

Electromagnetic Probes in Heavy-Ion Collisions I

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- 3 QCD and Chiral Symmetry
 - Chiral Symmetry
 - Chiral Symmetry and Hadron Phenomenology
 - Strongly interacting matter: QCD/hadronic models at finite T, μ
- 4 Fundamental theoretical tools
 - The McLerran-Toimela formula
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Plan of the Lectures

- Lecture I: Fundamentals
 - QCD, chiral symmetry, and electromagnetic probes
 - basic phenomenology of dilepton signals
 - model independent approach: QCD sum rules
 - literature: [DGH92, FHK⁺11, RW00, RWH09]
- Lecture II: Effective hadronic models
 - (effective) Lagrangians for hadrons; QED of hadrons
 - Transport model GiBUU at SIS energies (pp, pn, pA)
 - literature: [GS68, GS68, BGG⁺12, WHM12]
- Lecture III: Applications from SIS to RHIC energies
 - partonic and hadronic many-body theory
 - thermal/hydro evolution of the medium
 - understanding the dileptons at SPS, and RHIC
 - literature: [RW00, RWH09, HR06, HR08]
- Homepage of these lectures:
<http://fias.uni-frankfurt.de/~hees/dm12/>

Why Electromagnetic Probes?

- γ, l^\pm : only e. m. interactions
- reflect whole “history” of collision
- chance to see chiral symm. rest. directly?

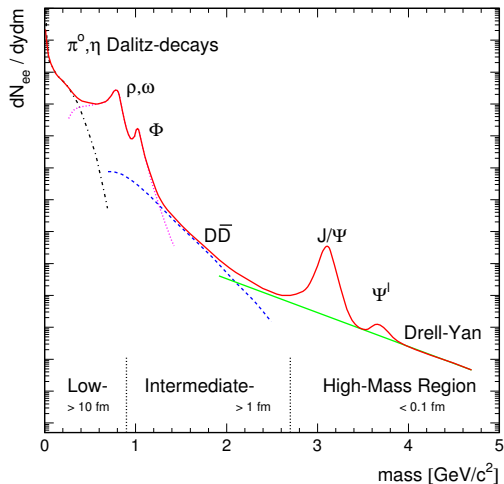
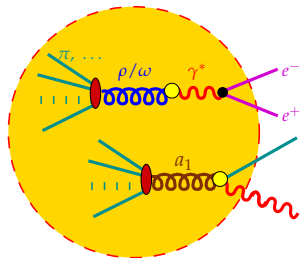
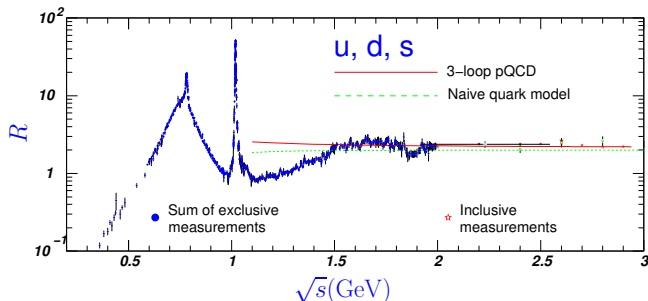


Fig. by A. Drees (from [RW00])

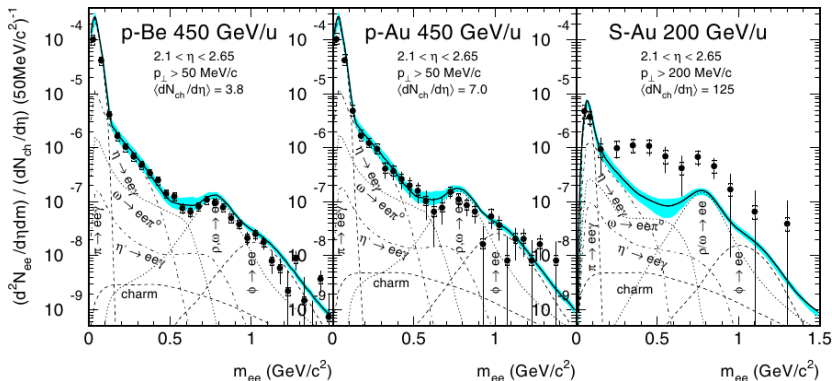
Vacuum Baseline: $e^+e^- \rightarrow \text{hadrons}$



$$R := \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}$$

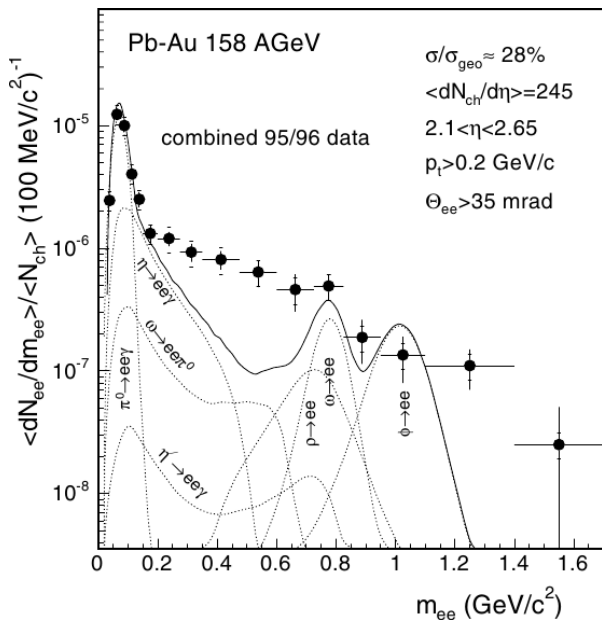
- probes all hadrons with quantum numbers of γ^*
- $R_{\text{QM}} = N_c \sum_{f=u,d,s} Q_f^2 = 3 \times [(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2$
- **Our aim** $pp \rightarrow \ell^+\ell^-$, $pA \rightarrow \ell^+\ell^-$, $AA \rightarrow \ell^+\ell^-$ ($\ell = e, \mu$)
- more to come in Lecture II!

The CERES findings: Dilepton enhancement



- pp (pBe): “elementary reactions”; baseline (mandatory to understand first!)
- pA: “cold nuclear matter effects”; next step (important as baseline for other observables like “ J/ψ suppression”)
- AA: “medium effects”; hope to learn something about **in-medium properties of vector mesons, fundamental QCD properties**

The CERES findings: Dilepton enhancement



QCD and (“accidental”) Symmetries

- Theory for strong interactions: QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \bar{\psi}(i\not{D} - \hat{M})\psi$$

- Particle content:

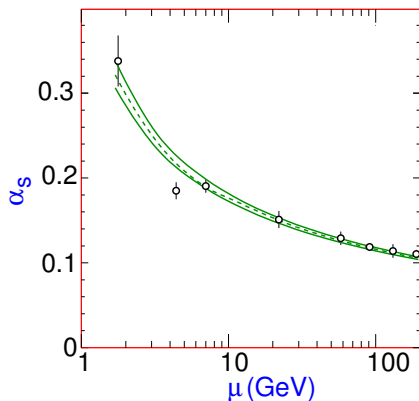
- ψ : Quarks, including flavor- and color degrees of freedom,
 $\hat{M} = \text{diag}(m_u, m_d, m_s, \dots) =$ current quark masses
- A_{μ}^a : gluons, gauge bosons of $\text{SU}(3)_{\text{color}}$

- Symmetries

- fundamental building block: local $\text{SU}(3)_{\text{color}}$ symmetry
- in light-quark sector: approximate chiral symmetry ($\hat{M} \rightarrow 0$)
- dilation symmetry (scale invariance for $\hat{M} \rightarrow 0$)

Features of QCD

- asymptotically free: at **large** momentum transfers $\alpha_s \rightarrow 0$
- running from renormalization group:
Nobel prize 2004 for Gross, Wilczek, Politzer



- quarks and gluons **confined in hadrons**
- theoretically not fully understood (nonperturbative phenomenon!)
- need of **effective hadronic models** at low energies: (Chiral) symmetry!

Chiral Symmetry

- Consider only **light** u, d quarks
- **iso-spin 1/2 doublet**: $\psi = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- NB: ψ has three “indices”: Dirac spinor, color, flavor iso-spin!
- γ matrices: $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\mathbb{1}$, $\gamma_5 := i\gamma_0\gamma_1\gamma_2\gamma_3$, $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$, $\gamma_5^\dagger = \gamma_5$, $\gamma_5^2 = \mathbb{1}$
- Diracology of **left and right-handed components**

$$\psi_L = \frac{\mathbb{1} - \gamma_5}{2}\psi = P_L\psi, \quad \psi_R = \frac{\mathbb{1} + \gamma_5}{2}\psi = P_R\psi,$$

$$P_R^2 = P_L^2 = \mathbb{1}, \quad P_R P_L = P_L P_R = 0, \quad P_{L/R}\gamma_5 = \gamma_5 P_{L/R} = -P_{L/R}$$

$$P_{L/R}\gamma_\mu = \gamma_\mu P_{R/L}, \quad \overline{P_L\psi} = \overline{\psi}P_R, \quad \overline{P_R\psi} = \overline{\psi}P_L$$

$$\overline{\psi}\gamma_\mu\psi = \overline{\psi}_L\gamma_\mu\psi_L + \overline{\psi}_R\gamma_\mu\psi_R, \quad \overline{\psi}\psi = \overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L$$

- $\overline{\psi} := \psi^\dagger\gamma_0$, $\overline{\gamma_5\psi} = \psi^\dagger\gamma_5^\dagger\gamma_0 = -\overline{\psi}\gamma_5$
- in the massless limit ($m_u = m_d = 0$)

$$\mathcal{L}_{u,d} = \overline{\psi}i\not{D}\psi = \overline{\psi}_L i\not{D}\psi_L + \overline{\psi}_R i\not{D}\psi_R$$

Chiral Symmetry

- in the massless limit ($m_u = m_d = 0$)
- a lot of global **chiral symmetries**:
 - change of **independent** phases for **left** and **right** components:

$$\psi_L(x) \rightarrow \exp(i\phi_L)\psi_L(x), \quad \psi_R(x) \rightarrow \exp(i\phi_R)\psi_R(x)$$

- symmetry group $U(1)_L \times U(1)_R$
- independent “iso-spin rotations”

$$\psi_L(x) \rightarrow \exp(i\vec{\alpha}_L \cdot \vec{T})\psi_L(x), \quad \psi_R(x) \rightarrow \exp(i\vec{\alpha}_R \cdot \vec{T})\psi_R(x)$$

- $\vec{T} = \vec{\tau}/2$, $\vec{\tau}$: **Pauli matrices**; symmetry group $SU(2)_L \times SU(2)_R$
- alternative notation scalar-pseudoscalar phases/iso-spin rotations

$$\psi \rightarrow \exp(i\phi_s)\psi, \quad \psi \rightarrow \exp(i\gamma_5\phi_a)\psi$$

$$\psi \rightarrow \exp(i\vec{\alpha}_V \cdot \vec{T})\psi, \quad \psi \rightarrow \exp(i\gamma_5\vec{\alpha}_A \cdot \vec{T})\psi$$

- **Caveat**: Symmetry group is **NOT** $U(1)_s \times U(1)_a$ or $SU(2)_V \times SU(2)_A$
- $U(1)_s$ and $SU(2)_V$ **are subgroups** that are **symmetries** even if $m_u = m_d \neq 0 \Rightarrow$ Heisenberg’s iso-spin symmetry!

Currents: relation to mesons

- based on [Koc97, Sch03, Din11]
- Noether: each global symmetry leads to a **conserved current** ($\partial_\mu j^\mu = 0$)
- from **chiral symmetries**

$$j_s^\mu = \bar{\psi}\gamma^\mu\psi, \quad j_a^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$
$$\vec{j}_V^\mu = \bar{\psi}\gamma^\mu\vec{T}\psi, \quad \vec{j}_A^\mu = \bar{\psi}\gamma^\mu\gamma_5\vec{T}\psi$$

- Link to mesons: Build Lorentz-invariant objects with corresponding quantum numbers
 - σ : $\bar{\psi}\psi$ (scalar and iso-scalar)
 - π 's: $i\bar{\psi}\vec{T}\gamma_5\psi$ (pseudoscalar and iso-vector)
 - ρ 's: $\bar{\psi}\gamma_\mu\vec{T}\psi$ (vector and iso-vector)
 - a_1 's: $\bar{\psi}\gamma_\mu\gamma_5\vec{T}\psi$ (axialvector and iso-vector)
- in nature: σ and π 's; ρ 's and a_1 's **do not have same mass!**
- reason: QCD ground state **not symmetric** under pseudoscalar and pseudovector trasfos since $\langle \text{vac} | \bar{\psi}\psi | \text{vac} \rangle \neq 0$

E.m. Current: relation to mesons

- $Q = t_3 + Y/2$, t_3 : iso-spin-3-comp, $Y = s + c + b + t + B$
 - quarks: $t_{3u} = -t_{3d} = 1/2$, $t_{3c} = t_{3s} = t_{3t} = t_{3b} = 0$,
 $Y_u = Y_d = 1/3$, $Y_c = Y_t = 4/3$, $Y_s = Y_b = -2/3$,
 $B_f = 1/3$, $Q_u = Q_c = Q_t = 2/3$, $Q_d = Q_s = Q_b = -1/3$
- electromagnetic current of quarks (including sum over 3 colors!)

$$J_{\text{em}}^\mu = \sum_f \bar{\psi}_f (\hat{T}_3 + \hat{Y}/2) \psi_f = \sum_f Q_f \bar{\psi}_f \gamma^\mu \psi_f = \frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} (\bar{d} \gamma^\mu d + \bar{s} \gamma^\mu s)$$

- split into flavor-iso-spin states:

$$\omega \quad (T = 0) : j_{\text{em}\omega}^\mu = 1/6 (\bar{u} \gamma^\mu u + \bar{d} \gamma^\mu d)$$

$$\phi \quad (T = 0) : j_{\text{em}\phi}^\mu = -1/3 \bar{s} \gamma^\mu s$$

$$\rho^0 \quad (T = 1) : j_{\text{em}\rho}^\mu = 1/2 (\bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d)$$

- expressed in **normalized** hadronic basis

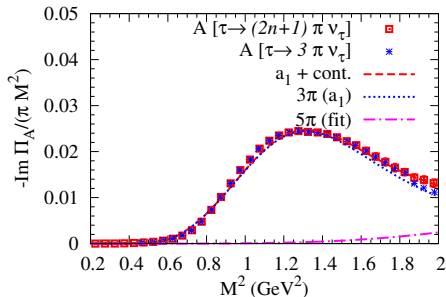
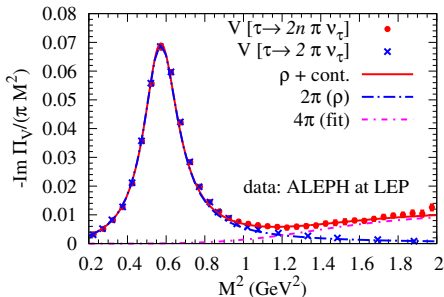
$$j_{\text{em}}^\mu = \frac{1}{\sqrt{2}} \left[\frac{\bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d}{\sqrt{2}} + \frac{1}{3} \frac{\bar{u} \gamma^\mu u + \bar{d} \gamma^\mu d}{\sqrt{2}} - \frac{\sqrt{2}}{3} \bar{s} \gamma^\mu s \right]$$

Spontaneous symmetry breaking

- **spontaneously broken symmetry**: ground state not symmetric
- vacuum necessarily **degenerate**
- vacuum invariant under scalar and vector transformations:
 $U(1)_L \times U(1)_R$ broken to $U(1)_s$; $SU(2)_L \times SU(2)_R$ broken to $SU(2)_V$
- for each broken symmetry **massless scalar Goldstone boson**
- there are three pions which are very light compared to other hadrons (finite masses due to **explicit** breaking through m_u, m_d !)
- **but no pseudoscalar isoscalar light particle!** ($m_\eta \simeq 548$ MeV)
- **reason: $U(1)_a$ anomaly**
 - pseudoscalar $U(1)_a$ symmetry doesn't survive quantization!
 - good for explanation of correct decay rate for $\pi_0 \rightarrow \gamma\gamma$
 - axialscalar current not conserved $\partial_\mu j_a^{\mu 5} = 3/8\alpha_s \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$
- explicit breaking due to quark masses
 - can be treated perturbatively \Rightarrow **chiral perturbation theory**
 - axial-vector current only approximately conserved \Rightarrow **PCAC**
 - a lot of low-energy properties of hadrons derivable

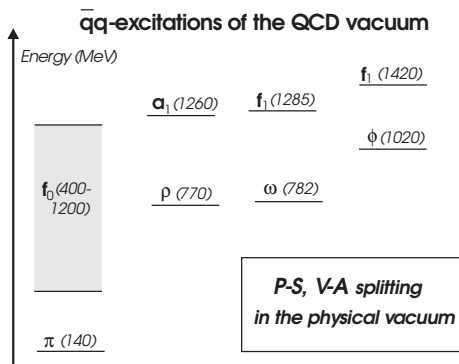
Most accurate experiment related to χ SB

- weak decay $\tau \rightarrow \nu + n \cdot \pi$
- weak interactions: **currents** $\propto j_V^\mu - j_A^\mu$
 - ew. sector in standard model: gauged+Higgsed chiral model $SU(2)_L \times U(1)_Y$
 - **no anomaly** in gauge symmetry due to particle content!
- n even: must go through **vector** current
 n odd: must go through **axialvector** current



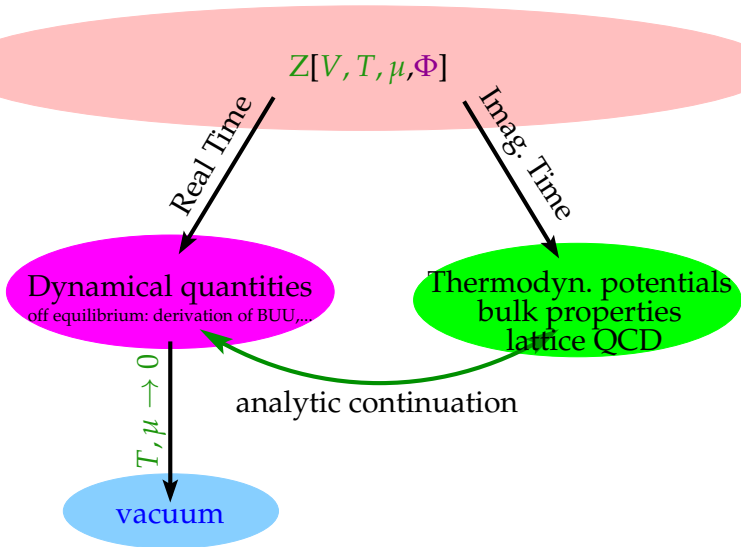
Phenomenology from Chiral Symmetry

- Use (approximate) **chiral symmetry** to build effective models
- **Ward identities**
 - PCAC: $\langle 0 | \partial^\mu J_{A\mu}^k | \pi^j(\vec{k}) \rangle = iF_\pi^2 m_\pi^2 \delta^{kj}$
 - $m_\pi^2 F_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u | 0 \rangle$
(Gell-Mann-Oakes-Renner relation)
- Spontaneous breaking causes splitting of chiral partners:



Finite Temperature/Density: Idealized theory picture

- partition sum: $Z(V, T, \mu_q, \Phi) = \text{Tr}\{\exp[-(\mathbf{H}[\Phi] - \mu_q \mathbf{N})/T]\}$



Finite Temperature

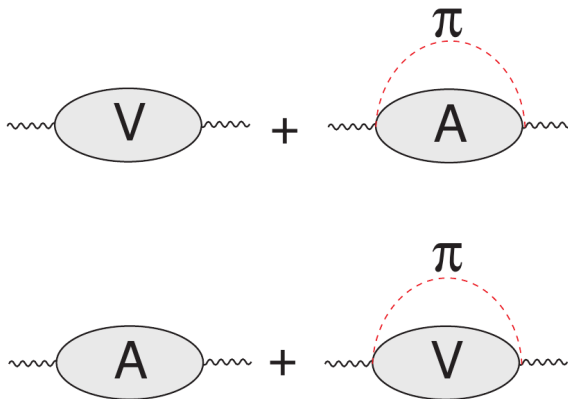
- Asymptotic freedom
 - **quark condensate melts** at high enough **temperatures/densities**
- all bulk properties from **partition sum**:

$$Z(V, T, \mu_q) = \text{Tr}\{\exp[-(\mathbf{H} - \mu_q \mathbf{N})/T]\}$$

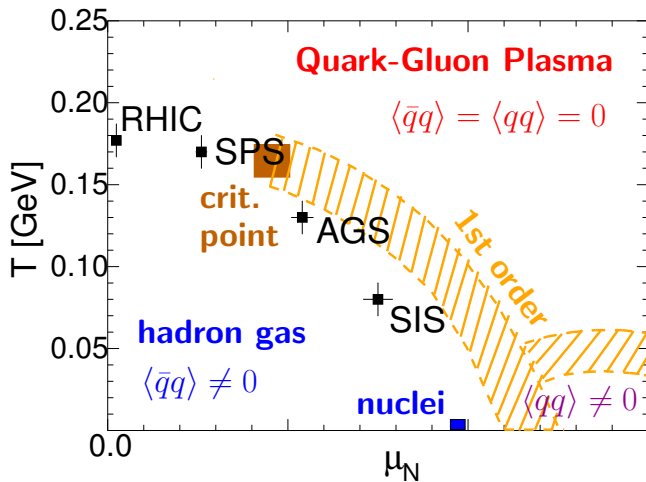
- Free energy: $\Omega = -\frac{T}{V} \ln Z = -P$
- **Quark condensate**: $\langle \bar{\psi}_q \psi_q \rangle_{T, \mu_q} = \frac{V}{T} \frac{\partial P}{\partial m_q}$
- Lattice QCD (at $\mu_q = 0$)
 - **chiral symmetry** $\Leftrightarrow \langle \bar{\psi} \psi \rangle$
 - **deconfinement transition** \Leftrightarrow Polyakov Loop $\text{tr} \left\langle P \exp(i \int_0^\beta d\tau A^0) \right\rangle$
 - **Chiral symmetry restoration** and **deconfinement transition** at same T_c

Vector-Axialvector Mixing in the Medium

- **in the medium**: vector-axialvector currents mix
- due to **thermal pions**
- possible mechanism for χ SR!
- in low-density / temperature approximation: **model independent**
- see [DEI90a, DEI90b, UBW02, SYZ96, SYZ97]



The QCD Phase Diagram



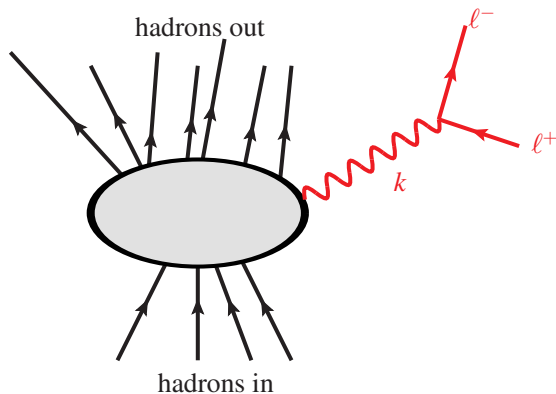
The McLerran-Toimela formula

- radiation of **dileptons** from **thermalized strongly interacting particles**
- **dileptons** escape fireball without any final-state interactions
- calculation exact concerning **strong interactions**
- leading-order $\mathcal{O}(\alpha^2)$ in **QED**

$$\mathbf{H}_{\text{em}} = e \int d^3\vec{x} \mathbf{J}_\mu(t, \vec{x}) A^\mu(t, \vec{x}), \quad A^\mu(t, \vec{x}) = \frac{\epsilon^\mu}{2\omega V} \exp(ik \cdot x)$$

- \mathbf{J}_μ : exact Heisenberg em. current operator of quarks or hadrons
- $e = \sqrt{4\pi\alpha}$, $\alpha \simeq 1/137$

The McLerran-Toimela formula



- Fermi's golden rule \Rightarrow transition-matrix element for process $|i\rangle \rightarrow |f'\rangle = |f\rangle + |e^+e^-(k)\rangle$
- QED Feynman rules

$$S_{f'i} = \left\langle f \left| \int d^4x \mathbf{J}_\mu(x) \right| i \right\rangle D_\gamma^{\mu\nu}(x, x') e\bar{u}_\ell(x') \gamma_\mu v_\ell(x')$$

The McLerran-Toimela formula

- Fourier transformation: energy-momentum conservation
 $|f'\rangle = |f, \ell^+ \ell^-(k)\rangle$

$$S_{fi} = T_{fi} (2\pi)^4 \delta^{(4)}(P_f + k - P_i)$$

- Fermi's trick: Rate

$$R_{f'i} = \frac{|S_{f'i}|^2}{\tau V} = (2\pi)^4 \delta^{(4)}(P_f + k - P_i) |T_{f'i}|^2$$

- summing over $|f\rangle$ and polarizations of **dilepton states**
- averaging over initial hadron states: heat bath (grand canonical)

$$\rho = \frac{1}{Z} \exp[-\beta(\mathbf{H}_{\text{QCD}} - \mu_B \mathbf{Q}_{\text{baryon}})]$$

The McLerran-Toimela formula

- result (derivation see [GK91], Appendices)

$$\frac{dR_{ll}}{d^4k} = -\frac{\alpha^2}{3\pi^3} \frac{k^2 + 2m_\ell^2}{(k^2)^2} \sqrt{1 - \frac{4m_\ell^2}{k^2}} g_{\mu\nu} n_B(k^0) \text{Im} \Pi_{\text{ret}}^{\mu\nu}(k)$$

- **em. current-current correlator**

$$i\Pi_{\text{ret}}^{\mu\nu}(k) := \int d^4x \exp(ik \cdot x) \langle [J^\mu(x), J^\nu(0)] \rangle_{T, \mu_B} \Theta(x^0)$$

- in principle measurable: in **linear response approximation** Green's function for em. current running through medium
- $k^2 = M^2 > 0$ **invariant mass of dilepton**
- probing medium with photons: **same correlator** for $k^2 = M^2 = 0$
- then correlator \Leftrightarrow dielectric function $\epsilon(\omega)$ in electrodynamics!

The McLerran-Toimela formula

- for **real photons**

$$\omega \frac{dR}{d^3\vec{k}} = -\frac{\alpha g_{\mu\nu}}{2\pi^2} \text{Im} \Pi_{\text{ret}}^{\mu\nu}(k), \quad \omega = k^0 = |\vec{k}|$$

- NB: Phenomenological **effective hadronic model**:
vector-meson dominance model
- em. current $\propto V^\mu$ (with $V \in \{\rho, \omega, \phi\}$)

$$\Sigma_{\mu\nu}^\gamma = \text{---} G_\rho \text{---}$$

- Dilepton/photon rates: $\propto A_V = -2 \text{Im} D_V^{(\text{ret})}$
(**vector-meson spectral function!**)
- measuring **in-medium vector-meson** spectral function!?!
- \rightarrow Lecture III

- based on [LPM98]
- calculate current correlator, e.g., the vector part of the **em. current**

$$j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

- corresponds to the **ρ** meson!
- use **pQCD** to determine correlator

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi(k^2)$$

in deep spacelike region, $Q^2 = -k^2 \gg \Lambda_{\text{QCD}}$

- related to **time-like** region \Rightarrow **sum rule**

$$\Pi(k^2) = \Pi(0) + cQ^2 + \frac{Q^4}{\pi} \int_0^\infty ds \frac{\text{Im } \Pi(s)}{s^2(s + Q^2 - i\epsilon)}$$

- dispersion relation: **spectral function** $\text{Im } \Pi!$

- left-hand side of **sum rule**
- pQCD + chiral models for baryon-pion interactions [see, e.g., [DGH92]]

$$R(Q^2) := \frac{\Pi(k^2 = -Q^2)}{Q^2} = -\frac{1}{8\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{Q^4} m_q \langle \bar{q}q \rangle + \frac{1}{24Q^4} \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu} \right\rangle - \frac{112}{81Q^6} \kappa \langle \bar{q}q \rangle^2$$

- additional cold-nuclear matter contributions

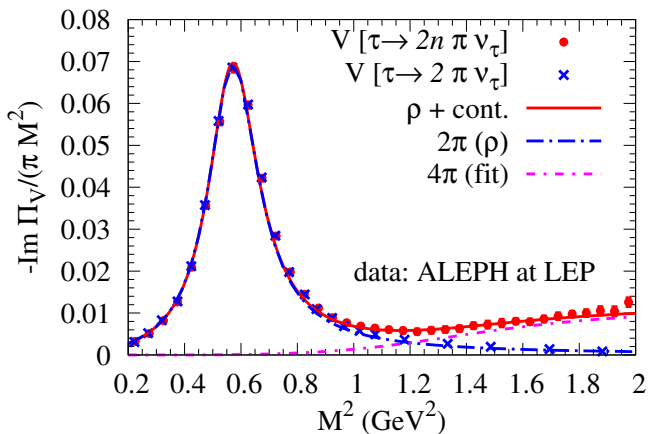
$$\Delta R(Q^2) = \frac{m_N}{4Q^4} A_2 \rho_N - \frac{5m_N^3}{12Q^6} A_4 \rho_N$$

- $A_{2,4}$ from parton-distribution functions
- also condensates corrected

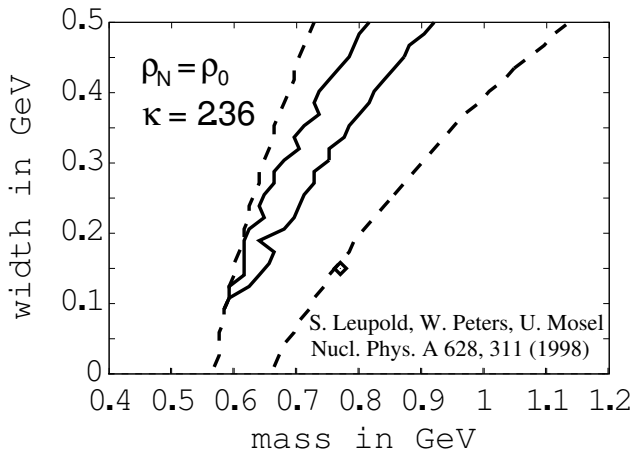
$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_q} \rho_N,$$

$$\left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu} \right\rangle = \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu} \right\rangle_{\text{vac}} - \frac{8}{9} m_N^{(0)} \rho_N$$

- right-hand side of **sum rule**
- use hadronic models to fit measured **vector-current correlator**
- e.g., ALEPH/OPAL data of $\tau \rightarrow \nu + 2n\pi$



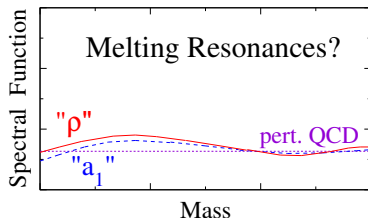
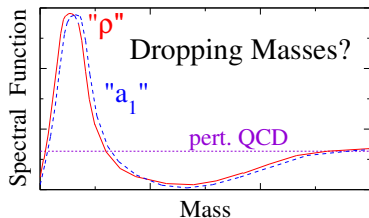
- typical result from [LPM98]



- possible **medium effects** on **ρ meson**
 - dropping mass, unchanged/small width
 - unchanged mass, broadened spectrum (large width)

Scenarios for chiral symmetry restoration

- hadron spectrum must become **degenerate** between chiral partners



- models alone of little help (realization of χS not unique!)
 - “vector manifestation” $\rho_{\text{long}} = \chi$ partner of $\pi \Rightarrow$ dropping mass
 - “standard realization” $\rho = \chi$ partner of a_1 , extreme broadening + little mass shifts
- theory “shopping list”
 - effective hadronic models (well constrained in vacuum!)
 - and concise evaluation in the medium!**
 - models for **fireball evolution**
 - must include partonic \rightarrow phase transition \rightarrow hadronic evolution
- precise l^+l^- (γ) data from HICs mandatory!**

- Motivation for dilepton measurements in HICs
 - leptons are **penetrating probes**
 - **invariant-mass spectra of l^+l^-** undistorted by FSIs
 - give spectral properties of **electromagnetic current correlator**
 - **related to vector-meson spectral function**
 - related to (approximate) **chiral symmetry** \Leftrightarrow **chiral phase transition**
 - **chiral-symmetry restoration** (perhaps) observable!?!)
 - one key observation in HICs:
enhancement of dileptons in low-mass region compared to pp collisions
- QCD and chiral symmetry
 - fundamental symmetry: local **color-gauge symmetry** $SU(3)_c$
 - a lot of “accidental” global symmetries in light-quark sector
 - **chiral** and scaling symmetry in light-quark sector
 - $U(1)_A$ symmetry and scaling symmetry **anomalously broken**
 - axialvector-iso-vector symmetry **spontaneously broken**
 - pions as **Goldstone bosons**
 - slightly **explicitly broken** by light-quark masses

- Fundamental results from theory
 - Dilepton spectrum \Leftrightarrow em. current-correlation function
 - model-independent approach: QCD sum rules
 - relate pQCD + measurable condensates at $Q^2 = -q^2 \gg \Lambda^2$ to measurable spectral functions at $q^2 = s > 0$
 - dropping mass and resonance melting as mechanism for χ SR possible
 - cannot be decided theoretically from first principles
- Need for
 - hadronic effective models in vacuum and in medium
 - evolution models of fireball (quantum transport, QMD, hydrodynamics)
 - high-precision dilepton data

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