Helmholtz International Summer School

Dense Matter in Heavy Ion Collisions and *Astrophysics*

JINR, Dubna, Russia, August 28 – September 8, 2012

Spinodal Instabilities in Phase Transitions

Jørgen Randrup (LBNL)

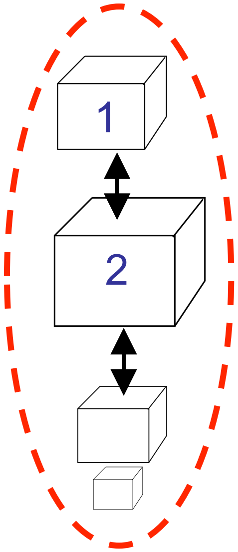
Lecture I: Phase coexistence (equilibrium) TUE 11:30-12:30

Lecture II: Phase separation (non-equilibrium) THU 10:00-11:00

Seminar: Problem Solving, Discussion FRI 17:00-18:00

Lecture III: Nuclear collisions (fresh results) SAT 11:30-12:30

Basic thermodynamics



$\boldsymbol{X}_1 = \{E_1, N_1, V_1, \ldots\} \Rightarrow S_1(\boldsymbol{X}_1)$

$\boldsymbol{X}_2 = \{E_2, N_2, V_2, \ldots\} \Rightarrow S_2(\boldsymbol{X}_2)$

$\boldsymbol{X} = \{E, N, V, \ldots\} = \boldsymbol{X}_1 + \boldsymbol{X}_2 + \ldots$

$$\left\{ \begin{array}{l} E = E_1 + E_2 + \ldots \\ N = N_1 + N_2 + \ldots \\ V = V_1 + V_2 + \ldots \end{array} \right.$$

$S = S_1 + S_2 + \ldots$

The combined system is in equilibrium provided S has a local *maximum* - which requires $\delta S = 0$ and $\delta^2 S < 0$:

δS:

$$0 \doteq \delta S = \sum_i \delta S_i = \sum_i \left(\sum_\ell \frac{\partial S_i}{\partial X_i^\ell} \delta X_i^\ell \right) = \sum_\ell \left(\sum_i \lambda_i^\ell \delta X_i^\ell \right)$$

$$\lambda_i^\ell \equiv \frac{\partial S_i}{\partial X_i^\ell} \left\{ \begin{array}{l} \lambda_i^E = \dfrac{\partial S_i}{\partial E_i} = \beta_i = \dfrac{1}{T_i} \\ \lambda_i^N = \dfrac{\partial S_i}{\partial N_i} = \alpha_i = -\dfrac{\mu_i}{T_i} \\ \lambda_i^V = \dfrac{\partial S_i}{\partial V_i} = \pi_i = \dfrac{p_i}{T_i} \end{array} \right.$$

$$\delta X^\ell = \sum_i \delta X_i^\ell \doteq 0 \quad \left\{ \begin{array}{l} \delta E = \sum_i \delta E_i \doteq 0 \\ \delta N = \sum_i \delta N_i \doteq 0 \\ \delta V = \sum_i \delta V_i \doteq 0 \end{array} \right.$$

=> $\lambda_1^\ell \doteq \lambda_2^\ell \doteq \ldots$

$$\left\{ \begin{array}{l} T_1 = T_2 = \ldots \\ \mu_1 = \mu_2 = \ldots \\ p_1 = p_2 = \ldots \end{array} \right.$$

$\delta^2 S$:

$$0 \dot{>} \delta^2 S = \sum_i \delta^2 S_i = \sum_i \left(\sum_{\ell_1 \ell_2} \frac{\partial^2 S_i}{\partial X_i^{\ell_1} \partial X_i^{\ell_2}} \delta X_i^{\ell_1} \delta X_i^{\ell_2} \right)$$

=> The entropy curvature matrices $\dfrac{\partial^2 S_i}{\partial X_i^{\ell_1} \partial X_i^{\ell_2}}$ have only *negative* eigenvalues

Microcanonical thermodynamics: E and N are specified:

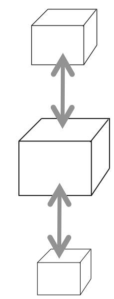
entropy density $\sigma(\varepsilon,\rho)$ $\Rightarrow \begin{cases} \beta(\varepsilon,\rho) = \partial_\varepsilon \sigma(\varepsilon,\rho) = 1/T(\varepsilon,\rho) & \text{temperature} \\ \alpha(\varepsilon,\rho) = \partial_\rho \sigma(\varepsilon,\rho) = -\mu(\varepsilon,\rho)/T(\varepsilon,\rho) & \text{chemical potential} \end{cases}$

$\begin{bmatrix} \partial_\varepsilon^{\,2}\sigma & \partial_\rho\partial_\varepsilon\sigma \\ \partial_\varepsilon\partial_\rho\sigma & \partial_\rho^{\,2}\sigma \end{bmatrix} < 0 \Rightarrow$ *stability*

$p(\varepsilon,\rho) = \sigma T - \varepsilon + \mu\rho$ — *pressure*

$h(\varepsilon,\rho) = p + \varepsilon$ — *enthalpy density*

Canonical thermodynamics: <E> and N are specified:

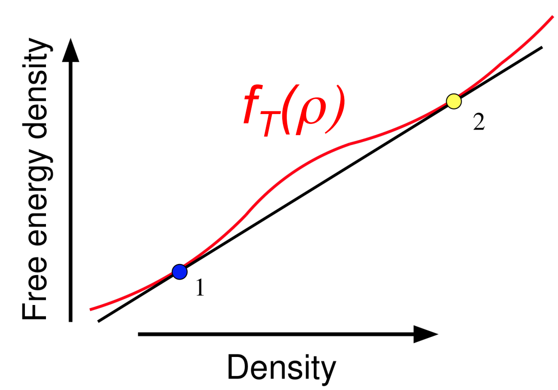


Same:
- Then replace S by $S' = S - \beta E$ and require $\delta S'=0$ & $\delta^2 S'<0$
- or consider $F = -TS' = E - TS$ and require $\delta F=0$ & $\delta^2 F>0$

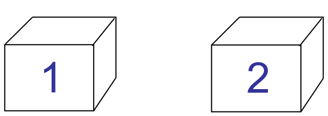
free energy density $f_T(\rho) \equiv \varepsilon_T(\rho) - T\sigma_T(\rho) = \mu_T(\rho)\rho - p_T(\rho)$

$$\mu_T(\rho) = \partial_\rho f_T(\rho)$$
$$\sigma_T(\rho) = -\partial_T f_T(\rho)$$

$\partial_\rho^{\,2} f_T(\rho) > 0 \Rightarrow$ *stability*



Phase coexistence ⇔ *$f_T(\rho)$ has common tangent!*

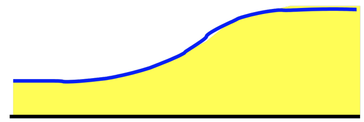


Thermodynamics of non-uniform matter: microcanonical

Non-uniform charge density $\tilde{\rho}(\boldsymbol{r})$

Non-uniform energy density $\tilde{\varepsilon}(\boldsymbol{r})$

Non-uniform entropy density $\tilde{\sigma}[\tilde{\rho}(\boldsymbol{r}),\tilde{\varepsilon}(\boldsymbol{r})](\boldsymbol{r})$



$$N = \int \tilde{\rho}(\boldsymbol{r})d\boldsymbol{r}$$
$$E = \int \tilde{\varepsilon}(\boldsymbol{r})d\boldsymbol{r}$$
$$S = \int \tilde{\sigma}(\boldsymbol{r})d\boldsymbol{r}$$

$$\boxed{\delta S = \int [\tilde{\beta}(\boldsymbol{r})\delta\tilde{\varepsilon}(\boldsymbol{r}) + \tilde{\alpha}(\boldsymbol{r})\delta\tilde{\rho}(\boldsymbol{r})]d\boldsymbol{r}}$$

$$\begin{cases} \tilde{T}(\boldsymbol{r}) = 1/\tilde{\beta}(\boldsymbol{r}) \\ \tilde{\mu}(\boldsymbol{r}) = -\tilde{\alpha}(\boldsymbol{r})\tilde{T}(\boldsymbol{r}) \end{cases}$$

$$\forall\,\delta\tilde{\varepsilon}(\boldsymbol{r}), \forall\,\delta\tilde{\rho}(\boldsymbol{r}):\; 0 \doteq \delta S - \beta_0\delta E - \alpha_0\delta N = \int [\underbrace{(\tilde{\beta}(\boldsymbol{r})-\beta_0)}\delta\tilde{\varepsilon}(\boldsymbol{r}) + \underbrace{(\tilde{\alpha}(\boldsymbol{r})-\alpha_0)}\delta\tilde{\rho}(\boldsymbol{r})]d\boldsymbol{r}$$

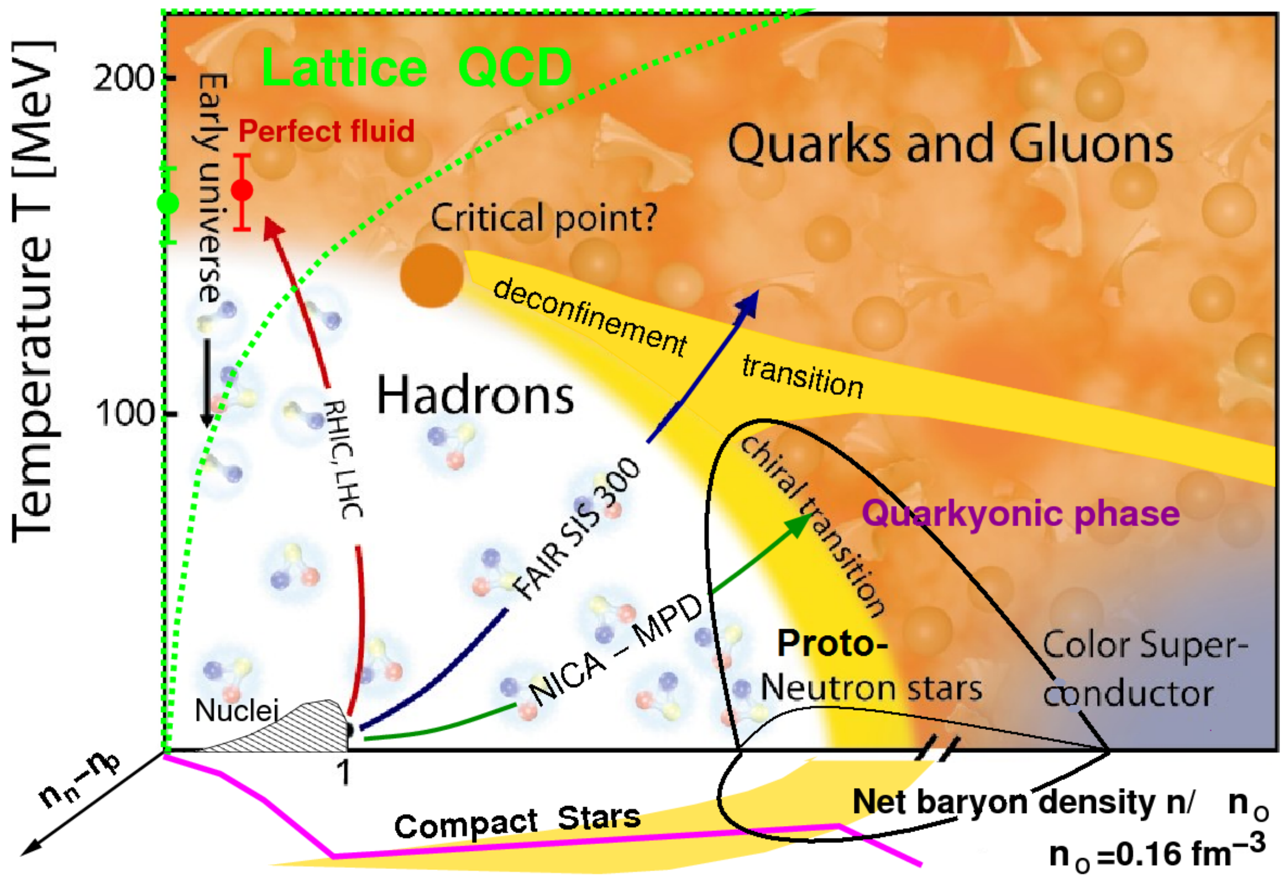
Constant temperature: $\forall\,\boldsymbol{r}:\; \tilde{\beta}(\boldsymbol{r}) \doteq \beta_0 \;\Rightarrow\; \boldsymbol{\nabla}\tilde{\beta} \doteq \boldsymbol{0}$

Constant chemical potential: $\forall\,\boldsymbol{r}:\; \tilde{\alpha}(\boldsymbol{r}) \doteq \alpha_0 \;\Rightarrow\; \boldsymbol{\nabla}\tilde{\alpha} \doteq \boldsymbol{0}$

Constant pressure:

$$\delta\pi = -\varepsilon\delta\beta - \rho\delta\alpha \qquad \pi \equiv p/T = \sigma - \beta\varepsilon - \alpha\rho$$

$$\boldsymbol{\nabla}\tilde{\pi} = -\tilde{\varepsilon}\boldsymbol{\nabla}\tilde{\beta} - \tilde{\rho}\boldsymbol{\nabla}\tilde{\alpha} \quad => \quad \tilde{p}(\boldsymbol{r}) = p_0$$



Temperature T [MeV]
200
100
Lattice QCD
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
chiral transition
Quarkyonic phase
Proto-
Neutron stars
Color Super-
conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_o
n_o=0.16 fm^{-3}

Application to relativistic nuclear collisions

Equations of state

Two-phase EoS => Interface tension

One-phase EoS (Maxwell partner)

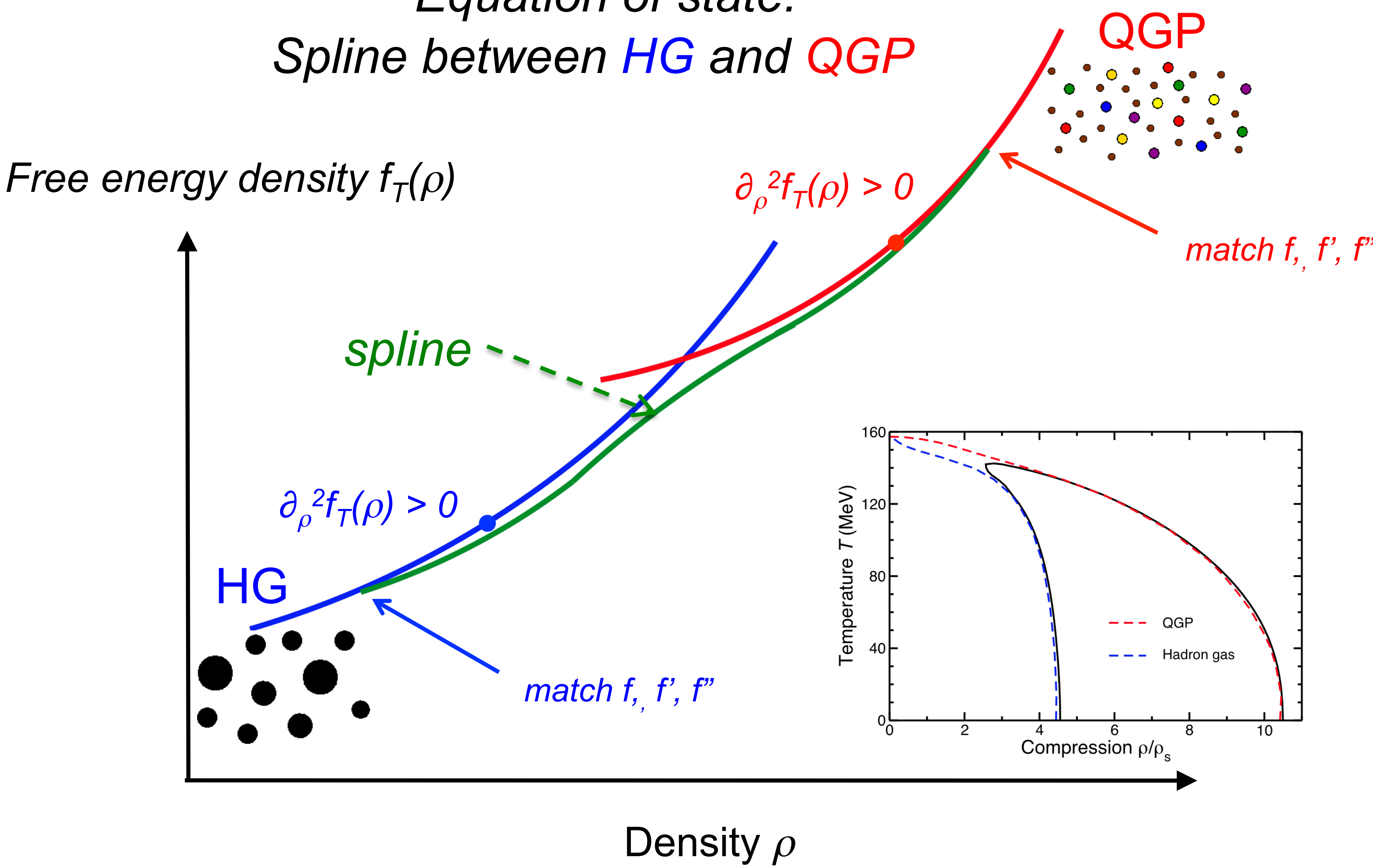
Fluid dynamics

Gradient term => Spinodal amplification

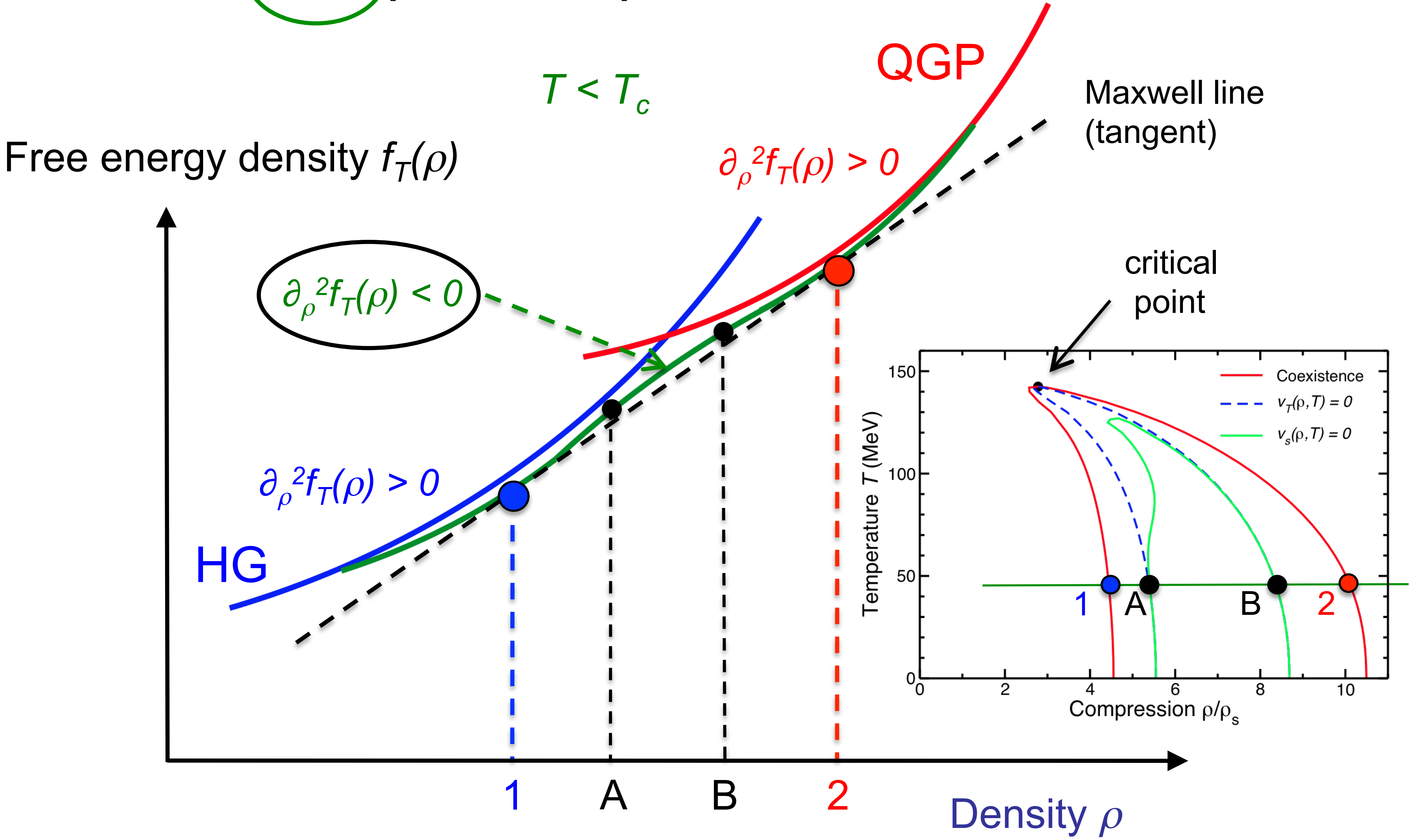
Collisions

Density moments: Enhancement

Equation of state: Spline between HG and QGP

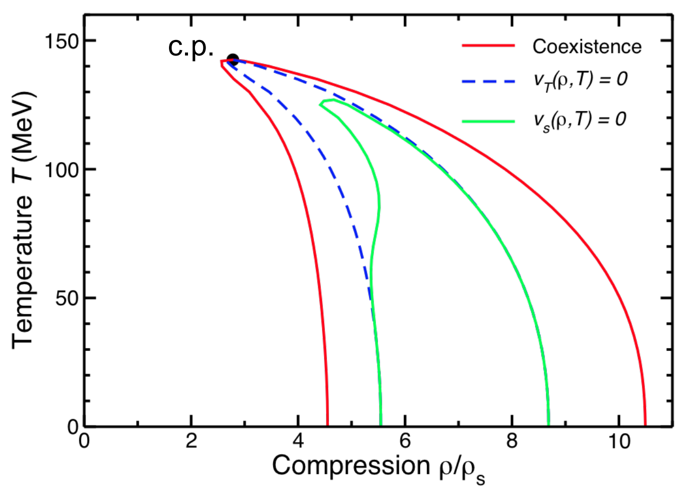


Two-phase Equation of State

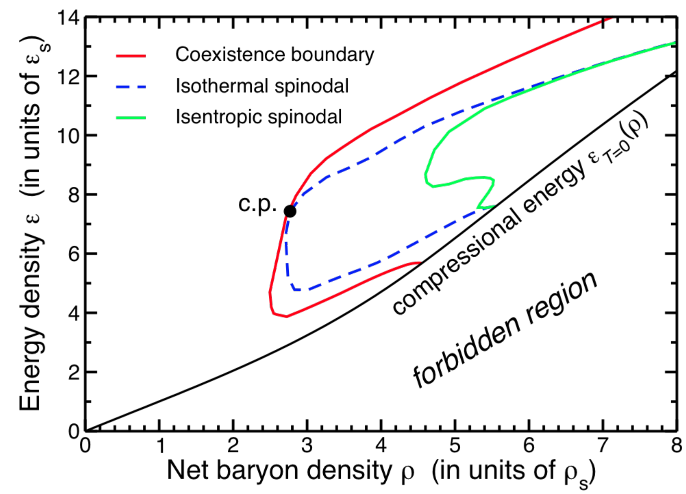


Phase diagrams

(ρ,T)



(ρ,ε)



The construction yields $p_T(\rho)$ ⟹ Fluid dynamics needs $p(\rho,\varepsilon)$

First tabulate $p_T(\rho)$ and $\varepsilon_T(\rho)$,
then get $p(\rho,\varepsilon)$ by interpolation

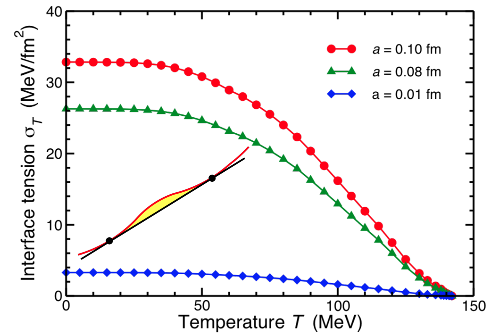
Finite-range (ideal) fluid dynamics

Gradient term in free energy density:

$$\tilde{f}_T(\boldsymbol{r}) \;=\; f_T(\tilde{\rho}(\boldsymbol{r})) + \tfrac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$$

$$\gamma_T = \int_{\rho_1}^{\rho_2} \left\{2C[f_T(\rho) - f_T^M(\rho)]\right\}^{1/2} d\rho$$

Interface tension $\gamma(T)$



=> gradient term in the pressure:

Laplace operator

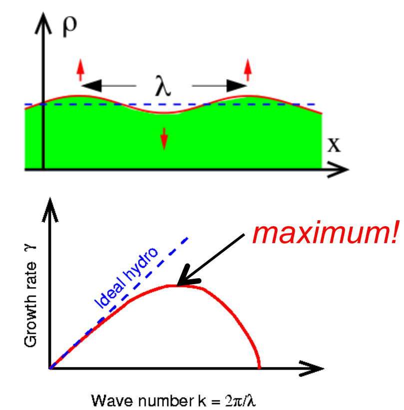
$$\tilde{p}(\boldsymbol{r}) = p_0(\tilde{\rho}(\boldsymbol{r}), \tilde{\varepsilon}(\boldsymbol{r})) - C\tilde{\rho}(\boldsymbol{r})\boldsymbol{\Delta}\tilde{\rho}(\boldsymbol{r})$$

$$\rho(r,t) = \rho_0 + \delta\rho(x,t) \doteq \;\rho_0 + \rho_k \mathrm{e}^{ikx - i\omega t}$$

$$p_k \;\longrightarrow\; p_k + C\rho_0 k^2 \rho_k$$

$$\gamma_k^2 = |v_s^2| k^2 - C\frac{\rho_0^2}{\varepsilon_0 + p_0} k^4$$

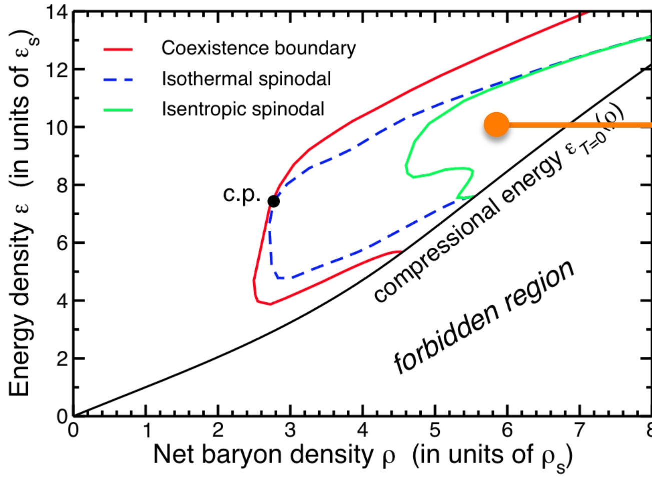
Spinodal dispersion relation



- easy to insert into a fluid dynamic transport code

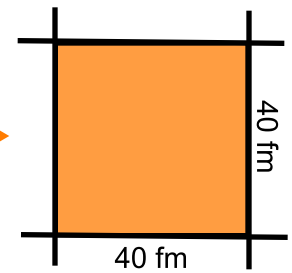
Unstable matter in a box

Phase diagram in (ρ,ε) plane



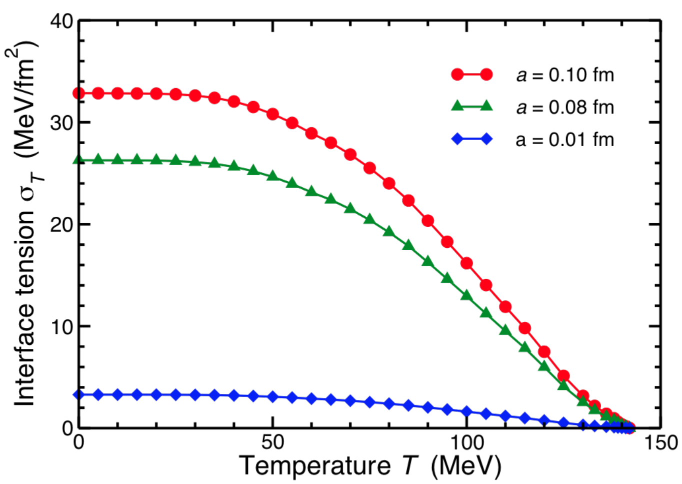
$$\tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2 \quad =>$$

$$\tilde{p}(\boldsymbol{r}) = p_0(\tilde{\rho}(\boldsymbol{r}),\tilde{\varepsilon}(\boldsymbol{r})) - C\tilde{\rho}(\boldsymbol{r})\boldsymbol{\Delta}\tilde{\rho}(\boldsymbol{r})$$

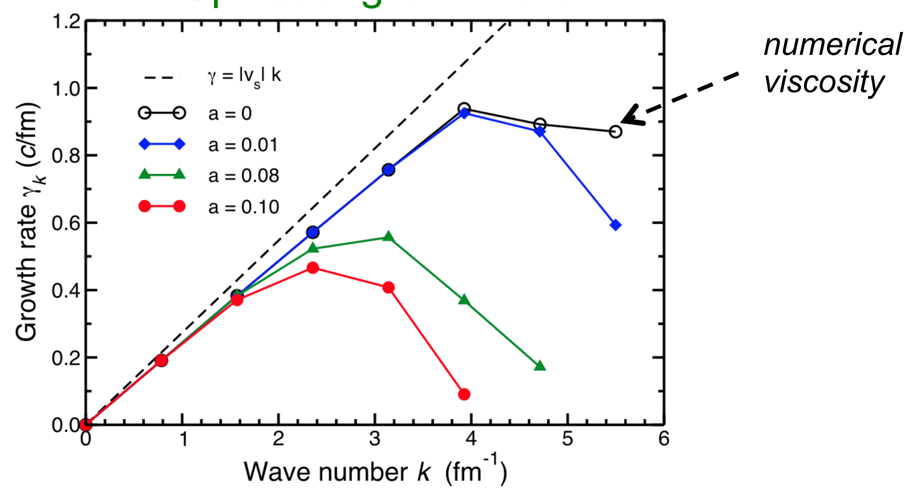


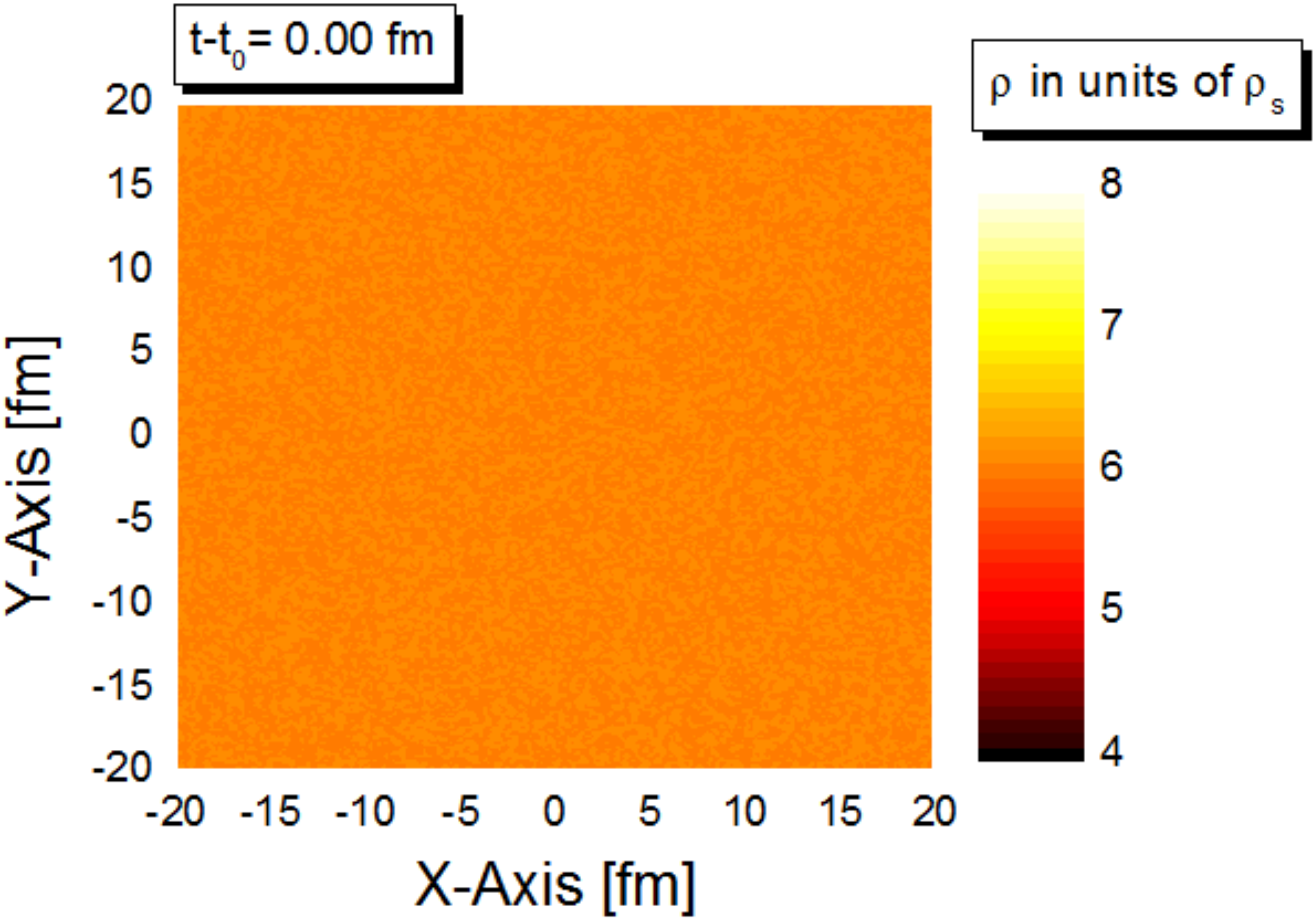
$$C = \frac{\varepsilon_s}{\rho_s^2}\, a^2$$

Interface tension



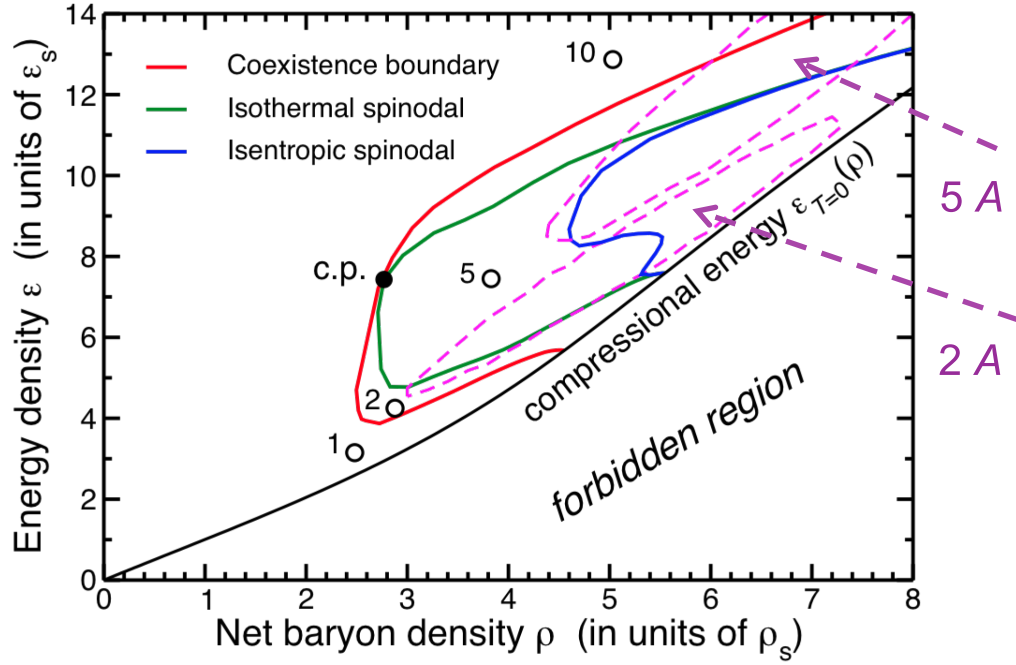
Spinodal growth rate





t-t0= 0.00 fm
ρ in units of ρs
20
15
10
5
0
-5
-10
-15
-20
Y-Axis [fm]
-20 -15 -10 -5 0 5 10 15 20
X-Axis [fm]
8
7
6
5
4

Maximum compression (500 events)

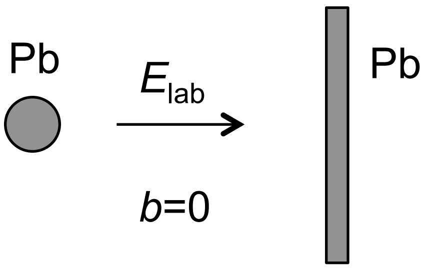


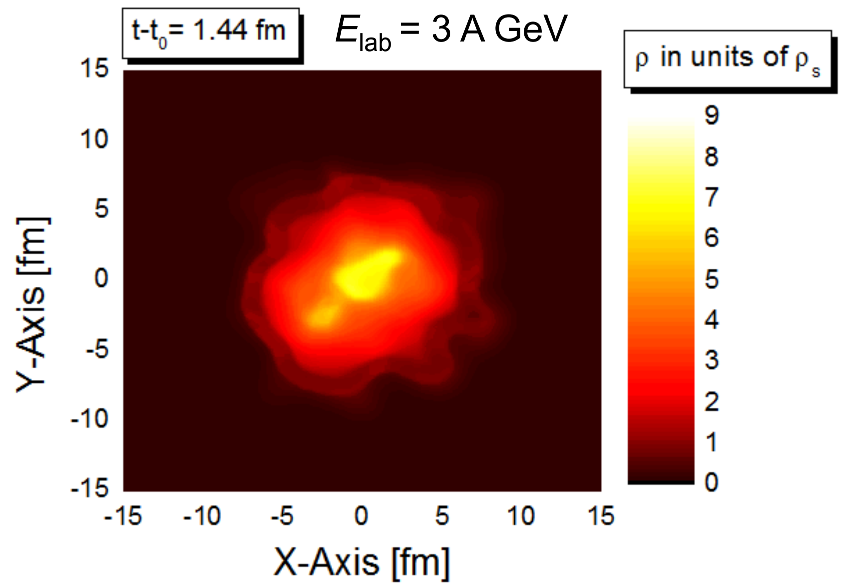
5 *A* GeV Pb + Pb (*b*=0)

2 *A* GeV Pb + Pb (*b*=0)

=> Energy range: 2-4 *A* GeV?

Evolution of the (net) baryon density $\rho(\mathbf{r},t)$



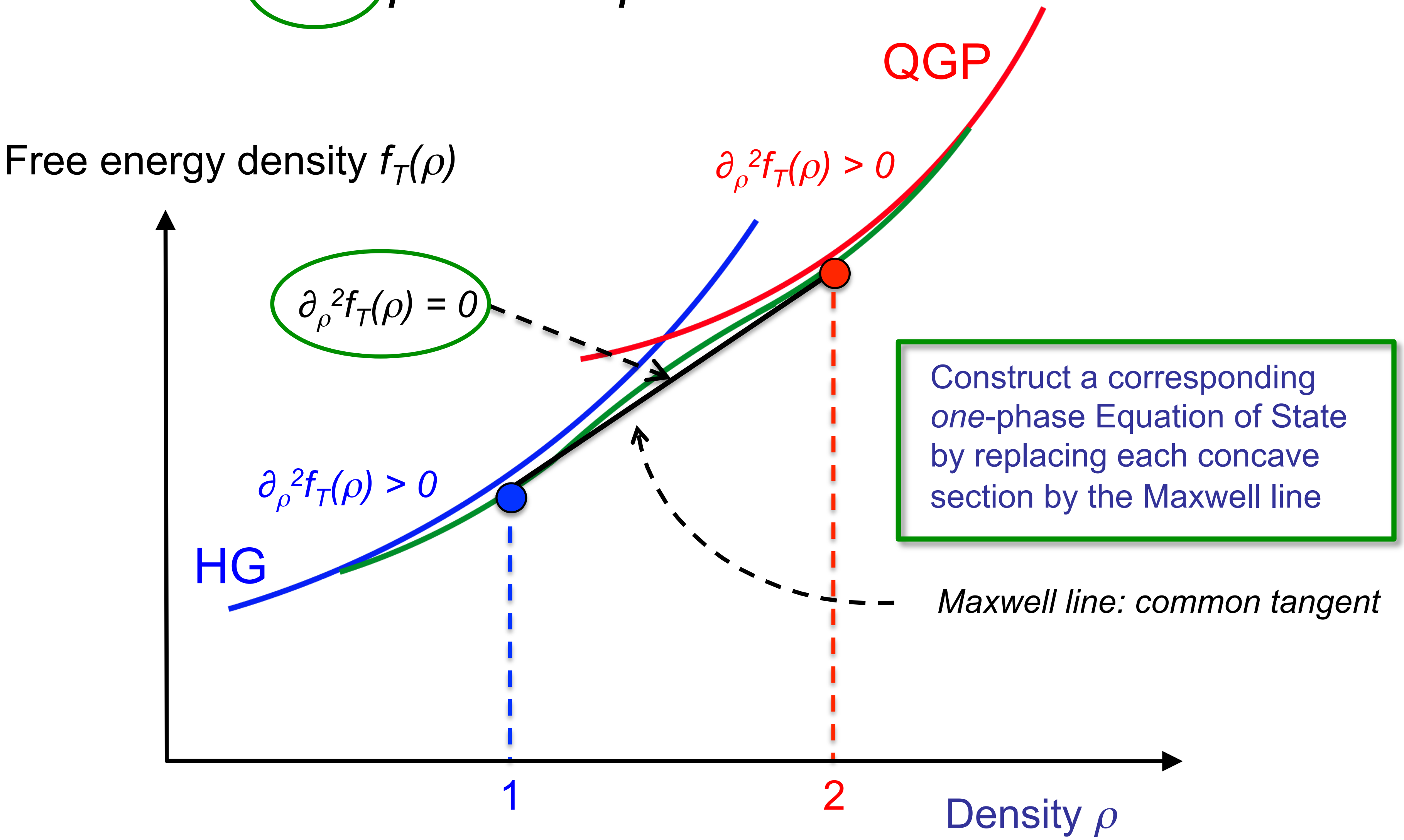


Total baryon number: $$A = \int \rho(\boldsymbol{r})\, d^3\boldsymbol{r}$$

N^{th} density moment: $$\langle \rho^N \rangle \equiv \frac{1}{A^N} \int \rho(\boldsymbol{r})^N \rho(\boldsymbol{r})\, d^3\boldsymbol{r}$$

Mean baryon density: $$\langle \rho \rangle \equiv \frac{1}{A} \int \rho(\boldsymbol{r})\, \rho(\boldsymbol{r})\, d^3\boldsymbol{r} = \langle \rho^{N=1} \rangle$$

One-phase Equation of State

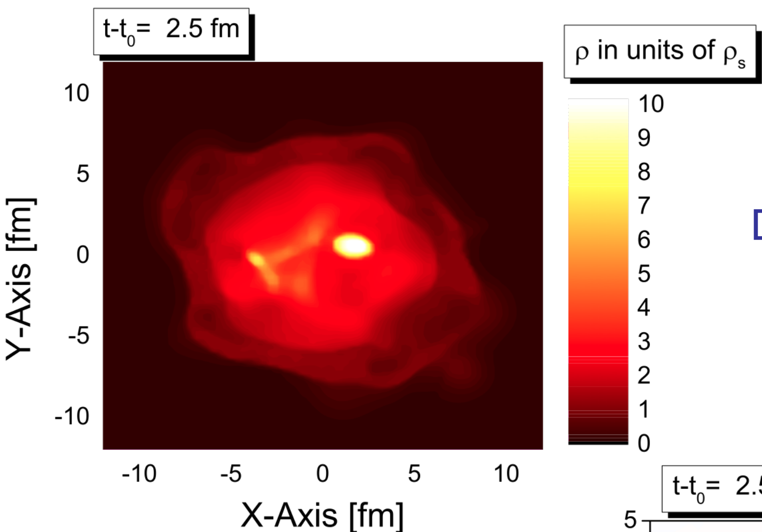


3 *A* GeV Pb + Pb (*b*=0)

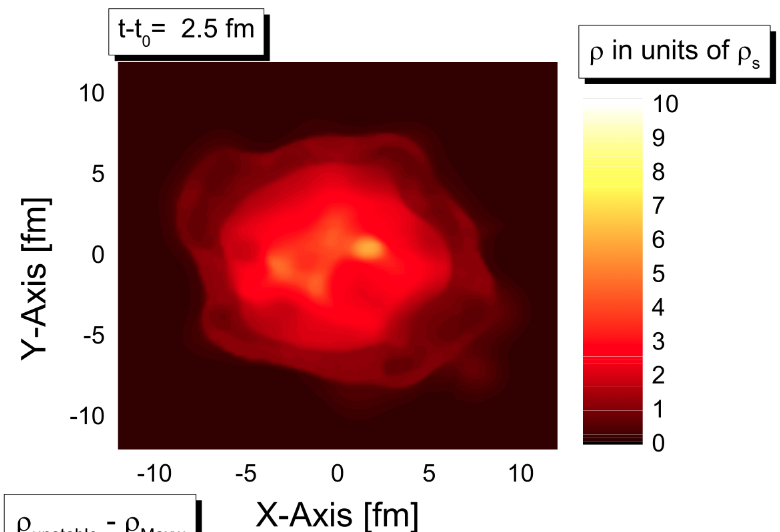
Identical initial conditions

Two-phase EoS:
unstable

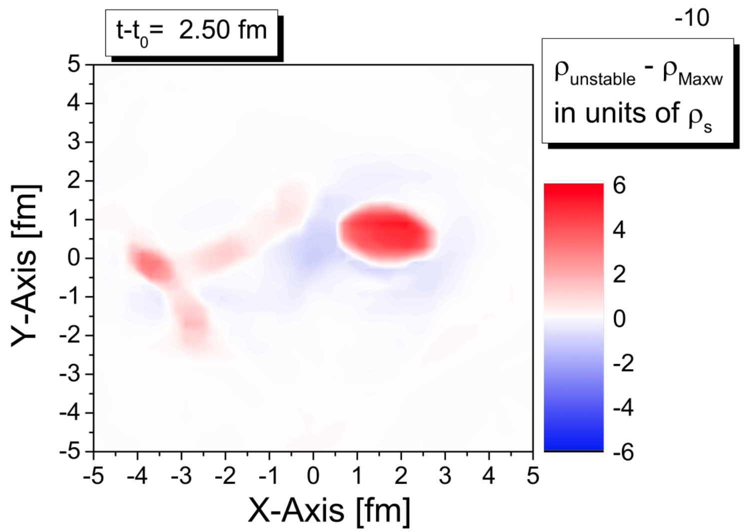
Maxwell EoS:
stable



Density r(x,y,z=0)



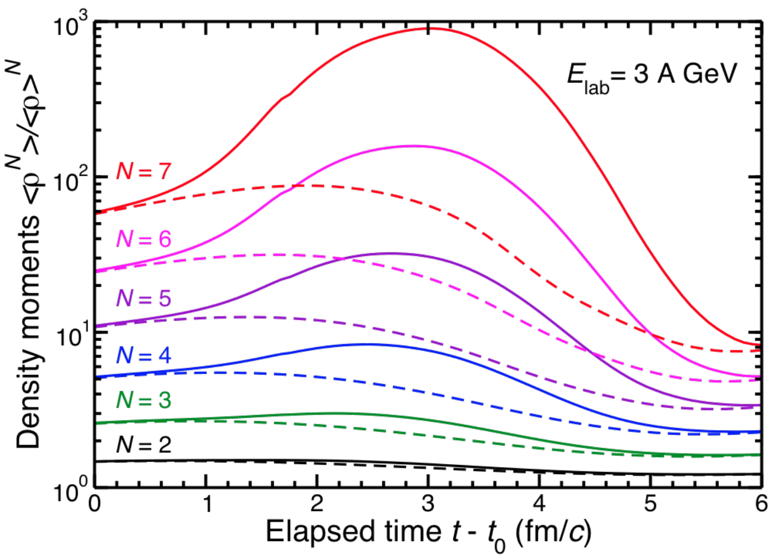
Difference:
unstable - *stable*

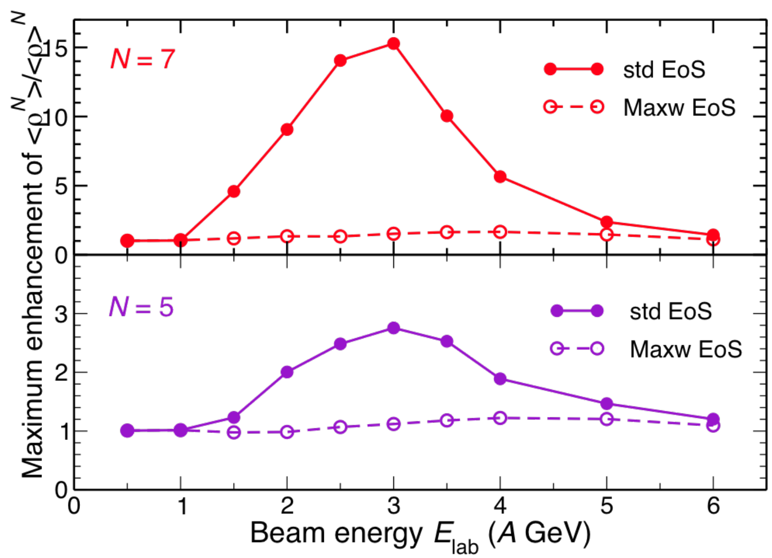


Evolution of the density moments

Density moment: $\langle \rho^N \rangle \equiv \frac{1}{A^N} \int \rho(\boldsymbol{r})^N \rho(\boldsymbol{r})\, d^3\boldsymbol{r}$ $\quad A = \int \rho(\boldsymbol{r})\, d^3\boldsymbol{r}$

Normalized moment: $\langle \rho^N \rangle / \langle \rho \rangle^N$ (dimensionless)





SUMMARY

I: 1st-order phase transition

Thermodynamics

Interface tension

II: Spinodal instabilities

Equation of state

Spinodal growth rates

finite-range dissipative fluid dynamics

III: Nuclear collisions

Finite-range fluid dynamics

Collisions: spinodal clumping