Dubna, 1./3. 9. 2012

HISS: Dense Matter in Heavy-Ion Collisions and Astrophysics

Cluster Formation and Liquid-Gas Transition in Nuclear Matter

Gerd Röpke, Rostock

Outline

- Nuclear matter - a strongly interacting quantum liquid

  where it occurs, what do we know: nuclei, stars, HIC

- Many-particle theory: Equation of state

  QCD? Effective interactions, Green functions, spectral functions

- Low-density limit: cluster formation

  Mass action law, nuclear statistical equilibrium, virial expansion

- Near saturation: medium effects

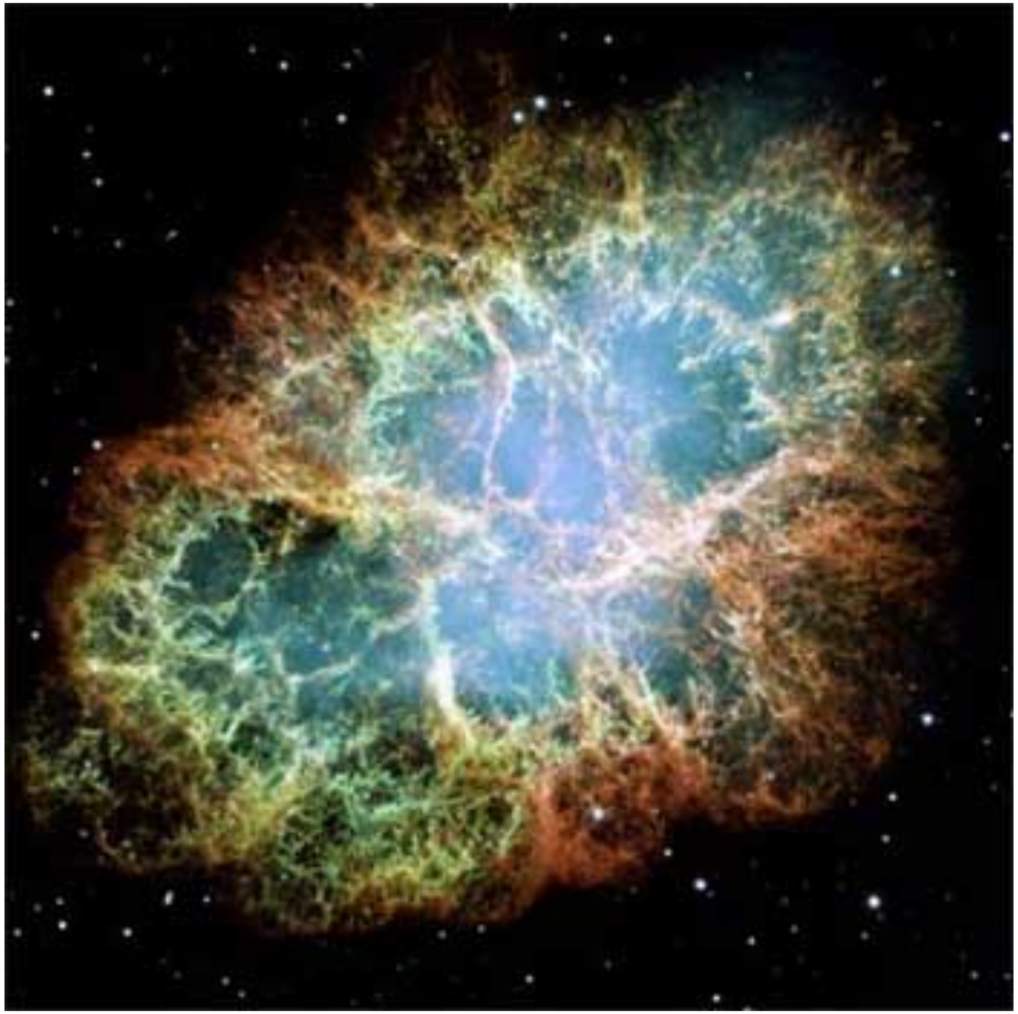
  mean-field and quasiparticles, dissolution of bound states

- Quantum condensates:

  transition from BEC to BCS, Hoyle states, pairing and quartetting

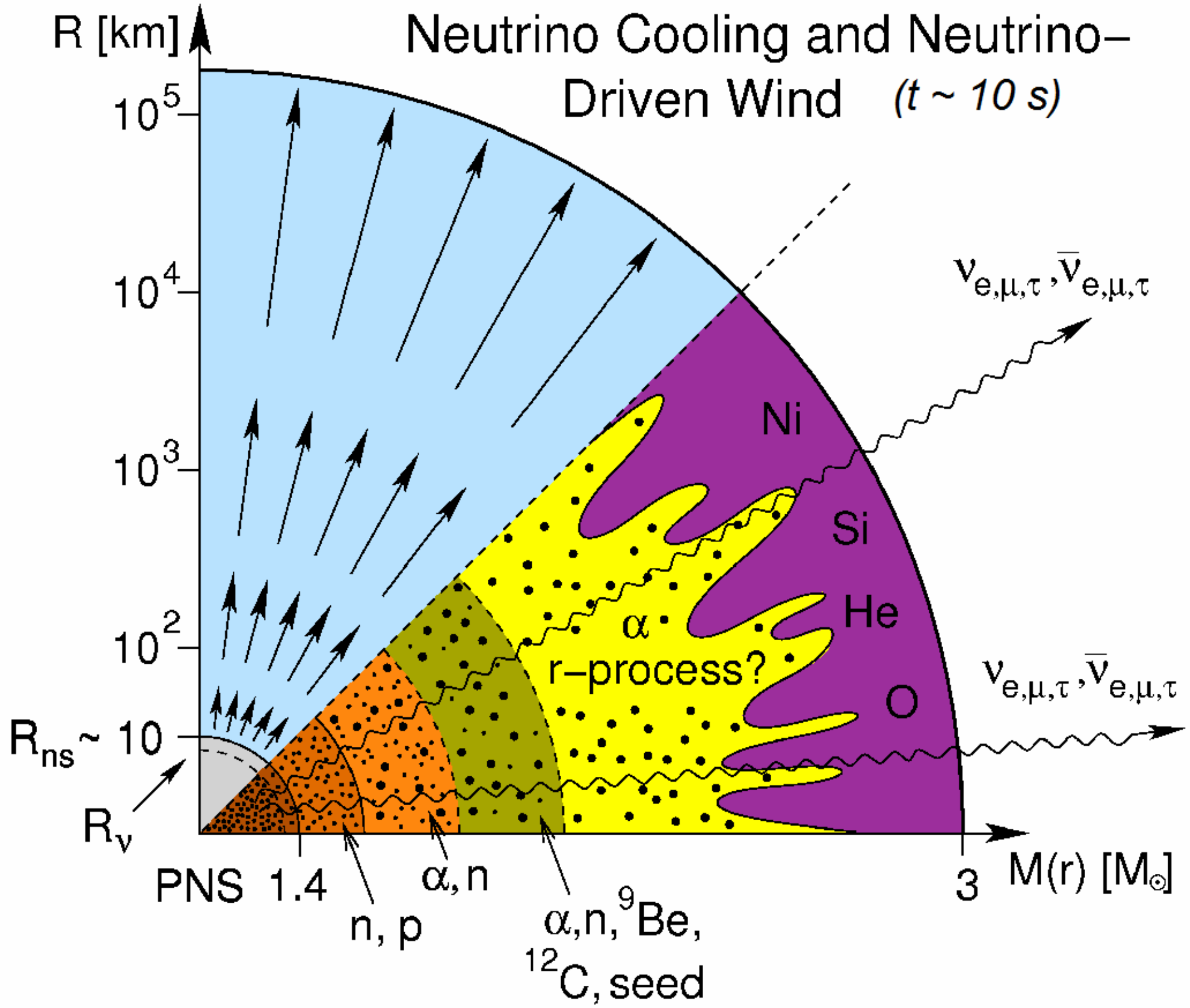
Supernova

Crab nebula, 1054 China, PSR 0531+21



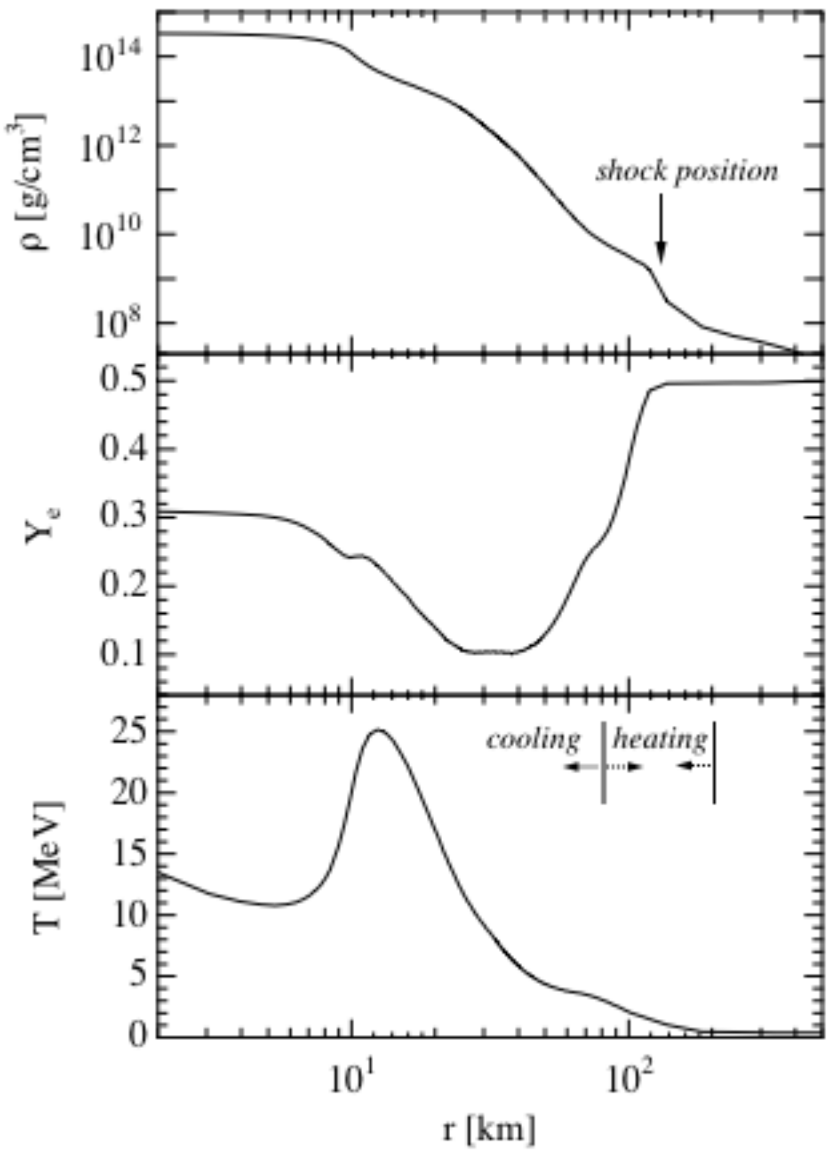
M1, the Crab Nebula. Courtesy of NASA/ESA

Supernova explosion



T.Janka

Core-collapse supernovae



Density.

electron fraction, and

temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

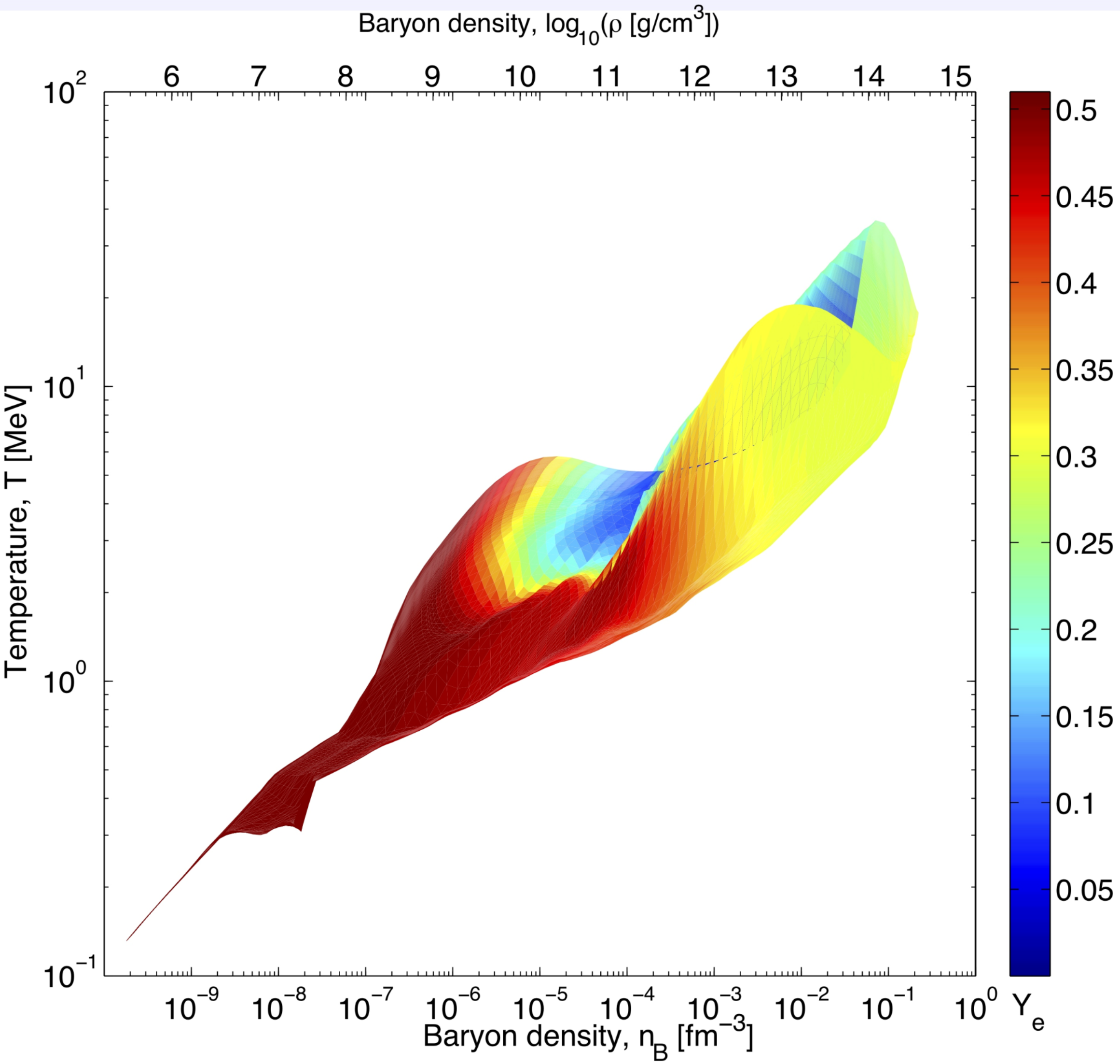
K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Nuclear matter phase diagram

Core collapse supernovae

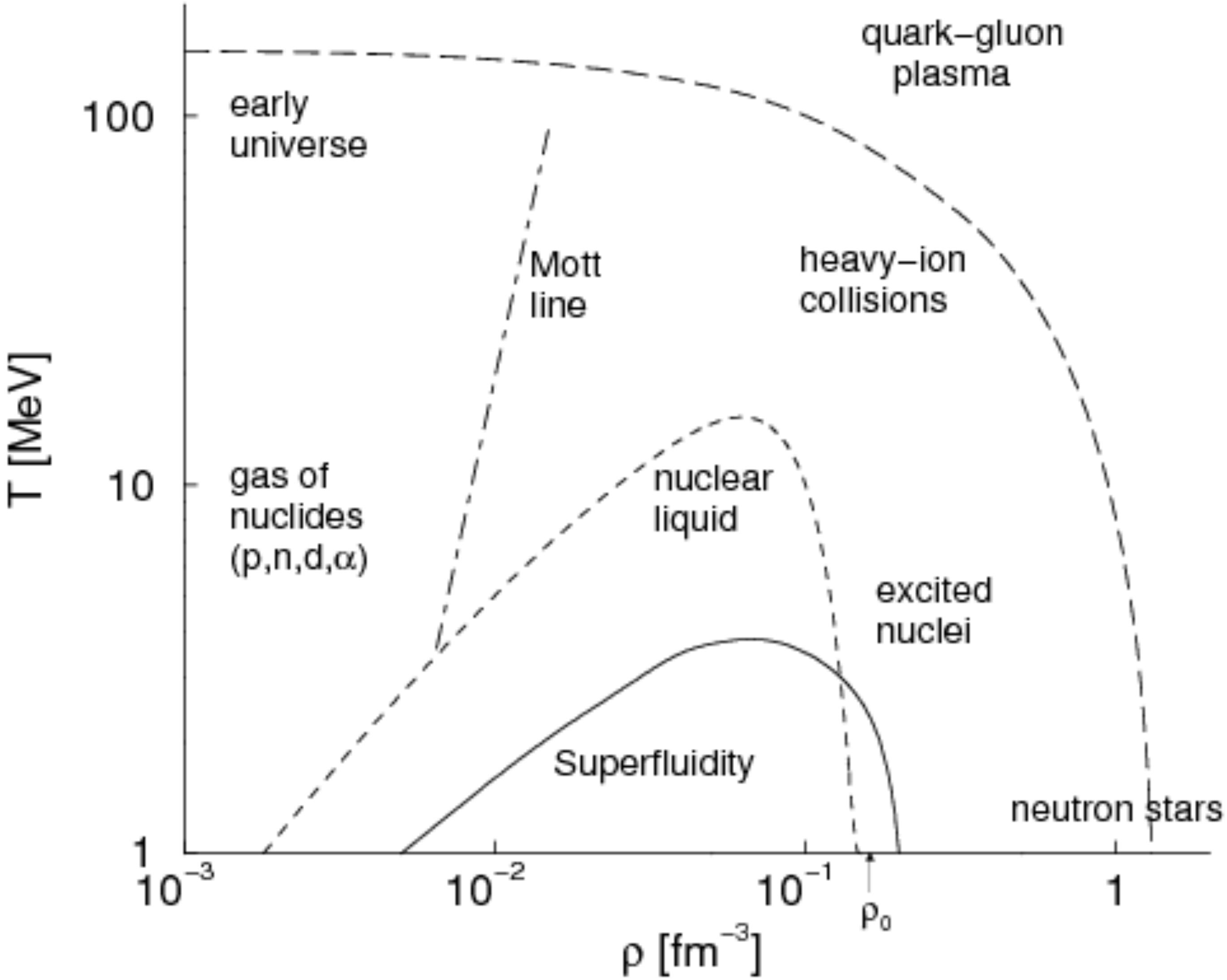
Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$

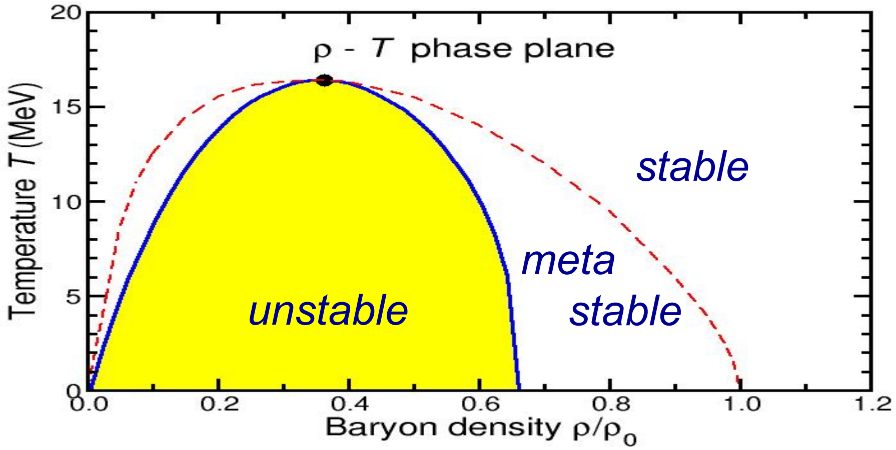


T. Fischer, GSI Darmstadt

Symmetric nuclear matter: Phase diagram

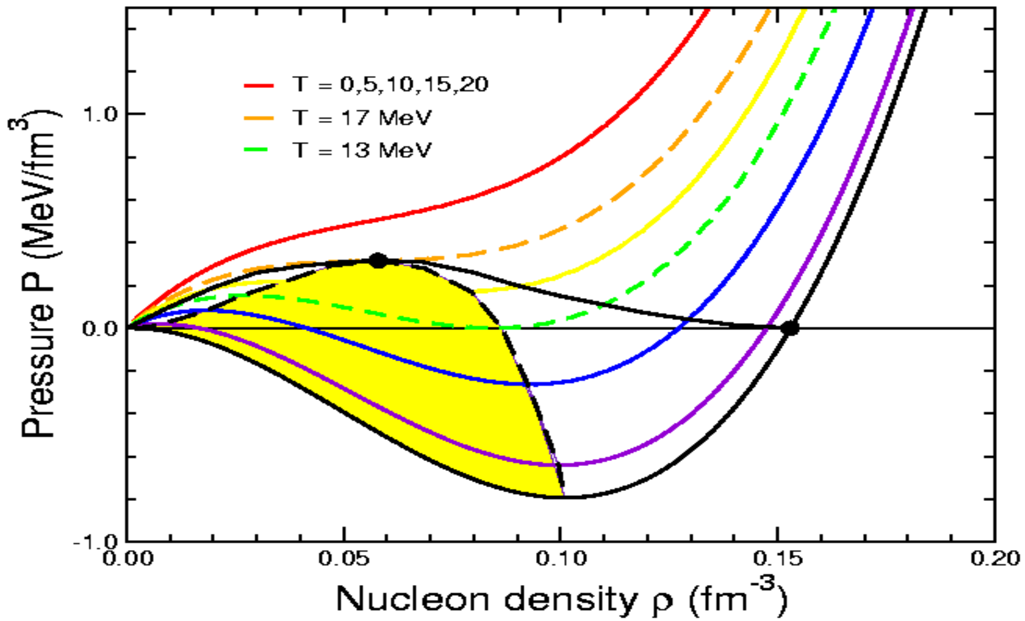


Nuclear matter



Phase diagram

$$\varepsilon(T;\rho) = \varepsilon_{\rm FG}(T;\rho) + w(\rho)$$



Equation of state:
$p_T(r)$

JRandrup: Dubna School 2010

Nucleon-nucleon interaction

QCD? Effective Lagrangians, interaction potentials

singlet (nn, pp): a = -23.678 fm, r = 1.726 fm

triplet (pn): a = 5.396 fm, r = 2.729 fm, E = -2.225 MeV

$$k \cot \delta = -\frac{1}{a} + r_0 \frac{k^2}{2}$$

Separable interaction

- general form:

$$V_\alpha(p, p') = \sum_{i,j=1}^{N} w_{\alpha i}(p) \lambda_{\alpha ij} w_{\alpha j}(p') \qquad \text{uncoupled}$$

and

$$V_\alpha^{LL'}(p, p') = \sum_{i,j=1}^{N} w_{\alpha i}^{L}(p) \lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \qquad \text{coupled}$$

p, p'	in- and outgoing relative momentum
α ...	channel
N ...	rank
$\lambda_{\alpha ij}$.	coupling parameter
L, L'	orbital angular momentum

Weak interaction - beta equilibrium? Coulomb interaction?

Many-particle theory

- equilibrium correlation functions
  e.g. equation of state $n(\beta,\mu) = \frac{1}{\Omega_0}\sum_1 \langle a_1^\dagger a_1 \rangle$

  density matrix $\langle a_1^\dagger a_{1'} \rangle = \int \frac{d\omega}{2\pi}\, e^{-i\omega t} f_1(\omega) A(1,1',\omega)$

- Spectral function

$$A(1,1',\omega) = \mathrm{Im}\left[G(1,1',\omega+i\eta) - G(1,1',\omega-i\eta)\right]$$

- Matsubara Green function

$$G(1,1',iz_\nu), \qquad z_\nu = \frac{\pi\nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \cdots$$

$$1 \equiv \{\mathbf{p}_1, \sigma_1, c_1\}, \quad f_1(\omega) = \frac{1}{e^{\beta(\omega-\mu)}+1}, \quad \Omega_0 - \text{volume}$$

Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of $\Sigma(1, iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1,\omega) = \frac{2\text{Im}\,\Sigma(1,\omega + i0)}{[\omega - E(1) - \text{Re}\,\Sigma(1,\omega)]^2 + [\text{Im}\,\Sigma(1,\omega + i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

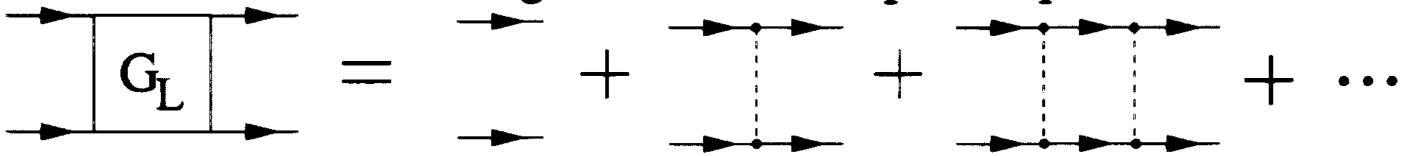
- Expansion for small Im $\Sigma(1, \omega + i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{\text{d}}{\text{d}z}\text{Re }\Sigma(1,z)|_{z=E^{\text{quasi}}-\mu_1}} - 2\text{Im }\Sigma(1,\omega+i\eta)\frac{\text{d}}{\text{d}\omega}\frac{P}{\omega+\mu_1 - E^{\text{quasi}}(1)}$$

quasiparticle energy $E^{\text{quasi}}(1) = E(1) + \text{Re }\Sigma(1,\omega)|_{\omega=E^{\text{quasi}}}$

- chemical picture: bound states $\hat{=}$ new species

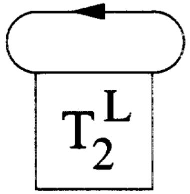
summation of ladder diagrams, Bethe-Salpeter equation



Different approximations

low density limit:

$$G_2^L(12, 1'2', i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi^*_{n\mathbf{P}}(12)$$

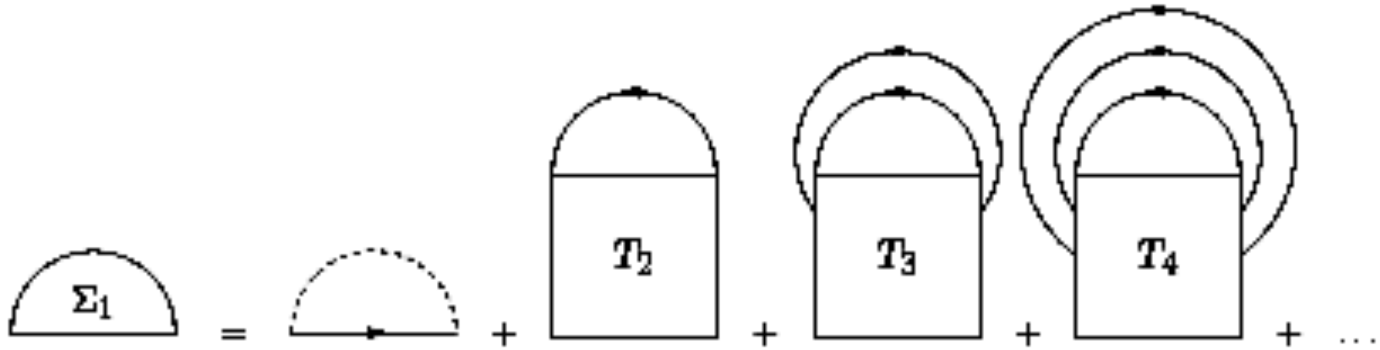
$\Sigma =$ 

$$n(\beta, \mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+ \sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k \; \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2)) 2 \sin^2 \delta_n(k) \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}k} \delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

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Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

Different approximations

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Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

continuum contribution

Second virial coefficient:

account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Different approximations

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Second virial coefficient:

account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Generalized Beth-Uhlenbeck formula:

medium modified binding energies,
medium modified scattering phase shifts

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
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Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
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Chemical equilibrium with quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

Quasiparticle cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs of clusters

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Medium effects: Quasiparticle approximation

- Skyrme / Gogny

- relativistic mean field (RMF)

  Lagrangian: non-linear sigma, TM1 parameters,
  single particle modifications, energy shift, effective mass

- DD-RMF [S.Typel, Phys. Rev. C **71**, 064301 (2007)]:

  expansion of the scalar field and the vector fields
  in powers of proton/neutron densities

- Dirac-Brueckner Hartree Fock (DBHF)

- Diffusion Monte Carlo EOS calculation [S. Gandolfi et al., Mon.Not.R.Astron.Soc., 2010]

Nuclear matter properties

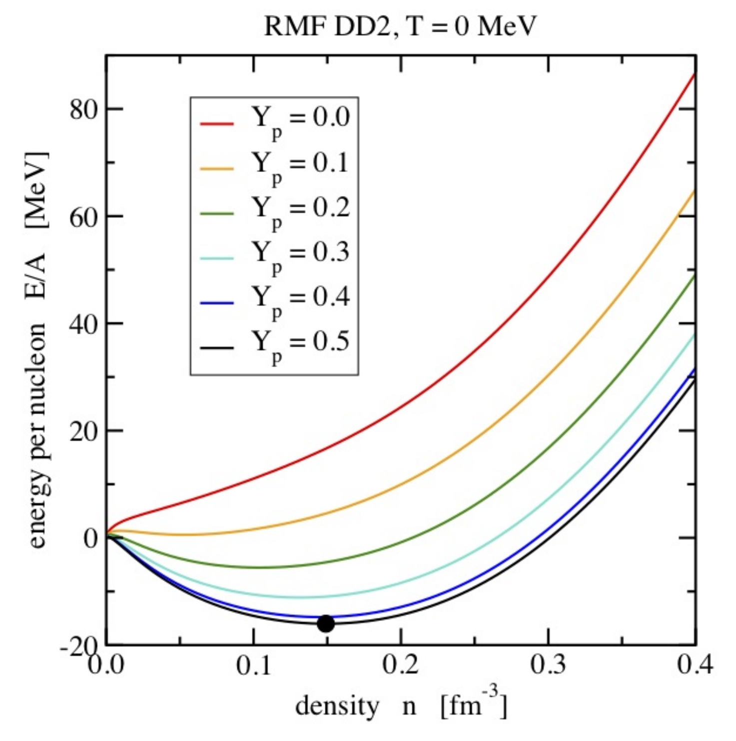
binding energy per nucleon near saturation:

$$\frac{E}{A}(n,\beta) = \frac{\varepsilon}{n} = a_V + \frac{K}{18}x^2 + \frac{K'}{162}x^3 + \beta^2\left(J + \frac{L}{3}x + \dots\right) + \dots$$

with $x = (n - n_{\rm sat})/n_{\rm sat}$, asymmetry $\beta = 1 - 2Y_p$ and

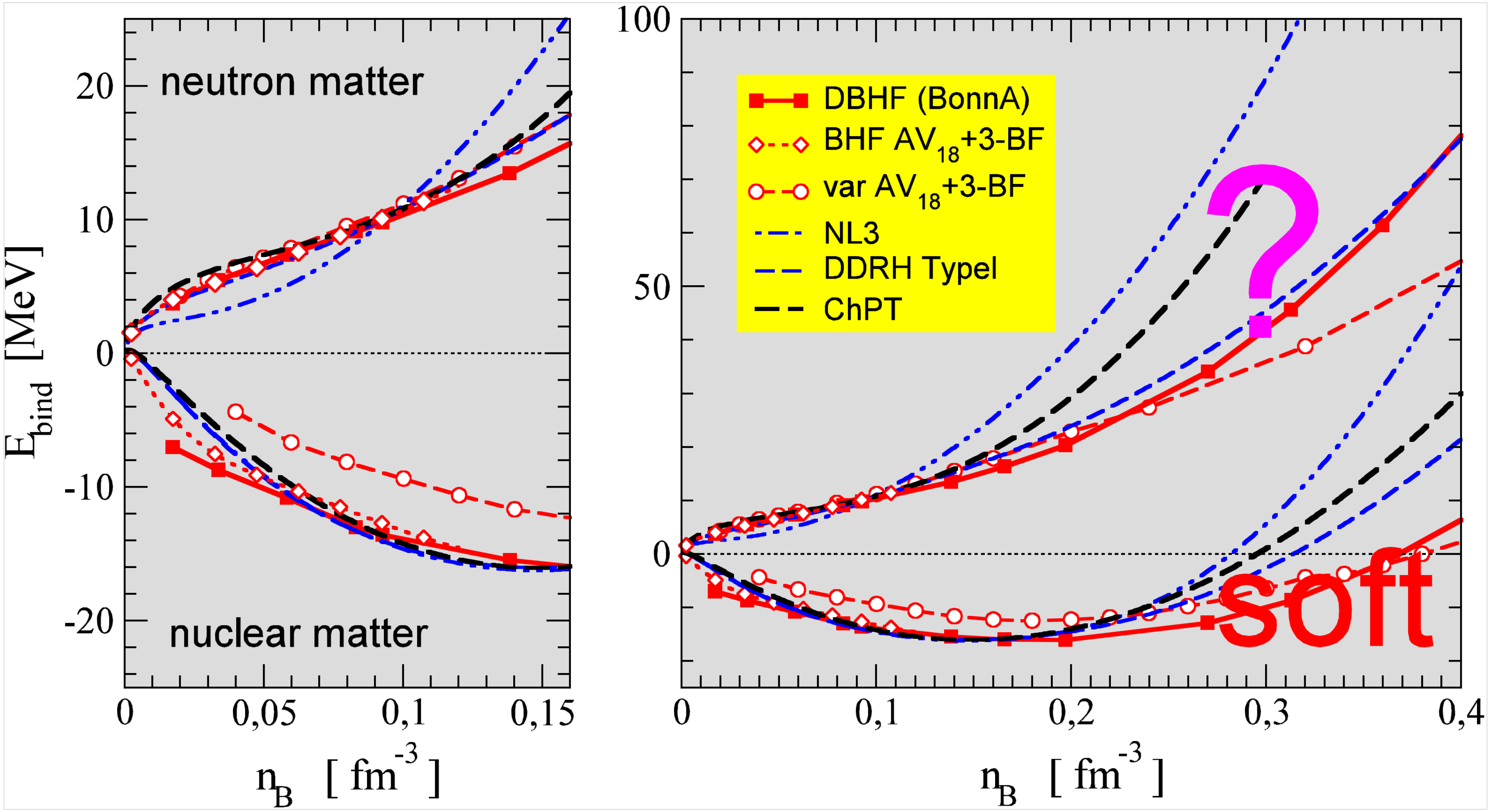
nuclear matter parameters

- $n_{\rm sat}$ saturation density
- a_V bulk energy
- K incompressibility
- K' skewness
- J bulk symmetry energy
- L symmetry energy slope



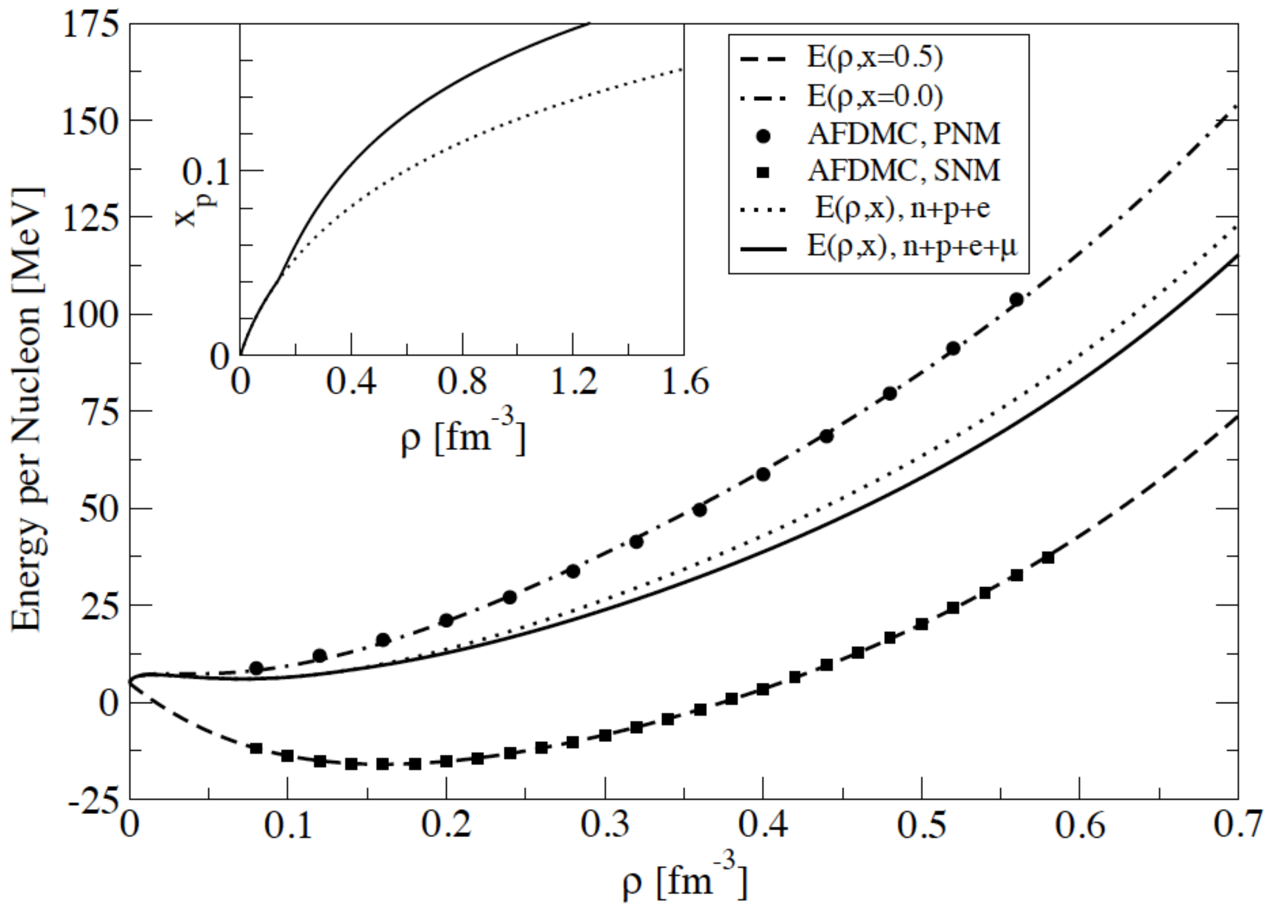
S. Typel, 2012

Quasiparticle picture: RMF and DBHF



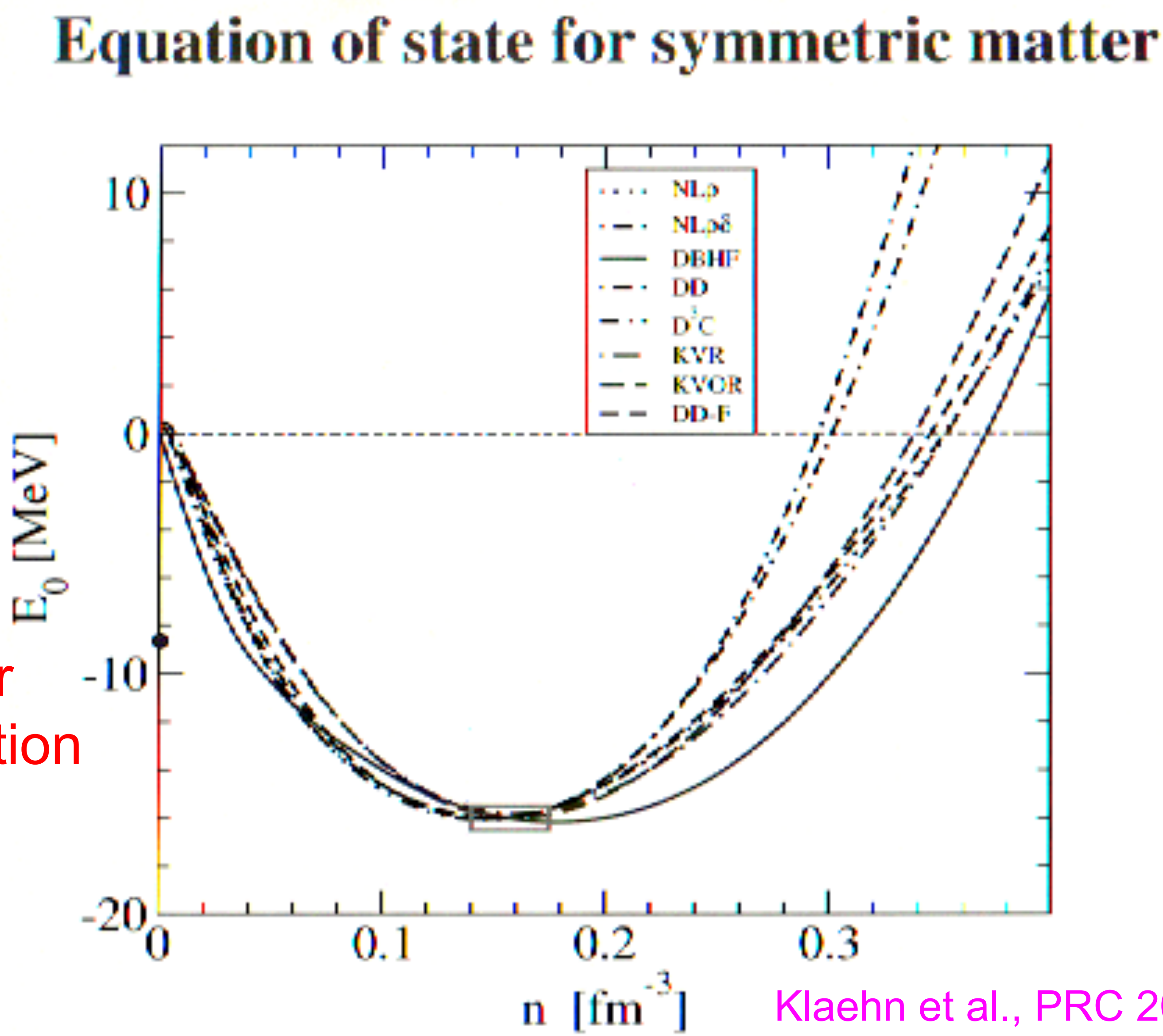
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Diffusion Monte Carlo EOS calculation



S. Gandolfi, A. Yu. Illarionov, et al., Mon.Not.R.Astron.Soc., 2010

Quasiparticle approximation for nuclear matter



But: cluster formation

Incorrect low-density limit

Klaehn et al., PRC 2006

Different approximations

medium effects

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

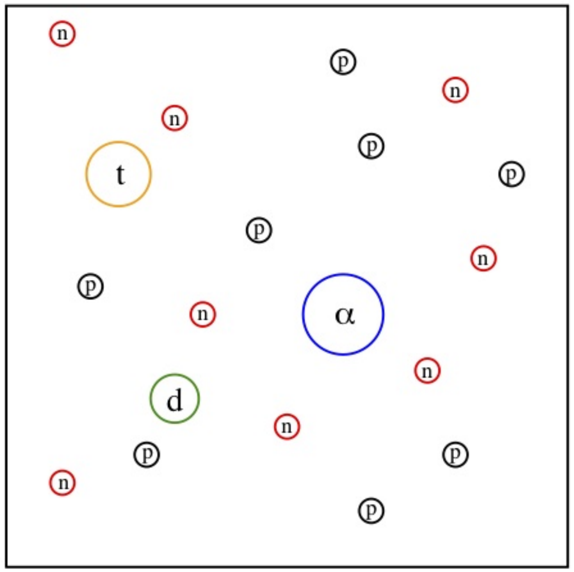
Inclusion of the light clusters (d,t,^{3}He,^{4}He)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



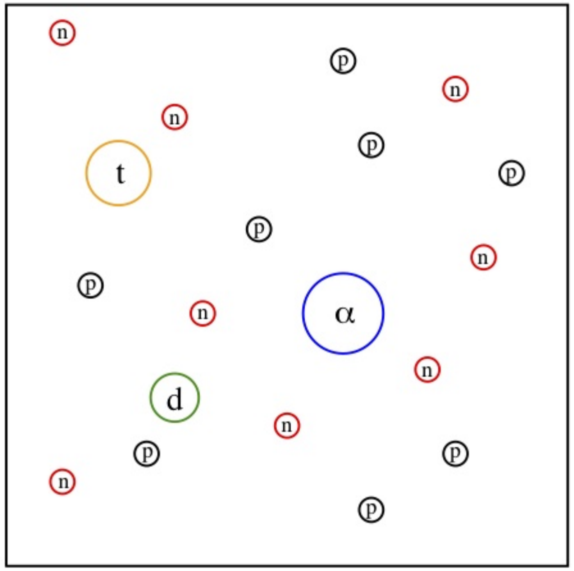
Interaction between the components
internal structure: Pauli principle

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

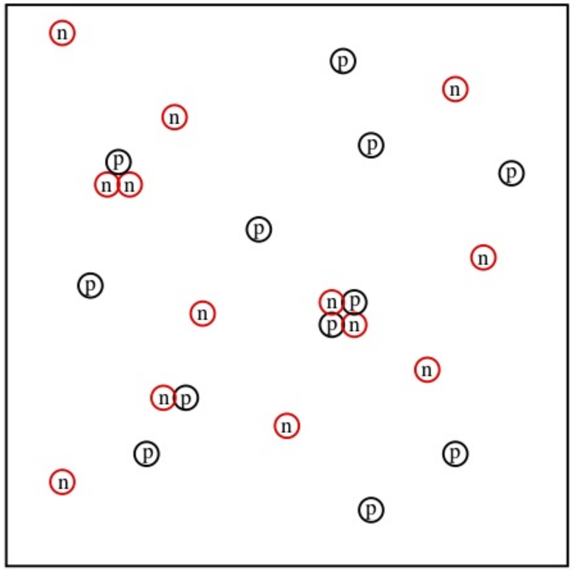


Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
with quasiparticle clusters:
self-energy and Pauli blocking

Ideal mixture of reacting nuclides

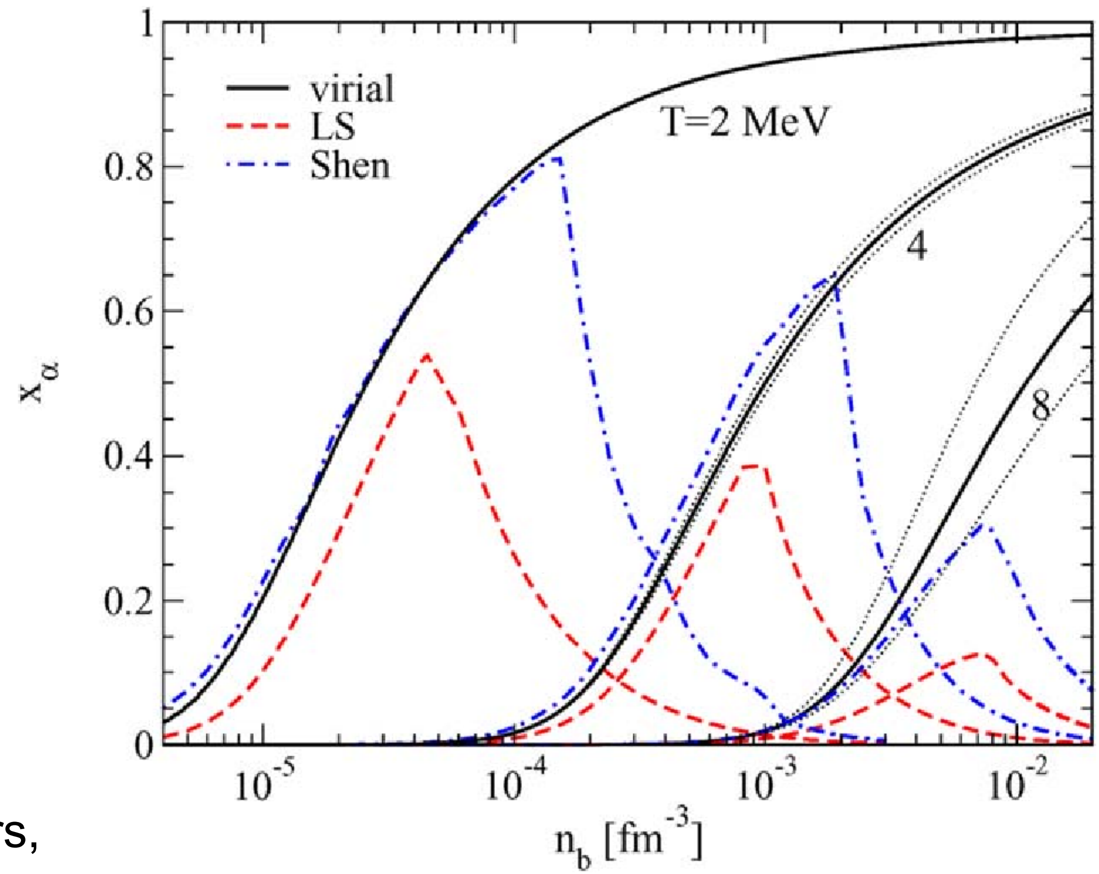
$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number *A*,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
~*K* center of mass momentum

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



LS, Shen:
higher clusters,
excluded volume

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

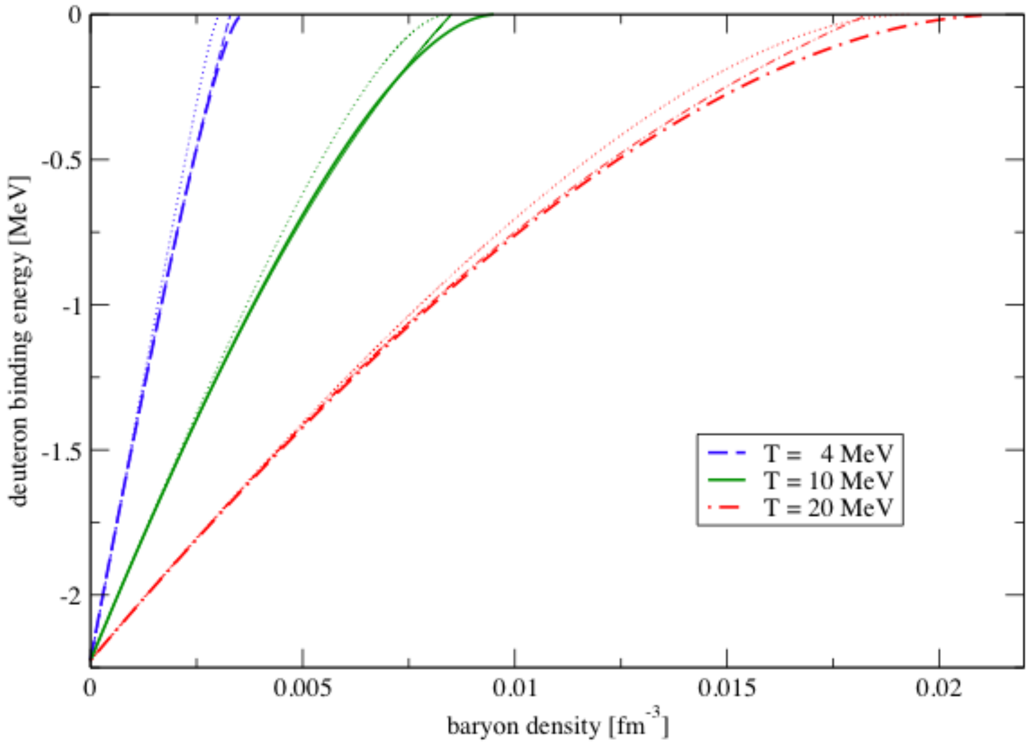
$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\{permutations\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Shift of the deuteron binding energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum



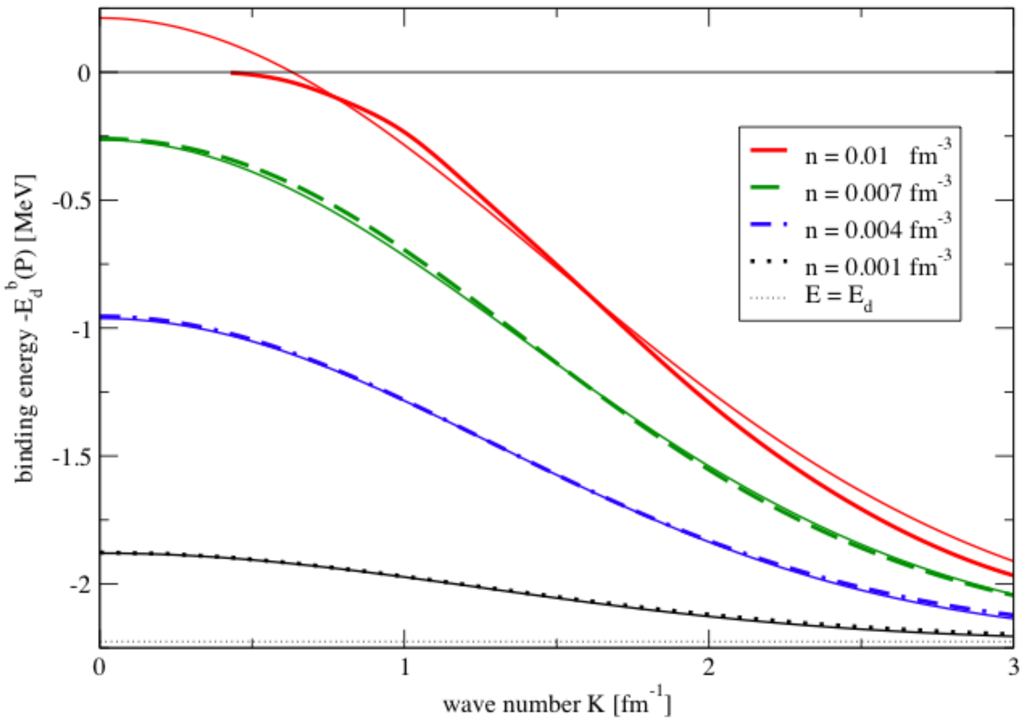
thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Shift of the deuteron binding energy

Dependence on center of mass momentum,
various densities, T=10 MeV



thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Different approximations

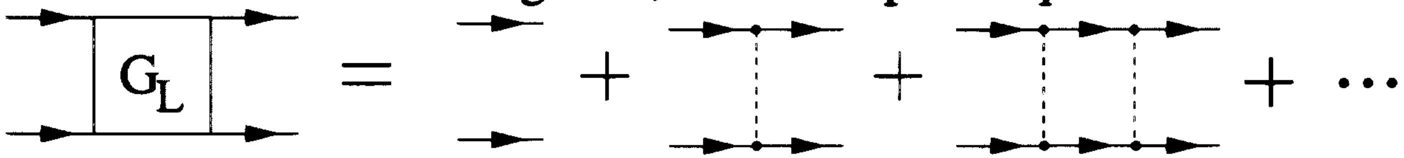
- Expansion for small Im $\Sigma(1, \omega + i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\mathrm{quasi}}(1))}{1 - \frac{\mathrm{d}}{\mathrm{d}z}\mathrm{Re}\ \Sigma(1,z)|_{z=E^{\mathrm{quasi}}-\mu_1}} - 2\mathrm{Im}\ \Sigma(1,\omega+i\eta)\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{P}{\omega+\mu_1 - E^{\mathrm{quasi}}(1)}$$

quasiparticle energy $E^{\mathrm{quasi}}(1) = E(1) + \mathrm{Re}\ \Sigma(1,\omega)|_{\omega=E^{\mathrm{quasi}}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation



Chemical picture and medium corrections

Nuclear matter at given temperature T,
baryon density n_B,
proton fraction (asymmetry) $Y_e = n_p/n_B$:   equation of state

- low-density limit: ideal mixture of reacting components:
  Nuclear statistical equilibrium (NSE)
- interactions: virial expansion (cluster virial expansion)
- higher densities: quasiparticle concept,
  medium modification of components (cluster mean-field approximation)

- nucleons as quasiparticles:
  Skyrme, relativistic mean-field (RMF), Dirac Brueckner Hartree-Fock
- light elements (d,t,h,alpha) as quasiparticles:
  shift of energy (self-energy, Pauli blocking), Mott effect.
  - excluded volume [M. Hempel, J. Schaffner-Bielich, Nucl. Phys. A 837 (2010) 210]
  - quantum statistical approach (QS): $E_{A,Z}(p;T,n_B,Y_e)$

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

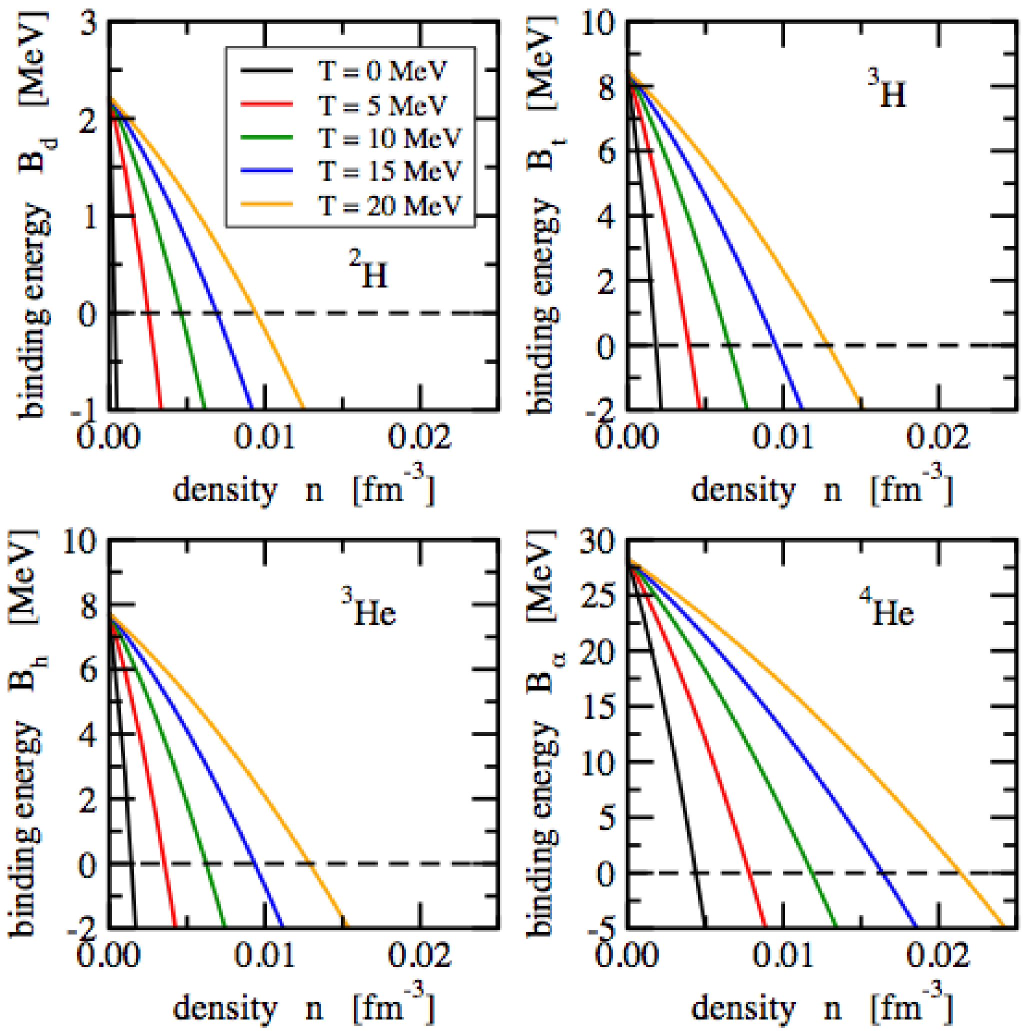
$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

- Bose-Einstein condensation

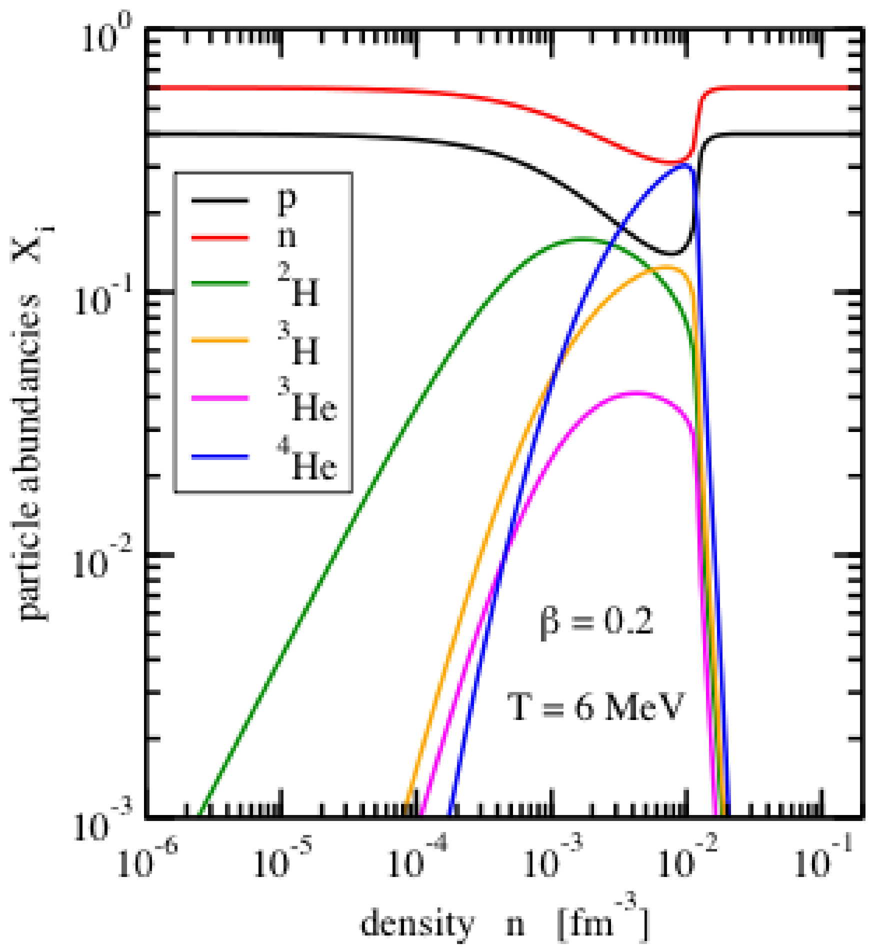
Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Light Cluster Abundances



S. Typel et al.,
PRC **81,** 015803 (2010)

Application to Heavy Ion Reactions

- Test the EOS
  (NSE, virial,… at low densities,
  Skyrme, DBHF, RMF,… near saturation)
- Unifying quantum statistical approach, medium effects, Mott effect
- Symmetry energy
- Bose enhancement?

Nimrod @ TAMU,
40Ar + 112,124Sn,
64Zn + 112,124Sn;   47 A MeV

Open questions: freeze-out model or dynamical transport models?
Identification of the source? - yields of p, (n), d, t, 3He, 4He,…

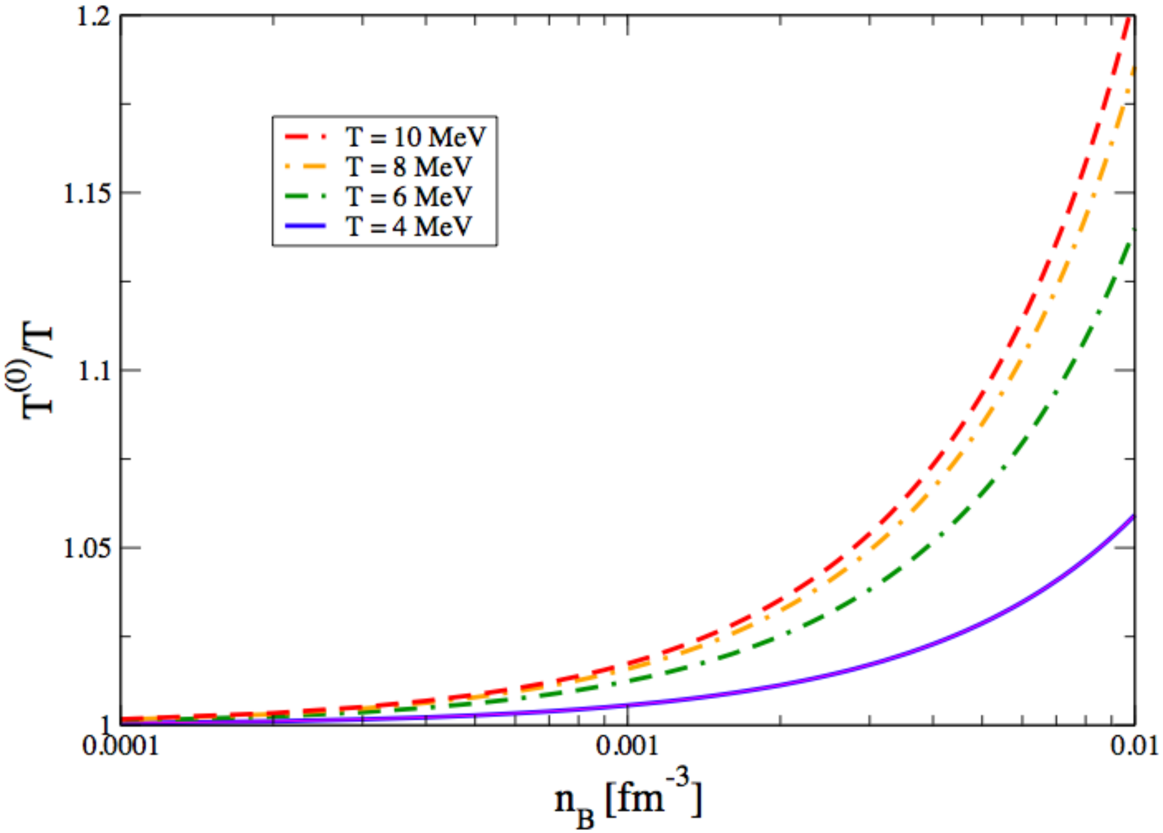
Data analysis

- asymmetry: $^3H/^3He$ fraction (Y_t/Y_h)
- temperature: Albergo thermometer (Y_aY_d/Y_tY_h)
- density: Natowitz densitometer

$$K_c(A, Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

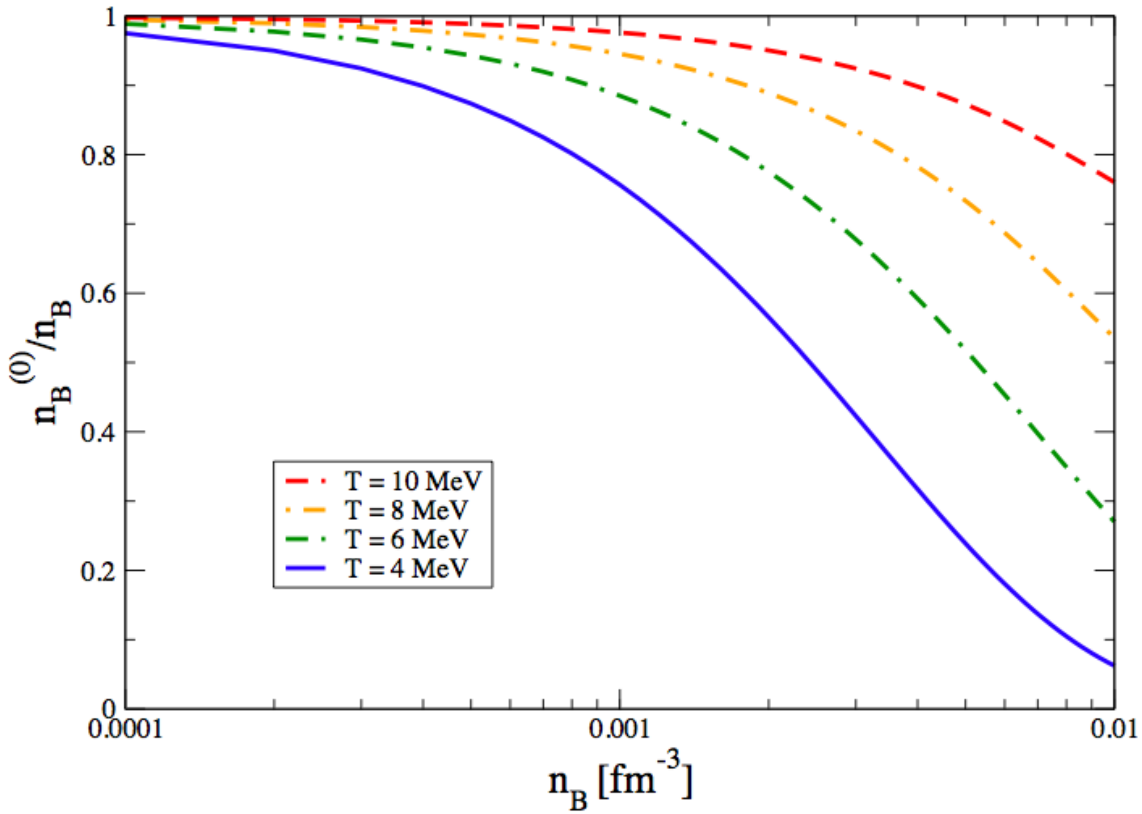
This is motivated from NSE. Medium effects?

Albergo Temperature Misfit



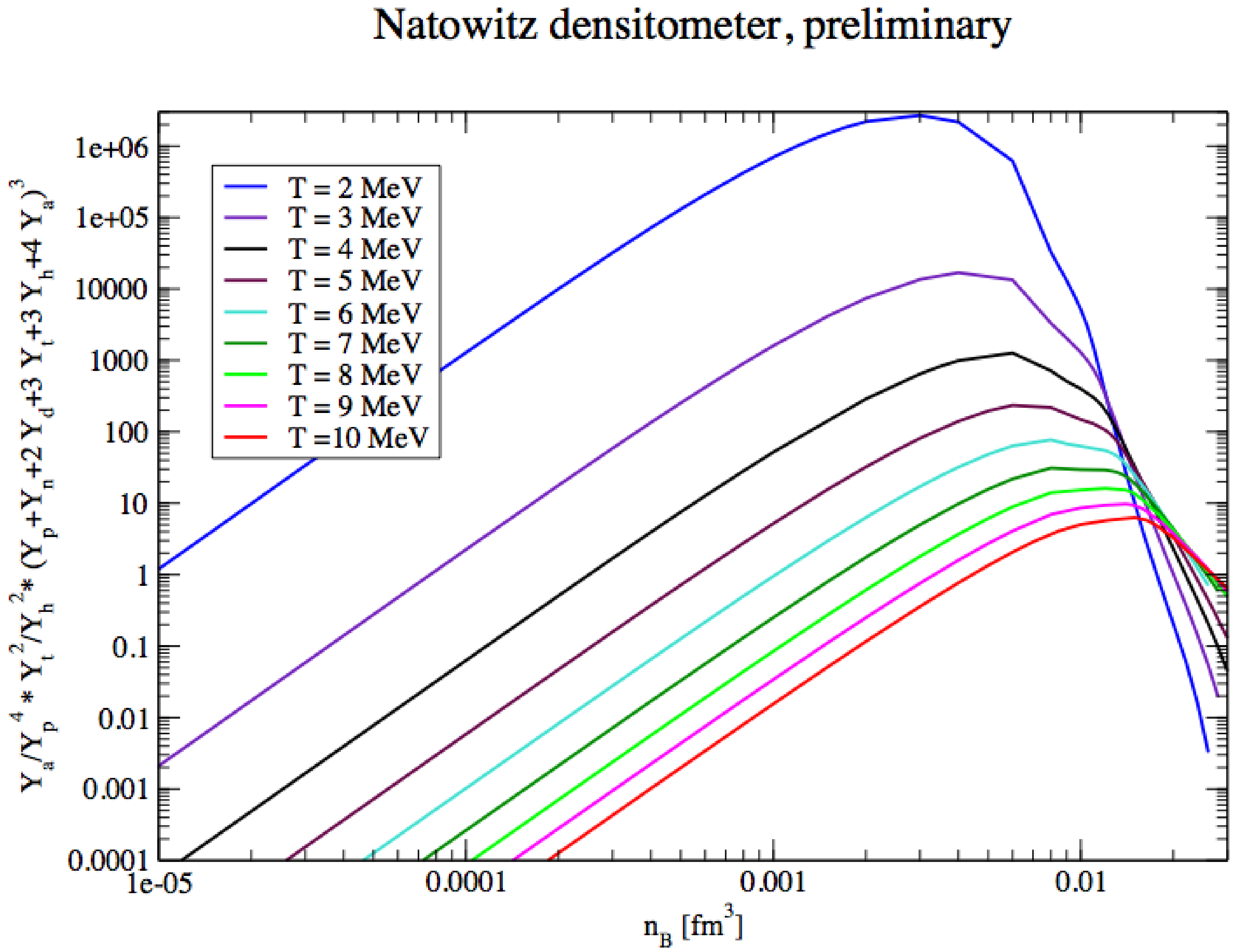
S. Shlomo, G. R., J.B. Natowitz,
PRC **79**, 034604 (2009)

Albergo Density Misfit



S. Shlomo, G. R., J.B. Natowitz,
PRC **79**, 034604 (2009)

Density determination from light cluster yields



EOS at low densities

PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

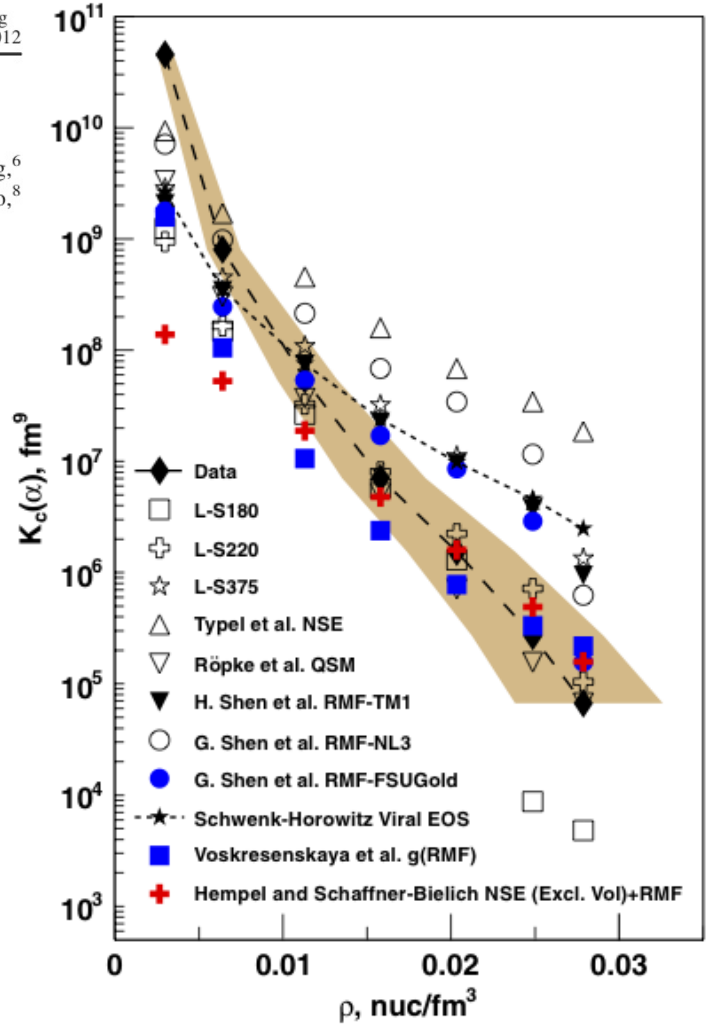
Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

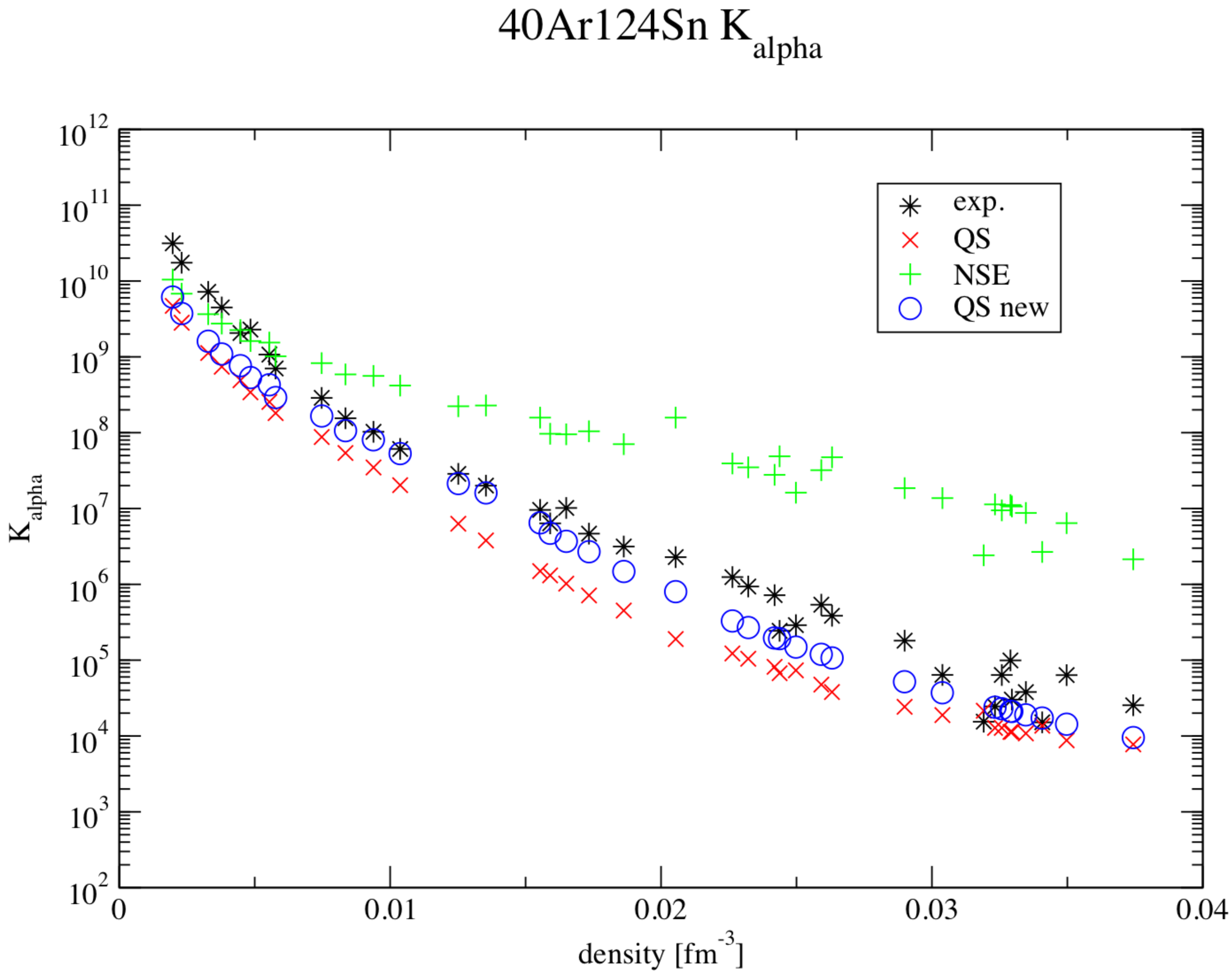
chemical constants

$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

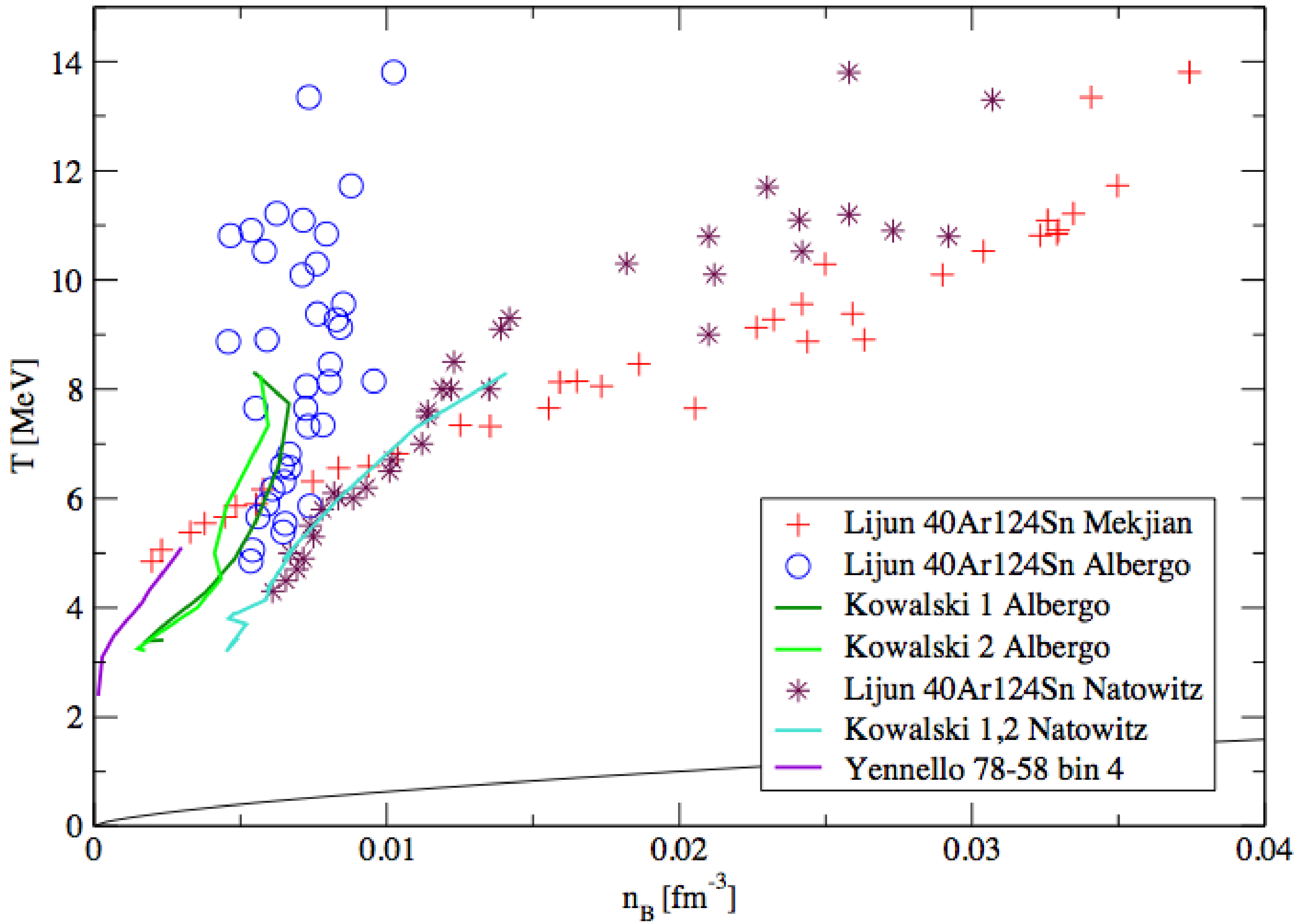
Bose enhancement?



QS versus NSE: comparison with data



Determination of thermodynamic parameters from light cluster yields

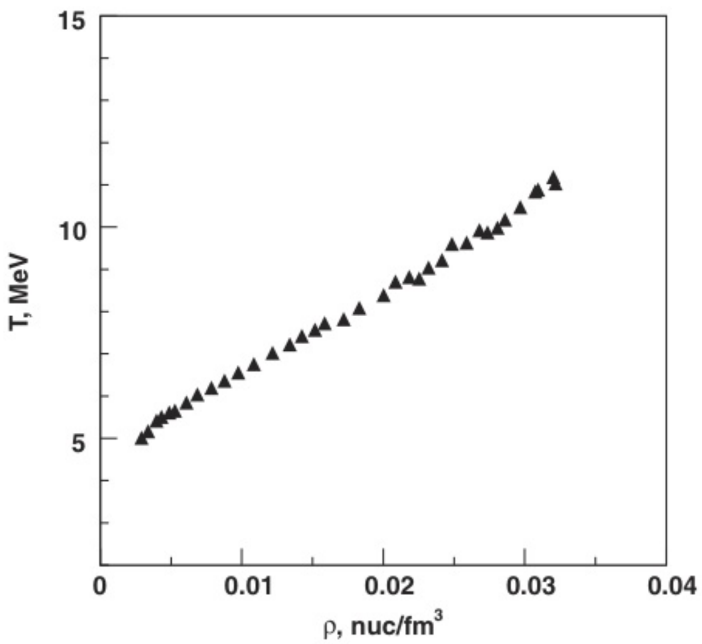


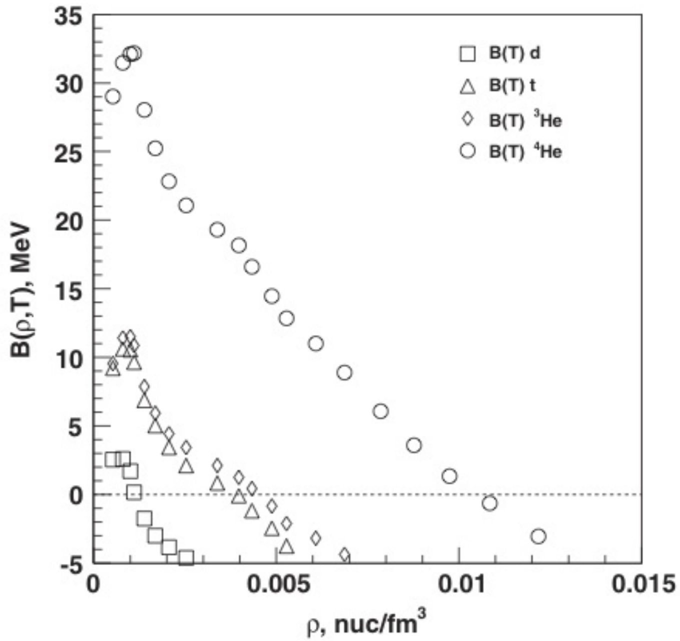
Cluster yields in HIC

PRL 108, 062702 (2012) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2012

Experimental Determination of In-Medium Cluster Binding Energies and Mott Points in Nuclear Matter

K. Hagel,[1] R. Wada,[2,1] L. Qin,[1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[2] M. Huang,[2] J. Wang,[2] H. Zheng,[1] S. Kowalski,[6] C. Bottosso,[1] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[7] M. Lunardon,[7] S. Moretto,[7] G. Nebbia,[7] S. Pesente,[7] V. Rizzi,[7] G. Viesti,[7] M. Cinausero,[8] G. Prete,[8] T. Keutgen,[9] Y. El Masri,[9] and Z. Majka[10]





in-medium binding energies

Mott points from cluster yields

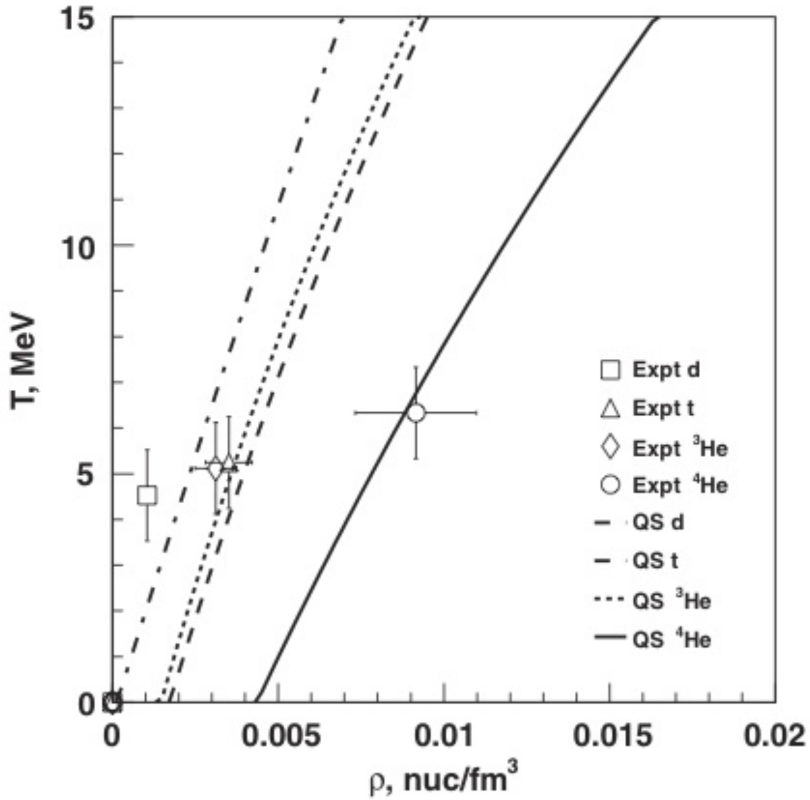


FIG. 3. Comparison of experimentally derived Mott point densities and temperatures with theoretical values. Symbols represent the experimental data. Estimated errors on the temperatures are 10% and on the densities 20%. Lines show polynomial fits to the Mott points presented in Ref. [1].

K. Hagel et al., PRL **108**, 062702 (2012)

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

- Bose-Einstein condensation

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Chemical equilibrium
with quasiparticle clusters:
self-energy and Pauli blocking

Generalized Beth-Uhlenbeck
formula:
medium modified binding energies,
medium modified scattering phase shifts

M. D. Voskresenskaya, S. Typel, Nucl. Phys. A 887 (2012) 42

Beth-Uhlenbeck formula

rigorous results at low density: virial expansion

Beth-Uhlenbeck formula

$$
\begin{aligned}
n(T,\mu) &= \frac{1}{V}\sum_p e^{-(E(p)-\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{nP} e^{-(E_{nP}-2\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{\alpha P}\int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_BT}\frac{d}{dE}\delta_\alpha(E) \\
&+ \dots
\end{aligned}
$$

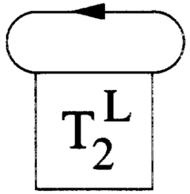
$\delta_\alpha(E)$: scattering phase shifts, channel α

Different approximations

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12)\frac{1}{i\omega_\lambda - E_{n\mathbf{P}}}\Psi^*_{n\mathbf{P}}(12)$$

$\Sigma =$



$$n(\beta,\mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+\sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k\ \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2))2\sin^2\delta_n(k)\frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}k}\delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

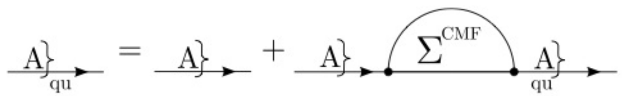
Cluster virial expansion within a quasistatistical

Generalized Beth-Uhlenbeck approach

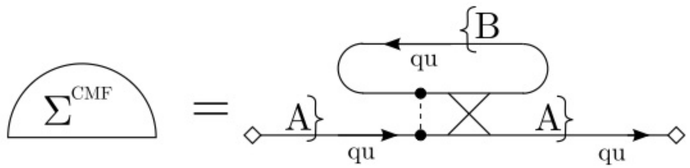
$$n_1^{\mathrm{qu}}(T,\mu_p,\mu_n) = \sum_{A,Z,\nu} \frac{A}{\Omega} \sum_{\substack{\vec{P} \\ P>P_{\mathrm{Mott}}}} f_A\big(E_{A,Z,\nu}(\vec{P};T,\mu_p,\mu_n),\mu_{A,Z,\nu}\big)$$

$$\begin{aligned} n_2^{\mathrm{qu}}(T,\mu_p,\mu_n) \;=\; & \sum_{A,Z,\nu}\sum_{A',Z',\nu'} \frac{A+A'}{\Omega} \sum_{\vec{P}}\sum_{c} g_c \frac{1+\delta_{A,Z,\nu;A',Z',\nu'}}{2\pi} \\ & \times \int_0^{\infty} dE\, f_{A+A'}\left(E_c(\vec{P};T,\mu_p,\mu_n)+E,\mu_{A,Z}+\mu_{A',Z'}\right) 2\sin^2(\delta_c)\,\frac{d\delta_c}{dE} \end{aligned}$$

Avoid double counting



Generating functional



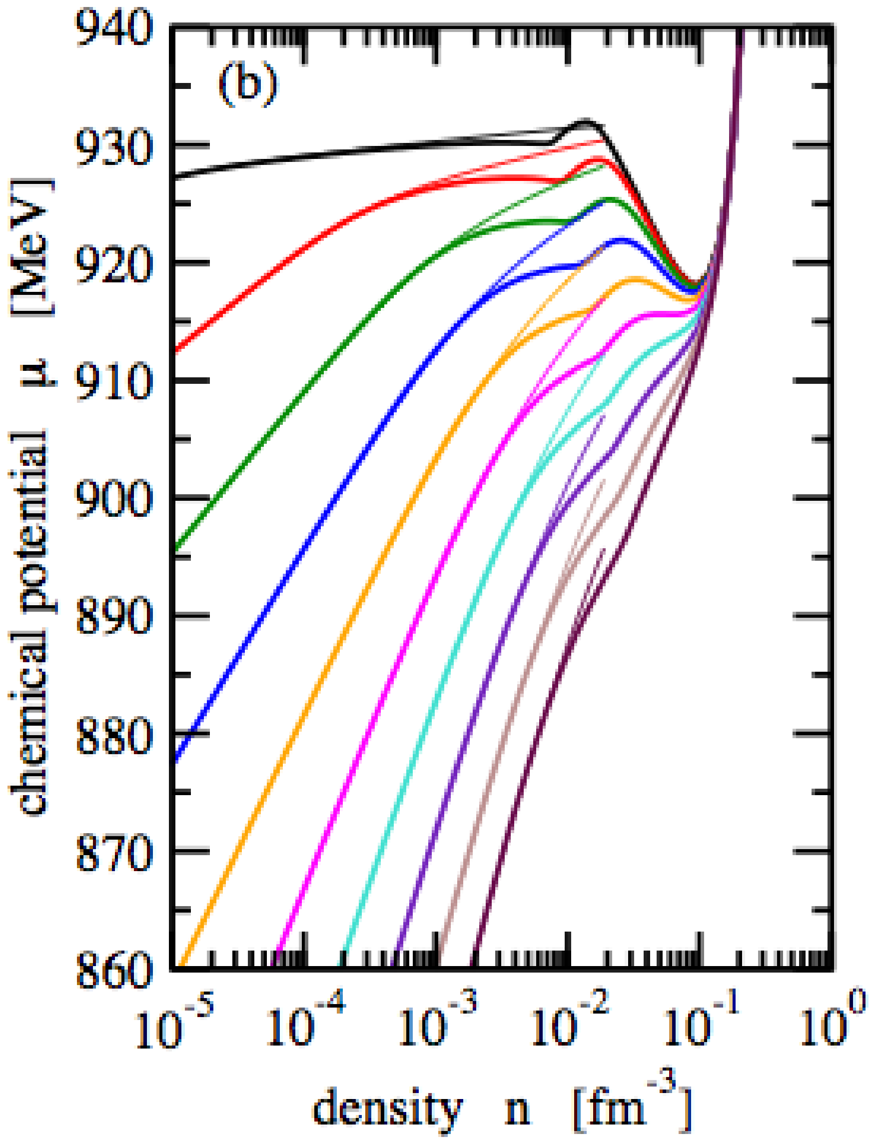
G.R., N. Bastian, D. Blaschke, et al., in preparation

Chemical potential of symmetric matter

Isotherms

T[MeV]
2
4
6
8
10
12
14
16
18
20

thin lines: NSE

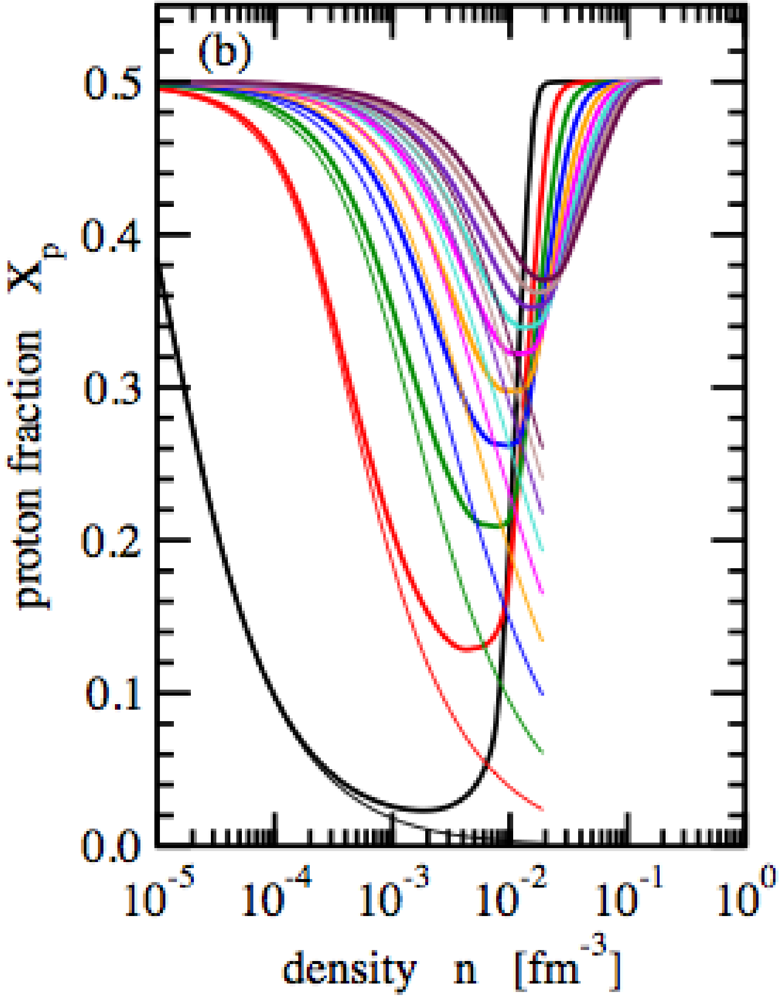


Proton fraction in symmetric matter

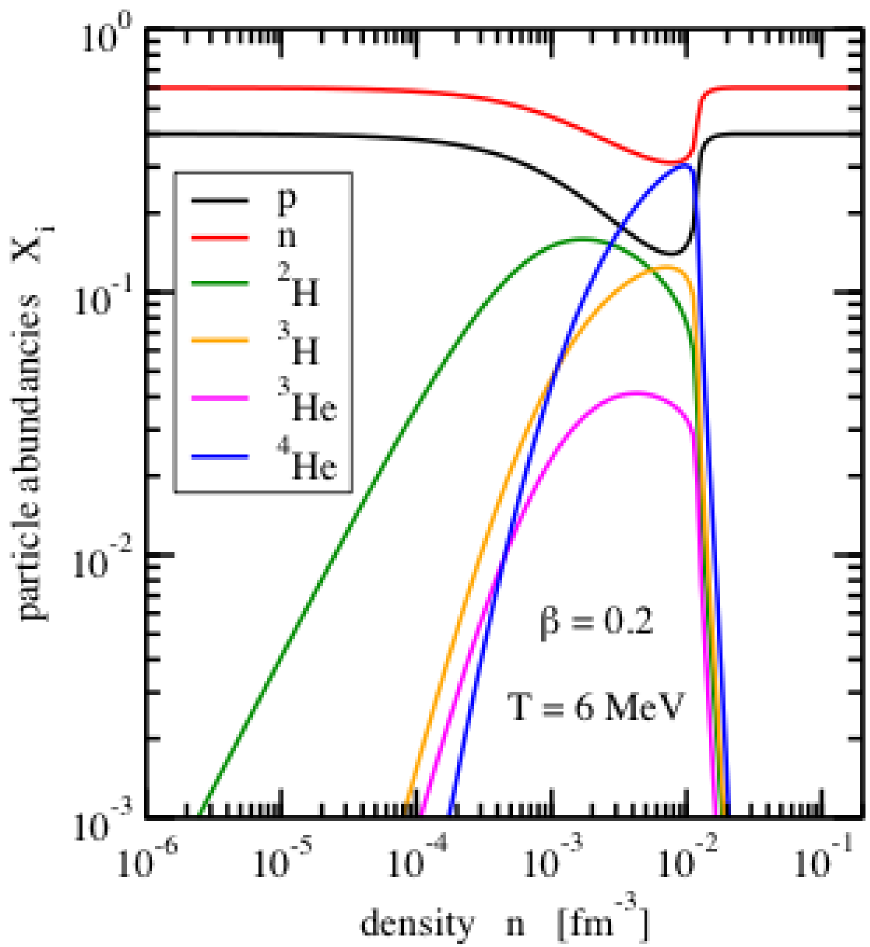
Isotherms

T[MeV]
20
18
16
14
12
10
8
6
4
2

thin lines: NSE



Light Cluster Abundances



S. Typel et al.,
PRC **81,** 015803 (2010)

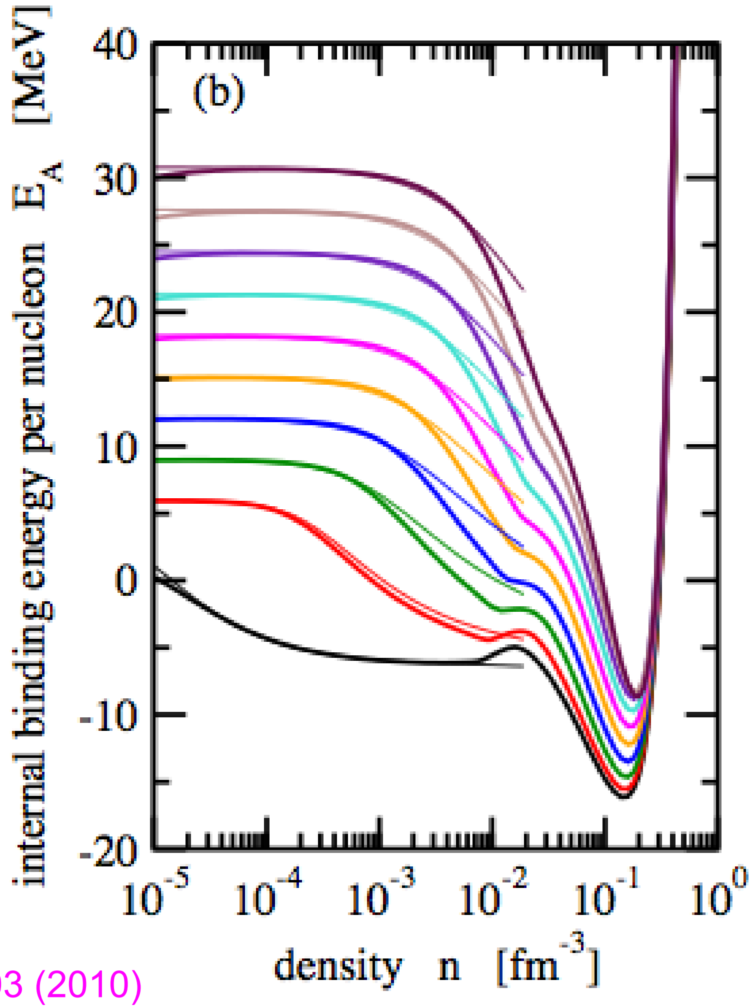
Internal energy per nucleon

Isotherms

T[MeV]

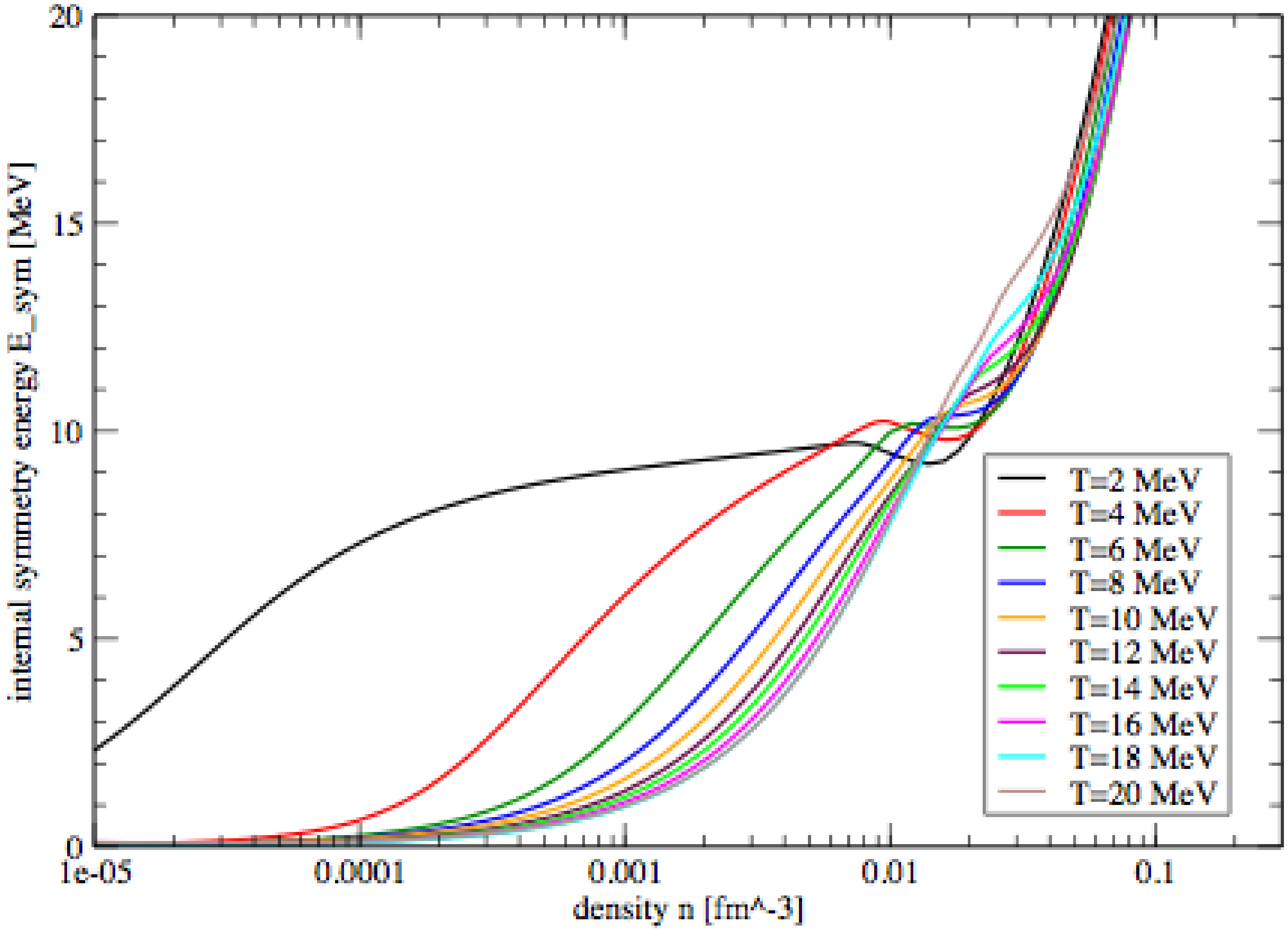
20
18
16
14
12
10
8
6
4
2

thin lines: NSE



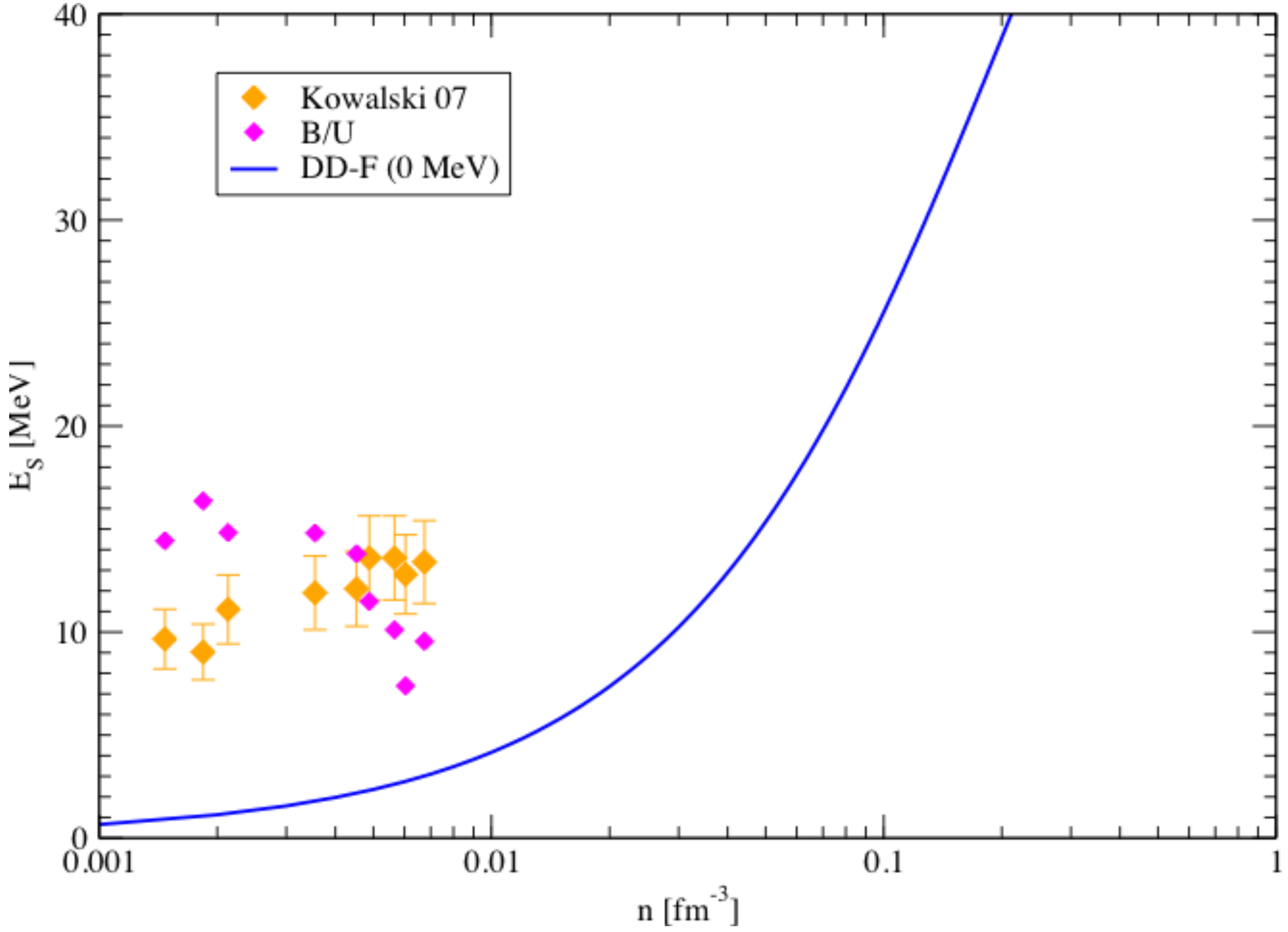
S. Typel et al., PRC **81**, 015803 (2010)

Internal symmetry energy



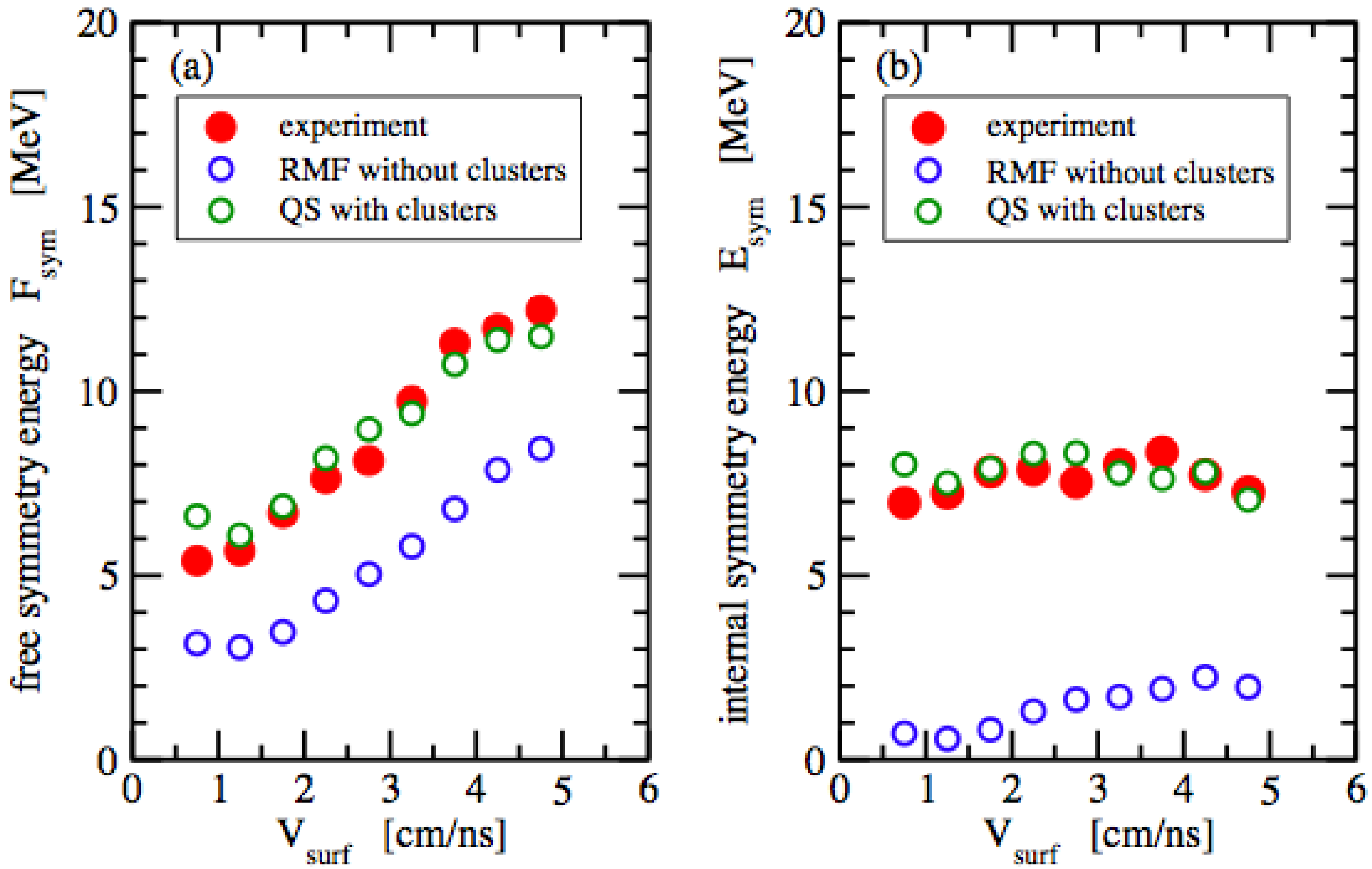
Symmetry energy

Heavy-ion collisions, spectra of emitted clusters,
temperature (3 - 10 MeV), free energy



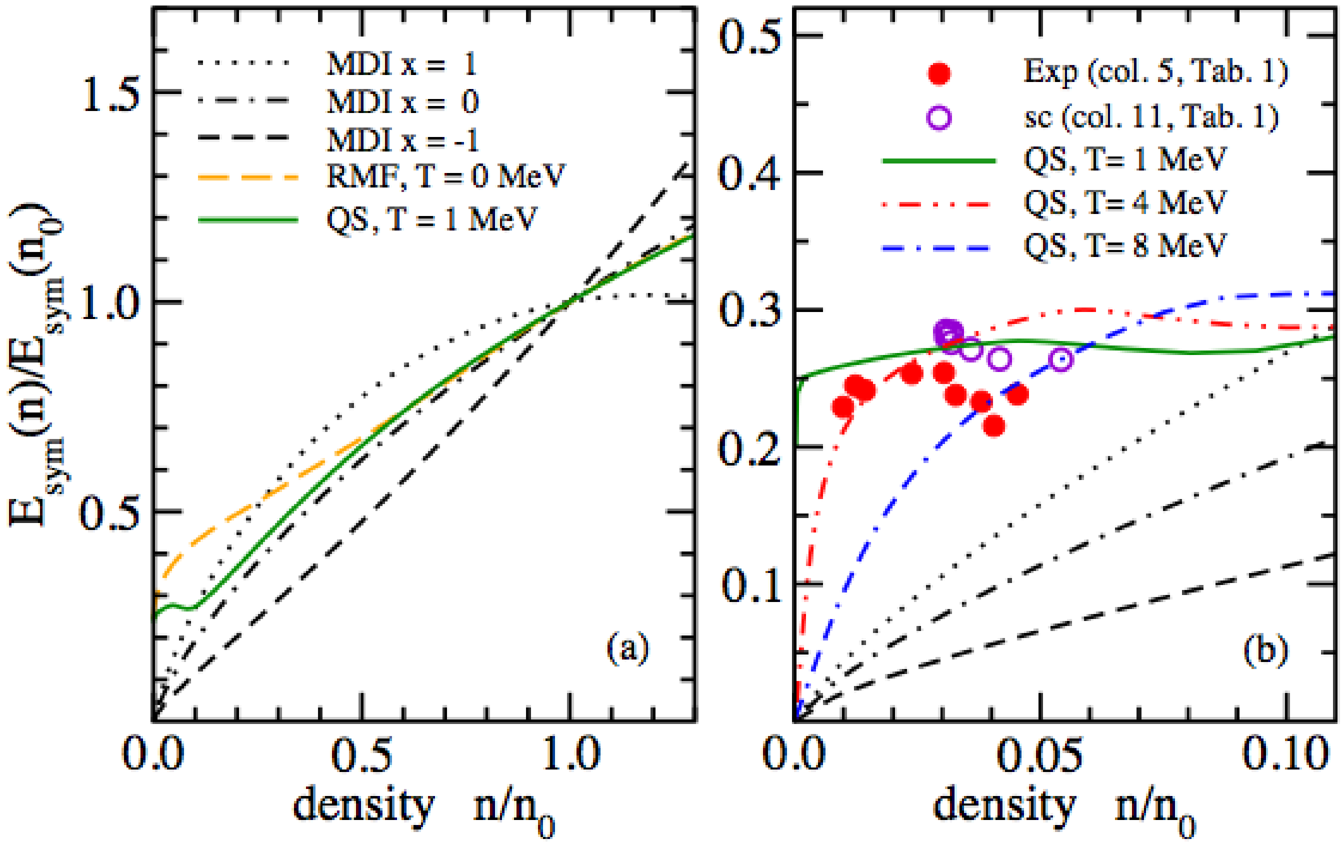
S. Kowalski et al., PRC **75**, 014601 (2007)

Symmetry energy, comparison experiment with theories



J.Natowitz et al., PRL 2010

Symmetry Energy



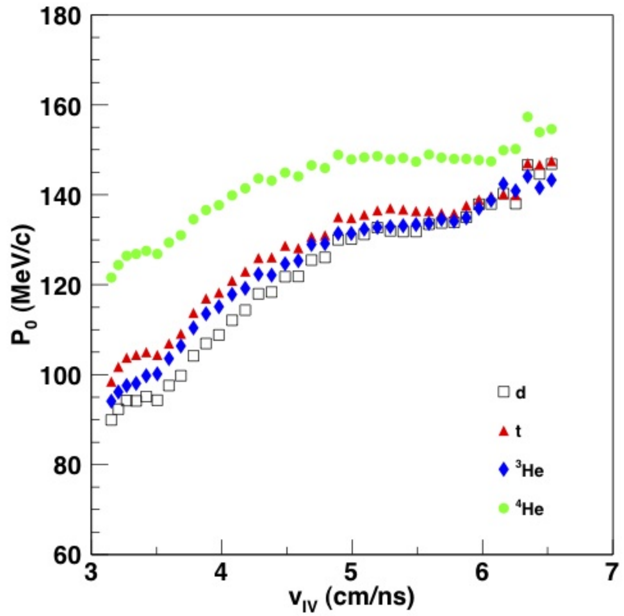
Scaled internal symmetry energy as a function of the scaled total density.
MDI: Chen et al., QS: quantum statistical, Exp: experiment at TAMU

J.Natowitz et al. PRL, May 2010

Symmetry energy at medium densities

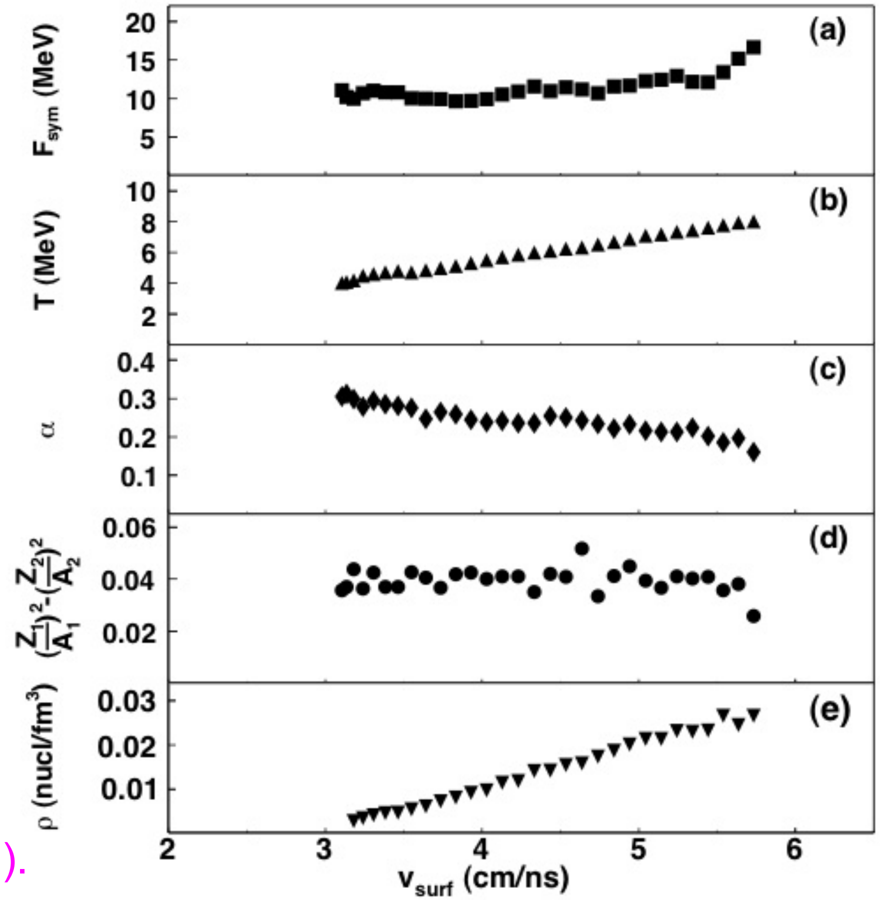
The Nuclear Matter Symmetry Energy at $0.03 \leq \rho/\rho_0 \leq 0.2$

R. Wada,[1,2] K. Hagel,[2] L. Qin,[2] J. B. Natowitz,[2] Y. G. Ma,[3] G. Röpke,[4] S. Shlomo,[2] A. Bonasera,[2,5] S. Typel,[6] Z. Chen,[2,1] M. Huang,[2,1] J. Wang,[2,1] H. Zheng,[2] S. Kowalski,[7] C. Bottosso,[2] M. Barbui,[2] M. R. D. Rodrigues,[2] K. Schmidt,[2] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] and Z. Majka[11]

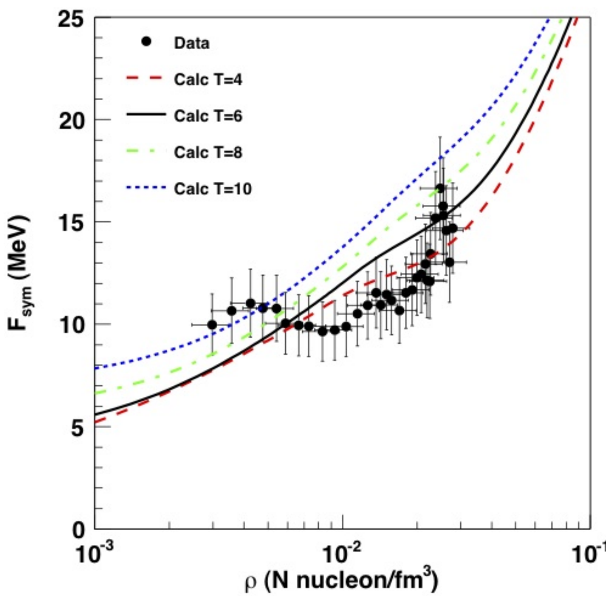


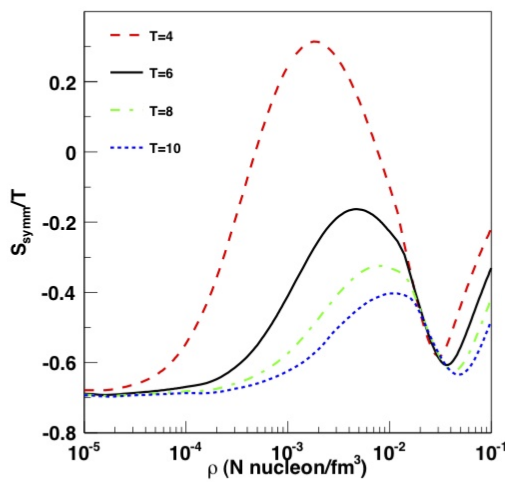
Coalescence parameter,
Mekjian model

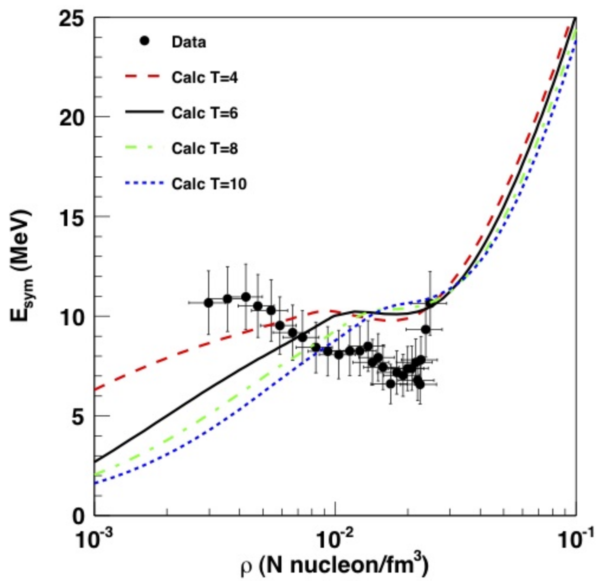
R. Wada et al., Phys. Rev. C 85, 064618 (2012).



Free symmetry energy





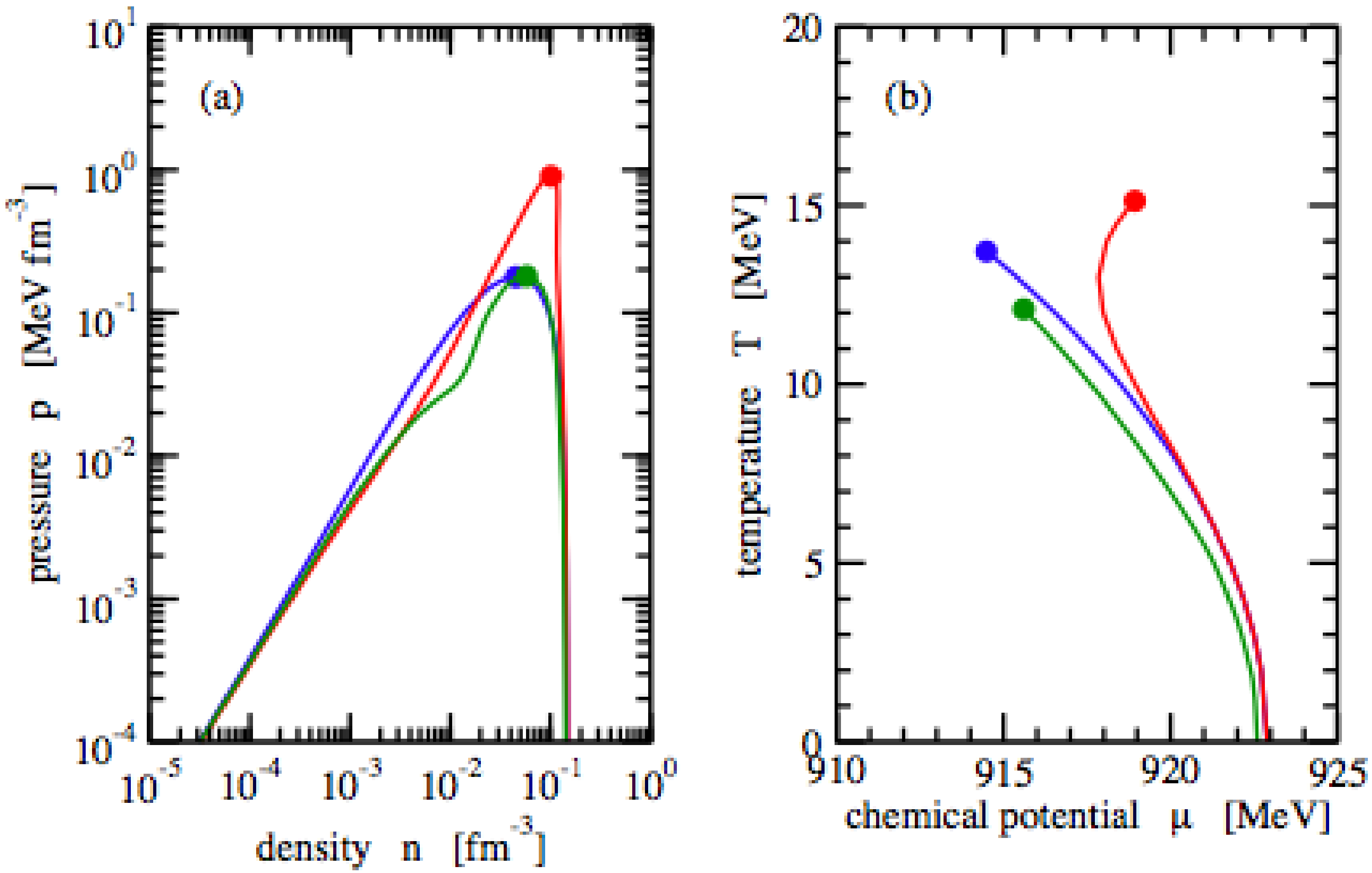


symmetry entropy

Internal symmetry energy

R. Wada et al., Phys. Rev. C 85, 064618 (2012).

Liquid-vapor phase transition



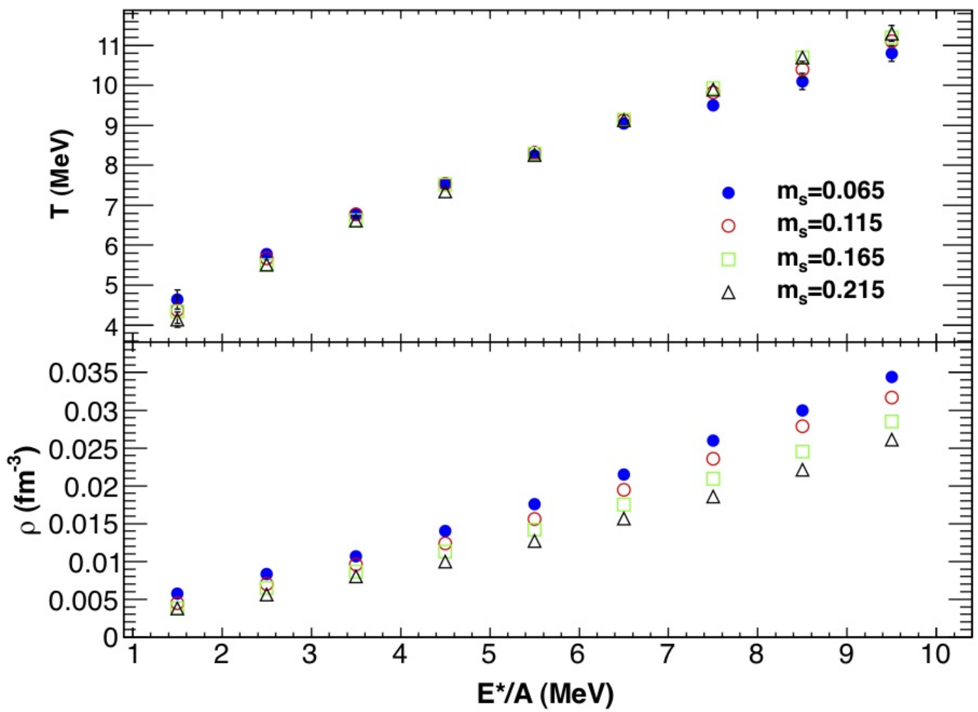
blue: no light cluster, green: with light clusters, QS, red: cluster-RMF

S. Typel et al., PRC **81**, 015803 (2010)

Experimental determination of critical temperature and density

TABLE I: Critical values and thermodynamic quantities for the four m_s bins.

m_s	T_c (MeV)	ρ_c (fm^{-3})	P_c (MeV/fm^3)	$P_c/\rho_c T_c$	ΔH (MeV)
0.065	12.12 ± 0.39	0.070 ± 0.006	0.211 ± 0.002	0.25 ± 0.02	31.50 ± 1.01
0.115	12.51 ± 0.35	0.066 ± 0.005	0.209 ± 0.001	0.25 ± 0.02	32.53 ± 0.90
0.165	13.11 ± 0.30	0.064 ± 0.004	0.232 ± 0.001	0.27 ± 0.02	31.46 ± 0.71
0.215	13.39 ± 0.21	0.061 ± 0.002	0.258 ± 0.002	0.31 ± 0.01	32.13 ± 0.50



Critical Scaling of
Two-component Systems
from Quantum Fluctuations

J. Mabiala, et al., arXiv:12083280v1 [nucl-ex]

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

Quasiparticle cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs of clusters

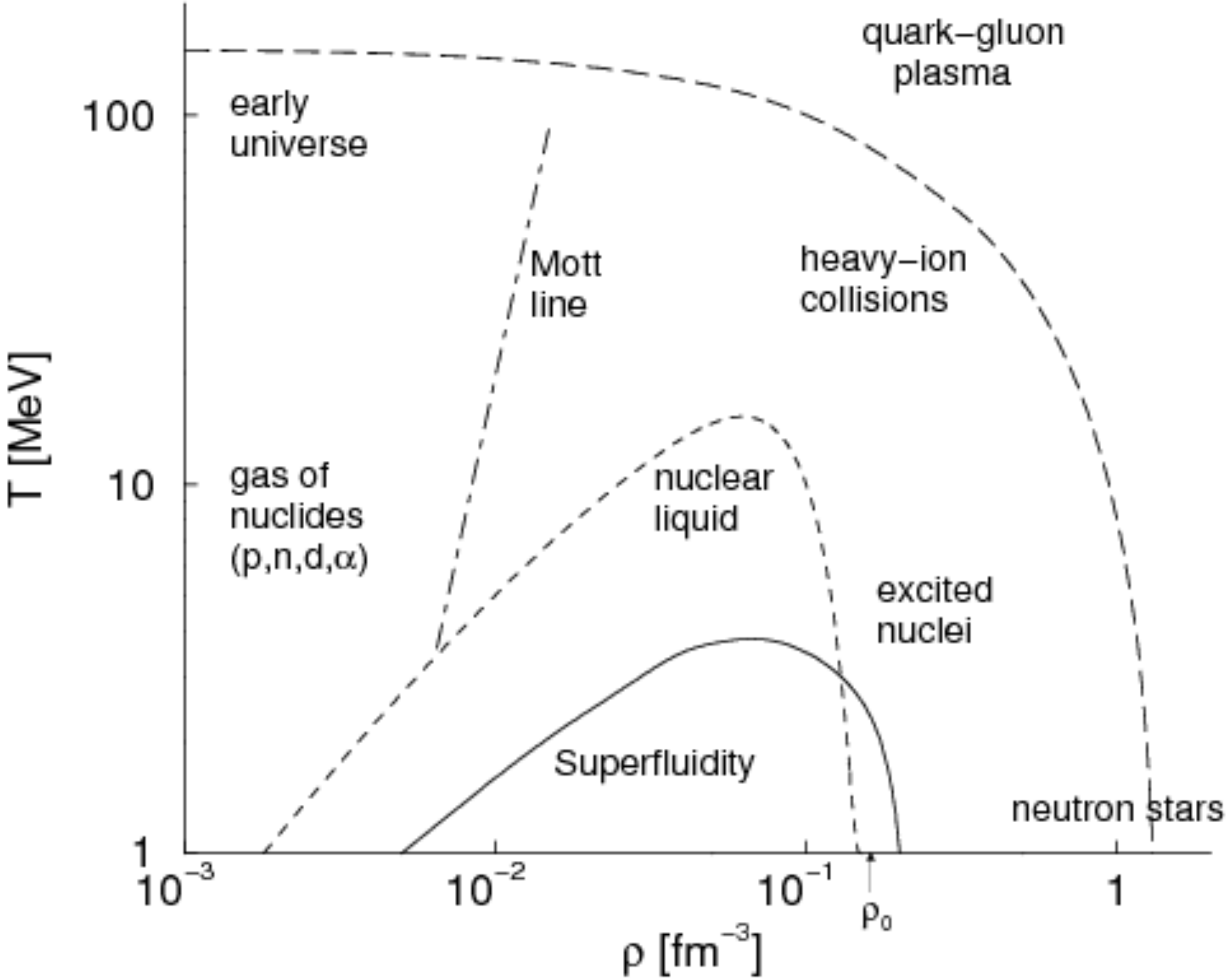
Conclusion I

• Due to the interaction, cluster are formed in nuclear matter that are of significance in the low-density limit. Here, the nuclear statistical equilibrium or cluster virial expansions can be used to describe the thermodynamic properties.

• Medium effects become of relevance for densities > 10^{-4} fm^{-3}. Single nucleon quasiparticle energies can be introduced. In addition, Pauli blocking modifies the cluster properties so that they are dissolved with increasing density.

• Properties of nuclear matter such as the symmetry energy are determined in the low-density region by the formation of bound states.

Symmetric nuclear matter: Phase diagram



Dubna, 1./3. 9. 2012

HISS: Dense Matter in Heavy-Ion Collisions and Astrophysics

Cluster Formation and Liquid-Gas Transition in Nuclear Matter

Gerd Röpke, Rostock

Outline

- Nuclear matter - a strongly interacting quantum liquid

  where it occurs, what do we know: nuclei, stars, HIC

- Many-particle theory: Equation of state

  QCD? Effective interactions, Green functions, spectral functions

- Low-density limit: cluster formation

  Mass action law, nuclear statistical equilibrium, virial expansion

- Near saturation: medium effects

  mean-field and quasiparticles, dissolution of bound states

- Quantum condensates:

  transition from BEC to BCS, Hoyle states, pairing and quartetting

Conclusion I

• Due to the interaction, cluster are formed in nuclear matter that are of significance in the low-density limit. Here, the nuclear statistical equilibrium or cluster virial expansions can be used to describe the thermodynamic properties.

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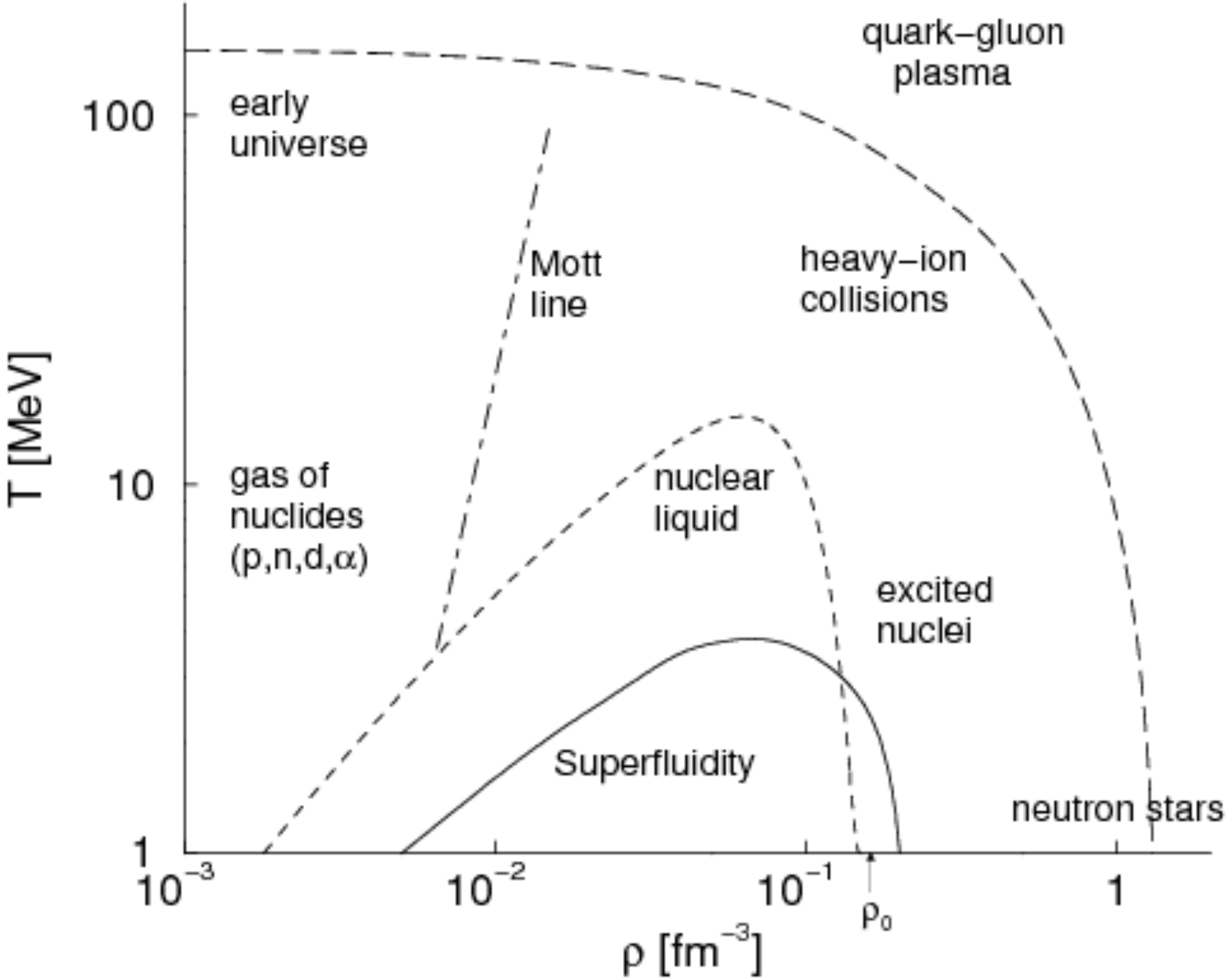
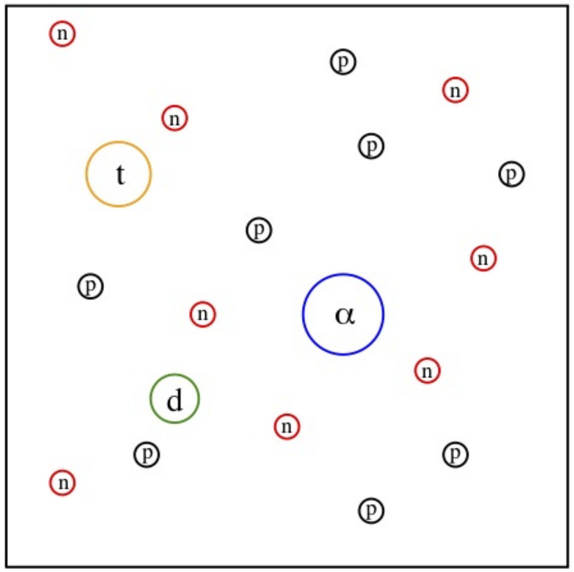


Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

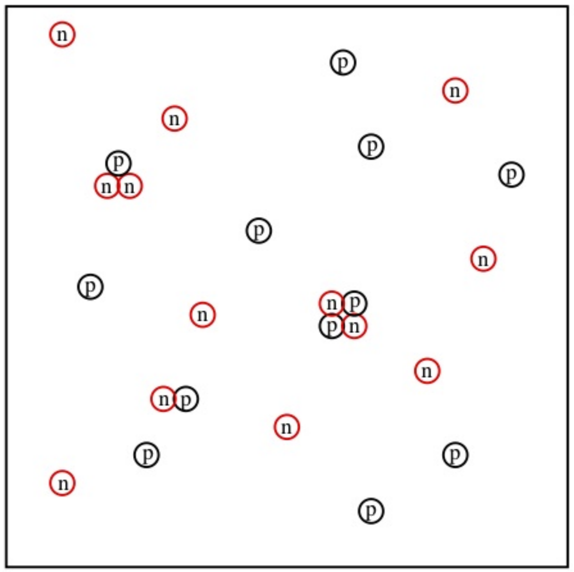


Interaction between the components
internal structure: Pauli principle

Physical picture:

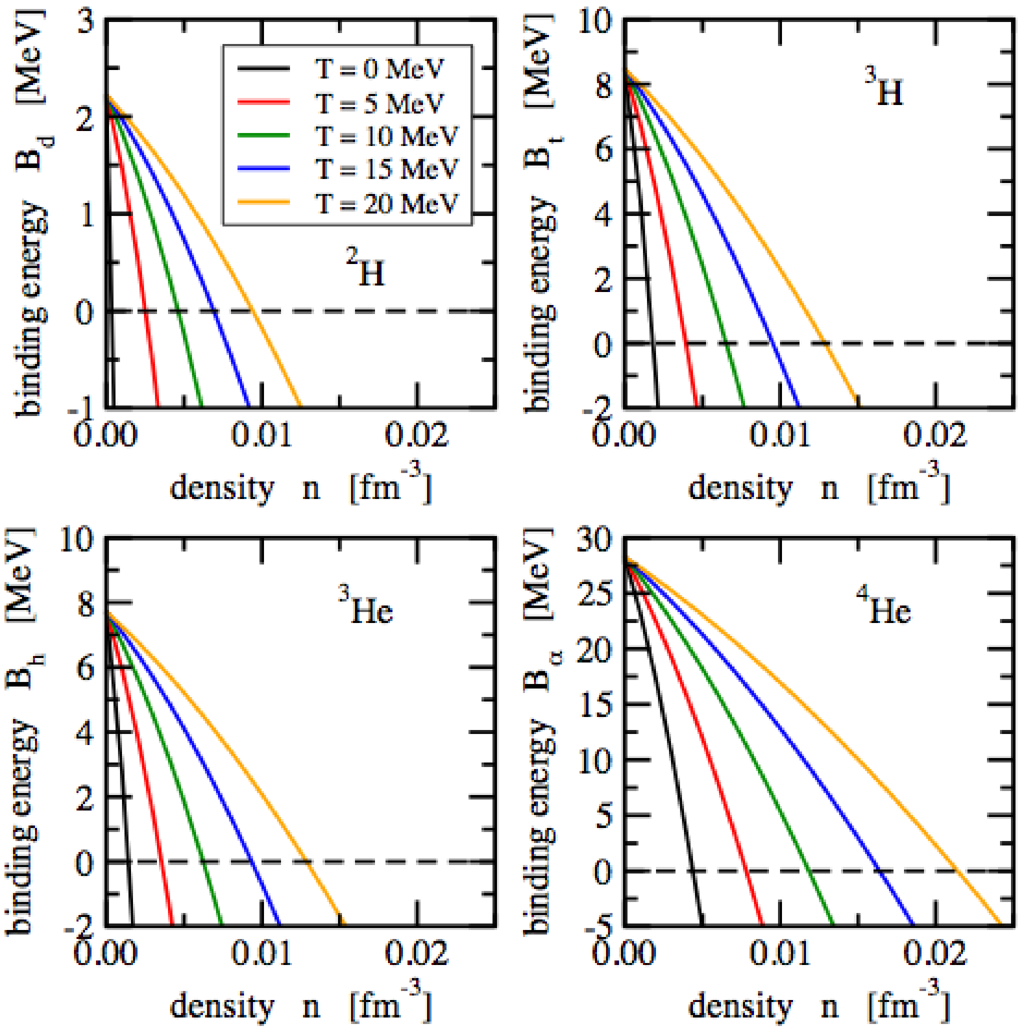
"elementary" constituents

and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Different approximations

medium effects

Ideal Fermi gas:
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(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

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all bound states (clusters)
scattering phase shifts of all pairs of clusters

Pauli blocking and Mott effect

Two different fermions (a,b: proton,neutron) form a bound state (c: deuteron).

$$c_q = \sum_p F(q,p) a_p b_{q-p}$$

Is the bound state a boson? Commutator relation

$$\left[c_q, c_{q'}^+\right]_- = \sum_{p,p'} F(q,p) F^*(q',p') \left[a_p b_{q-p}, b_{q'-p'}^+ a_{p'}^+\right]_-$$

$$\underline{a_p b_{q-p} b_{q'-p'}^+ a_{p'}^+} + a_p b_{q'-p'}^+ b_{q-p} a_{p'}^+ - a_p b_{q'-p'}^+ b_{q-p} a_{p'}^+ - b_{q'-p'}^+ a_p b_{q-p} a_{p'}^+$$

$$+ b_{q'-p'}^+ a_p b_{q-p} a_{p'}^+ + b_{q'-p'}^+ a_p a_{p'}^+ b_{q-p} - b_{q'-p'}^+ a_p a_{p'}^+ b_{q-p} \underline{- b_{q'-p'}^+ a_{p'}^+ a_p b_{q-p}}$$

$$= a_p a_{p'}^+ \delta_{q-p,q'-p'} - b_{q'-p'}^+ b_{q-p} \delta_{p,p'} = \left(\delta_{p,p'} - a_{p'}^+ a_p\right) \delta_{q-p,q'-p'} - b_{q'-p'}^+ b_{q-p} \delta_{p,p'}$$

$$\left[c_q, c_{q'}^+\right]_- = \sum_{p,p'} F(q,p) F^*(q',p') \left[\left(\delta_{p,p'} - a_{p'}^+ a_p\right) \delta_{q-p,q'-p'} - b_{q'-p'}^+ b_{q-p} \delta_{p,p'}\right]$$

$$= \sum_p F(q,p) F^*(q,p) \delta_{q,q'} - \sum_{p,p'} F(q,p) F^*(q',p') \left[\left(a_{p'}^+ a_p\right) \delta_{q-p,q'-p'} + \left(b_{q'-p'}^+ b_{q-p}\right) \delta_{p,p'}\right]$$

averaging

$$\left\langle \left[c_q, c_{q'}^+\right]_- \right\rangle = \delta_{q,q'} \left[1 - \sum_p F(q,p) F^*(q,p) \left(\left\langle a_p^+ a_p \right\rangle + \left\langle b_{q-p}^+ b_{q-p} \right\rangle\right)\right]$$

Fermionic substructure: phase space occupation, “excluded volume”

Many-particle system

- Hamiltonian (non-relativistic)

$$H = \sum_1 E(1) a_1^+ a_1 + \frac{1}{2} \sum_{12,1'2'} V(12,1'2') a_1^+ a_2^+ a_{2'} a_{1'}$$

- fermions in states $\{1\} = \{p, \sigma, \tau\}$
- interaction: Coulomb, nuclear, …
- bound states (bosons)
- quantum condensates
- homogeneous system in equilibrium: $\rho = \exp[-S/k_B]$
- variation in time and space

  (finite systems? non-equilibrium?)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

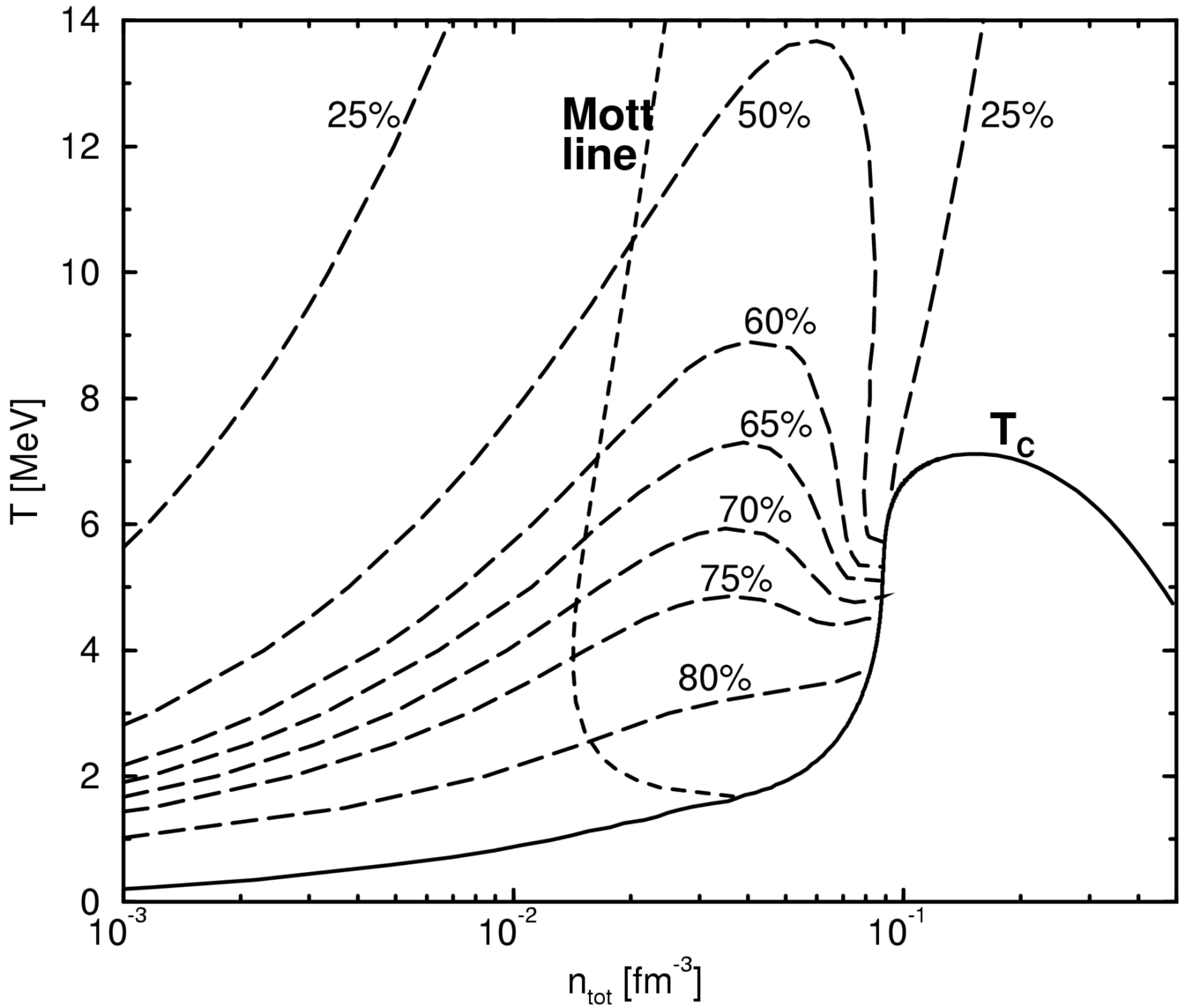
Quantum condensates: Outline

1. Many-particle system
   - Mean field approach: self-energy and Pauli blocking
   - Generalized Beth-Uhlenbeck equation
   - Mott effect and transition from BEC to BCS
   - Self-consistent solutions and pseudogap
2. Correlations: account of higher clusters
   - Cluster expansion of the self-energy
   - Cluster - mean field approximation
   - Quantum condensates: Pairing and quartetting
3. Finite systems: 4-n nuclei
   - Cluster formation in dilute nuclei
   - BEC states: Hoyle state and THSR wave function
   - Suppression of the condensate at increasing density

Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

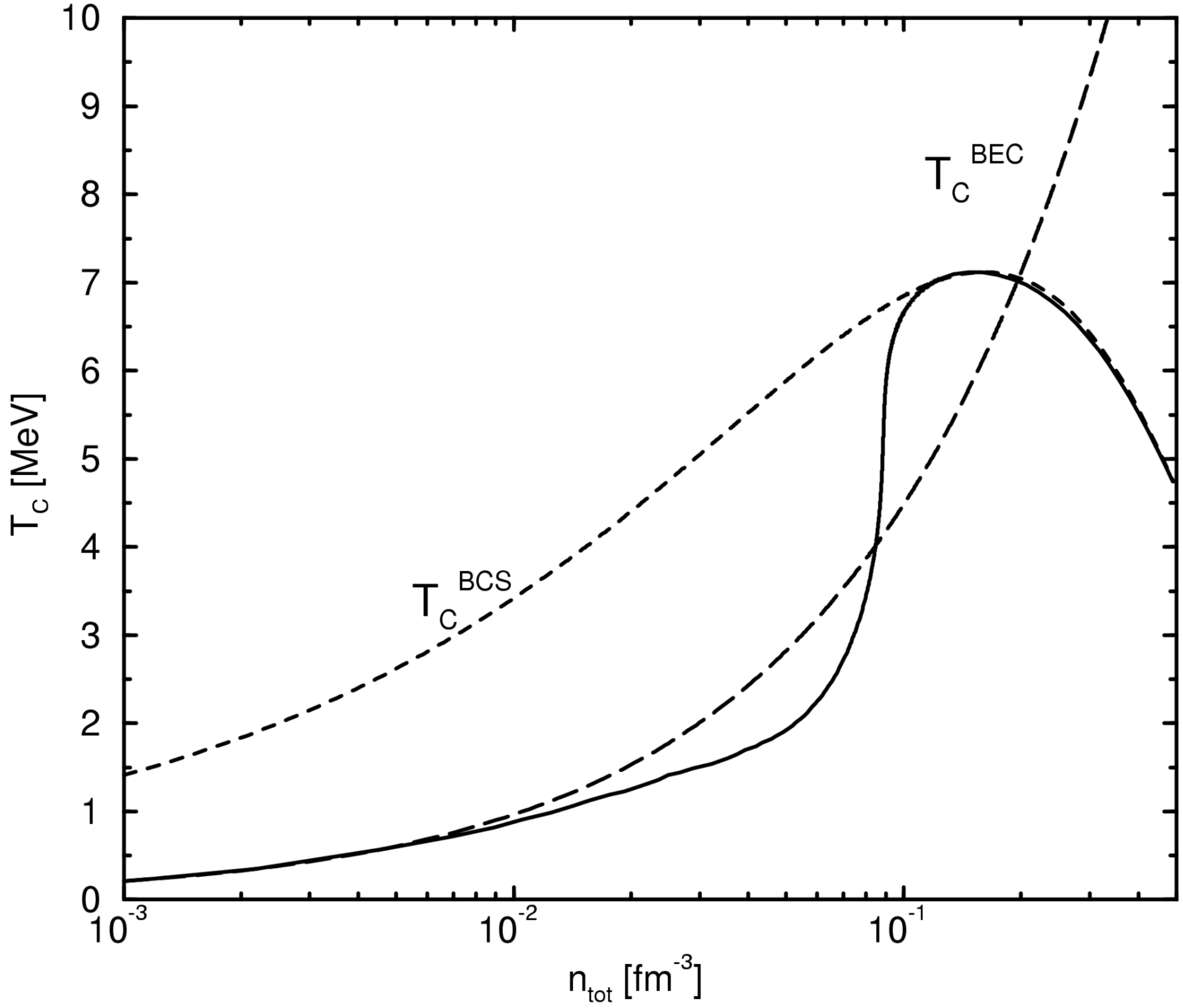
H. Stein et al.,
Z. Phys. **A351**, 259 (1995)

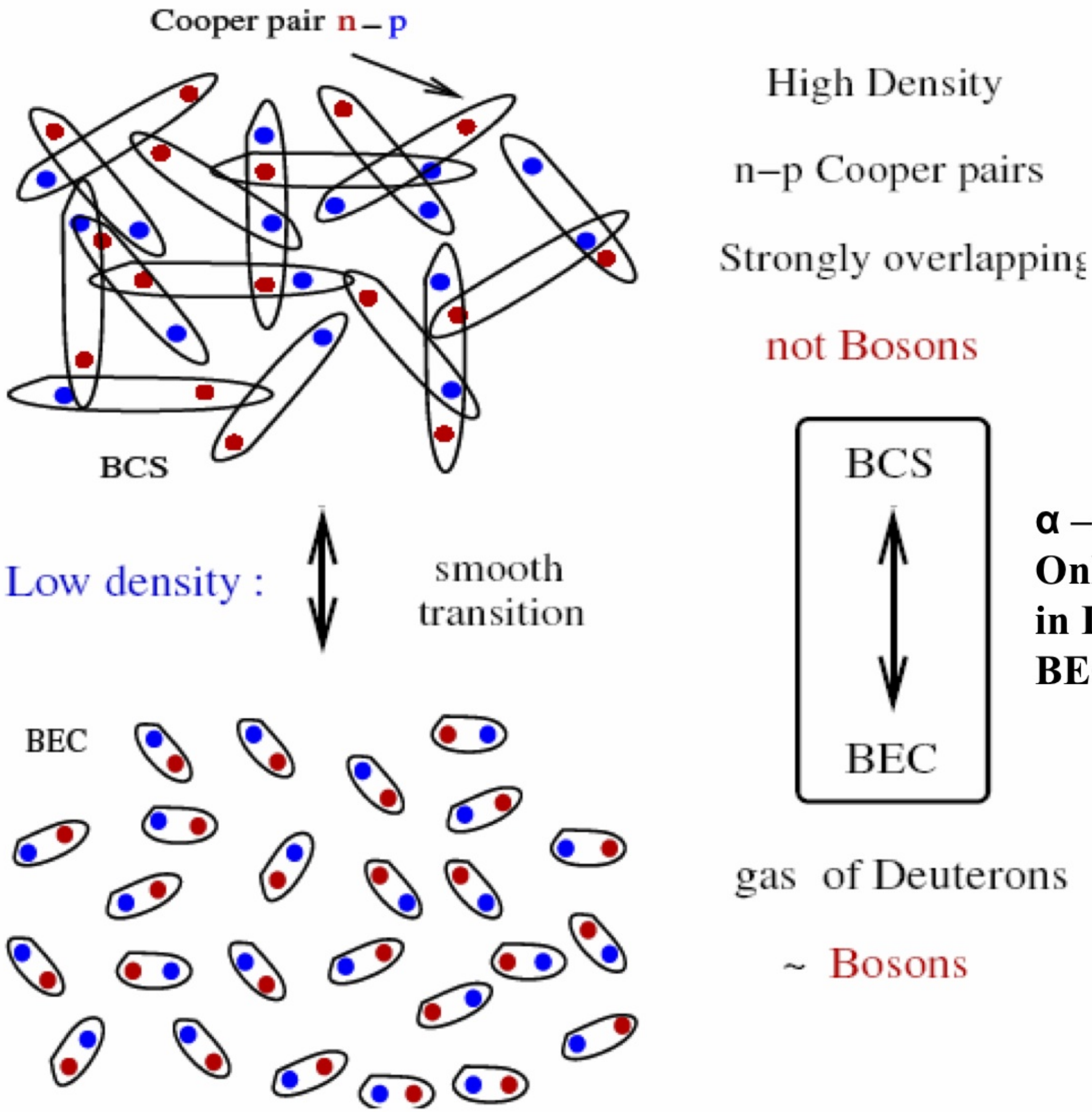


Quantum condensate

Bose-Einstein-
Condensation
of deuterons
(BEC)

Bardeen-Cooper
Schrieffer
pairing
(BCS)





α – Particles
Only Exist
in Low Density
BEC Phase

D. W. Snoke and G. Baym

Table 1. *Bosons under study*

Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	metals	superconductivity
Cooper pair	h^+, h^+	copper oxides	high-T_c superconductivity
exciton	e^-, h^+	semiconductors	luminescence and drag-free transport in Cu_2O
biexciton	$2(e^-, h^+)$	semiconductors	luminescence and optical phase coherence in CuCl
positronium	e^-, e^+	crystal vacancies	(proposed)
hydrogen	e^-, p^+	magnetic traps	(in progress)
^4He	$^4He^{2+}, 2e^-$	He-II	superfluidity
^3He pairs	$2(^3He^{2+}, 2e^-)$	^3He-A,B phases	superfluidity
cesium	$^{133}Cs^{55+}, 55e^-$	laser traps	(in progress)
interacting bosons	nn or pp	nuclei	excitations
nucleonic pairing	nn or pp	nuclei neutron stars	moments of inertia superfluidity and pulsar glitches
chiral condensates	$\langle \bar{q}q \rangle$	vacuum	elementary particle structure
meson condensates	pion condensate $= \langle \bar{u}d \rangle$, etc. kaon condensate $= \langle \bar{u}s \rangle$	neutron star matter	neutron stars, supernovae (proposed)
Higgs boson	$\langle \bar{t}t \rangle$ condensate (proposed)	vacuum	elementary particle masses

Bose - Einstein Condensation

Int. Workshop BEC 93
Levico Terme
Ed.: Griffin, Snoke, Stringari
Cambridge Univ. Press, 1995

"This is the first book devoted to Bose - Einstein Condensation (BEC) as an interdisciplinary subject, covering atomic and molecular physics, laser physics, low temperature physics, nuclear physics and astrophysics."

Many-particle system

- Hamiltonian (non-relativistic)

$$H = \sum_{1} E(1) a_1^+ a_1 + \frac{1}{2} \sum_{12,1'2'} V(12,1'2') a_1^+ a_2^+ a_{2'} a_{1'}$$

- Entropy $\rho(t) = \exp[-S(t)/k_B]$
- cluster decomposition, non-equilibrium

$$S(t) = S_0(t) + S_1(t) + S_2(t)$$

$$S_1(t) = \sum_{1,2} \xi(12,t) a_2^+ a_1 + \frac{1}{2} \sum_{1,2} \psi(12,t) a_1^+ a_2^+ + c.c.$$

$$S_2(t) = \frac{1}{2} \sum_{12,1'2'} \omega(12,1'2',t) a_1^+ a_2^+ a_{2'} a_{1'}$$

Lagrange parameter ξ, ψ, ω are determined by $\langle a_2^+ a_1 \rangle^t, \langle a_2 a_1 \rangle^t, \ldots$

Many-particle system

- Hamiltonian (non-relativistic)

$$H = \sum_1 E(1) a_1^+ a_1 + \frac{1}{2} \sum_{12,1'2'} V(12,1'2') a_1^+ a_2^+ a_{2'} a_{1'}$$

- Entropy $\rho(t) = \exp[-S(t)/k_B]$
- cluster decomposition, equilibrium

$$S = S_0 + S_1 + S_2$$

$$S_1 = \sum_1 (E(1) - \mu)/T a_1^+ a_1$$

$$S_2 = \frac{1}{2} \sum_{12,1'2'} V(12,1'2')/T a_1^+ a_2^+ a_{2'} a_{1'}$$

Lagrange parameter T, μ are determined by $\langle H \rangle, \langle N \rangle$

BEC - BCS crossover in mean-field approximation

only single-particle contributions to the entropy

$$S_1(t) = \sum_{1,2} \xi(12,t) a_2^+ a_1 + \frac{1}{2} \sum_{1,2} \psi(12,t) a_1^+ a_2^+ + c.c.$$

Lagrange multipliers are determined by the given mean values

$$\left\langle a_2^+ a_1 \right\rangle^t = \delta_{12} n(1,t) \qquad \left\langle a_2 a_1 \right\rangle^t = F(12,t) = \delta_{p_1+p_2,2q} \chi(12) e^{i\alpha_p(t)} F(p,t)$$

diagonalization by Bogoliubov-Valatin transformation

$$a_{p+q,\uparrow} = u_p b_{q+p,\uparrow} + v_p b^+_{q-p,\downarrow} \qquad 2\,|u_p|^2 = 1 + (1+\theta_p^2)^{-1/2}$$

$$\theta_p = \frac{\sqrt{2}\,|F(12)|}{1 - n(1) - n(2)}$$

the anomalous mean values $\left\langle b_2 b_1 \right\rangle^t$ vanish

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$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:
Alm et al.,1993

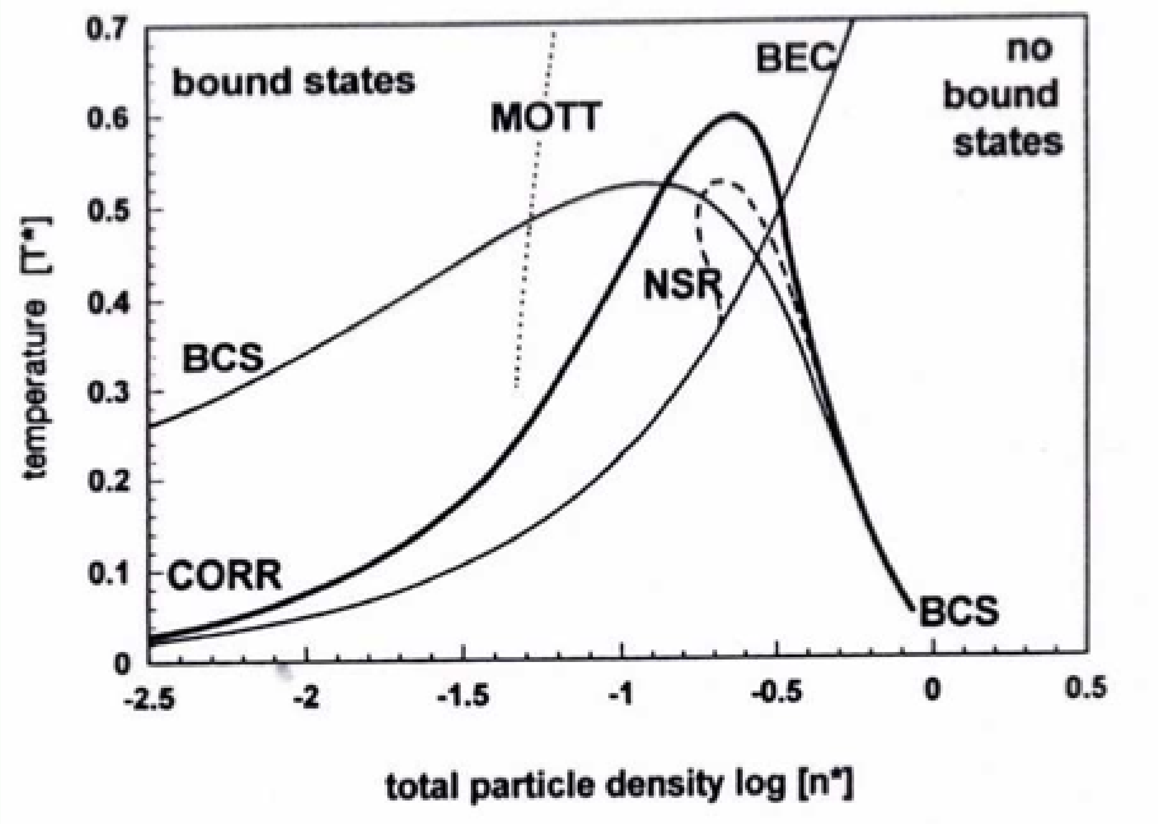
Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

Crossover from BEC to BCS

Phase transition to the superfluid state

fermionic model system with separable interaction, $T^* = T/E_0, n^* = n(\hbar^2/mE_0)^{3/2}$



NSR[a]: blocking by single-particle distribution function

thick line[b]: including the interaction with the correlated component of the medium

a P. Nozieres, S. Schmitt-Rink, J. Low Temp. Phys. **59**, 159 (1985)

b G. Röpke, Ann. Phys. (Leipzig) **3**, 145 (1994)

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

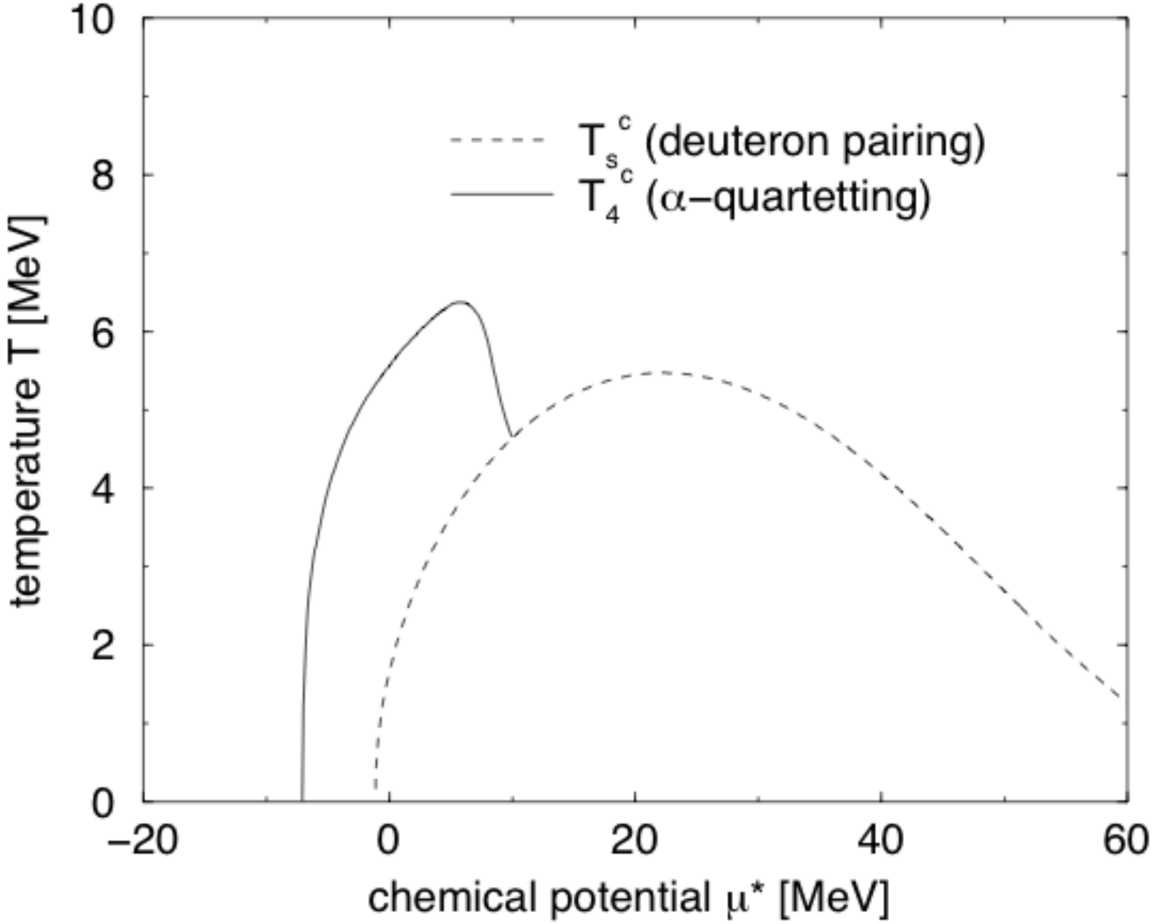
$$+\left\{permutations\right\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

quartetting: $E_{n,0}(T,\mu)=4\mu$

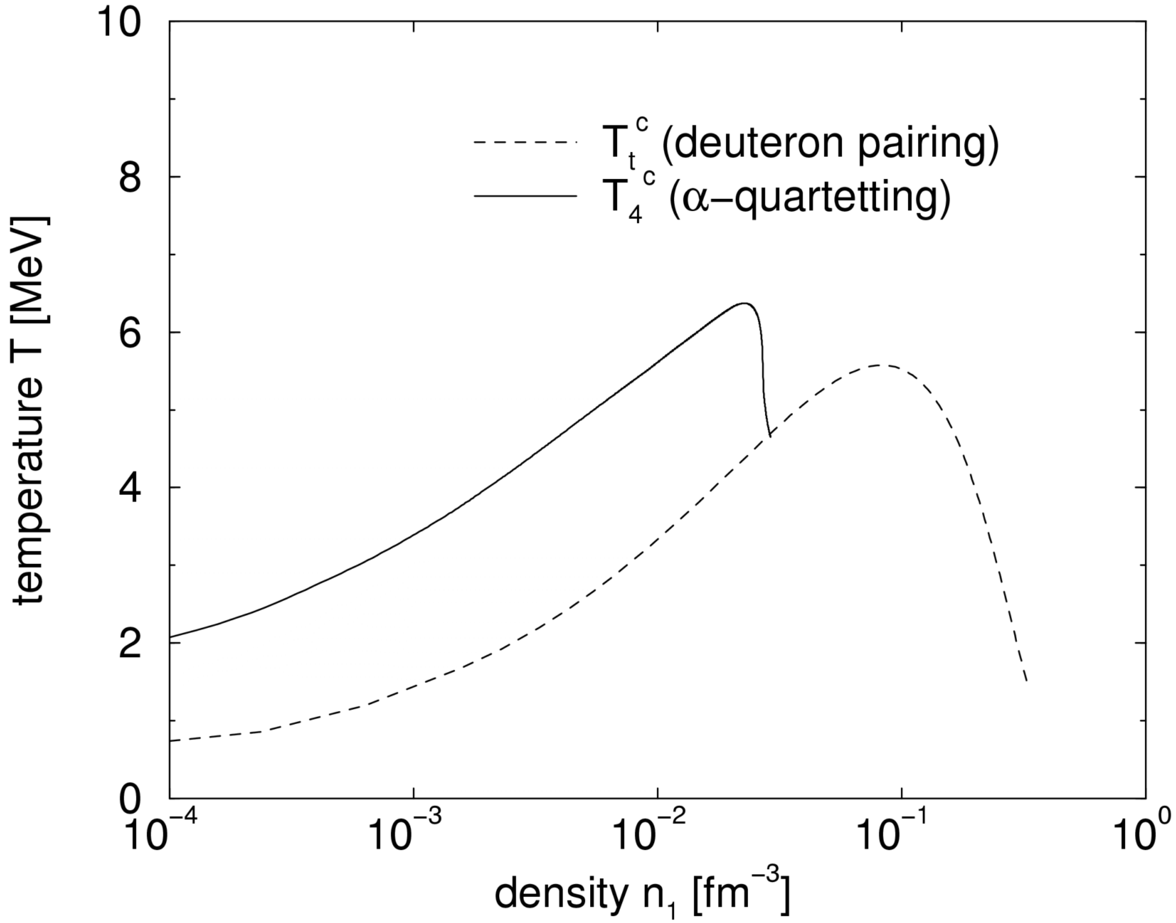
α-cluster-condensation

(quartetting)



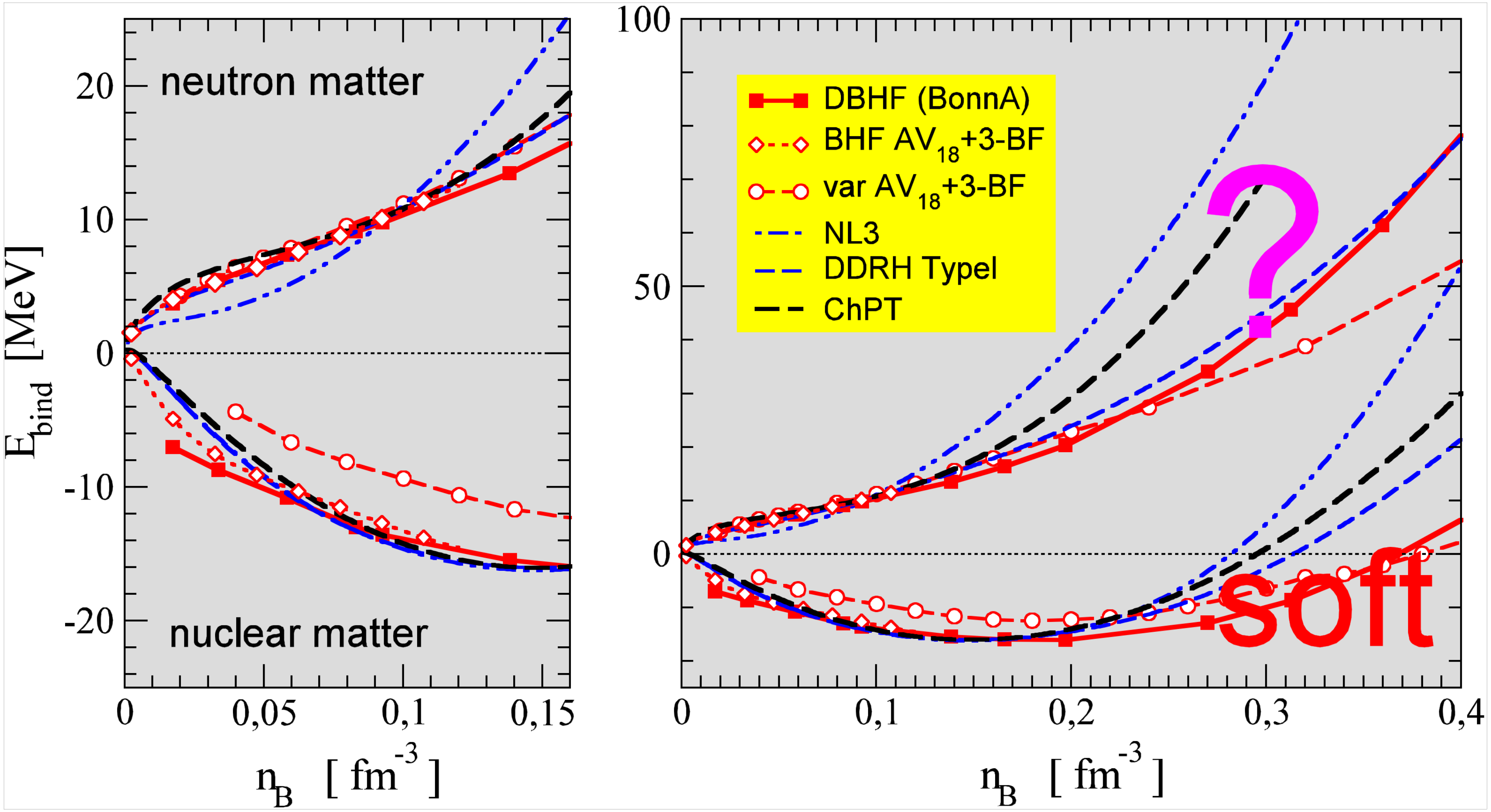
α-cluster-condensation

(quartetting)



G.Röpke, A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

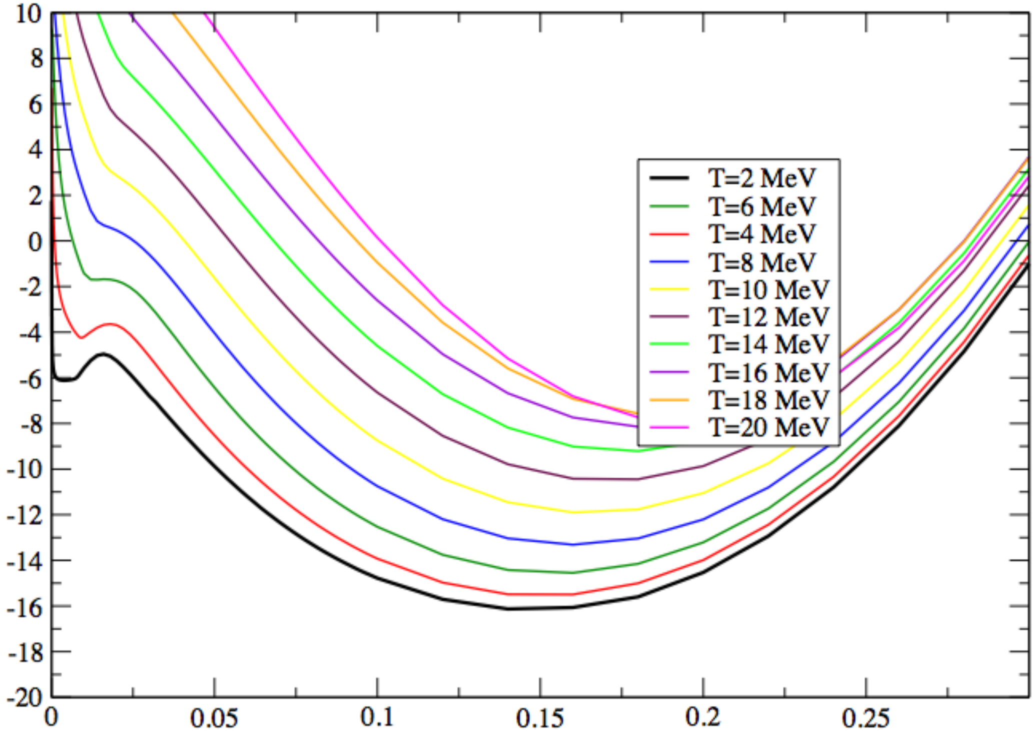
Quasiparticle picture: RMF and DBHF



J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

C.Ducoin, J.Margueron, C.Providencia,I. Vidana
arXiv:1102.1283 (7 Feb 2011)

Internal energy per nucleon



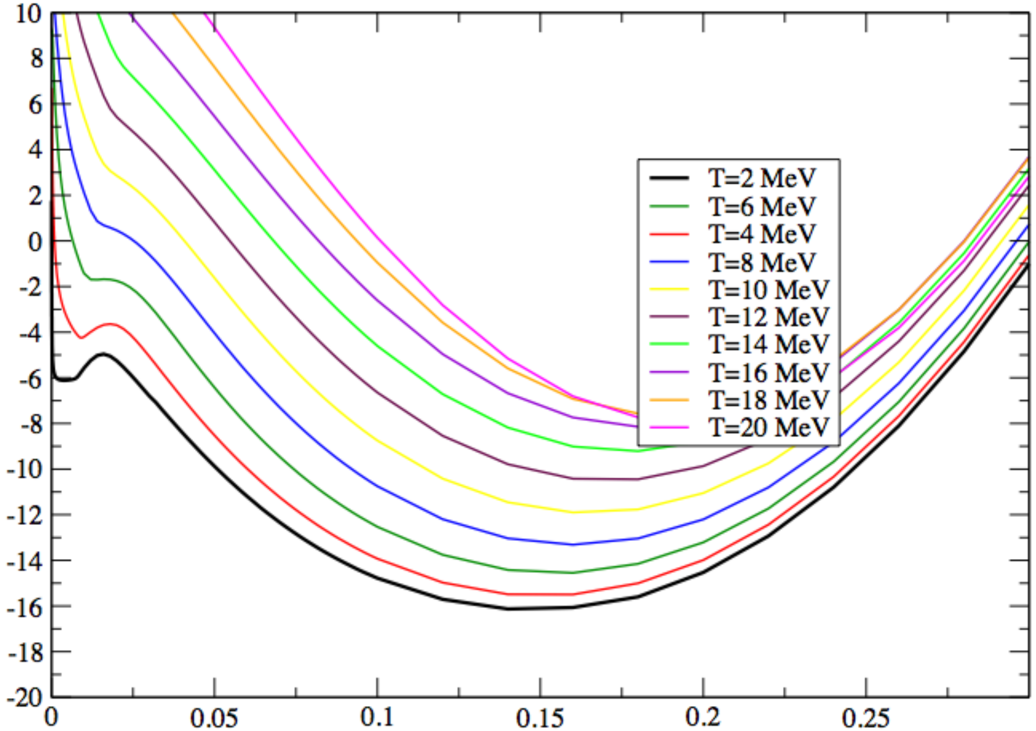
Quantum statistical approach:

Cluster ?

Condensate?

EOS for symmetric matter - low density region?

Internal energy per nucleon



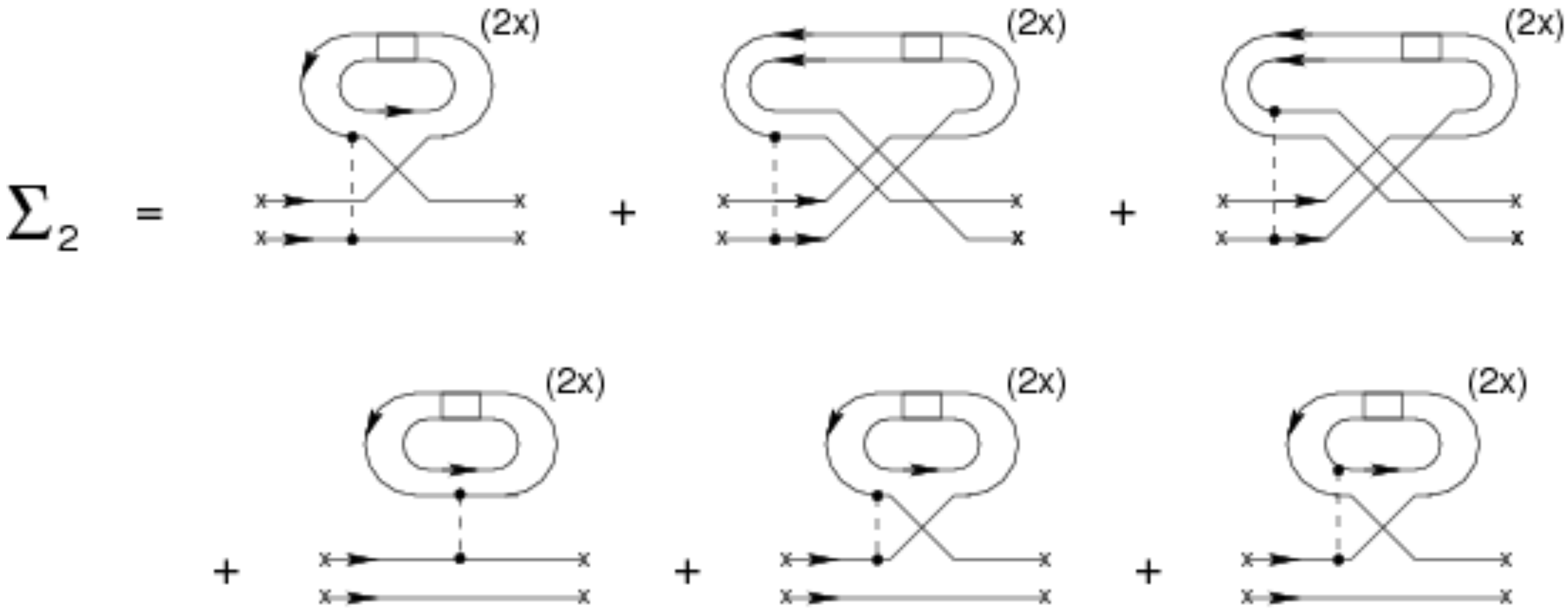
Quantum statistical approach:

Cluster ?

Condensate?

EOS for symmetric matter - low density region?

Correlations in the medium



Cluster - mean field approximation

Cluster (A) interacting with a distribution of clusters (B) in the medium, fully antisymmetrized

$$\sum_{1'\ldots A'} \{H_A^0(1\ldots A,1'\ldots A') + \sum_i \Delta_i^{A,mf}\delta_{k,k'} + \frac{1}{2}\sum_{i,j}\Delta V_{ij}^{A,mf}\delta_{l,l'} - E_{A\nu P}\delta_{k,k'}\}\psi_{A\nu P}(1'\ldots A') = 0$$

self-energy

$$\Delta_1^{A,mf}(1) = \sum_2 V(12,12)_{ex} f^*(2) + \sum_{B\nu P}\sum_{2\ldots B'} f_B(E_{B\nu P})\sum_i V_{1i}(1i,1'i')\psi^*_{B\nu P}(1\ldots B)\psi_{B\nu P}(1'\ldots B')$$

effective interaction

$$\Delta V_{12}^{A,mf} = -\frac{1}{2}[f^*(1) + f^*(2)]V(12,1'2') - \sum_{B\nu P}\sum_{2^*\ldots B''} f_B(E_{B\nu P})\sum_i V_{1i}\psi^*_{B\nu P}(22^*\ldots B^*)\psi_{B\nu P}(2'2''\ldots B'')$$

phase space occupation $\quad f^*(1) = f_1(1) + \sum_{B\nu P}\sum_{2\ldots B} f_B(E_{B\nu P})\,|\psi_{B\nu P}(1\ldots B)|^2$

Self-consistent RPA

Two-time cluster Matsubara Green's functions

$$
\begin{aligned}
G_{\alpha\beta}^{\tau-\tau'} &= -\langle Tr A_\alpha(\tau) A_\beta^\dagger(\tau')\rangle \\
&= -Tr\left[\rho_G T_\tau e^{\tau K} A_\alpha e^{-(\tau-\tau')K} A_\beta^\dagger e^{-\tau' K}\right]
\end{aligned}
$$

Equation of motion method

$$
\begin{aligned}
-\frac{\partial}{\partial\tau} G_{\alpha\beta}^{\tau-\tau'} &= \delta_{\tau-\tau'}\langle[A_\alpha, A_\beta^\dagger]\rangle - \langle Tr[A_\alpha, K]^\tau A_\beta^\dagger(\tau')\rangle \\
&= \delta_{\tau-\tau'} N_{\alpha\beta} + \sum_\gamma \int d\tau_1' \mathcal{H}_{\alpha\gamma}^{\tau-\tau_1'} G_{\gamma\beta}^{\tau_1'-\tau'} \ .
\end{aligned}
$$

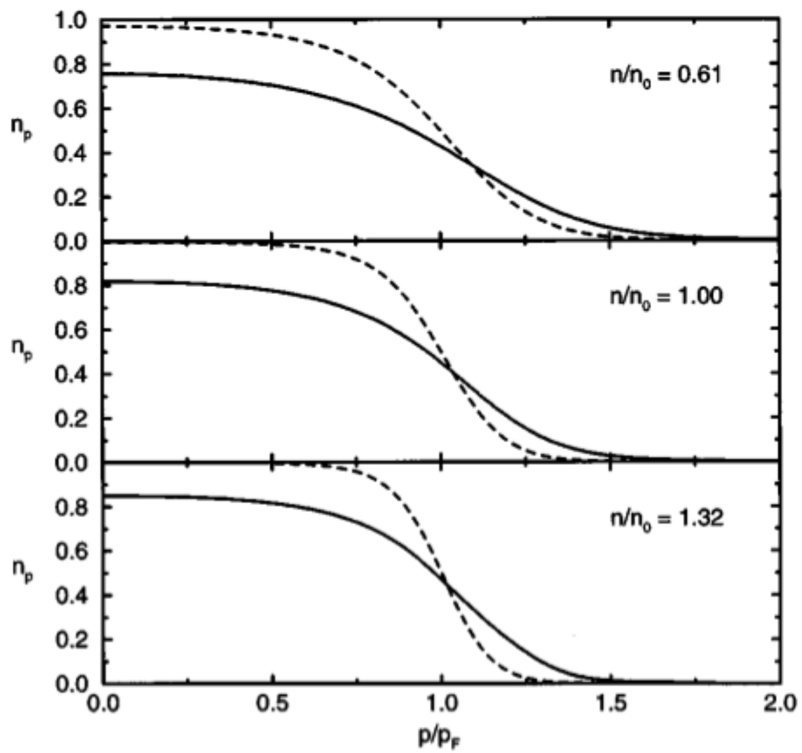
Effective Hamiltonian is split into an instantaneous and a dynamic part

$$
\begin{aligned}
\mathcal{H}_{\alpha\beta}^{\tau-\tau'} &= \sum_{\beta'} \left\{\delta_{\tau-\tau'}\langle[[A_\alpha, K], A_{\beta'}^\dagger]\rangle - \langle Tr[A_\alpha, K]^\tau [K, A_{\beta'}^\dagger]^{\tau'}\rangle_{irr}\right\} N_{\beta'\beta}^{-1} \\
&\equiv \mathcal{H}_{\alpha\beta}^{(0)}\delta_{\tau-\tau'} + \mathcal{H}_{\alpha\beta}^{(r)\tau-\tau'} \ .
\end{aligned}
$$

J.Dukelsky, G. Roepke, and P.Schuck, NPA **628**, 17 (1998)
P. Schuck, D.S. Delion, J.Dukelsky, and G. Roepke, in preparation

Single nucleon distribution function

Dependence on density

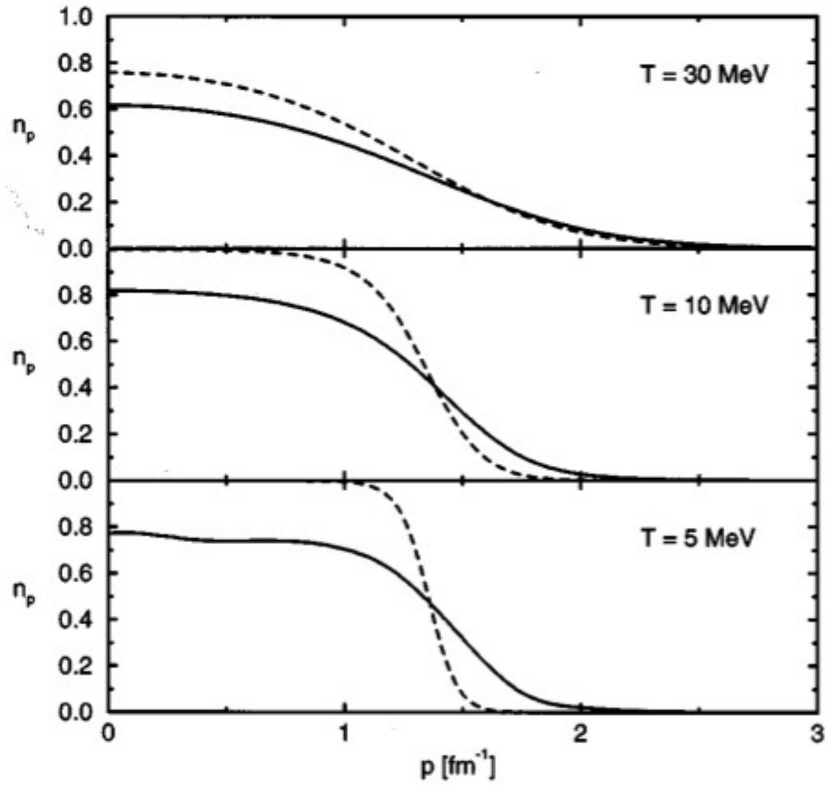


T = 10 MeV

Alm et al., PRC 53, 2181 (1996)

Single nucleon distribution function

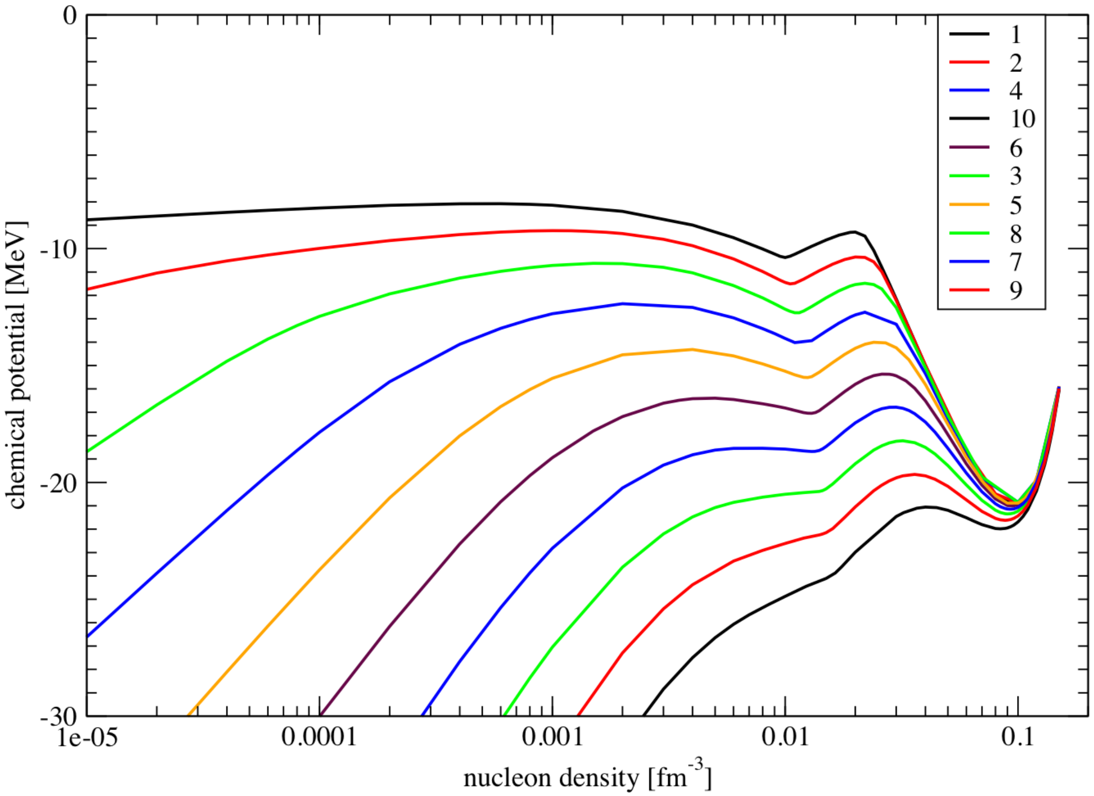
Dependence on temperature



saturation density

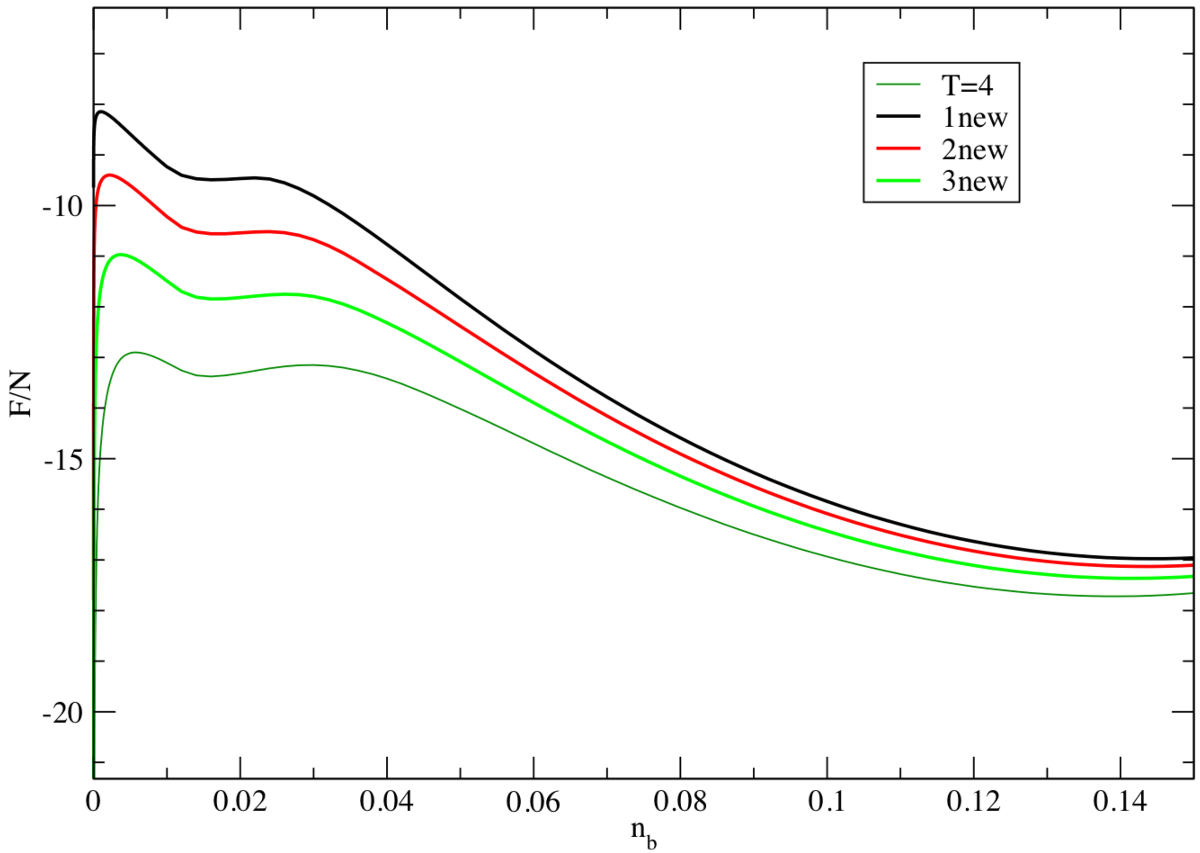
Alm et al., PRC 53, 2181 (1996)

Chemical potential



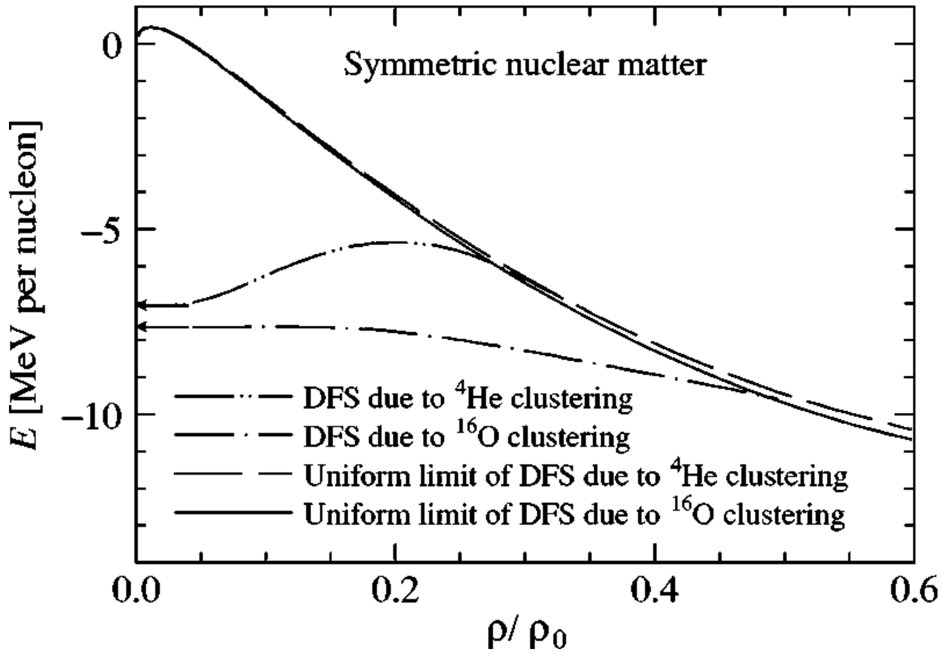
preliminary

Free energy per nucleon



preliminary

Clustering phenomena in nuclear matter below the saturation density



- FIG. 8. Energy curves of DFSs due to a and 16O clustering in
- the symmetric nuclear matter by the use of the BB s*B*4d force. The
- density of matter is normalized by the saturation density of the
- uniform matter with the Fermi sphere, r0=0.206 fm−3. The presentation
- of the curves is similar to that in Fig. 4.

Hiroki Takemoto et al.,
PR C **69**, 035802 (2004)

Alpha matter and quartetting

Where it appears?

(Pairing is well understood:

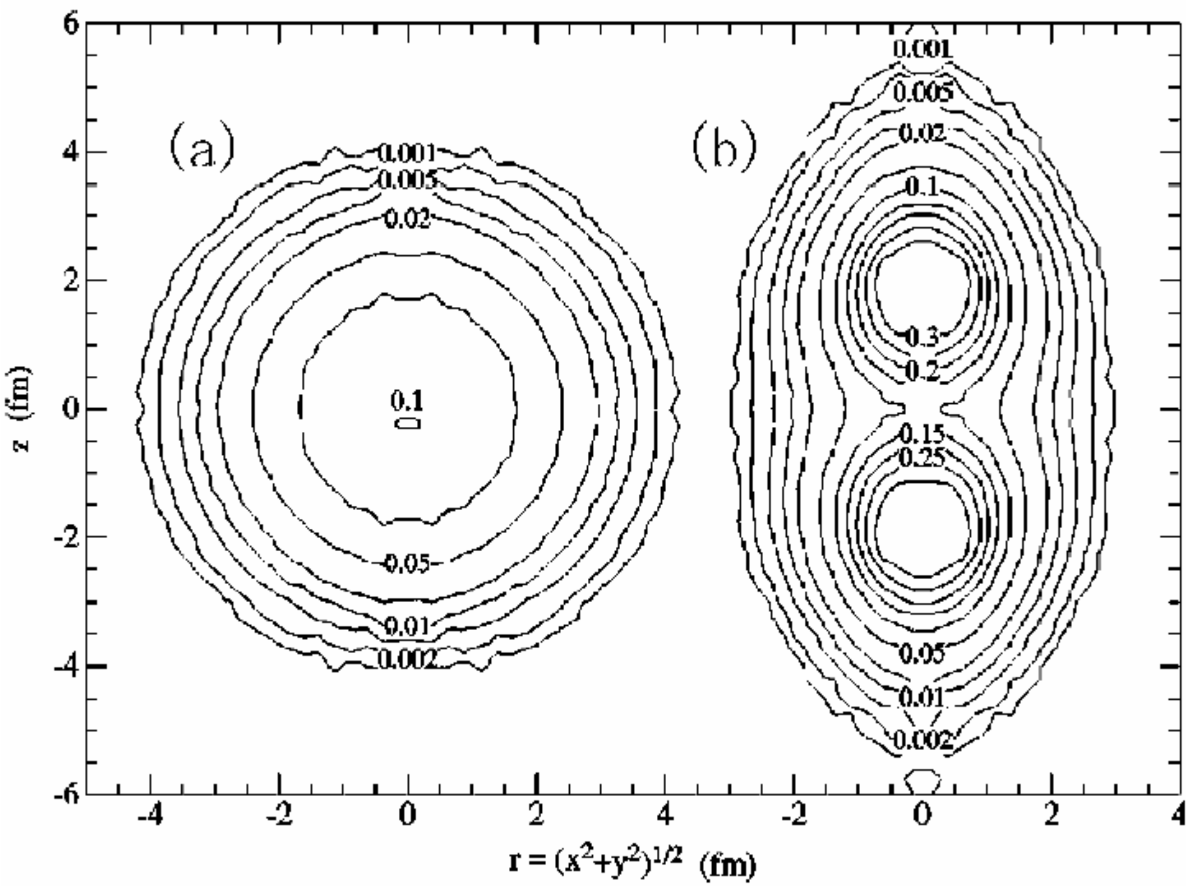
nuclear structure: Bethe-Weizsaecker formula, surface…

neutron stars)

- Matter at low densities, low temperatures:
- Quartetting in nuclei
- Condensate in heavy ion collisions
- Neutron star crust

- Suppression of the condensate with increasing density
- Dissolution of clusters with increasing density
- Formation of larger clusters (C, O, Si,…Fe, Ni, …)

Alpha cluster structure of Be 8



R.B. Wiringa et al.,
PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for 8Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Self-conjugate 4n nuclei

^{12}C:

0^+ state at 0.39 MeV above the 3α threshold energy:
α cluster interact predominantly in relative S waves,
gaslike structure

α-particle condensation in low-density nuclear matter
$(\rho \leq \rho_0/5)$

$n\alpha$ cluster condensed states
– a general feature in $N = Z$ nuclei?

Self-conjugate 4n nuclei

$n\alpha$ nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ...

Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$ break up at the threshold energy $E_{n\alpha}^{\mathrm{thr}} = nE_\alpha$

Quantum condensate

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α-particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r-space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha C} \rangle = \mathcal{A}\{\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)\Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8)\cdots\Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n})\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \mathrm{BCS} \rangle = \mathcal{A}\{\Phi(\vec{r}_1, \vec{r}_2)\Phi(\vec{r}_3, \vec{r}_4)\cdots\}$$

A. Tohsaki et al., PRL **87**, 192501 (2001)

Variational ansatz

Variational ansatz for $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$: $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = e^{-\frac{2}{B^2}\vec{R}^2}\phi_\alpha(\vec{r}_i-\vec{r}_j)$

Center of mass : $\vec{R} = \frac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)$

Intrinsic α-wave function :

$$\phi_\alpha(\vec{r}_i-\vec{r}_j) = e^{-\frac{1}{8b^2}\left\{(\vec{r}_4-\vec{r}_1)^2+(\vec{r}_4-\vec{r}_2)^2+(\vec{r}_4-\vec{r}_3)^2+\cdots\right\}}$$

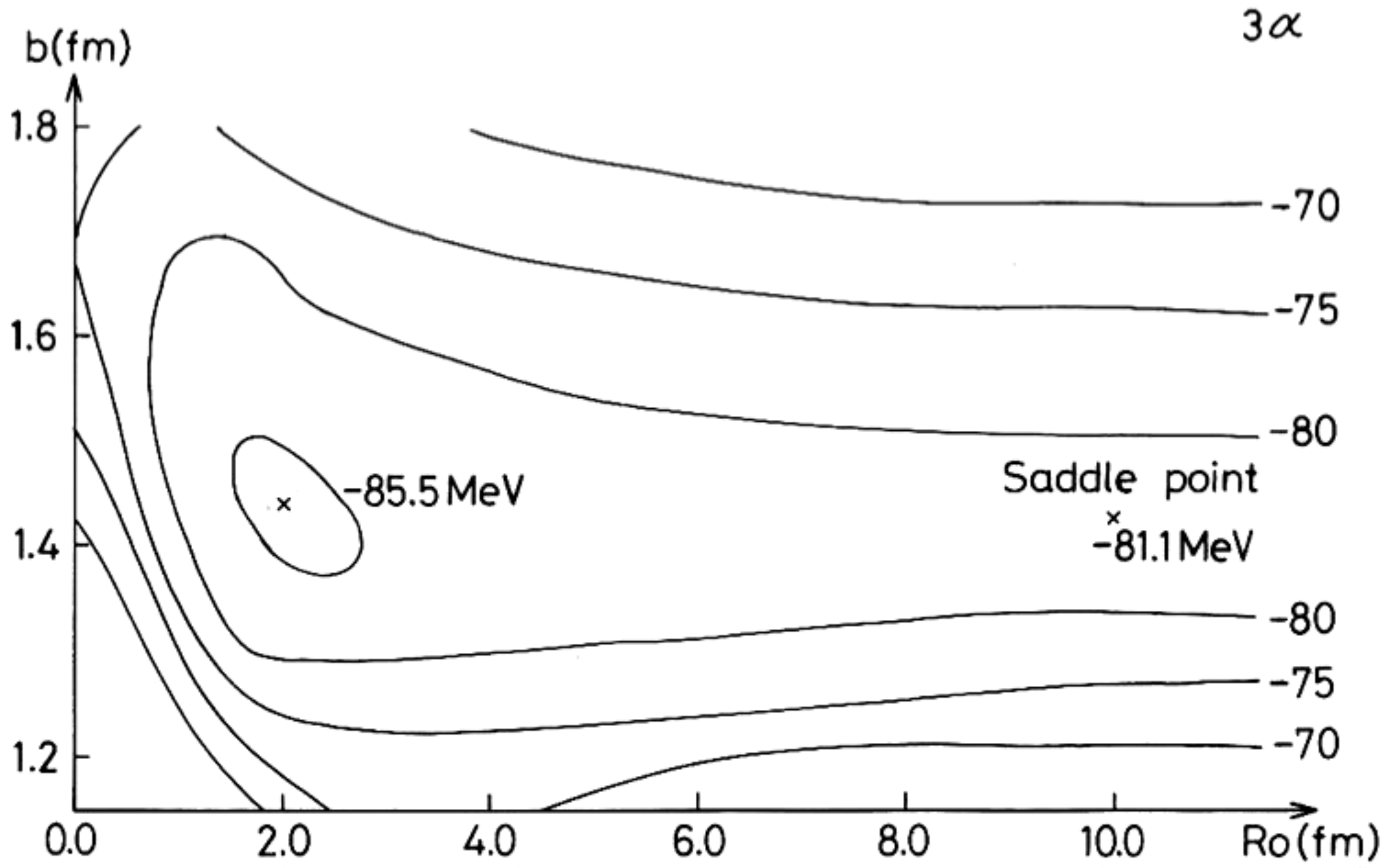
Two variational parameters : $B,\ b$

Two limits : $B=b$ $\quad |\Phi_{\alpha C}\rangle =$ Slater determinant

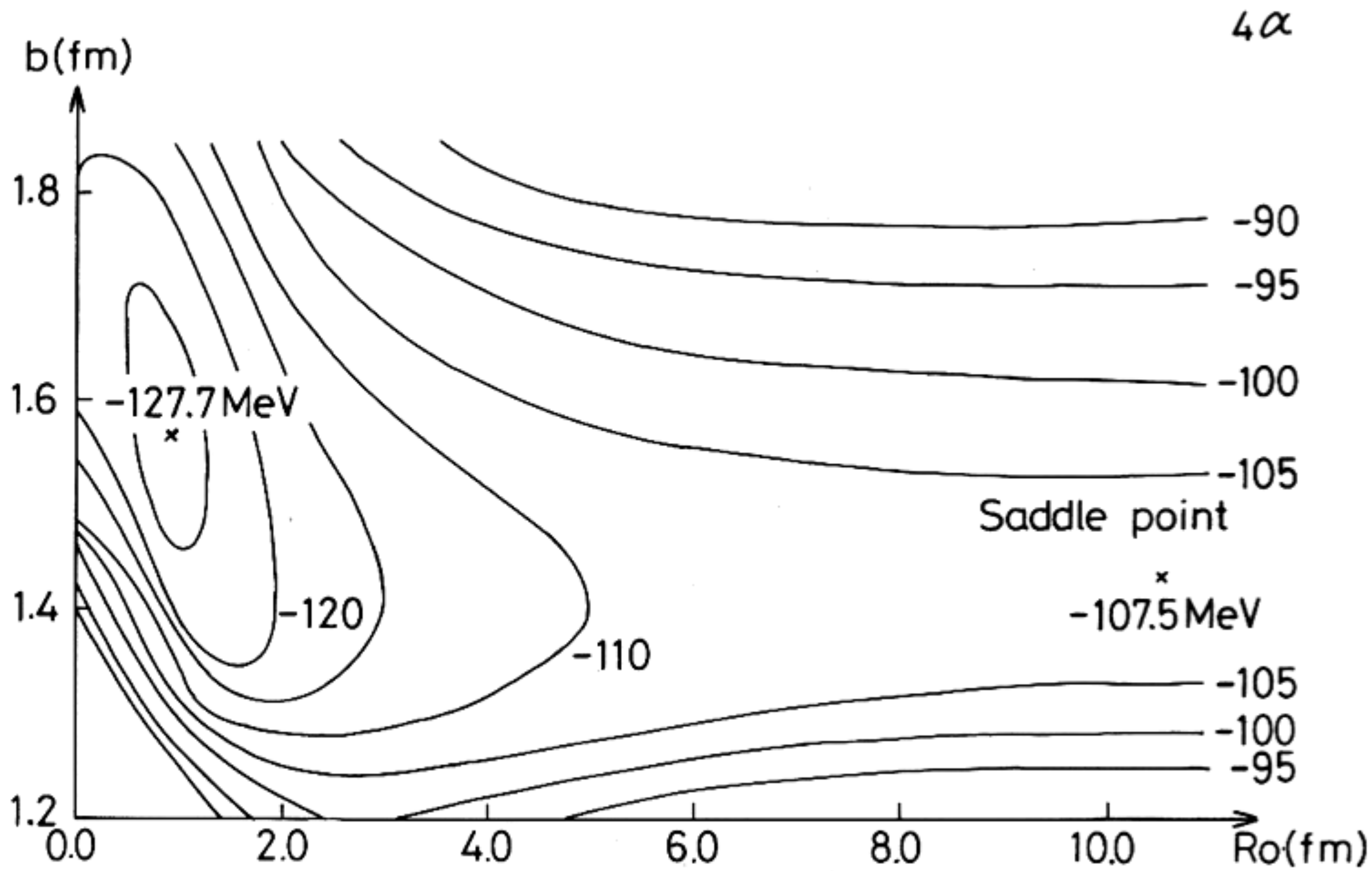
$B \gg b$ $\quad |\Phi_{\alpha C}\rangle =$ gas of independent α-particles

Two dimensional surface : $E(B,b) = \dfrac{\langle\Phi_{\alpha C}|H|\Phi_{\alpha C}\rangle}{\langle\Phi_{\alpha C}|\Phi_{\alpha C}\rangle}$

3 α variational energy

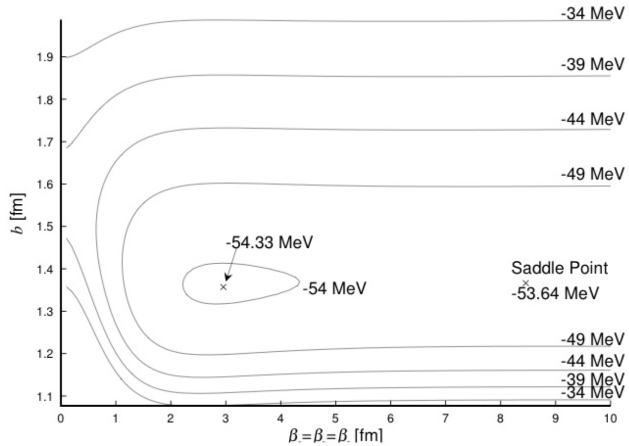


4 α variational energy

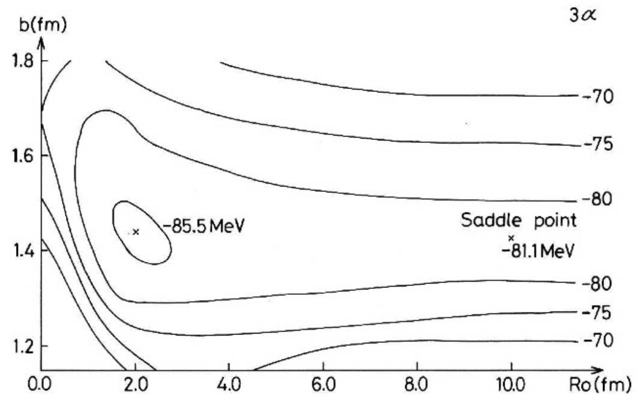


A. Tohsaki et al., PRL **87**, 192501 (2001)

Energy surface, variational ansatz

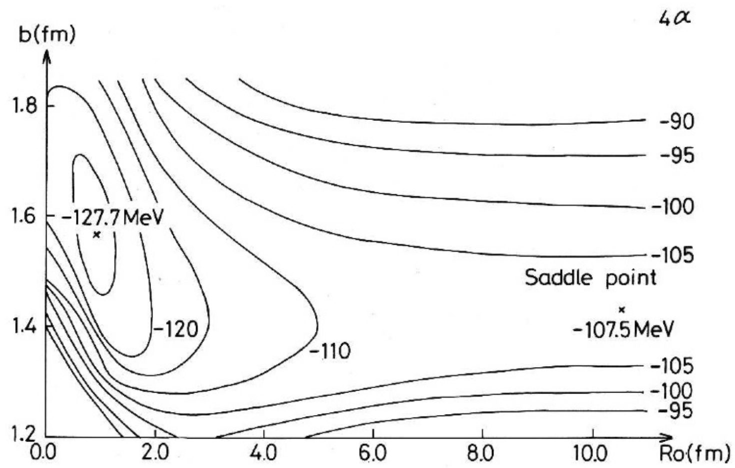


^8B



^{12}C

^{16}O



Results

		E_k (MeV)	E_{exp} (MeV)	$E_k - E_{n\alpha}^{thr}$ (MeV)	$(E - E_{n\alpha}^{thr})_{exp}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{exp}$ (fm)
^{12}C	$k = 1$	-85.9	-92.16 (0_1^+)	-3.4	-7.27	2.97	2.65
	$k = 2$	-82.0	-84.51 (0_2^+)	+0.5	0.38	4.29	
	$E_{3\alpha}^{thr}$	-82.5	-84.89				
^{16}O	$k = 1$	-124.8	-127.62 (0_1^+)	-14.8	-14.44	2.59	2.73
		(−128.0)*		(−18.0)*			
	$k = 2$	-116.0	-116.36 (0_3^+)	-6.0	-3.18	3.16	
	$k = 3$	-110.7	-113.62 (0_5^+)	-0.7	-0.44	3.97	
	$E_{4\alpha}^{thr}$	-110.0	-113.18				
^{8}Be				-0.17	+0.1		

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E_{n\alpha}^{thr} = nE_\alpha$ denotes the threshold energy for the decay into α-clusters, the values marked by * correspond to a refined mesh.

M. Chernykh et al., PRL **98**, 032501 (07); Y. Funaki et al., PRL **101**, 082502 (08)

Estimation of condensate fraction in zero temperature α-matter

α-cluster condensate in ^{12}C, ^{16}O:
resonating group method
occupation numbers of α-orbits in ^{12}C

	RMS radii	S-orbit	D-orbit	G-orbit
0_1^+ (g.s.)	2.44 fm	1.07	1.07	0.82
0_2^+	4.31 fm	2.38	0.29	0.16

80 % condensate at 1/8 nuclear matter density

T. Yamada, P. Schuck: (2.16 − normal)/3 ≃ 60%

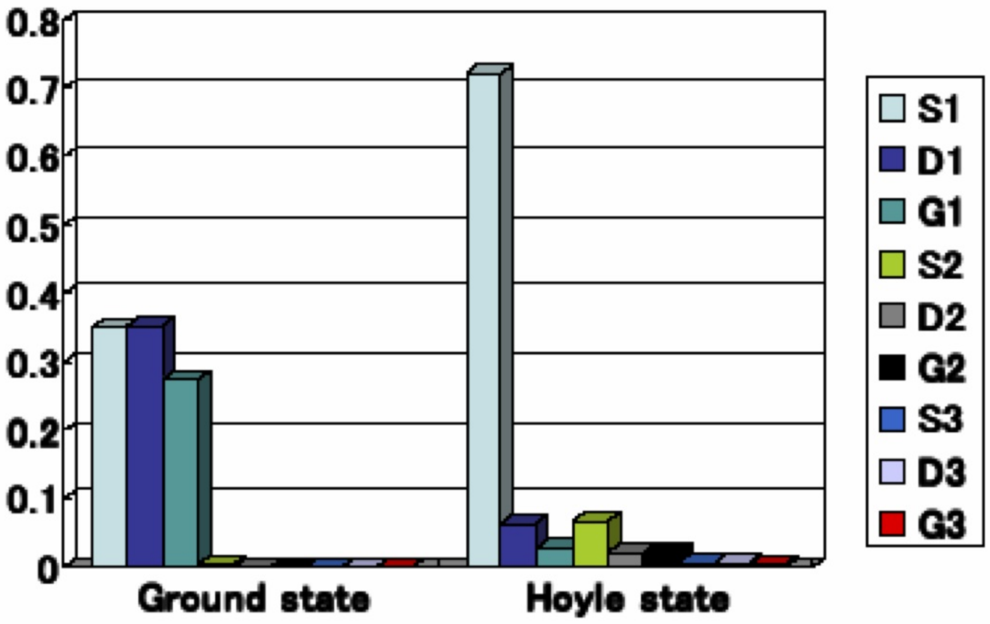
BEC of α clusters in the same S-orbit?

α-particle density matrix :

$$\rho_\alpha(\vec{R}, \vec{R}'), \quad \vec{R} \,:\, \text{c.m. of } \alpha$$

Diagonalization :

^{12}C : O_2^+ 70% S-wave occupancy



Estimation of condensate fraction in zero temperature α-matter

$$n_0 = \frac{\langle \Psi | a_0^\dagger a_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

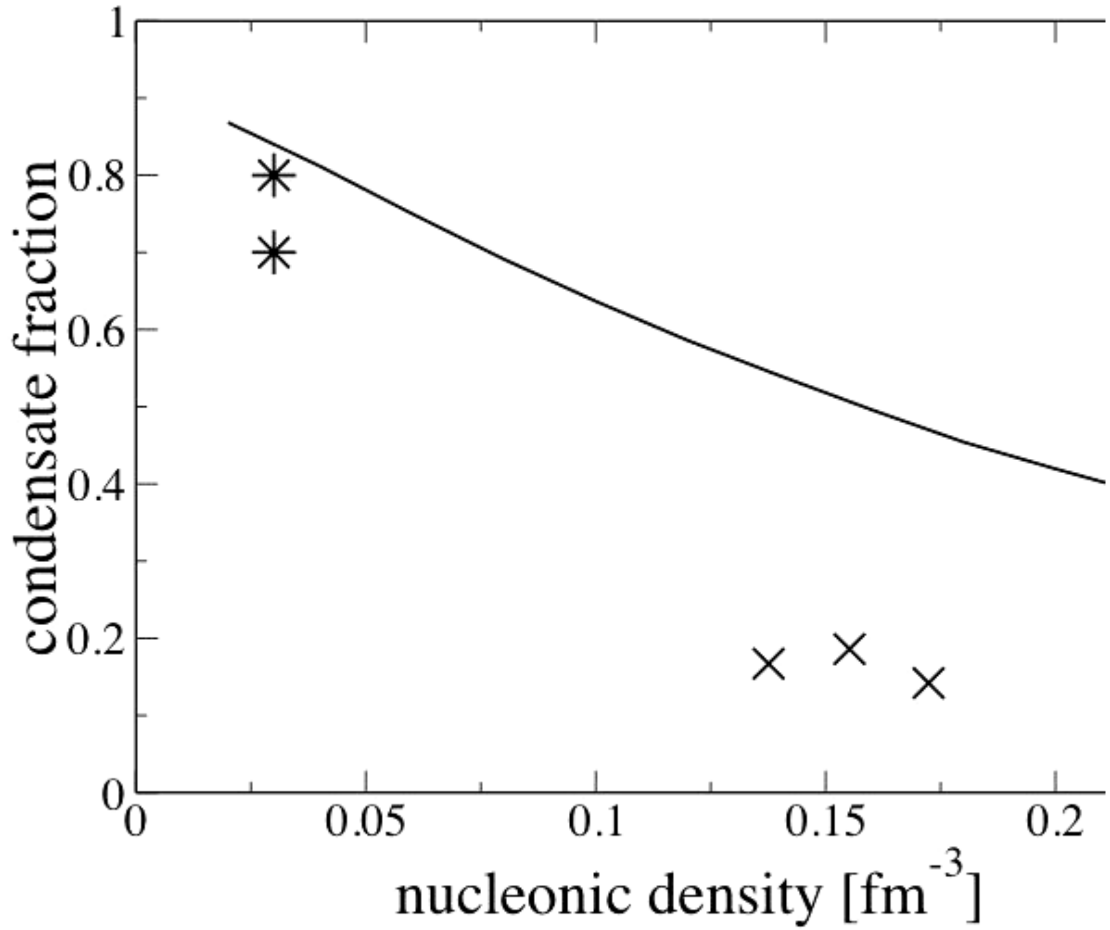
destruction of the BEC of the ideal Bose gas:
thermal excitation, but also correlations

"excluded" volume for α-particles ≈ 20 fm^3
at nucleon density $\rho = 0.048$ fm^{-3} filling factor ≈ 28 %
(liquid ^{4}He: 8 % condensate),
destruction of the condensate at $\approx \rho_0/3$

Suppresion of condensate fraction

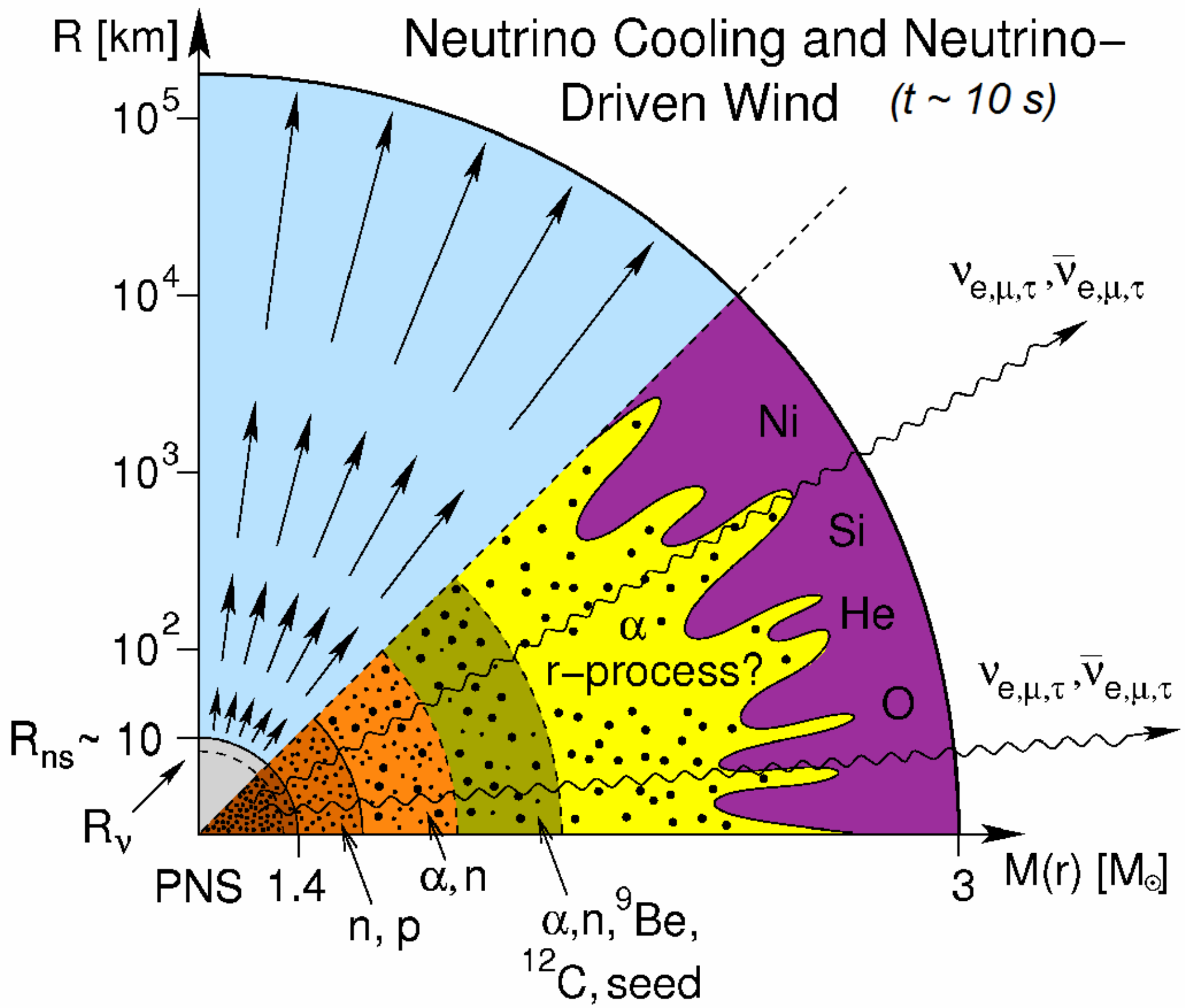
- Alpha-alpha interaction (Ali/Bodmer),
  no Pauli blocking:
- Variational calculation (Clark/Jastrow approach to the alpha-particle condensate amplitude)
  (crosses)
- First order approximation
  (full line)

- Yamada/Schuck's result for condensate in C12 - O2+
  (stars)



Y. Funaki et al., PRC **77**, 064312 (08)

Supernova explosion



T.Janka

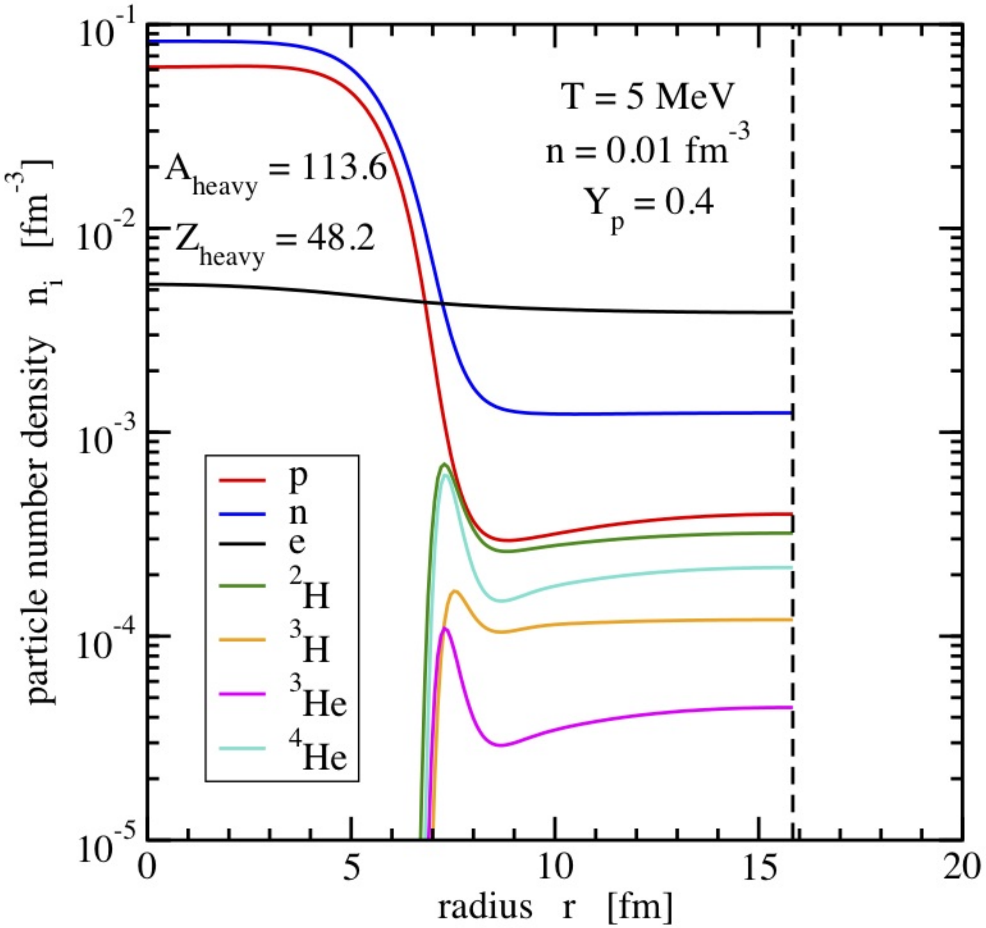
α cluster in astrophysics

Crust of neutron stars

Protons in droplets (heavy nuclei)

α−cluster outside, at the surface, condensate?

S. Typel, GSI



Summary

- Correlations (cluster formation, quantum condensates) are essential in low-density matter. They are suppressed with increasing density (Pauli blocking).

- The low-density limit of the nuclear matter EoS can be rigorously treated. The Beth-Uhlenbeck virial expansion is a benchmark.

- An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self- energy and Pauli blocking is included. An interpolation between low and high densities is possible.

- Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation (quartetting), and the behavior of the symmetry energy.

Problems

- Quantum statistical approach to correlations (cluster formation) in dense matter. Larger clusters? Droplets?
- Condensation energy for Bose condensate state (compare pairing)?
  „strict“ solution of the 4-nucleon equation including Pauli blocking?
- Clusters in a clusterized medium: cluster mean-field,
  correlations in quantum condensates,
  transport with cluster formation
- Thermodynamics - finite systems, nonequilibrium
- Instability of homogeneous matter at low temperatures
- α–decay of heavy elements

Thanks

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A. Sedrakian, K. Sumiyoshi, A. Tohsaki, S. Typel,
H. Wolter, T. Yamada

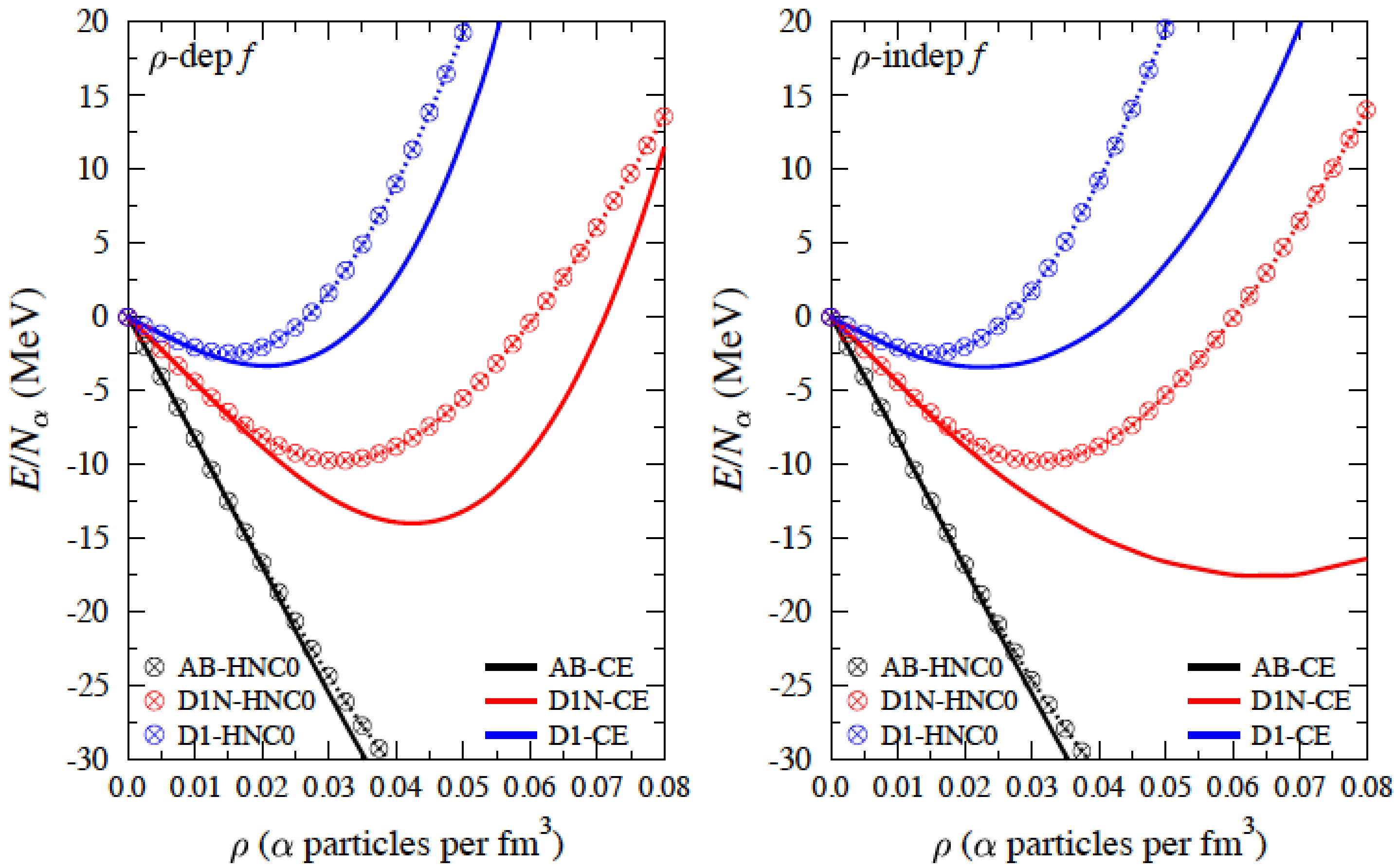
for collaboration

to you

for attention

D.G.

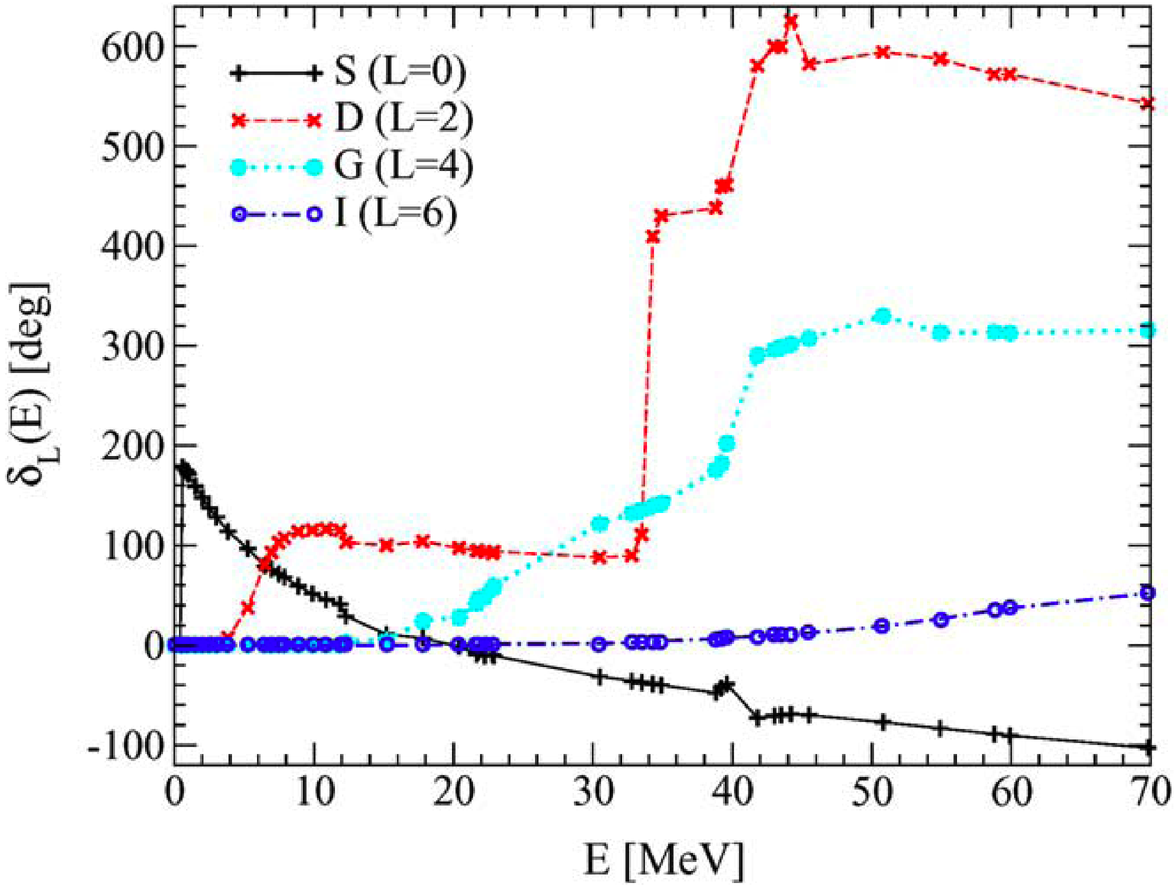
Energy of α-Matter at T=0



Total energy calculated with the cluster expansion
within the HNC/0 (circles) and HNC/4 (solid lines) approximation.
Different interaction potentials

F.Carstoiu, S.Misicu, PLB, 2009

α-α scattering phase shifts



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Clusters in nuclei

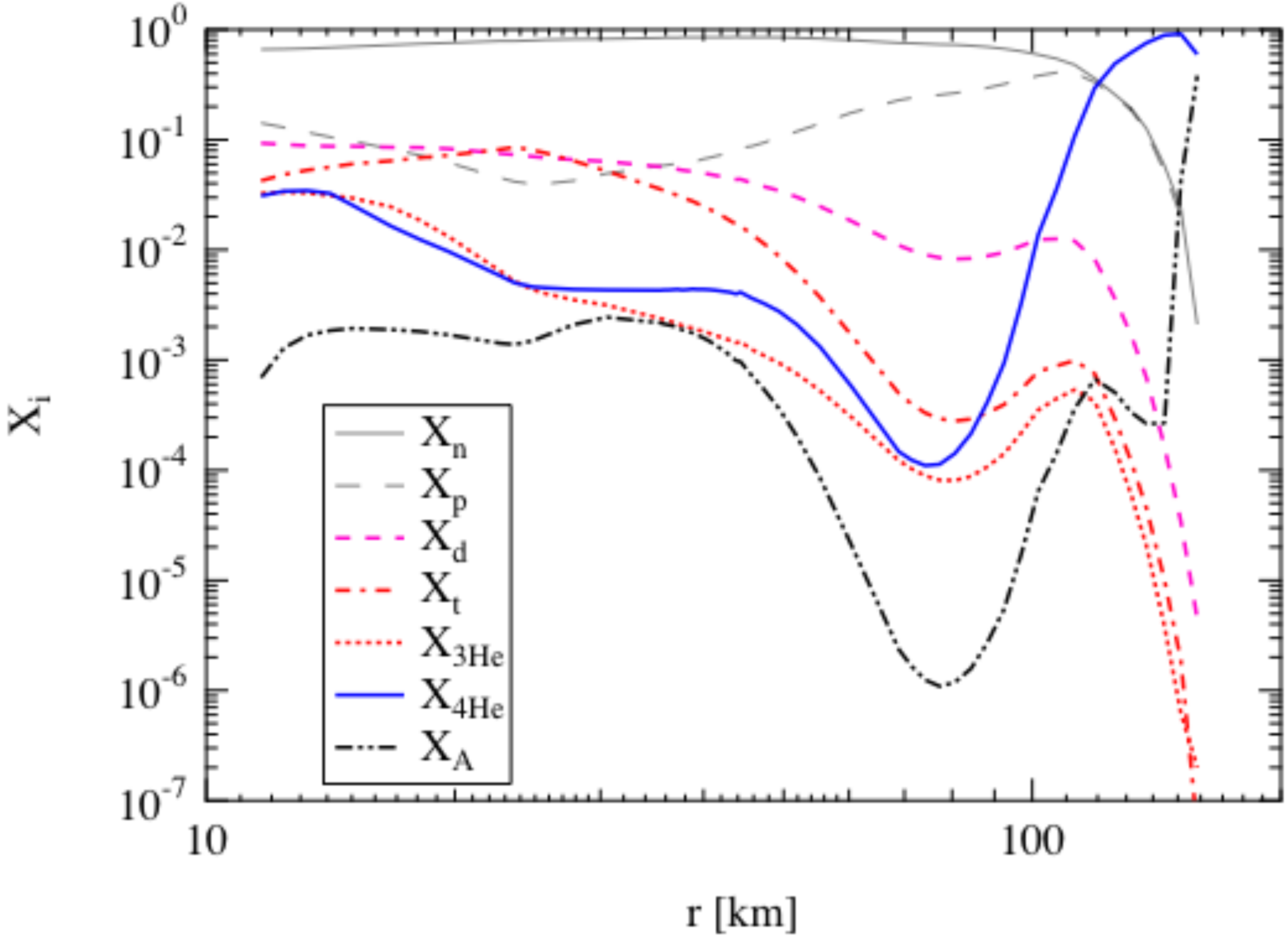
- Low-density isomers
- Alpha matter at low densities
- Quartetting
- Condensate wave function

- Suppression of the condensate with increasing density
- Dissolution of clusters with increasing density

Astrophysical Applications

- Supernova explosions
- Neutrino transport
- Neutron star structure
- Equation of state (EOS)
- Composition
- Transport properties (cross sections)

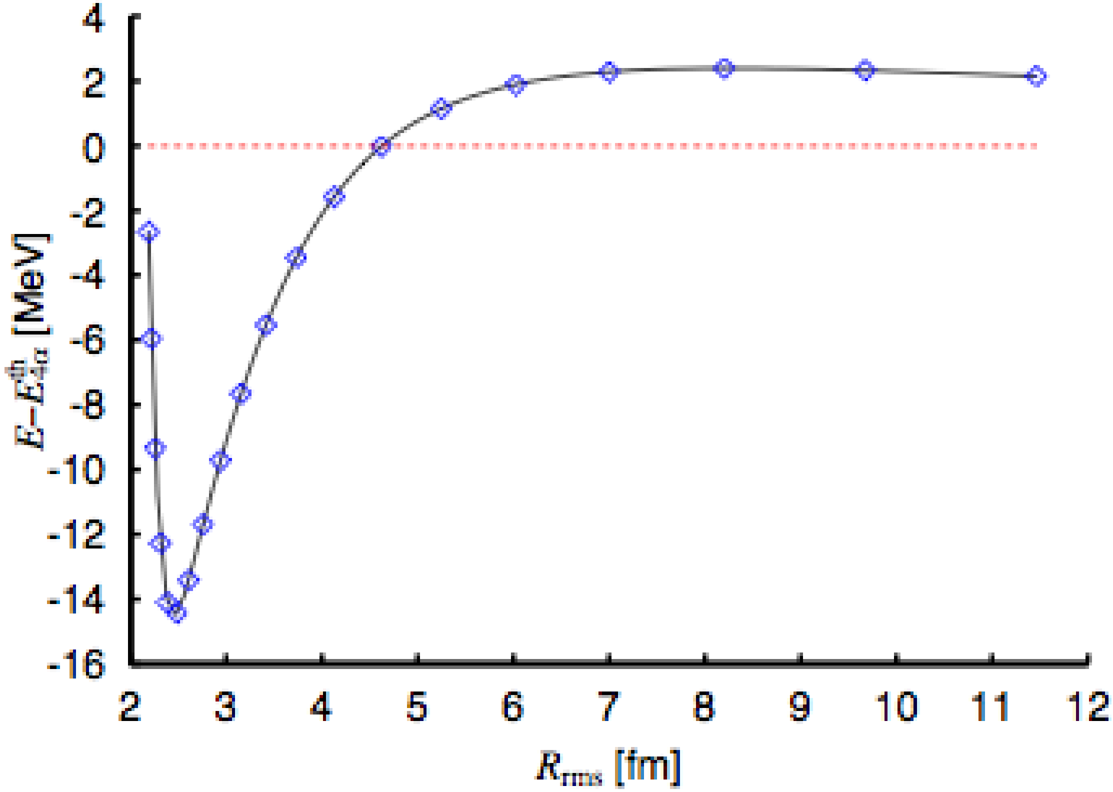
Composition of supernova core



K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

Mass fraction X of light clusters for a post-bounce supernova core

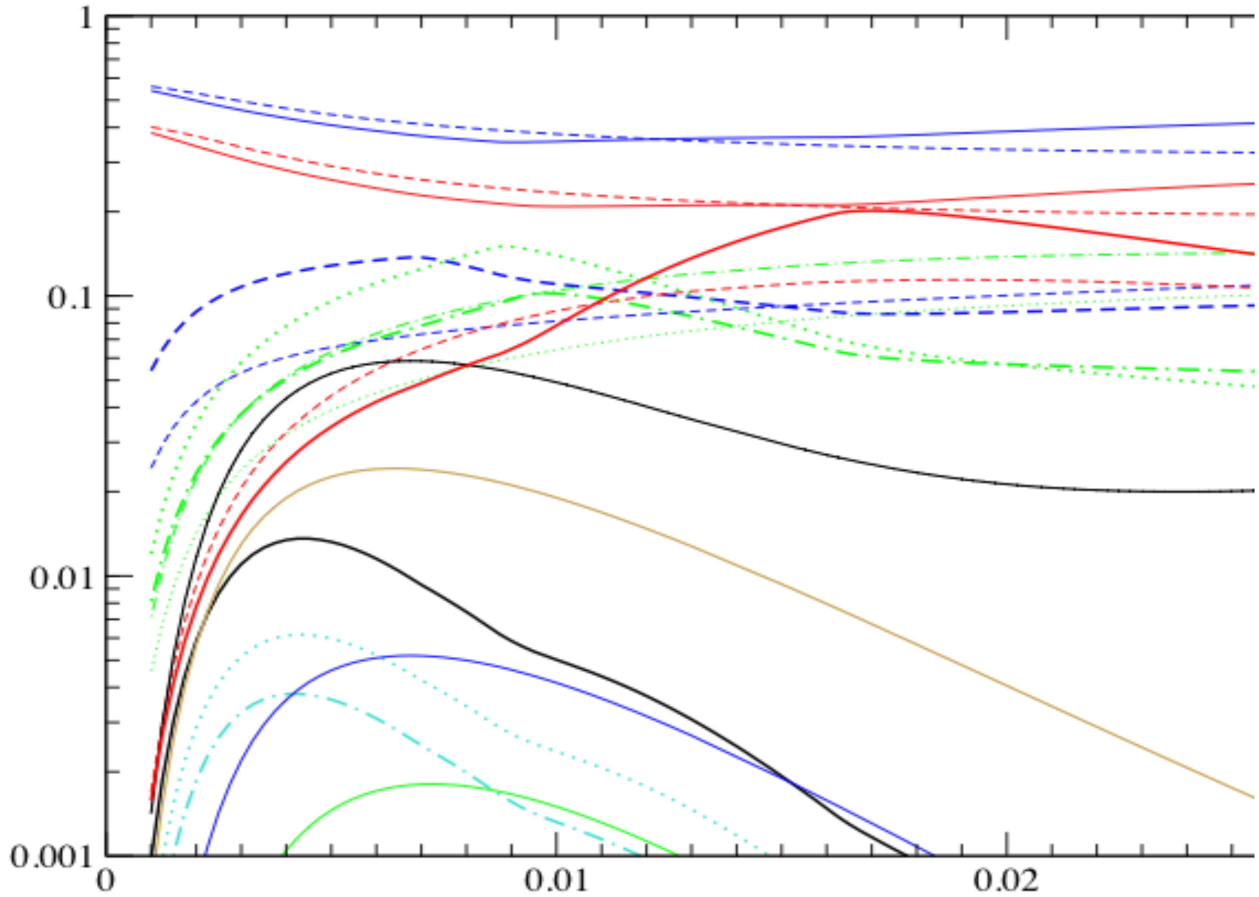
Rms-dependence of condensate energies



Variational energy for the Gaussian condensate of 4 alpha

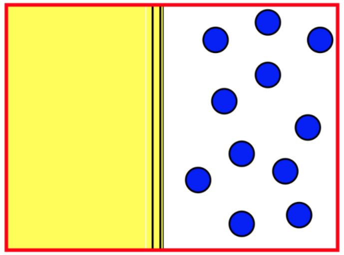
Heavy nuclei abundances in nuclear matter

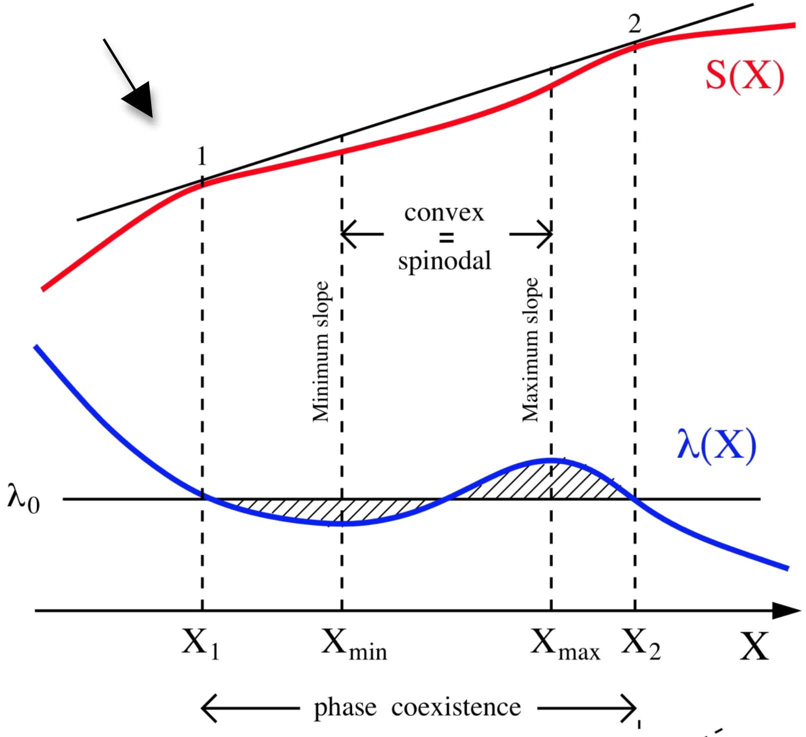
T=10 MeV, asymmetry 0.42, as function of baryon density



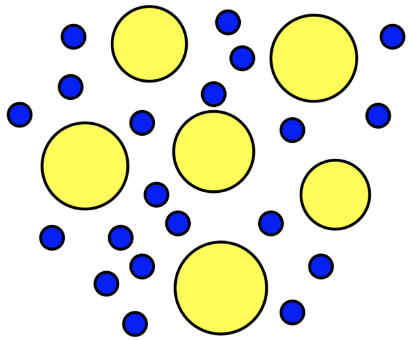
n, p, d, t, He3, He4, Li5,…

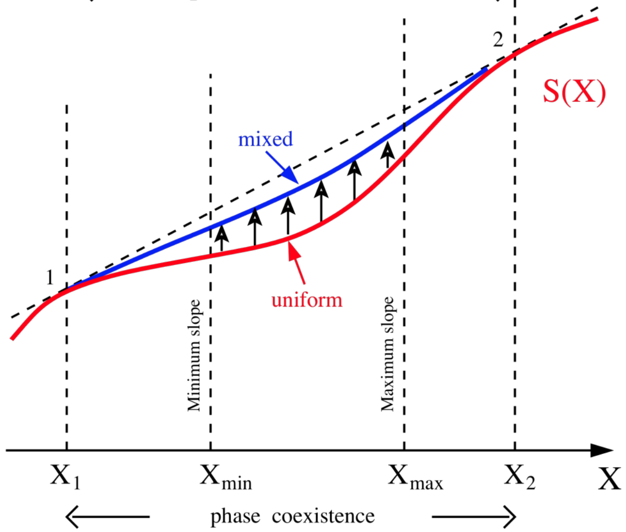
Separated phases:

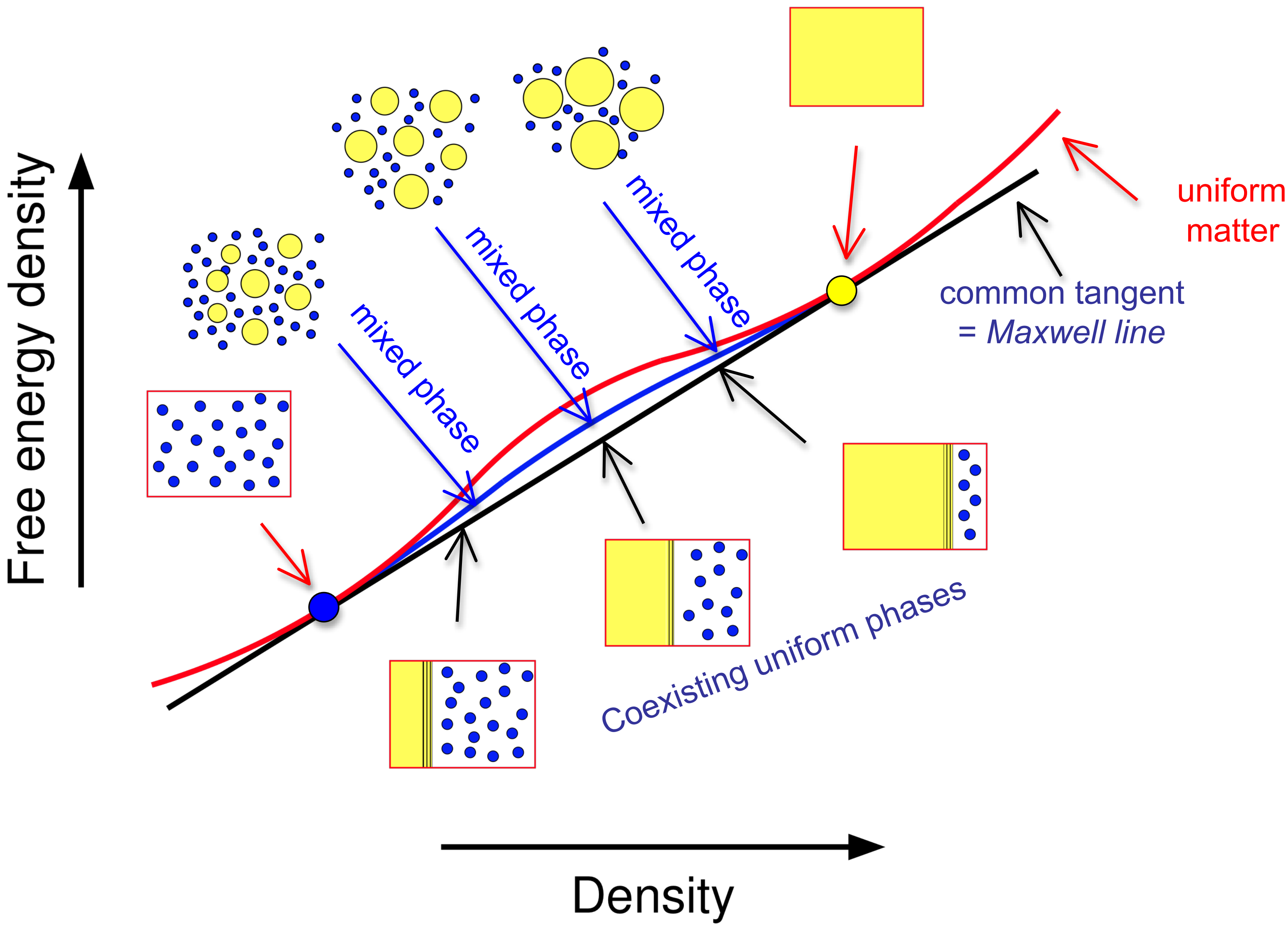




Mixed phase:







JRandrup: Dubna School,
2010

Hadron Gas versus Quark-Gluon Plasma

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

$$p_\pi(T) = -g_\pi \int_{m_\pi}^\infty \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 - \mathrm{e}^{-\beta\epsilon}]$$

$$p_N(T,\mu_0) = g_N \int_{m_N}^\infty \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon-\mu_0)}]$$

$$p_{\bar{N}}(T,\mu_0) = g_N \int_{m_N}^\infty \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon+\mu_0)}]$$

$$p_w(\rho) = \rho\partial_\rho w(\rho) - w(\rho)$$

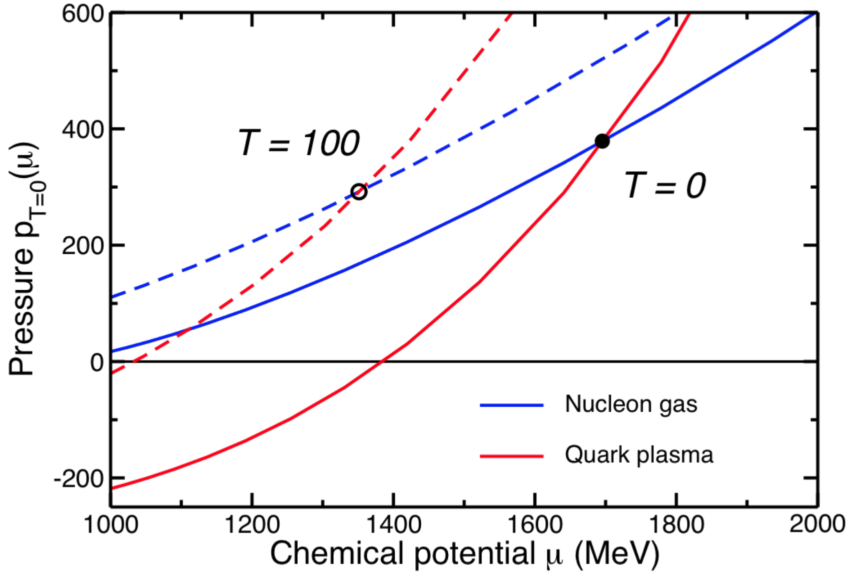
$$w(\rho) = \left[-A\left(\frac{\rho}{\rho_s}\right)^\alpha + B\left(\frac{\rho}{\rho_s}\right)^\beta\right]\rho$$

$$\mu = \mu_0 + \partial_\rho w = 3\mu_q$$

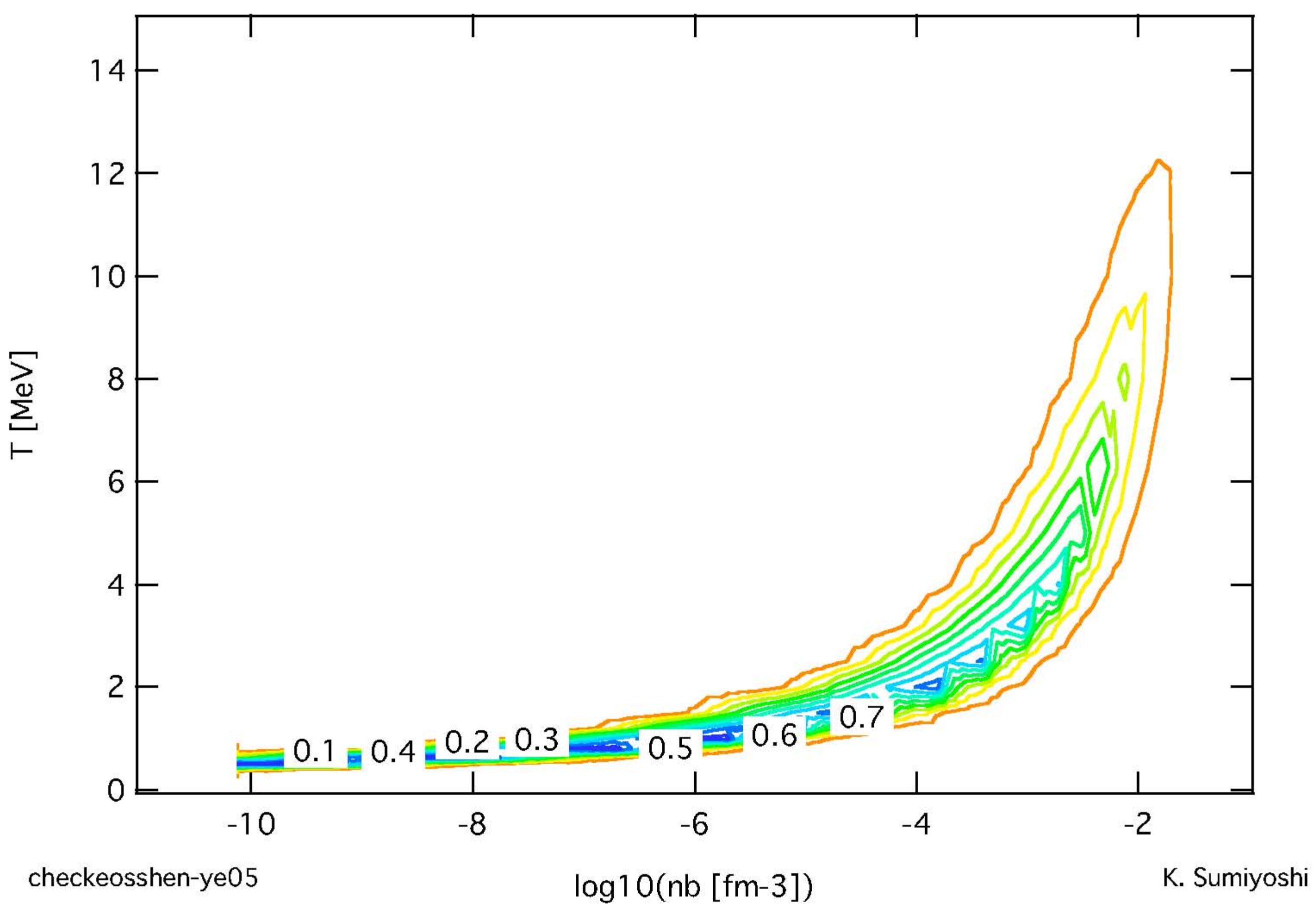
$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

$$p_g = g_g \frac{\pi^2}{90} T^4$$

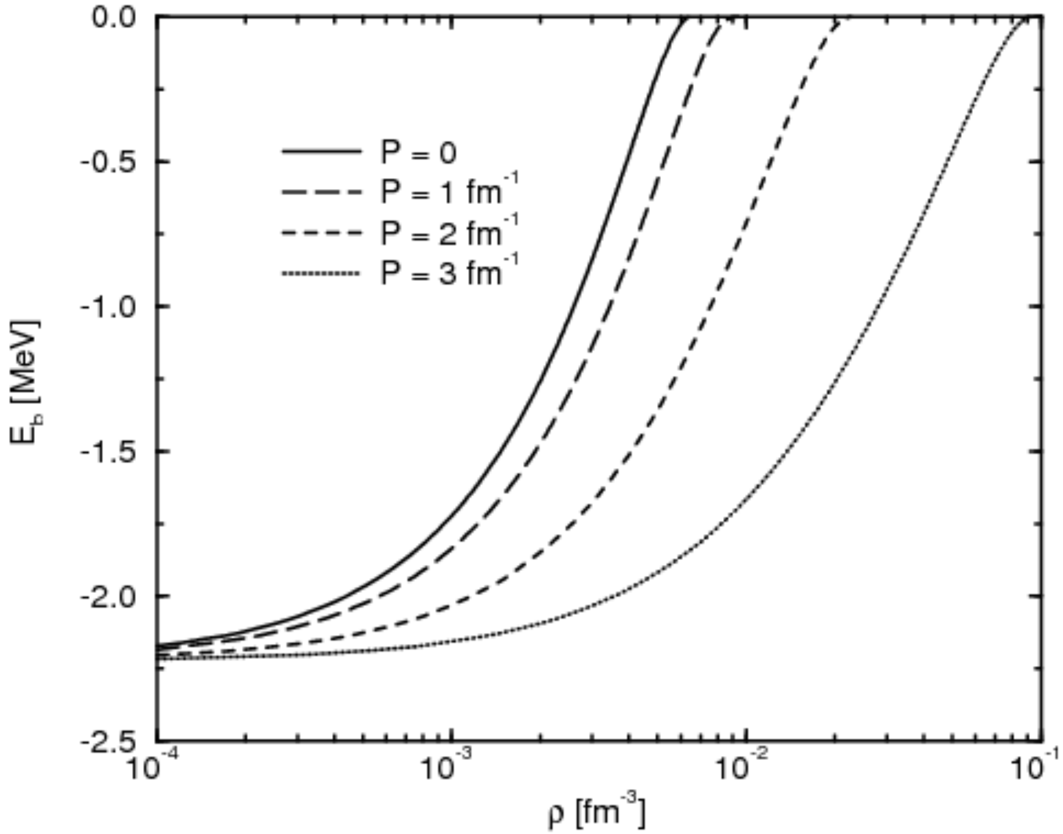
$$p_q + p_{\bar{q}} = g_q \left[\frac{7\pi^2}{360} T^4 + \frac{1}{12}\mu_q^2 T^2 + \frac{1}{24\pi^2}\mu_q^4\right]$$



alpha-fraction in symmetric matter

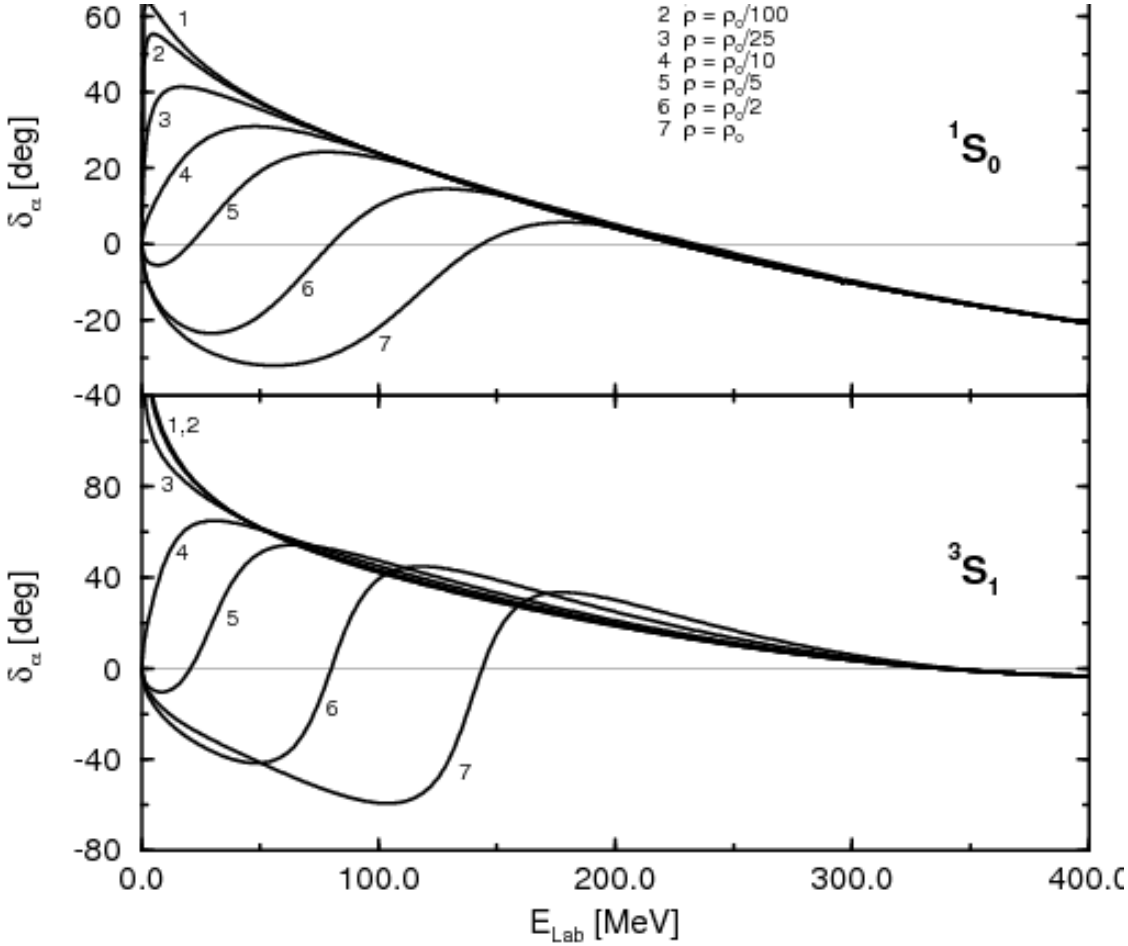


Deuterons in nuclear matter



T=10 MeV, P: center of mass momentum

Scattering phase shifts in matter



Deuteron quasiparticle properties

$$E_d^{\mathrm{qu}}(P) = E_d^{\mathrm{free}} + \Delta E_d + \frac{\hbar^2}{2m_d^*}P^2 + O(P^4)$$

$$\Delta E_d^{\mathrm{Pauli}}(T, n_B, \alpha) = \delta E_d^{(0)}(T, \alpha) n_B + O(n_B^2)$$

$$\frac{m_d^*}{m_d}(T, n_B, \alpha) = 1 + \delta m_d^{(0)}(T, \alpha) n_B + O(n_B^2)$$

T [MeV]	delta E [MeV fm³]	delta m* [fm³]
10	364.3	21.3
4	712.9	87.1

$$E_d^{\mathrm{free}} = -2.225\,\mathrm{MeV}$$

G.R.,PRC **79**, 014002 (2009)

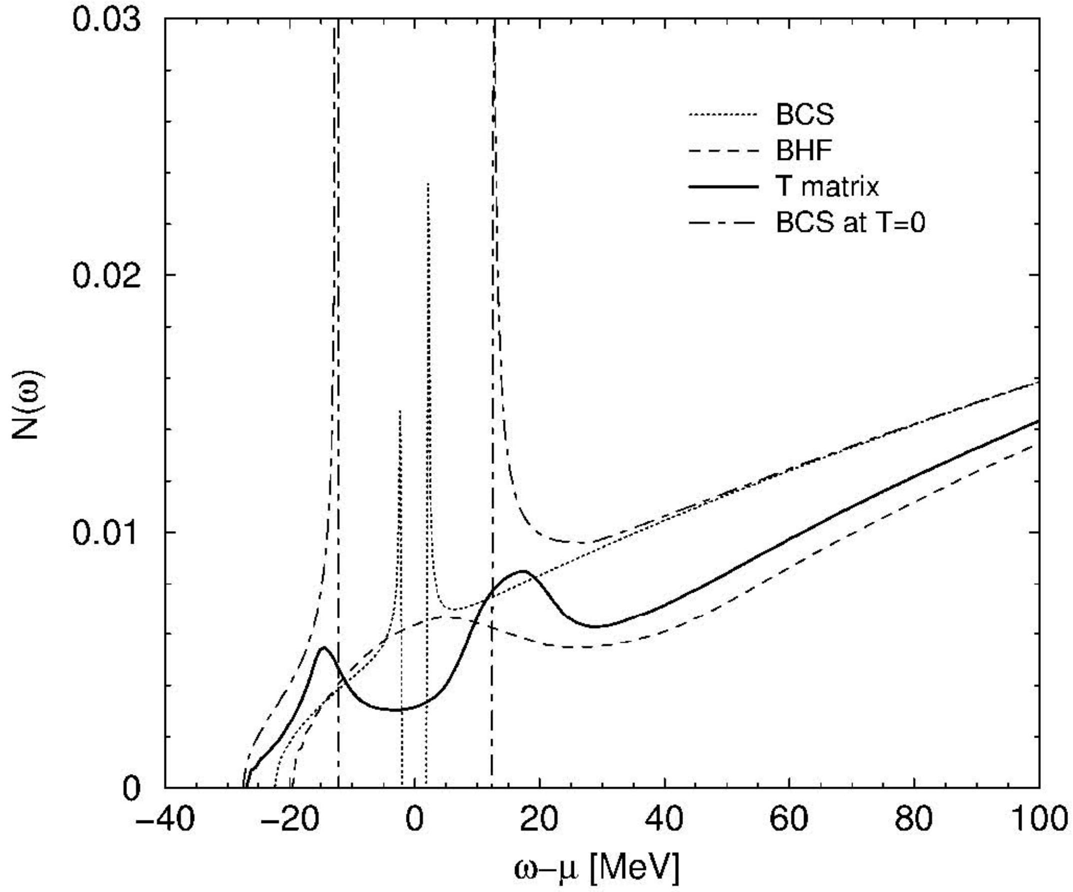
Pseudogap

Precritical Pair Fluctuations and Formation of a
Pseudogap in Low-Density Nuclear Matter

A. Schnell, G. Roepke, P. Schuck, PRL **83,** 1926 (1999)

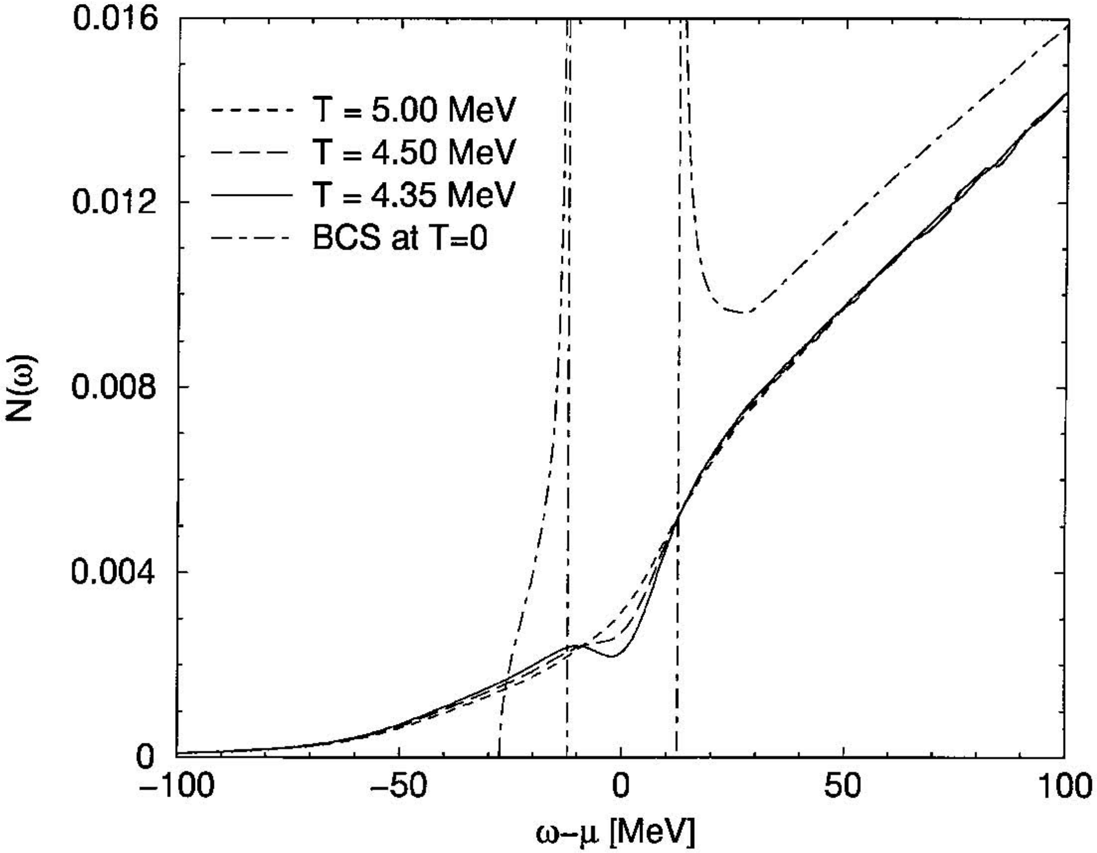
- Self-consistent solution of the two-nucleon Bethe Salpeter equation and evaluation of the density of states
- above the critical temperature: depletion near the chemical potential instead opening of the gap

Density of states near phase transition



T=5 MeV, rho=rho$_0$/3: T-matrix in quasiparticle approximation, compared with BCS and BHF. Also shown: BCS at T=0

Density of states near phase transition



rho=rho$_0$/3: T-matrix, self-consistent spectral function

Variational ansatz

$$|\Phi_{n\alpha}\rangle = \left(C_\alpha^\dagger\right)^n |\mathrm{vac}\rangle$$

α- particle creation operator

$$C_\alpha^\dagger = \int d^3R e^{-\vec{R}^2/R_0^2}$$

$$\times \int d^3r_1 \dots d^3r_4 \phi_{0s}(\vec{r}_1-\vec{R}) a^\dagger_{\sigma_1\tau_1}(\vec{r}_1) \dots \phi_{0s}(\vec{r}_4-\vec{R}) a^\dagger_{\sigma_4\tau_4}(\vec{r}_4)$$

with

$$\phi_{0s}(\vec{r}) = \frac{1}{(\pi b^2)^{3/4}} e^{-\vec{r}^2/(2b^2)}$$

Variational ansatz

total $n\alpha$ wave function

$$\langle \vec{r}_1\sigma_1\tau_1 \ldots \vec{r}_{4n}\sigma_{4n}\tau_{4n} | \Phi_{n\alpha} \rangle$$
$$\propto \mathcal{A}\left\{ e^{-\frac{2}{B^2}(\vec{X}_1^2 + \ldots \vec{X}_n^2)} \phi(\alpha_1) \ldots \phi(\alpha_n) \right\}$$

where $B^2 = (b^2 + 2R_0^2)$, $\vec{X}_i = \frac{1}{4}\Sigma_n \vec{r}_{in}$,
$\phi(\alpha_i) = e^{-\frac{1}{8b^2}\Sigma_{m>n}^4 (\vec{r}_{im} - \vec{r}_{in})^2}$ - internal α wave function

A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, PRL **87**, 192501 (2001)