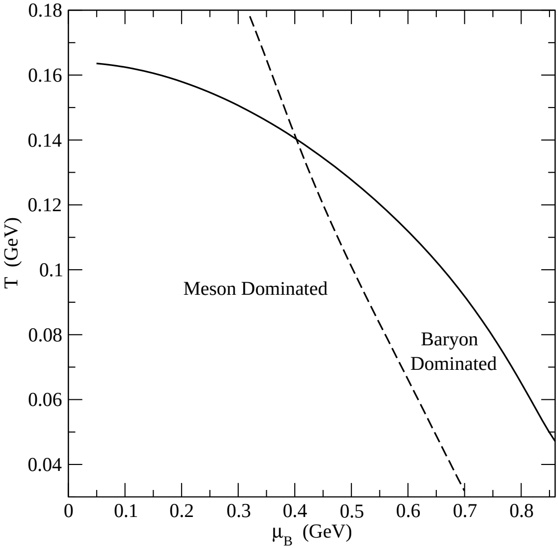


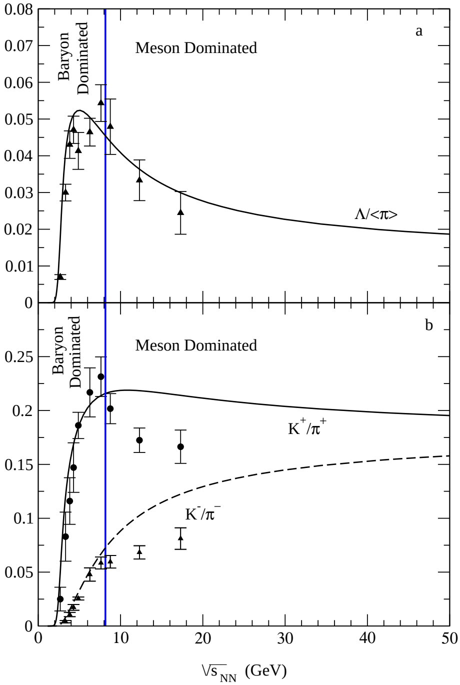

Total
Mesons
Baryons
s/T^3
0
2
4
6
8
10
10 20 30 40 50 60 70 80 90 100
√s_NN (GeV)

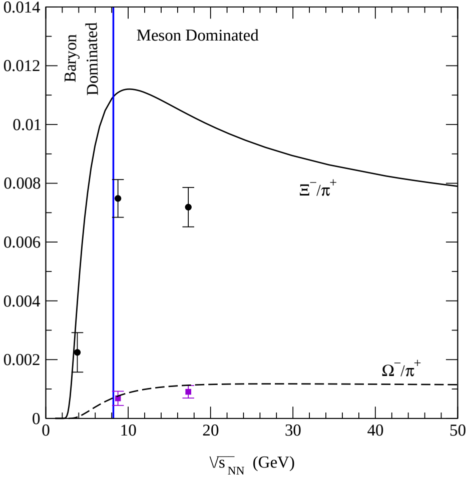
J.C., H. Oeschler, K. Redlich, S. Wheaton, Phys. Lett. B615 (2005) 50-54

In the statistical model a rapid change is expected as the hadronic gas undergoes a transition from a baryon-dominated to a meson-dominated gas. The transition occurs at a temperature T = 151 MeV and baryon chemical potential μ_B = 327 MeV corresponding to an incident energy of $\sqrt{s_{NN}}$ = 11 GeV.


0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
$n_B - n_{\bar{B}}$ (1/fm^3)
$\sqrt{s_{NN}}$ (GeV)


0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
T (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
μB (GeV)
Meson Dominated
Baryon
Dominated


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
a
Baryon Dominated
Meson Dominated
Λ/<π>
b
0.25
0.2
0.15
0.1
0.05
0
Baryon Dominated
Meson Dominated
K+/π+
K-/π−
0
10
20
30
40
50
√s_NN (GeV)

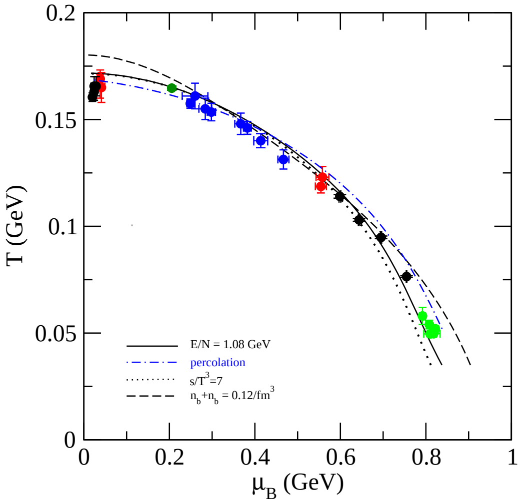


0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
Baryon Dominated
Meson Dominated
Ξ⁻/π⁺
Ω⁻/π⁺
0
10
20
30
40
50
√s_NN (GeV)

Maxima in Particle Ratios predicted by the Thermal Model.

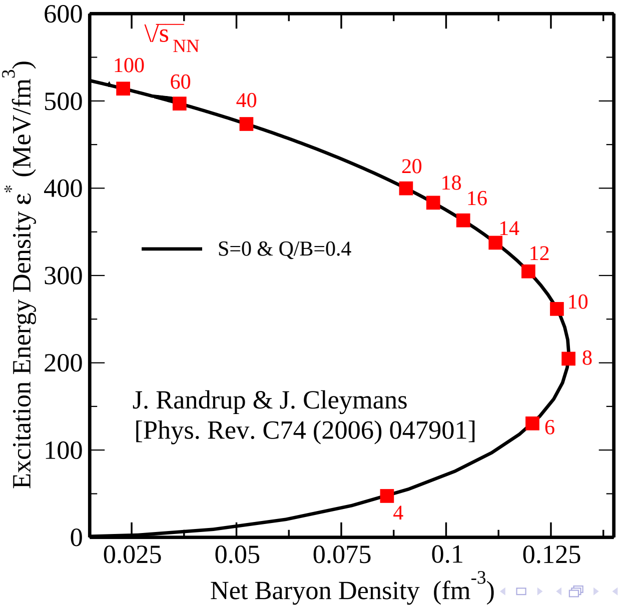
Ratio	Maximum at $\sqrt{s_{NN}}$ (GeV)	Maximum Value
$\Lambda / \langle\pi\rangle$	5.1	0.052
Ξ^- / π^+	10.2	0.011
K^+ / π^+	10.8	0.22
Ω^- / π^+	27	0.0012

J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.

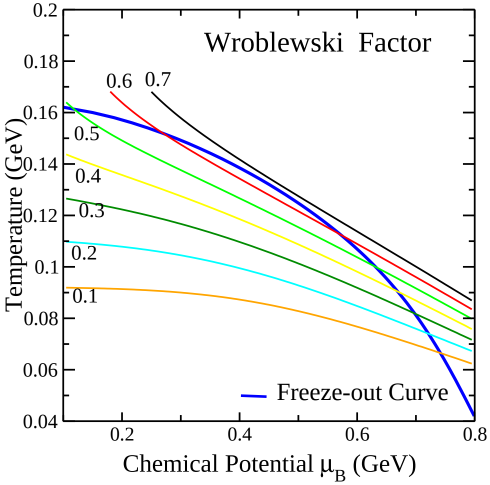

0.2
0.15
0.1
0.05
0
T (GeV)
0
0.2
0.4
0.6
0.8
1
μ_B (GeV)
E/N = 1.08 GeV
percolation
$s/T^3=7$
n_b+n_b = 0.12/fm^3

Hadronic Freeze-Out

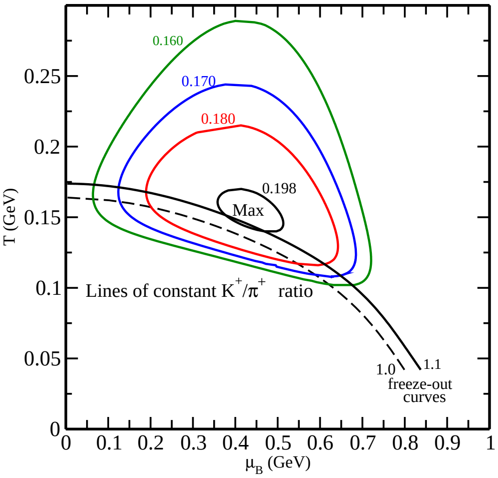
$\varepsilon^* = \varepsilon - m_N \rho$



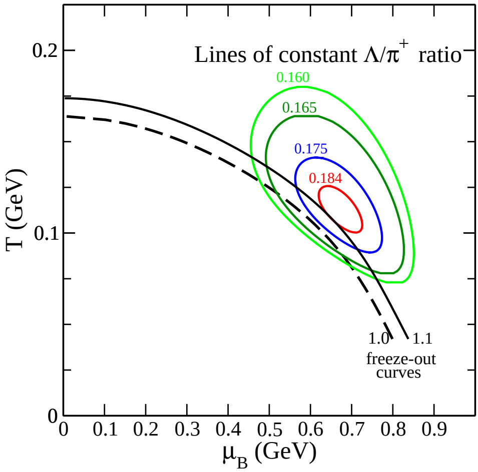
Maxima in particle ratios : K^+/π^+



Maxima in particle ratios : K^+/π^+



Maxima in particle ratios : K^+/π^+



GOOD NEWS FOR NICA.

Main goals of the project

1a) Heavy ion colliding beams $^{197}Au^{79+}$ x $^{197}Au^{79+}$ at

$\sqrt{s_{NN}} = 4 \div 11$ GeV (1 ÷ 4.5 GeV/u ion kinetic energy)

at

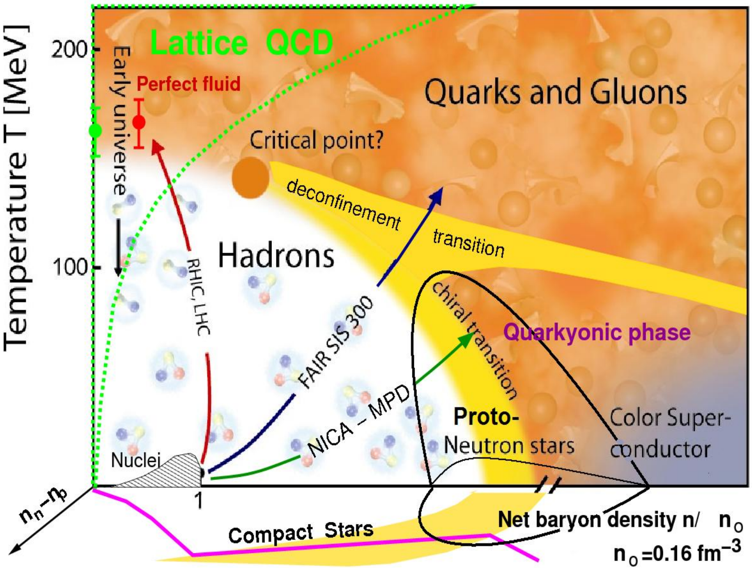
$L_{average} = 10^{27}$ cm$^{-2}\cdot$s^{-1} (at $\sqrt{s_{NN}}$ = 9 GeV)

1b) Light-Heavy ion colliding beams of the same energy range and luminosity

2) Polarized beams of protons and deuterons:

$p\uparrow p\uparrow$ $\sqrt{s_{NN}} = 12 \div 25$ GeV (5 ÷ 12.6 GeV kinetic energy)

$d\uparrow d\uparrow$ $\sqrt{s_{NN}} = 4 \div 13.8$ GeV (2 ÷ 5.9 GeV/u ion kinetic energy)


Temperature T [MeV]
200
100
Lattice QCD
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC, LHC
FAIR SIS 300
chiral transition
Quarkyonic phase
NICA – MPD
Proto-
Neutron stars
Color Super-
conductor
Nuclei
1
n_n-n_p
Compact Stars
Net baryon density n/ n_0
n_0=0.16 fm^{-3}

GOOD LUCK NICA.

NICA

Excluded Volume Corrections.

Why do we need them?
The pressure at zero temperature for a single degree of freedom is given by

$$P = \int_0^\mu \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E}$$

For massless particles, or at very high chemical potential (high density)

$$\begin{aligned} P &= \int_0^\mu p^2 dp \frac{4\pi}{(2\pi)^3} \\ &= \frac{1}{(24\pi)^2} \mu^4 \end{aligned}$$

which leads to:

$$P(\mathrm{quarks}) = 2x2x3x \frac{1}{(2\pi)^4} \left(\frac{\mu}{3}\right)^4 - B$$

and

$$P(\mathrm{nucleons}) = 2x2x \frac{1}{(2\pi)^4} (\mu)^4$$

i.e

$$P(\text{quarks}) < P(\text{nucleons})$$

and the system reverts back to the nucleon phase at very high densities.
Nucleon Phase -> Quark Phase -> Nucleon Phase
Excluded volume corrections prevent this from happening. This has been implemented in all the thermal model codes.

Relation between grand canonical and canonical ensembles:

$$Z_{GC}(T,V,\mu) = \sum_{N=0}^{\infty} e^{\frac{\mu N}{T}} Z_C(T,V,N)$$

Relation between grand canonical and pressure ensembles:

$$Z_p(T,P,\mu) = \int_0^{\infty} dV\ e^{\frac{PV}{T}} Z_{GC}(T,V,\mu)$$

Excluded Volume Corrections.

$$
\begin{aligned}
Z &= \exp\left\{ V \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}+\frac{\mu}{T}} \right\} \\
&= \sum_{N=0}^{\infty} \frac{V^N}{N!} e^{\mu N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N
\end{aligned}
$$

with excluded volume corrections

$$
\begin{aligned}
Z \rightarrow & \sum_{N=0}^{\infty} \frac{(V - V_0 N)^N}{N!} e^{\mu N/T} \\
& \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \theta(V - V_0 N)
\end{aligned}
$$

Excluded Volume Corrections.

It is more convenient to consider these corrections in the pressure ensemble:

$$Z_p \equiv \int_0^\infty dV\ e^{-PV/T} \sum_{N=0}^{\infty} \frac{V^N}{N!} e^{\mu N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N$$

$$Z_p \quad \rightarrow \quad \sum_{N=0}^{\infty} \int_0^\infty dV\ e^{-PV/T} \frac{(V - V_0 N)^N}{N!} e^{\mu N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \theta(V - V_0 N)$$

introduce $x \equiv V - V_0 N$.

Excluded Volume Corrections.

$$
\begin{aligned}
Z_p &= \sum_{N=0}^{\infty} \int_0^{\infty} dx\ e^{-Px/T} \\
& \frac{x^N}{N!} e^{-PV_0 N/T} e^{\mu N/T} \\
& \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N
\end{aligned}
$$

a new variable $\bar{\mu} \equiv \mu - PV_0$

Excluded Volume Corrections.

$$Z_p \rightarrow \sum_{N=0}^{\infty} \int_0^{\infty} dx\ e^{-Px/T} \frac{x^N}{N!} e^{\bar{\mu}N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N$$

which is the original partition function with the he replacement

$$\boxed{\bar{\mu} = \mu - P\ V_0}$$

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, ZfP C51, 485 (1991). J. C., M.I. Gorenstein, J. Stålnacke and E. Suhonen P. S. 48 277-280 (1993).

Excluded Volume Corrections.

The particle number density now becomes:

$$\begin{aligned} n &= \frac{T}{V}\frac{\partial}{\partial\mu}\ln\ Z \\ &= \frac{T}{V}\frac{\partial\bar{\mu}}{\partial\mu}\frac{\partial}{\partial\bar{\mu}}\ln\ Z \\ &= \frac{\partial\bar{\mu}}{\partial\mu}n_0 \\ &= [1 - V_0\ n]\ n_0 \end{aligned}$$

$$\boxed{n = \frac{n_0}{1 + V_0 n_0}}$$

Effects Cancel Out in Ratios.

J.C., K. Redlich, H. Satz, E. Suhonen, ZfP C33, 151, (1986)

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, ZfP C51, 485 (1991).

Canonical Corrections.

Exact Strangeness Conservation.

For a small system at low temperatures ($T \approx 50$ MeV), e.g. at GSI canonical corrections are necessary.
Instead of

$$N_K \approx \exp -M_K/T$$

one gets

$$N_K \approx \exp -2M_K/T$$

Extra suppression is due to strangeness conservation and lack of a large heat bath. This correction disappears quickly at higher energies and is already small at AGS energies.

Exact Strangeness Conservation.

$$Z = \mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}$$

Insert a Kronecker delta in the trace:

$$\sum_i n_i(S=1) + 2\sum_j n_j(S=2) + 3\sum_k n_k(3) =$$

$$\sum_i \bar{n}_i(S=-1) + 2\sum_j \bar{n}_j(S=-2) + 3\sum_k \bar{n}_k(S=-3)$$

and rewrite it as

$$\delta\left(\sum_i n_i(S=1) + ..., \sum_i \bar{n}_i(S=-1) + ...\right)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\phi$$

$$\exp\left(i\phi\sum_i n_i(S=1) + ..\ -i\phi\sum_i \bar{n}_i(S=-1)\right)$$

Exact Strangeness Conservation.

$$
\begin{aligned}
Z &= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left\{Z_1 e^{i\phi} + Z_{-1} e^{-i\phi}\right\} \\
&= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left\{\sqrt{Z_1 Z_{-1}}\left[\sqrt{\frac{Z_1}{Z_{-1}}} e^{i\phi} + \sqrt{\frac{Z_{-1}}{Z_1}} e^{-i\phi}\right]\right\}
\end{aligned}
$$

Z_1: sum of all particles with strangeness 1, e.g. K^+
Z_{-1}: sum of all particles with strangeness -1, e.g. Λ

Exact Strangeness Conservation.

Use

$$\exp\left\{\frac{x}{2}\left(t+\frac{1}{t}\right)\right\} = \sum_{n=-\infty}^{\infty} I_m(x)t^m$$

to obtain

$$Z = \frac{1}{2\pi}\int_0^{2\pi} e^{ip\phi} \sum_{p=-\infty}^{\infty} I_p(x_1)y_1^p$$

where

$$y_1 = \sqrt{\frac{Z_1}{Z_{-1}}} \qquad x_1 = 2\sqrt{Z_1 Z_{-1}}$$

$$Z = I_0(x_1)$$

Exact Strangeness Conservation.

In more detail, e.g. the multiplicity of K^+

$$N_{K^+} = \frac{T}{Z}\frac{\partial I_0(x_1)}{\partial \mu_{K^+}}\bigg|_{\mu_{K^+}=0}$$

Use

$$\frac{d}{dz} I_0(z) = I_1(z)$$

Exact Strangeness Conservation.

$$
\begin{aligned}
N_{K^+} &= \frac{T}{Z}\frac{\partial}{\partial \mu_{K^+}} I_0(x_1) \\
&= \frac{T}{I_0(x_1)} I_1(x_1) \frac{\partial x_1}{\partial \mu_{K^+}} \\
&= \frac{T}{I_0(x_1)} I_1(x_1) \frac{\partial 2\sqrt{Z_1 Z_{-1}}}{\partial \mu_{K^+}} \\
&= \frac{I_1(x_1)}{I_0(x_1)} \sqrt{\frac{Z_{-1}}{Z_1}} N^0_{K^+}
\end{aligned}
$$

where $N^0_{K^+}$ refers to the "unmodified" kaon multiplicity.

Exact Strangeness Conservation.

In the small volume limit this becomes

$$\lim_{z \to 0} I_0(z) = 1$$

and

$$\lim_{z \to 0} I_1(z) = \frac{z}{2}$$

$$\begin{aligned} \lim_{V \to 0} &= N^0_{K^+} Z_{-1} \\ \lim &= N^0_{K^+} Z_{-1} \\ &= N^0_{K^+} \left[N^0_{K^-} + N^0_{\Lambda} + \cdots \right] \end{aligned}$$

i.e., the particle multiplicity is

- proportional to V^2, and not V^1.
- proportional to $\exp(-2m_K/T)$ or to $\exp(-(m_K + m_\Lambda)/T)$ and not simply $\exp(-m_K/T)$, i.e. there is additional suppression of strange particles.

Exact Strangeness Conservation.

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Exact Strangeness Conservation

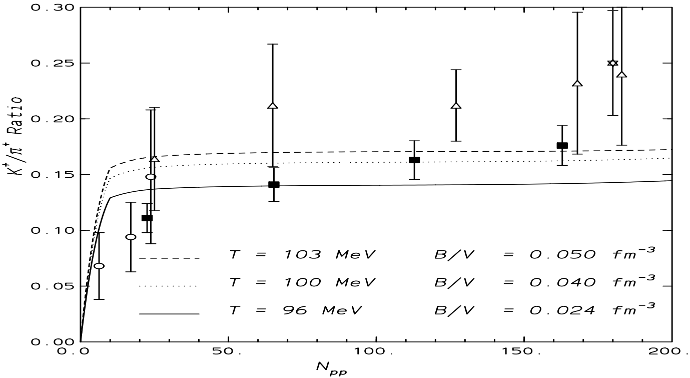


Figure 1

CANONICAL CORRECTIONS
are
IRRELEVANT.
Except at very low energies.

Transverse Momentum Distribution

STAR collaboration, B.I. Abelev at al.
arXiv:nucl-ex/0607033; Phys. Rev. C75, 064901 (2007)
PHENIX collaboration, A. Adare et al.
Phys. Rev. **C83**, 064903 (2011)
ALICE collaboration, K. Aamodt et al.
arXiv:1101.4110 [hep-ex]
CMS collaboration, V. Khachatryan et al.
arXiv: 1102.4282 [hep-ex]
ATLAS collaboration, G. Aad et al.
New J. Phys. **13** (2011) 053033.

All use the Tsallis distribution for $p-p$ collisions.

Tsallis Distribution

Possible generalization of Boltzmann-Gibbs statistics

Constantino Tsallis
Rio de janeiro, CBPF
J. Stat. Phys. 52 (1988) 479-487

Citations: 1 389
However:
Citations in HEP: 403

ISSN 0029-3865

MINISTÉRIO DA CIÊNCIA E TECNOLOGIA

CNPq

CBPF

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-062/87

POSSIBLE GENERALIZATION OF BOLTZMANN-GIBBS STATISTICS

by

Constantino TSALLIS

RIO DE JANEIRO
1987

Multifractal concepts and structures are quickly acquiring importance in many active areas (e.g., non-linear dynamical systems, growth models, commensurate/incommensurate structures). This is due to their utility as well as to their elegance. Within this framework, the quantity which is normally scaled is p_i^q, where p_i is the probability associated to an event and q any real number [1]. We shall use this quantity to generalize the standard expression of the entropy S in information theory, namely $S = -k \sum_{i=1}^{W} p_i \ln p_i$, where $W \in \mathbb{N}$ is the total number of possible (microscopic) configurations and $\{p_i\}$ the associated probabilities. We *postulate* for the entropy

$$S_q \equiv k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad (q \in \mathbb{R}) \tag{1}$$

where k is a conventional positive constant and $\sum_{i=1}^{W} p_i = 1$. We immediately verify that

$$S_1 \equiv \lim_{q \to 1} S_q = k \lim_{q \to 1} \frac{1 - \sum_{i=1}^{W} p_i e^{(q-1)\ln p_i}}{q-1} = -k \sum_{i=1}^{W} p_i \ln p_i \qquad (1')$$

where we have used the replica-trick type of expansion. We illustrate definition (1) in Fig. 1. S_q may be rewritten as follows:

$$S_q = \frac{k}{q-1} \sum_{i=1}^{W} p_i \left(1 - p_i^{q-1}\right) \tag{2}$$

Transverse Momentum Distribution

STAR, PHENIX, ALICE, CMS, ATLAS use:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = p_\mathrm{t} \times \frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left(1+\frac{m_\mathrm{t}-m_0}{nC}\right)^{-n}$$

Direct connection with Tsallis distribution.

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} ,$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} ,$$

which, in terms of the rapidity and transverse mass variables, becomes (for μ = 0)

$$\left.\frac{d^2N}{dp_t\, dy}\right|_{y=0} = gV\frac{p_t m_t}{(2\pi)^2} \left[1 + (q-1)\frac{m_t}{T}\right]^{q/(1-q)} ,$$

J.C. and D. Worku, arXiv:1106.3405[hep-ph]

Rewrite the Tsallis distribution using

$$[1+(q-1)x]^{1/(1-q)} = \exp\left(\frac{1}{1-q}\ln\left[1+(q-1)x\right]\right),$$

and consider the limit $q \to 1$

$$\begin{aligned}\lim_{q\to 1} \quad & [1+(q-1)x]^{1/(1-q)} \\ & = \exp\frac{1}{(1-q)}(q-1)x \\ & = \exp\left(-x\right), \end{aligned} \tag{1}$$

The Tsallis distribution reduces to the Boltzmann distribution in the limit where $q \to 1$

$$\lim_{q\to 1}\frac{d^2N}{dp_t\,dy} = gV\frac{p_t m_t\cosh y}{(2\pi)^2}\exp\left(-\frac{m_t\cosh y-\mu}{T}\right). \tag{2}$$

In all cases q is close to one, typically between 1.05 and 1.2.

Comparison of Tsallis with STAR, ALICE, CMS distributions

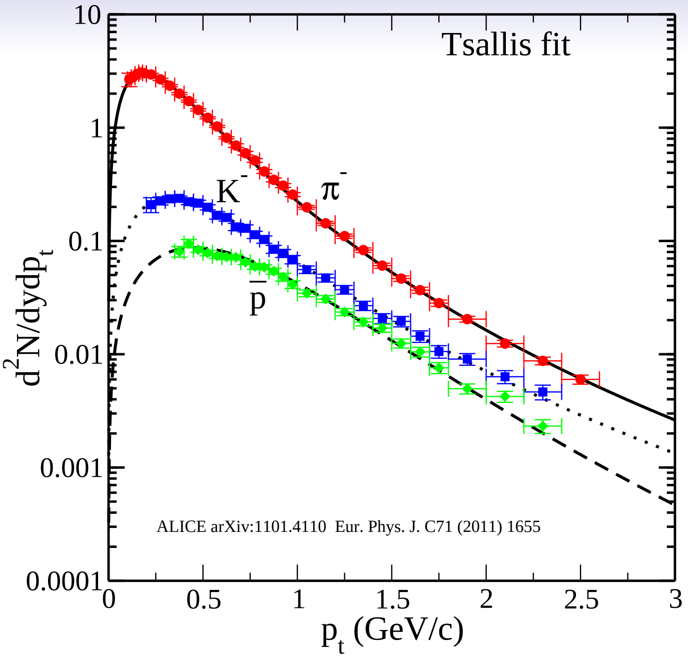
$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = gV\frac{p_t m_t}{(2\pi)^2}\left[1+(q-1)\frac{m_t}{T}\right]^{q/(1-q)}, \quad (3)$$

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = p_\mathrm{t}\times\frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left[1+\frac{m_\mathrm{t}-m_0}{nC}\right]^{-n} \quad (4)$$

$$n \rightarrow \frac{q}{q-1}$$

$$nC \rightarrow \frac{T}{q-1}\frac{m_t-m_0}{m_t}$$

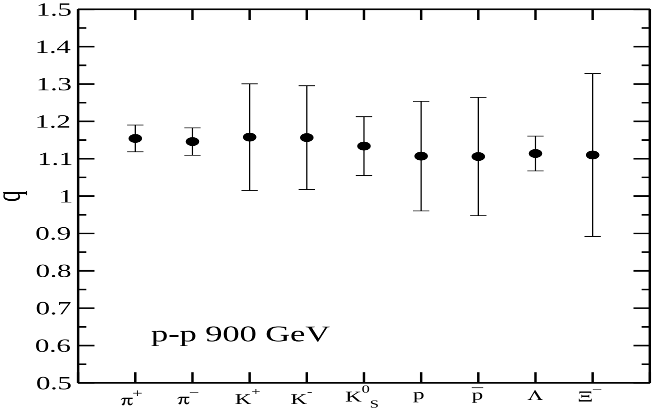
Only a factor of m_T differs!


Tsallis fit
K⁻
π⁻
p̄
d²N/dydp_t
p_t (GeV/c)
10
1
0.1
0.01
0.001
0.0001
0
0.5
1
1.5
2
2.5
3
ALICE arXiv:1101.4110 Eur. Phys. J. C71 (2011) 1655

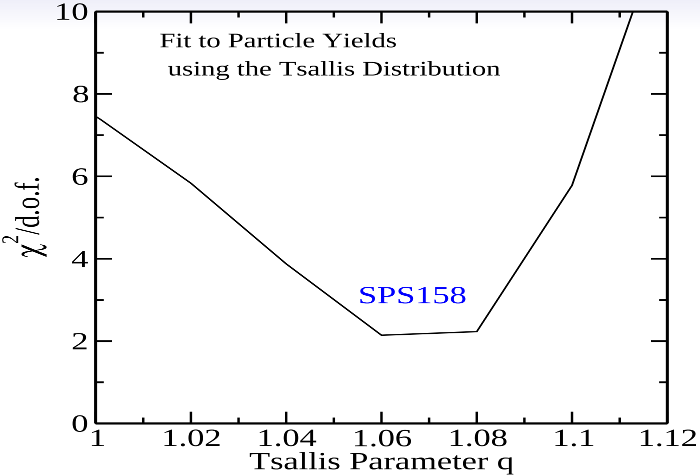
$p-p$ 900 GeV		
Particle	q	T
π^+	1.154 $\pm$0.036	0.0682 $\pm$0.0026
π^-	1.146 $\pm$0.036	0.0704 $\pm$ 0.0027
K^+	1.158 $\pm$0.142	0.0690 $\pm$0.0223
K^-	1.157 $\pm$0.139	0.0681 $\pm$ 0.0217
K_S^0	1.134 $\pm$0.079	0.0923 $\pm$0.0139
p	1.107 $\pm$0.147	0.0730 $\pm$ 0.0425
$\bar{p}$	1.106 $\pm$0.158	0.0764 $\pm$0.0464
Λ	1.114 $\pm$0.047	0.0698 $\pm$ 0.0148
Ξ^-	1.110 $\pm$0.218	0.0440 $\pm$ 0.0752

Table: Fitted values of the T and q parameters measured in $p-p$ collisions by the ALICE and CMS collaborations using the Tsallis form for the momentum distribution.

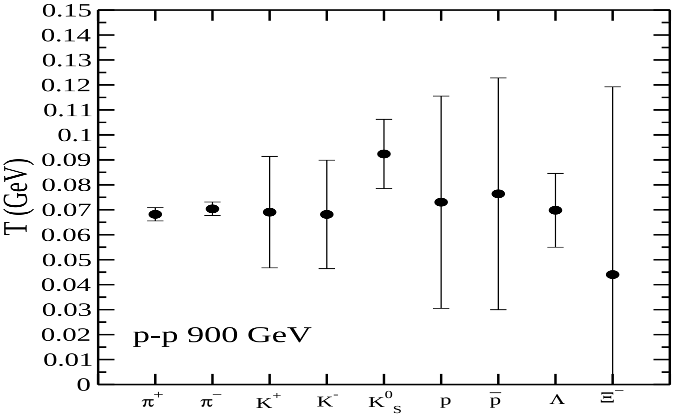
$p-p$ 900 GeV		
Particle	T Tsallis vs C ALICE (MeV)	q
π^+	70 (126)	1.147
K^+	70 (160)	1.156
p	73 (196)	1.110



Values of the Tsallis parameter *q* for different species of hadrons.



J. C., G. Hamar, P. Levai, S. Wheaton
Journal of Physics **G 36** (2009) 064018.



Values of the Tsallis temperature T for different species of hadrons.
J.C. and D. Worku e-Print: arXiv:1110.5526 [hep-ph]