Helmholtz International Summer School

Dense Matter in Heavy Ion Collisions and *Astrophysics*

JINR, Dubna, Russia, August 28 – September 8, 2012

Spinodal Instabilities in Phase Transitions

Jørgen Randrup (LBNL)

Lecture I:	*Phase coexistence (equilibrium)*	TUE 11:30-12:30
Lecture II:	*Phase separation (non-equilibrium)*	THU 10:00-11:00
Seminar:	*Problem Solving, Discussion*	FRI 17:00-18:00
Lecture III:	*Nuclear collisions (fresh results)*	SAT 11:30-12:30

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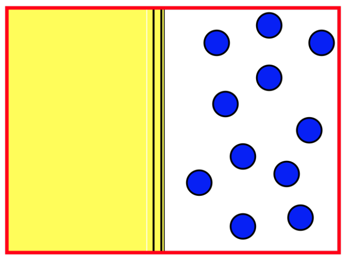
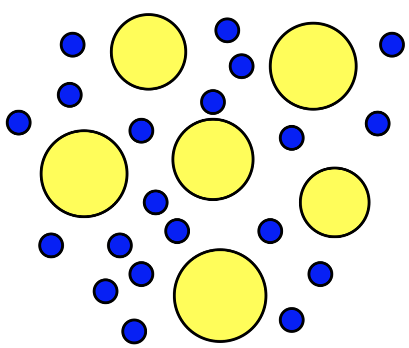
Lecture I: Phase coexistence (equilibrium)

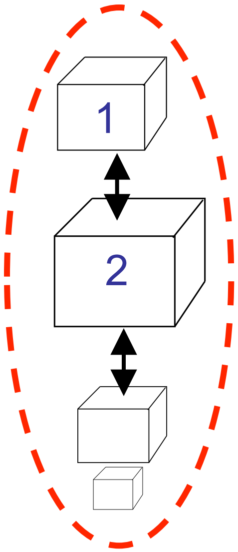


Thermodynamics:
statistical mechanics of large *uniform* systems



Non-uniform systems:
gradient effects, interface tension

Basic thermodynamics



$\boldsymbol{X}_1 = \{E_1, N_1, V_1, \ldots\} \Rightarrow S_1(\boldsymbol{X}_1)$

$\boldsymbol{X}_2 = \{E_2, N_2, V_2, \ldots\} \Rightarrow S_2(\boldsymbol{X}_2)$

$\boldsymbol{X} = \{E, N, V, \ldots\} = \boldsymbol{X}_1 + \boldsymbol{X}_2 + \ldots$

$$\begin{cases} E = E_1+E_2+\ldots \\ N = N_1+N_2+\ldots \\ V = V_1+V_2+\ldots \end{cases}$$

$S = S_1+S_2+\ldots$

The combined system is in equilibrium provided S has a local *maximum* - which requires $\delta S = 0$ and $\delta^2 S < 0$:

δS:
$$0 \doteq \delta S = \sum_i \delta S_i = \sum_i \left(\sum_\ell \frac{\partial S_i}{\partial X_i^\ell} \delta X_i^\ell \right) = \sum_\ell \left(\sum_i \lambda_i^\ell \delta X_i^\ell \right)$$

$$\lambda_i^\ell \equiv \frac{\partial S_i}{\partial X_i^\ell} \begin{cases} \lambda_i^E = \dfrac{\partial S_i}{\partial E_i} = \beta_i = \dfrac{1}{T_i} \\ \lambda_i^N = \dfrac{\partial S_i}{\partial N_i} = \alpha_i = -\dfrac{\mu_i}{T_i} \\ \lambda_i^V = \dfrac{\partial S_i}{\partial V_i} = \pi_i = \dfrac{p_i}{T_i} \end{cases}$$

$$\delta X^\ell = \sum_i \delta X_i^\ell \doteq 0 \quad \begin{cases} \delta E = \sum_i \delta E_i \doteq 0 \\ \delta N = \sum_i \delta N_i \doteq 0 \\ \delta V = \sum_i \delta V_i \doteq 0 \end{cases}$$

=> $\lambda_1^\ell \doteq \lambda_2^\ell \doteq \ldots$

$$\begin{cases} T_1 = T_2 = \ldots \\ \mu_1 = \mu_2 = \ldots \\ p_1 = p_2 = \ldots \end{cases}$$

$\delta^2 S$:
$$0 \dot{>} \delta^2 S = \sum_i \delta^2 S_i = \sum_i \left(\sum_{\ell_1 \ell_2} \frac{\partial^2 S_i}{\partial X_i^{\ell_1} \partial X_i^{\ell_2}} \delta X_i^{\ell_1} \delta X_i^{\ell_2} \right)$$

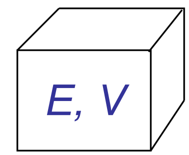
=> The entropy curvature matrices $\dfrac{\partial^2 S_i}{\partial X_i^{\ell_1} \partial X_i^{\ell_2}}$ have only *negative* eigenvalues

Thermodynamics with no conserved charge

Statistical equilibrium in bulk matter

Control parameter(s) $\{X\}$:

$$\begin{cases} \text{Energy } E = V\varepsilon \quad \varepsilon = E/V \\ \text{Volume } V \to \infty \end{cases}$$



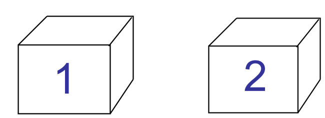
Entropy function $S\{X\}$:

$$S(E,V) = V\sigma(\varepsilon)$$

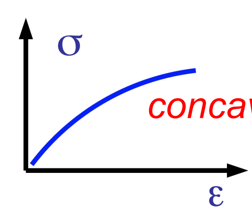
Derivative(s) $\lambda_X = \partial_X S$:

$$\begin{cases} \beta = 1/T = \partial_E S(E,V) = \partial_\varepsilon \sigma(\varepsilon) & \textit{temperature} \\ \pi = p/T = \partial_V S(E,V) = \sigma - \beta\varepsilon & \textit{pressure} \end{cases}$$

Thermodynamic *coexistence*:
=> $T_1 = T_2$ & $p_1 = p_2$



Thermodynamic (local) *stability*: $\delta^2 S_{tot} < 0$
=> Entropy curvature $\partial_\varepsilon^2 \sigma$ must be *negative*

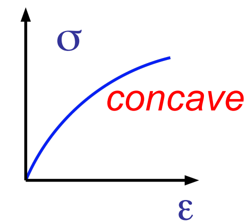


Single-phase system

Entropy density:

$$\partial_\varepsilon \sigma(\varepsilon) > 0$$

$$\partial_\varepsilon^2 \sigma(\varepsilon) < 0$$

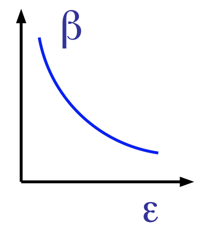


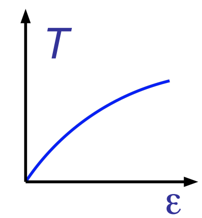
Temperature:

$$\beta(\varepsilon) = \partial_\varepsilon \sigma(\varepsilon) > 0$$

$$\partial_\varepsilon \beta(\varepsilon) = \partial_\varepsilon^2 \sigma(\varepsilon) < 0$$

$$\partial_\varepsilon^2 \beta(\varepsilon) > 0$$



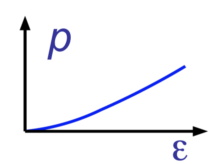


Pressure:

$$\pi(\varepsilon) = \sigma(\varepsilon) - \beta(\varepsilon)\varepsilon = \beta(\varepsilon)\, p(\varepsilon)$$

$$\begin{cases} \partial_\varepsilon \pi(\varepsilon) = \beta - \beta - \varepsilon\, \partial_\varepsilon \beta = -\varepsilon\, \partial_\varepsilon^2 \sigma(\varepsilon) > 0 \\ \partial_\varepsilon \pi(\varepsilon) = \partial_\varepsilon \beta p = \beta \partial_\varepsilon p + p\, \partial_\varepsilon \beta \end{cases}$$

$$\beta(\varepsilon)\, \partial_\varepsilon p(\varepsilon) = -[\varepsilon + p(\varepsilon)]\, \partial_\varepsilon \beta(\varepsilon) = -h(\varepsilon)\, \partial_\varepsilon \beta(\varepsilon) > 0$$



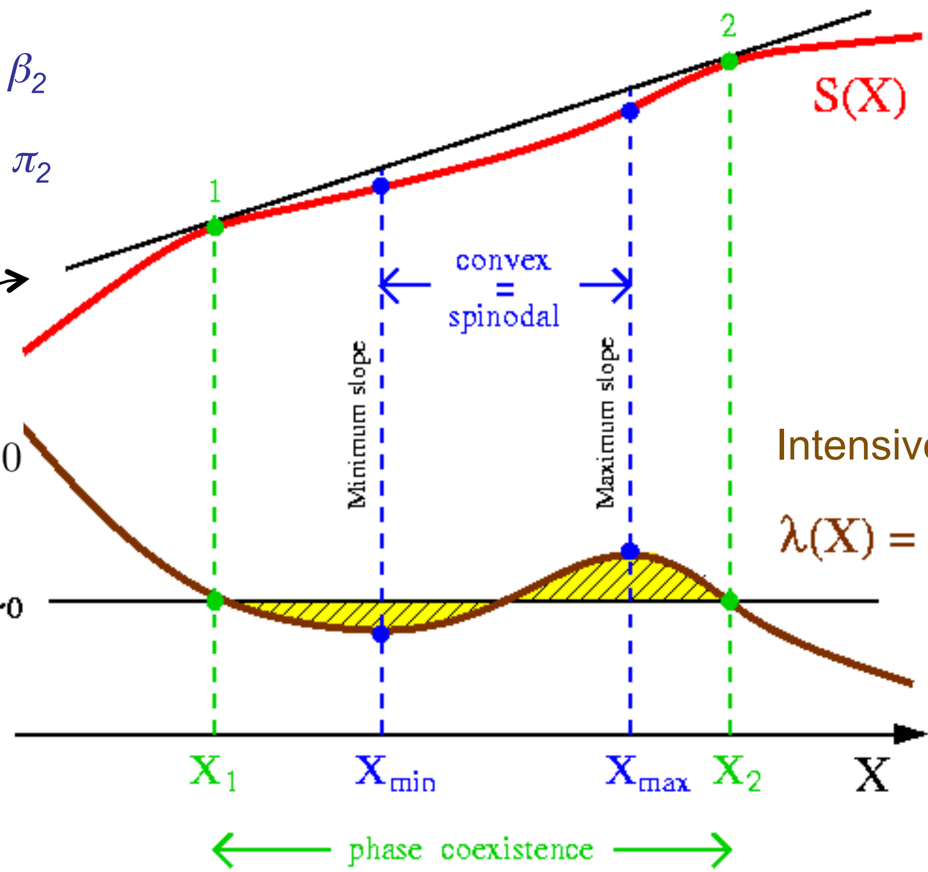
First order <=> Phase coexistence <=> Spinodal instability

Extensive variable X

Entropy function $S(X)$

… occur when S(X) is locally convex:

$X=E$: $\begin{cases} \partial_\varepsilon \sigma(\varepsilon) = \beta: \ \beta_1 = \beta_2 \\ \pi = \sigma - \beta\varepsilon: \ \pi_1 = \pi_2 \end{cases}$



Maxwell construction:

$$\int_{X_1}^{X_2} dX\,(\lambda(X) - \lambda_0) = 0$$

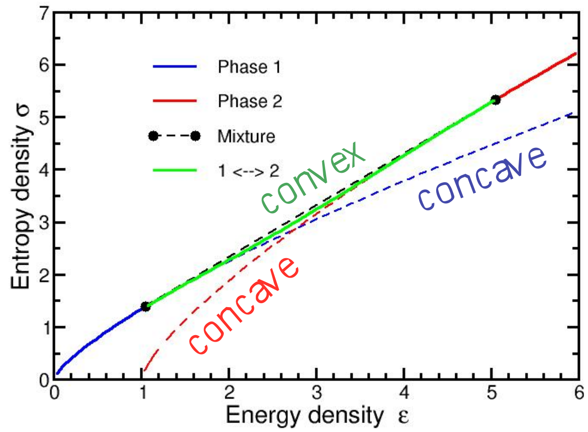
Intensive variable:

$\lambda(X) = -dS/dX$

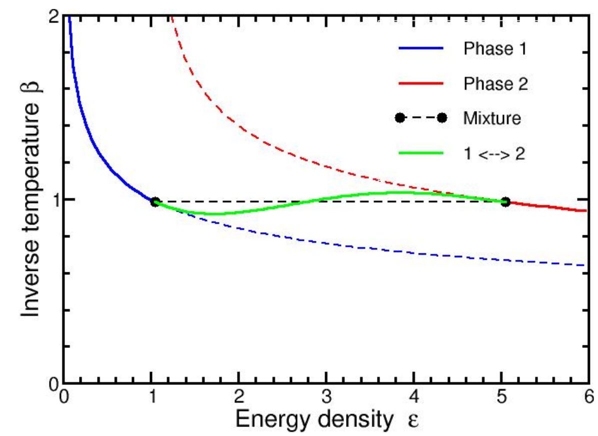
$X=E \Rightarrow \lambda=\beta$

Phase transformation with no conserved charge

Entropy density: $\sigma(\varepsilon)$

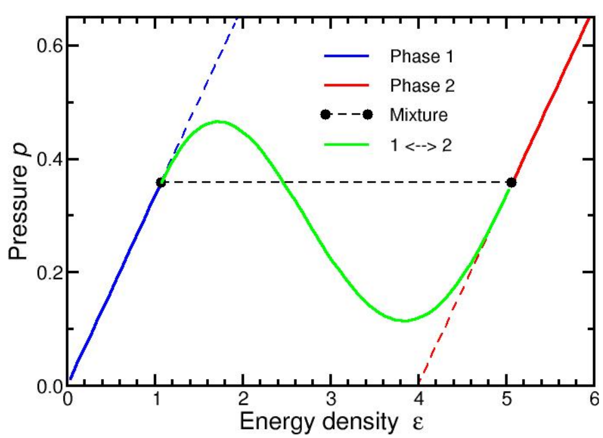


Inverse temperature: $\beta(\varepsilon) = \partial\sigma/\partial\varepsilon$

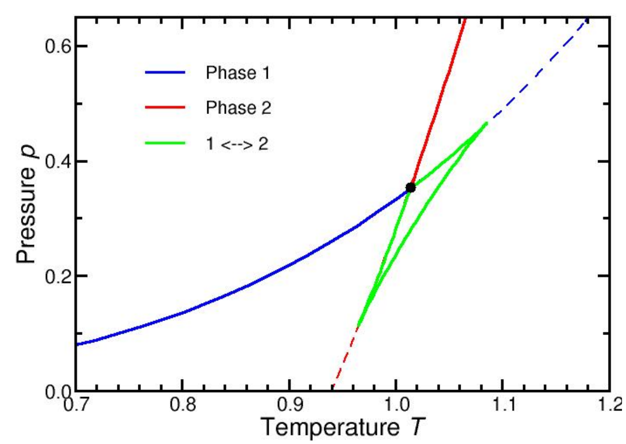


Equation of State

Pressure: $p(\varepsilon) = T\sigma - \varepsilon$



Pressure: $p(T)$

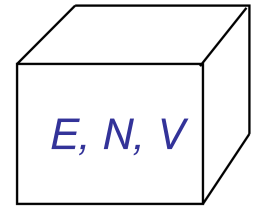


Thermodynamics with one conserved charge

Statistical equilibrium in bulk matter

Control parameter(s) $\{X\}$:

- Energy $E = V\varepsilon$ $\varepsilon = E/V$
- Charge $N = V\rho$ $\rho = N/V$
- Volume $V \to \infty$

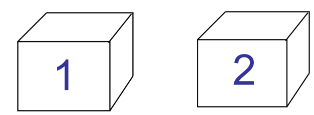


Entropy function $S\{X\}$: $S(E,N,V) = V\sigma(\varepsilon,\rho)$

Derivative(s) $\lambda_X = \partial_X S$:

- $\beta = 1/T = \partial_E S(E,N,V) = \partial_\varepsilon \sigma(\varepsilon,\rho)$
- $\alpha = -\mu/T = \partial_N S(E,N,V) = \partial_\rho \sigma(\varepsilon,\rho)$
- $\pi = p/T = \partial_V S(E,N,V) = \sigma - \beta\varepsilon - \alpha\rho$

Thermodynamic *coexistence*: $\delta S_{tot} = 0$
=> $T_1 = T_2$ & $\mu_1 = \mu_2$ & $p_1 = p_2$



Thermodynamic (local) *stability*: $\delta^2 S_{tot} < 0$
=> Curvature matrix $\{\partial_\chi \partial_{\chi'} \sigma(\varepsilon,\rho)\}$ has only *negative* eigenvalues:

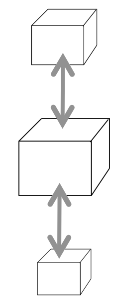
$$\begin{bmatrix} \partial_\varepsilon^2\sigma & \partial_\rho\partial_\varepsilon\sigma \\ \partial_\varepsilon\partial_\rho\sigma & \partial_\rho^2\sigma \end{bmatrix}$$

Microcanonical scenario: E and N are specified:

entropy density $\sigma(\varepsilon,\rho) \Rightarrow \begin{cases} \beta(\varepsilon,\rho) = \partial_\varepsilon \sigma(\varepsilon,\rho) = 1/T(\varepsilon,\rho) & \text{temperature} \\ \alpha(\varepsilon,\rho) = \partial_\rho \sigma(\varepsilon,\rho) = -\mu(\varepsilon,\rho)/T(\varepsilon,\rho) & \text{chemical potential} \end{cases}$

$\Rightarrow \begin{cases} p(\varepsilon,\rho) = \sigma T - \varepsilon + \mu\rho & \text{pressure} \\ h(\varepsilon,\rho) = p + \varepsilon & \text{enthalpy density} \end{cases}$

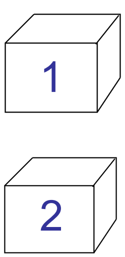
Canonical scenario: <E> and N are specified:



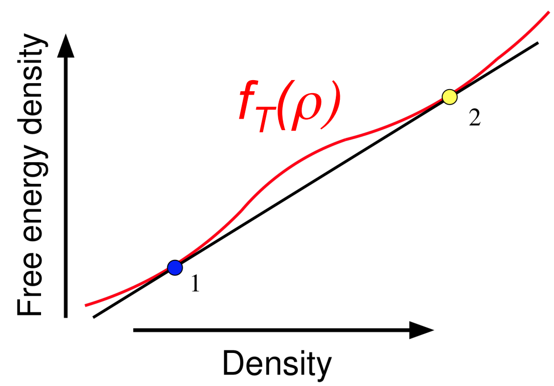
Same:
- Then replace S by $S' = S - \beta E$ and require $\delta S'=0$ & $\delta^2 S'<0$
- \- or consider $F = -TS' = E - TS$ and require $\delta F=0$ & $\delta^2 F>0$

free energy density $f_T(\rho) \equiv \varepsilon_T(\rho) - T\sigma_T(\rho) = \mu_T(\rho)\rho - p_T(\rho)$

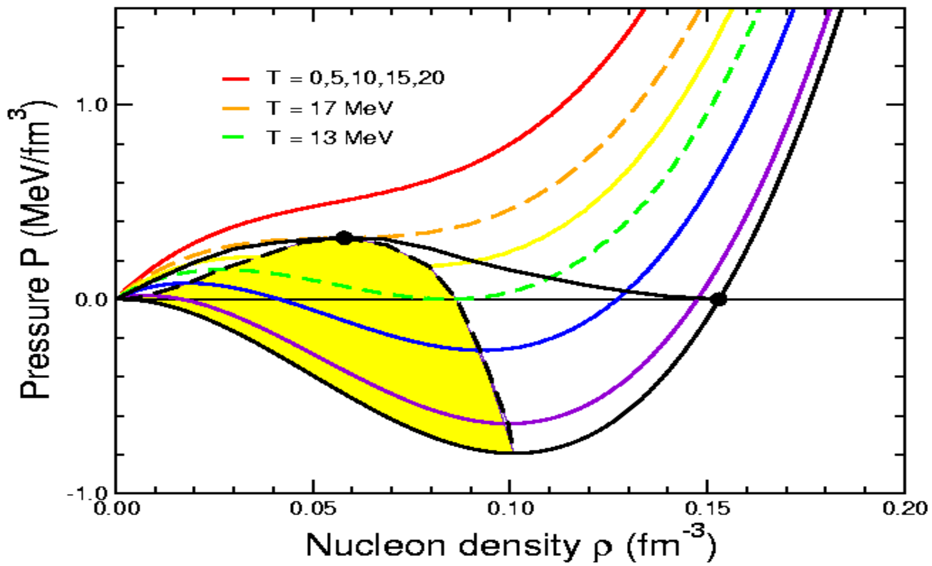
$$\Rightarrow \begin{cases} \mu_T(\rho) = \partial_\rho f_T(\rho) \\ \sigma_T(\rho) = -\partial_T f_T(\rho) \end{cases}$$



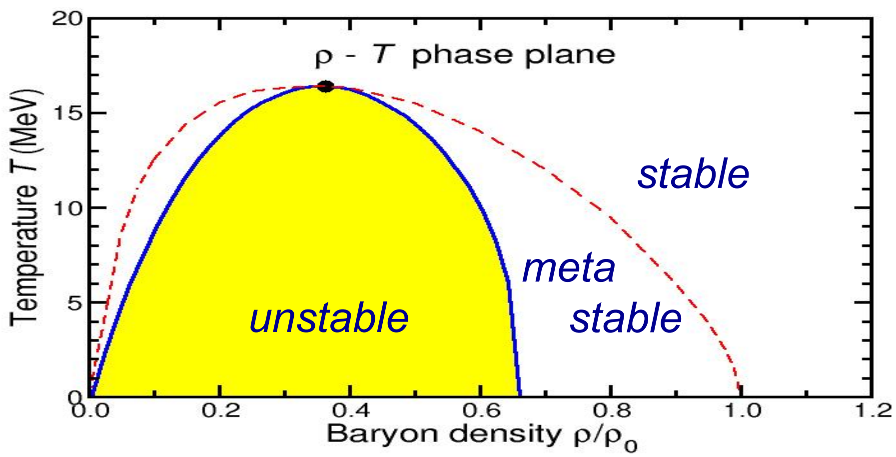
Phase coexistence ⇔ $f_T(\rho)$ has common tangent!



Example: Nuclear matter



Equation of state: $p_T(\rho)$

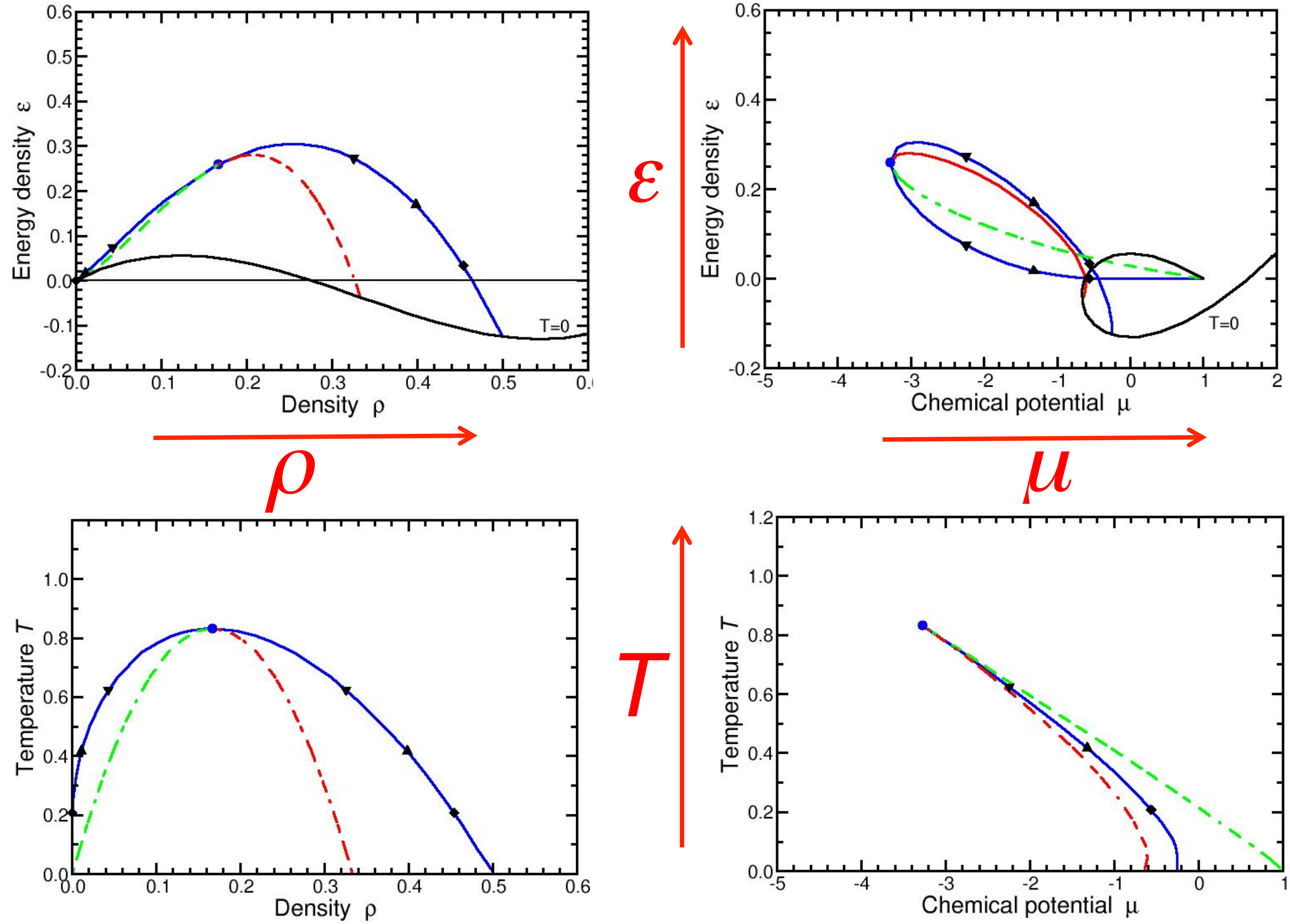


(ρ,T) phase diagram

The spinodal boundary occurs at $\partial_T p_T(\rho) = 0$ =>

$$v_T^2 \equiv \frac{\rho}{h}\partial_\rho p_T(\rho) = 0$$

Nuclear phase diagram in different representations



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Isentropic changes

Entropy density: $\sigma(\varepsilon,\rho)$

Energy density: ε

Net baryon density: ρ

Temperature: $T(\varepsilon,\rho) = 1/\sigma_\varepsilon$

Chemical potential: $\mu(\varepsilon,\rho) = -T\sigma_\rho$

Pressure: $p(\varepsilon,\rho) = T\sigma - \varepsilon + \mu\rho$

Enthalpy density: $h(\varepsilon,\rho) = p + \varepsilon$

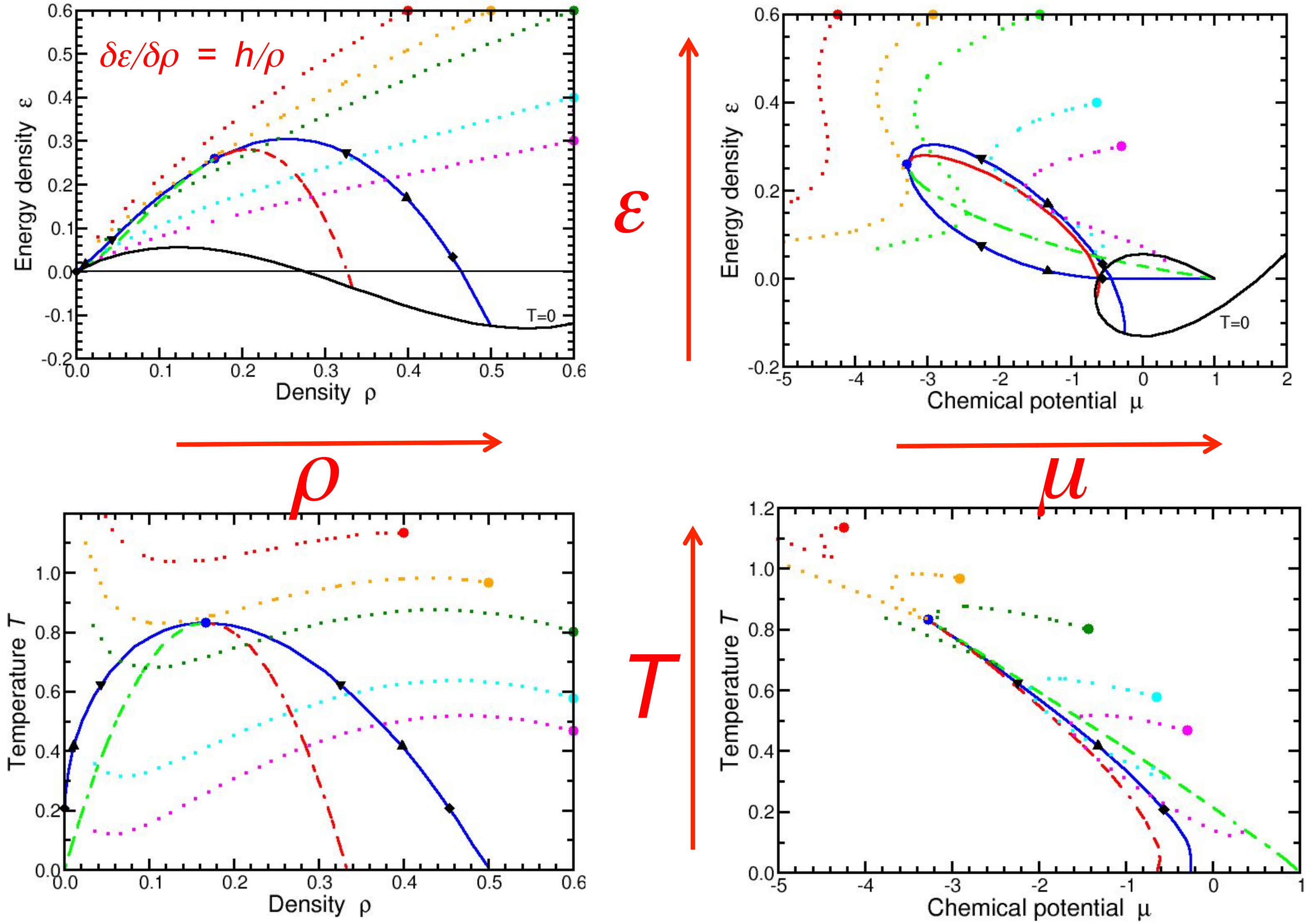
Entropy per (net) baryon: $s(\varepsilon,\rho) = \sigma/\rho$

Changes: $(\delta\varepsilon,\delta\rho)$ => δs :

$$\rho^2 T\delta s = \rho^2 T\delta(\sigma/\rho) = \rho T\delta\sigma - T\sigma\delta\rho = \rho\delta\varepsilon - \mu\rho\delta\rho - [h-\mu\rho]\delta\rho = \rho\delta\varepsilon - h\delta\rho$$

$\delta s = 0$ => $\rho\delta\varepsilon = h\delta\rho$ <=> $\delta\varepsilon/\delta\rho = h/\rho$

Isentropic phase trajectories in different representations



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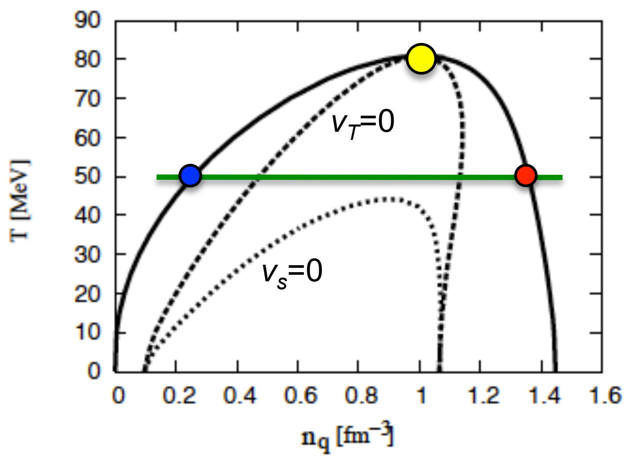
Example: Nambu – Jona-Lasino model

C. Sasaki, B. Friman, K. Redlich, Phys. Rev. Lett. 99, 232301 (2007):

Density fluctuations in the presence of spinodal instabilities

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + \mu\gamma_0)\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\vec{\tau}\gamma_5\psi)^2\right]$$

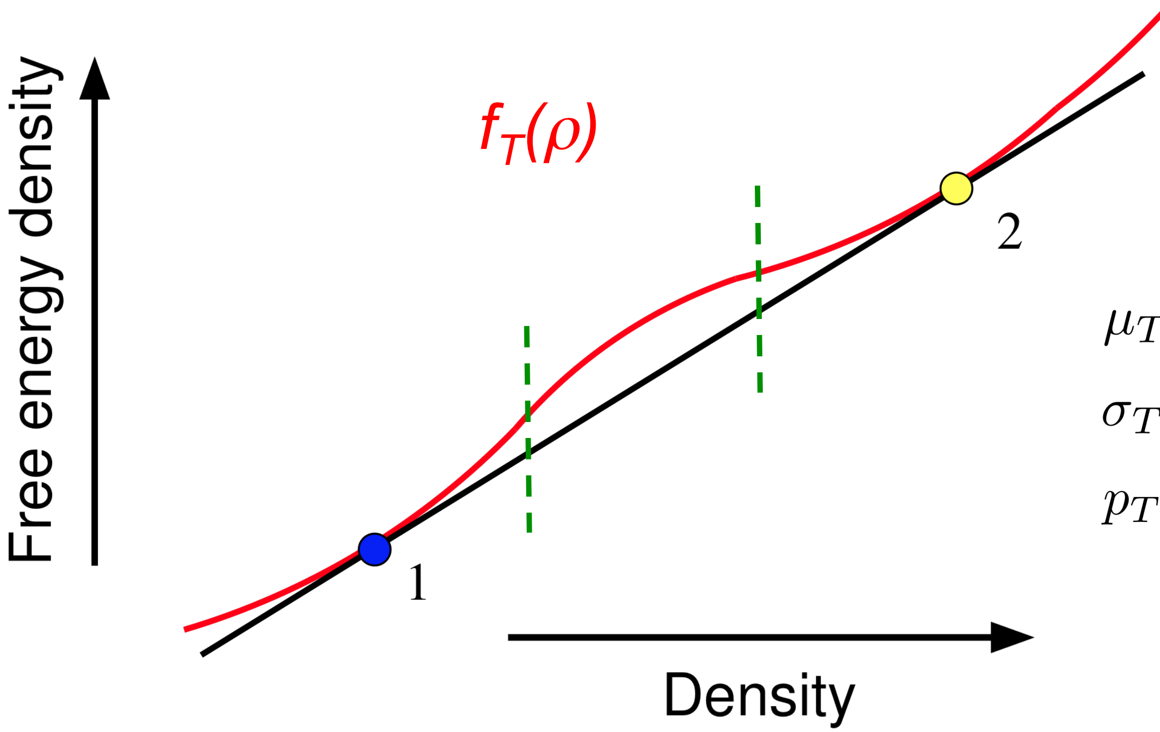


Canonical description: T specified

Free energy density

$$f_T(\rho) \equiv \varepsilon_T(\rho) - T\sigma_T(\rho) = \mu_T(\rho)\rho - p_T(\rho)$$

Phase coexistence ⇔ common tangent:

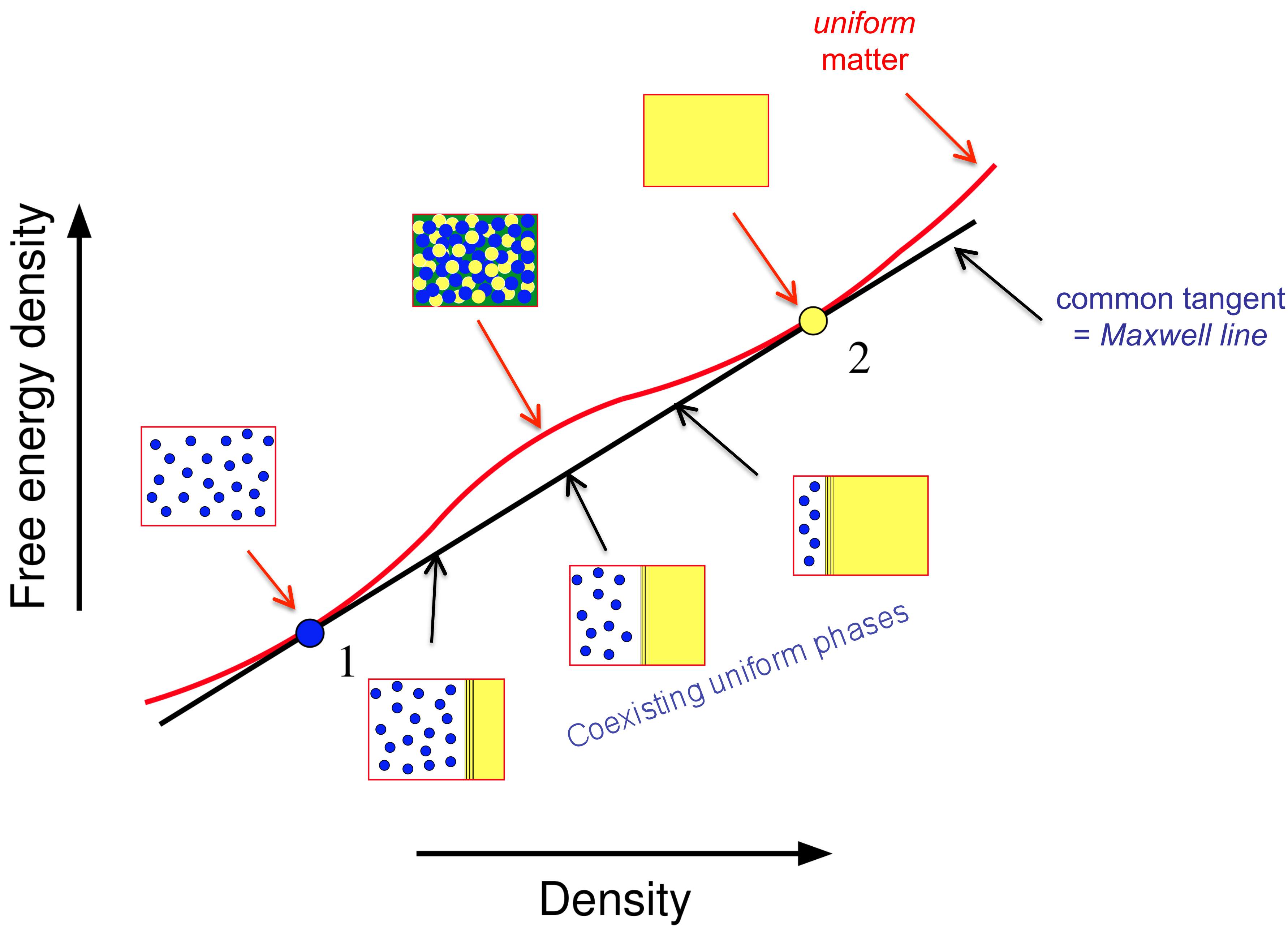


$$\mu_T(\rho) = \partial_\rho f_T(\rho)$$

$$\sigma_T(\rho) = -\partial_T f_T(\rho)$$

$$p_T(\rho) = \mu_T(\rho)\rho - f_T(\rho)$$

$$= \rho^2 \partial_\rho (f_T(\rho)/\rho)$$



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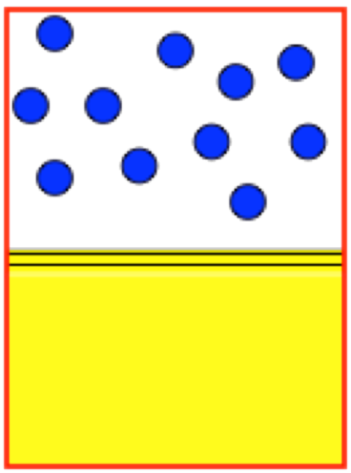
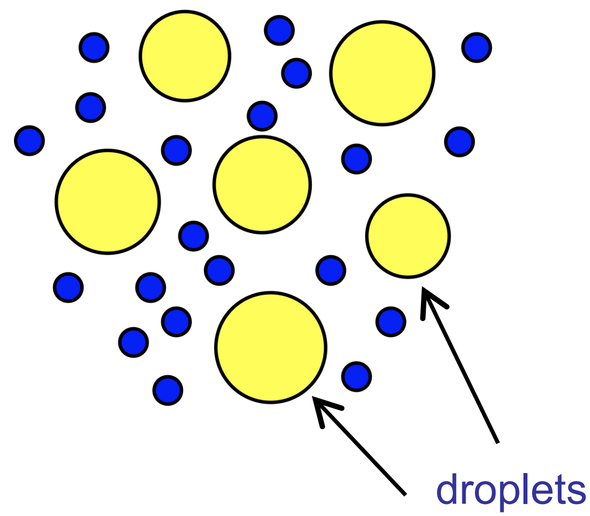
Liquid-gas phase coexistence

uniform gas phase



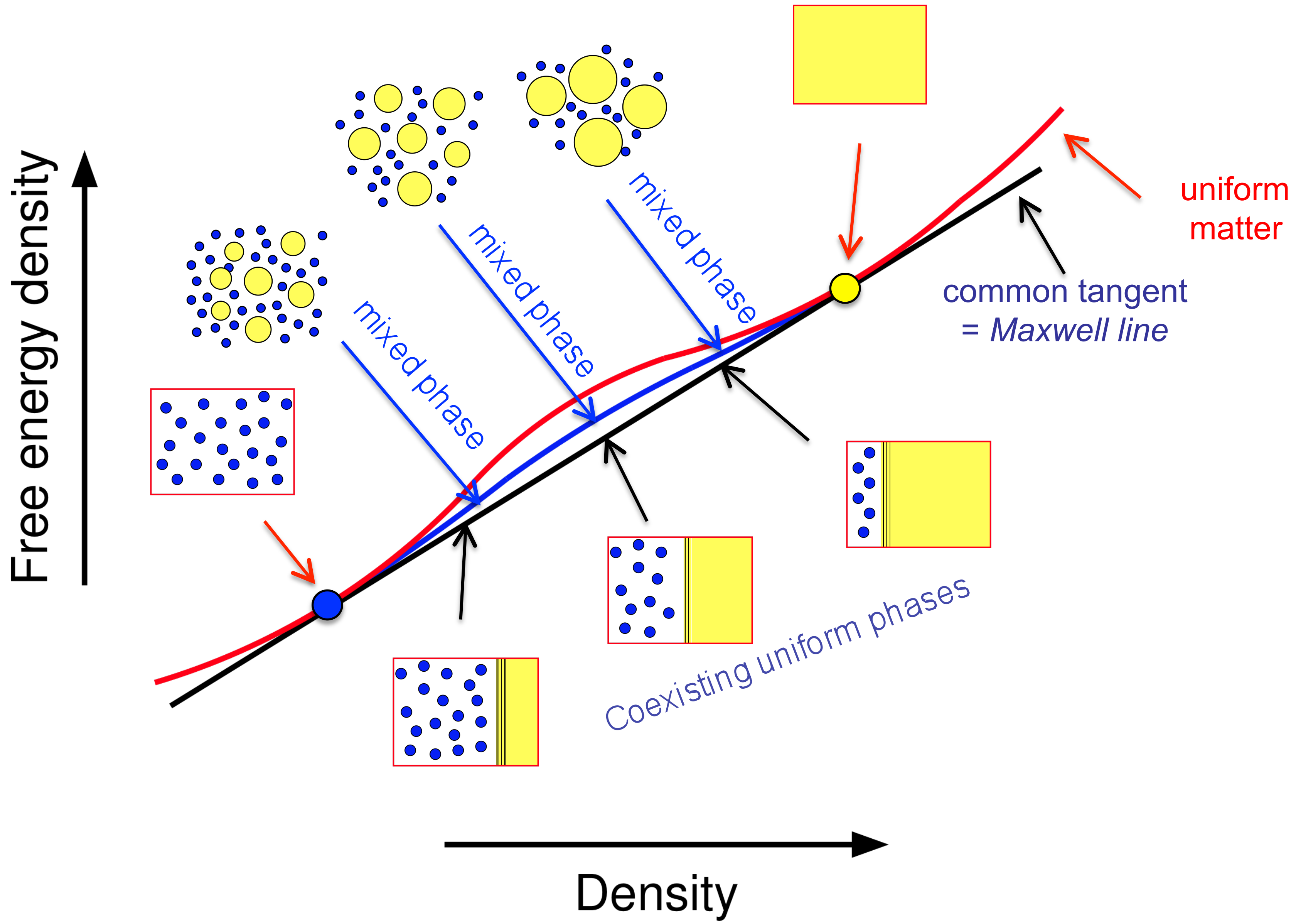
uniform liquid phase

≠



can *coexist* in mutual equilibrium

phase mixture



JRandrup: Dubna School, 2010

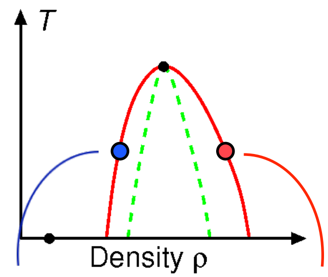
Thermodynamics of non-uniform matter

Consider two coexisting phases of bulk matter:

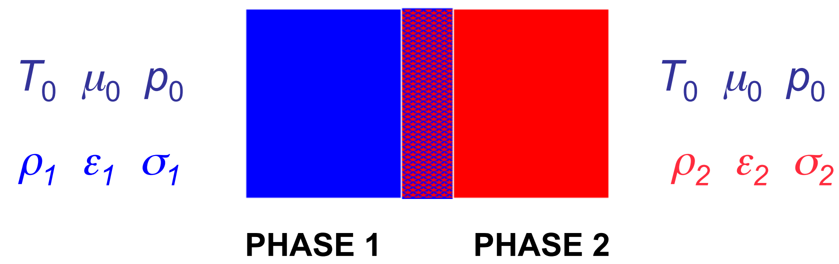
Same temperature T_0

Same chemical potential μ_0

Same pressure p_0

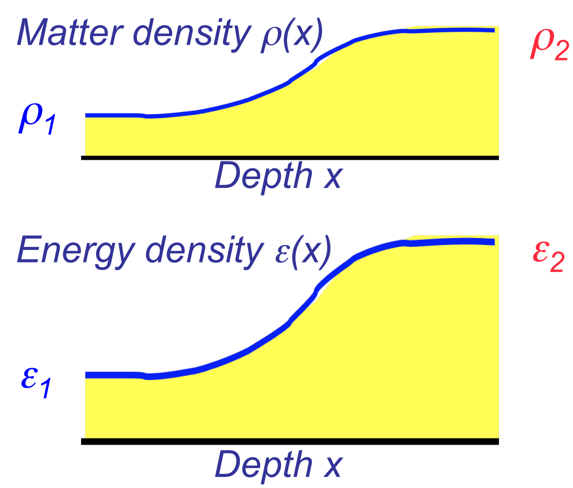


Place them in contact with a planar interface:



A diffuse interface will then develop:

The density profiles change smoothly through the interface region from one coexistence value to the other coexistence value:

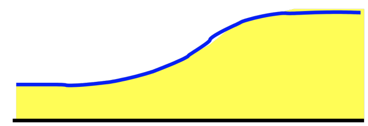


Thermodynamics of non-uniform matter: microcanonical

Non-uniform charge density $\tilde{\rho}(\boldsymbol{r})$

Non-uniform energy density $\tilde{\varepsilon}(\boldsymbol{r})$

Non-uniform entropy density $\tilde{\sigma}[\tilde{\rho}(\boldsymbol{r}),\tilde{\varepsilon}(\boldsymbol{r})](\boldsymbol{r})$



$$N = \int \tilde{\rho}(\boldsymbol{r})d\boldsymbol{r}$$
$$E = \int \tilde{\varepsilon}(\boldsymbol{r})d\boldsymbol{r}$$
$$S = \int \tilde{\sigma}(\boldsymbol{r})d\boldsymbol{r}$$

$$\delta S = \int [\tilde{\beta}(\boldsymbol{r})\delta\tilde{\varepsilon}(\boldsymbol{r}) + \tilde{\alpha}(\boldsymbol{r})\delta\tilde{\rho}(\boldsymbol{r})]d\boldsymbol{r}$$

$$\begin{cases} \tilde{T}(\boldsymbol{r}) = 1/\tilde{\beta}(\boldsymbol{r}) \\ \tilde{\mu}(\boldsymbol{r}) = -\tilde{\alpha}(\boldsymbol{r})\tilde{T}(\boldsymbol{r}) \end{cases}$$

$$\forall\,\delta\tilde{\varepsilon}(\boldsymbol{r}), \forall\,\delta\tilde{\rho}(\boldsymbol{r}):\; 0 \doteq \delta S - \beta_0\delta E - \alpha_0\delta N = \int [(\underbrace{\tilde{\beta}(\boldsymbol{r}) - \beta_0}_{})\delta\tilde{\varepsilon}(\boldsymbol{r}) + (\underbrace{\tilde{\alpha}(\boldsymbol{r}) - \alpha_0}_{})\delta\tilde{\rho}(\boldsymbol{r})]d\boldsymbol{r}$$

Constant temperature: $\forall\,\boldsymbol{r}:\; \tilde{\beta}(\boldsymbol{r}) \doteq \beta_0 \;\Rightarrow\; \boldsymbol{\nabla}\tilde{\beta} \doteq \boldsymbol{0}$

Constant chemical potential: $\forall\,\boldsymbol{r}:\; \tilde{\alpha}(\boldsymbol{r}) \doteq \alpha_0 \;\Rightarrow\; \boldsymbol{\nabla}\tilde{\alpha} \doteq \boldsymbol{0}$

Constant pressure:

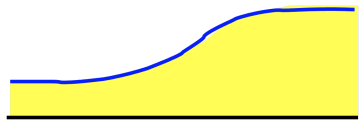
$$\delta\pi = -\varepsilon\delta\beta - \rho\delta\alpha \qquad \pi \equiv p/T = \sigma - \beta\varepsilon - \alpha\rho$$

$$\boldsymbol{\nabla}\tilde{\pi} = -\tilde{\varepsilon}\boldsymbol{\nabla}\tilde{\beta} - \tilde{\rho}\boldsymbol{\nabla}\tilde{\alpha}$$ => $$\tilde{p}(\boldsymbol{r}) = p_0$$

Thermodynamics of non-uniform matter: canonical

Constant temperature *T*

Non-uniform charge density $\tilde{\rho}(\boldsymbol{r})$



Non-uniform free-energy density $\tilde{f}_T[\tilde{\rho}(\boldsymbol{r})](\boldsymbol{r})$

$$N = \int \tilde{\rho}(\boldsymbol{r})d\boldsymbol{r}$$

$$F_T = \int \tilde{f}_T(\boldsymbol{r})d\boldsymbol{r}$$

$$\delta F_T = \int \delta\tilde{f}_T(\boldsymbol{r})d\boldsymbol{r} = \int \tilde{\mu}_T(\boldsymbol{r})\delta\tilde{\rho}(\boldsymbol{r})d\boldsymbol{r}$$

$$\forall\, \delta\tilde{\rho}(\boldsymbol{r}) :\ 0 \doteq \delta F_T - \mu_0\delta N = \int (\underbrace{\tilde{\mu}_T(\boldsymbol{r}) - \mu_0}) \delta\tilde{\rho}(\boldsymbol{r})d\boldsymbol{r}$$

Constant chemical potential: $\forall\, \boldsymbol{r} :\ \tilde{\mu}_T(\boldsymbol{r}) \doteq \mu_0 \ \Rightarrow\ \boldsymbol{\nabla}\tilde{\mu}_T(\boldsymbol{r}) \doteq \boldsymbol{0}$

Constant pressure:

$$\delta p = \rho\delta\mu$$

$$\boldsymbol{\nabla}\tilde{p}_T(\boldsymbol{r}) = -\tilde{\rho}(\boldsymbol{r})\boldsymbol{\nabla}\tilde{\mu}_T(\boldsymbol{r})$$ => $$\tilde{p}_T(\boldsymbol{r}) = p_0$$

J. Randrup, Phys. Rev. C 79, 054911 (2009)

H. Heiselberg *et al.*, Phys. Rev. Lett. 70, 1355 (1993)

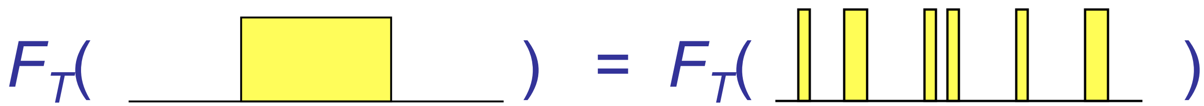
Finite range: gradient term

Free energy density for *uniform* matter at temperature *T*: $f_T(\rho)$

But we need to treat *non*-uniform systems: $\tilde{\rho}(\boldsymbol{r})$

Local density approximation: $\tilde{f}_T[\tilde{\rho}(\boldsymbol{r})](\boldsymbol{r}) \approx f_T(\tilde{\rho}(\boldsymbol{r}))$

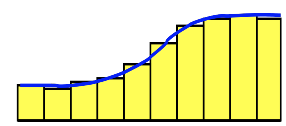
… implies:



No good! => Finite range *must* be taken into account

$$\tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \tfrac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$$

=>

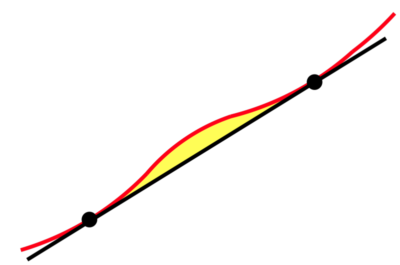


Interface profile

$$0 \doteq \delta F_T - \mu_0 \delta N = \delta \int \left[f_T(\tilde\rho(\boldsymbol{r})) + \tfrac{1}{2} C (\boldsymbol{\nabla} \tilde\rho(\boldsymbol{r}))^2 - \mu_0 \tilde\rho(\boldsymbol{r}) \right] d\boldsymbol{r}$$

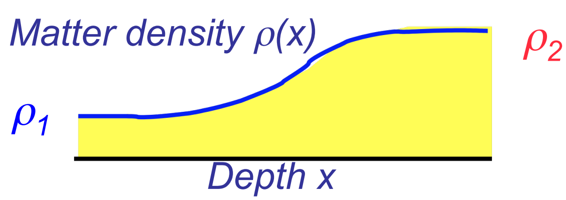
$$= \int \left[\underbrace{\partial_\rho f_T(\tilde\rho(\boldsymbol{r})) - C \boldsymbol{\Delta} \tilde\rho(\boldsymbol{r})} - \mu_0 \right] \delta\tilde\rho(\boldsymbol{r}) d\boldsymbol{r}$$

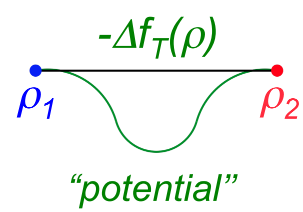
=> $$\tilde\mu_T(\boldsymbol{r}) = \partial_\rho f_T(\tilde\rho(\boldsymbol{r})) - C \boldsymbol{\Delta} \tilde\rho(\boldsymbol{r}) = \mu_T(\tilde\rho(\boldsymbol{r})) - C \boldsymbol{\Delta} \tilde\rho(\boldsymbol{r})$$



$$\tilde\mu_T(\boldsymbol{r}) \doteq \mu_0 \quad \Rightarrow \quad \boxed{C \partial_x^2 \tilde\rho(x) = \mu_T(\tilde\rho(x)) - \mu_0 = -\partial_\rho \Delta f_T(\tilde\rho(x))}$$

$$\Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho)$$

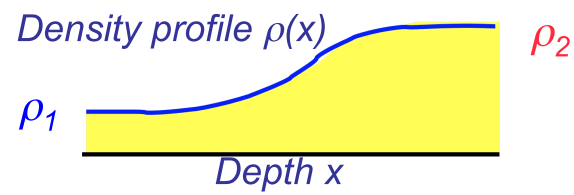




$$\Rightarrow \quad \tfrac{1}{2} C (\partial_x \tilde\rho(x))^2 = \Delta f_T(\tilde\rho(x))$$

"energy conservation"

Interface tension



$$\tilde{f}_T(x) = f_T(\tilde{\rho}(x)) + \tfrac{1}{2}C(\partial_x\tilde{\rho}(x))^2$$

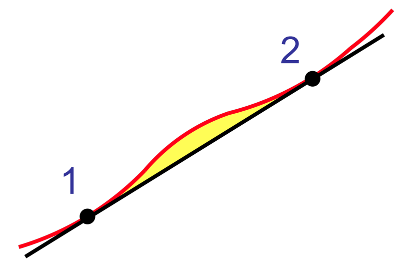
$$C\partial_x^2\tilde{\rho}(x) = \mu_T(\tilde{\rho}(x)) - \mu_0 = -\partial_\rho\Delta f_T(\tilde{\rho}(x))$$

The diffuse interface adds free energy (relative to a sharp interface);
The additional free energy (per unit area) is the *interface tension:*

$$\gamma_T^{12} = \int \tilde{f}_T^{12}(x)dx$$

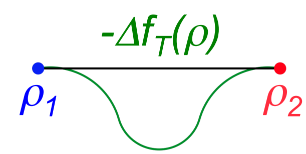
$$\tilde{f}_T^{12}(x) = \tilde{f}_T(x) - f_i - \frac{f_2 - f_1}{\rho_2 - \rho_1}(\tilde{\rho}(x) - \rho_i) = \tilde{f}_T(x) - f_T^M(\tilde{\rho}(x))$$

$$= f_T(\tilde{\rho}(x)) + \tfrac{1}{2}C(\partial_x\tilde{\rho}(x))^2 - f_T^M(\tilde{\rho}(x)) = 2\Delta f_T(\tilde{\rho}(x))$$



$$\Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho)$$

$$\gamma_T^{12} = \int_{-\infty}^{+\infty} 2\Delta f_T(\tilde{\rho}(x))dx = \int_{\rho_1}^{\rho_2} \left[2C\Delta f_T(\rho)\right]^{1/2} d\rho$$



*The interface profile is **not** needed,*
only the EoS for uniform matter!

$$dx = d\rho/\partial_x\rho$$

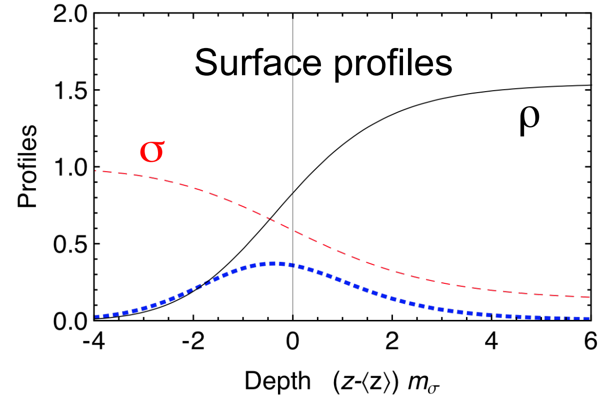
$$\Rightarrow \quad \tfrac{1}{2}C(\partial_x\tilde{\rho}(x))^2 = \Delta f_T(\tilde{\rho}(x))$$

Two recent examples:

Marcus B. Pinto, V. Koch, and JR: Phys. Rev. C 86, 025203 (2012):
Surface tension of quark matter in a geometric approach

Linear σ model:

$$\mathcal{L}_{\mathrm{LSM}} = \tfrac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 + \tfrac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma, \boldsymbol{\pi}) + \bar{\psi}[\mathrm{i}\gamma^\mu \partial_\mu - g(\sigma + \mathrm{i}\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})]\psi$$



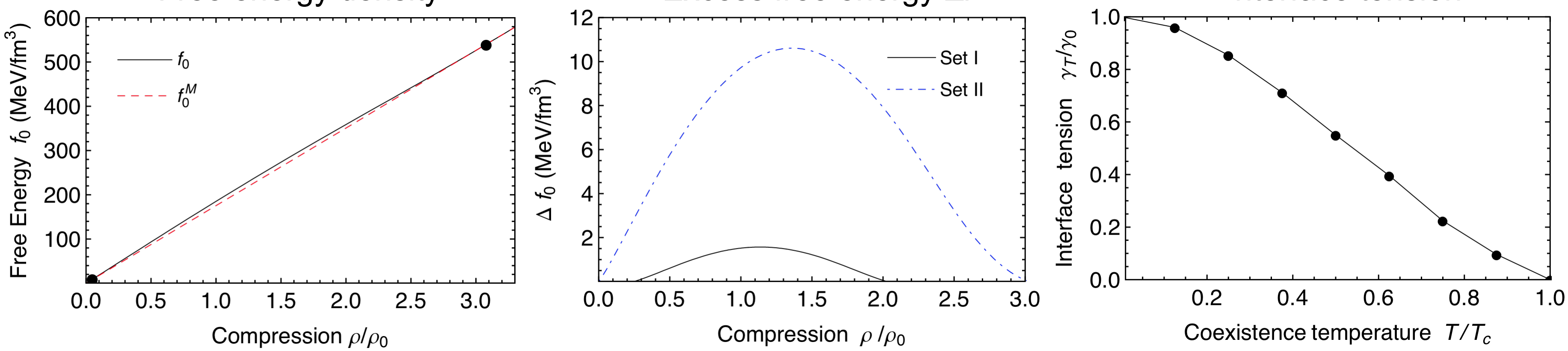
Nambu-Jona-Lasino model:

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2]$$

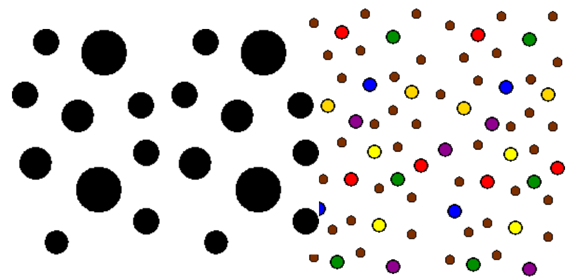
Free energy density

Excess free energy *Δf*

Interface tension



Interface tension between hadron gas and QGP



* Lecture II

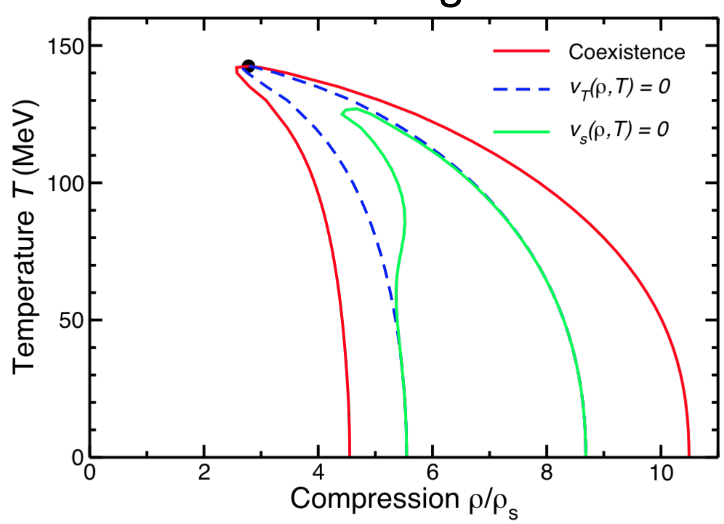
Construct* Equation of State
(spline between HG & QGP)

$$C = a^2 \frac{\varepsilon_s}{\rho_s^2}$$

Add gradient term in free energy density:

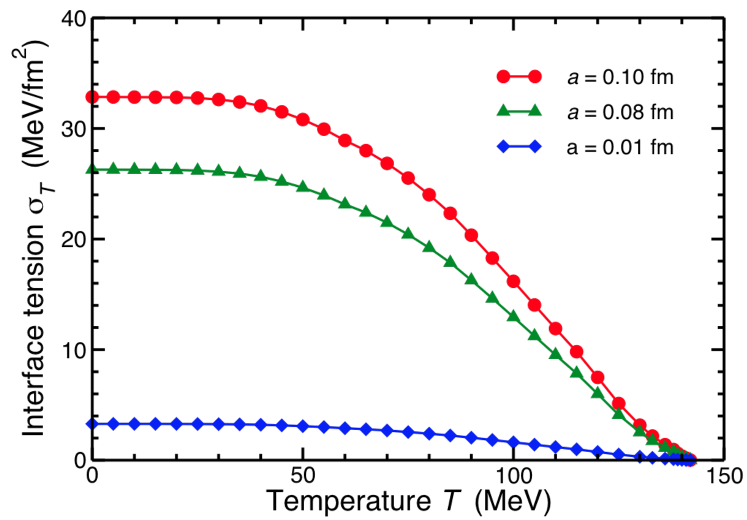
$$\tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$$

Phase diagram



JR: PRC 82 (2010) 034902

Interface tension



Jan Steinheimer & JR (2012)