Kinetics and Hydrodynamics of Phase Transitions (with application to HIC)

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Plan:

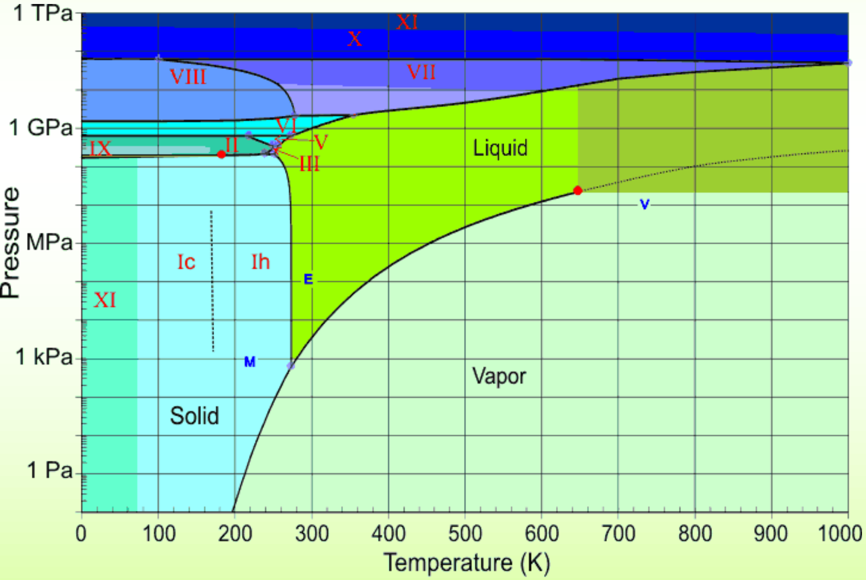
- Phase transitions in different systems including HIC. A general description.
- Mean field description. Relativistic bosons in external fields. II and I order phase transitions. Dynamical description.
- Phenomenological description of II and I order phase transitions in non-relativistic systems. Dynamical description.
- Role of fluctuations of the order parameter and of noise. *Example of color superconducting transition*.
- Manifestation of instabilities in solution of quasiparticle kinetic equation (*on example of Bose-Einstein condensation in pion gas with elastic collisions*) and in solution of Kadanoff-Baym equation in case of finite particle mass-width (*on example of pion condensation in nuclear matter*).
- Back to dynamics of mean field but now for phase transition to the state with finite momentum.
- Hydrodynamical description of I order phase transition of the liquid-gas type (*on example of the hadron-quark I order phase transition*). Demonstration of important role of non-zero viscosity and thermal conductivity.
- (if remains a time) Example of stationary state. Mixed phase vs. Maxwell construction in neutron stars. *Quark-hadron pasta, nuclear pasta, kaon pasta.*

Phase Transition Phenomena

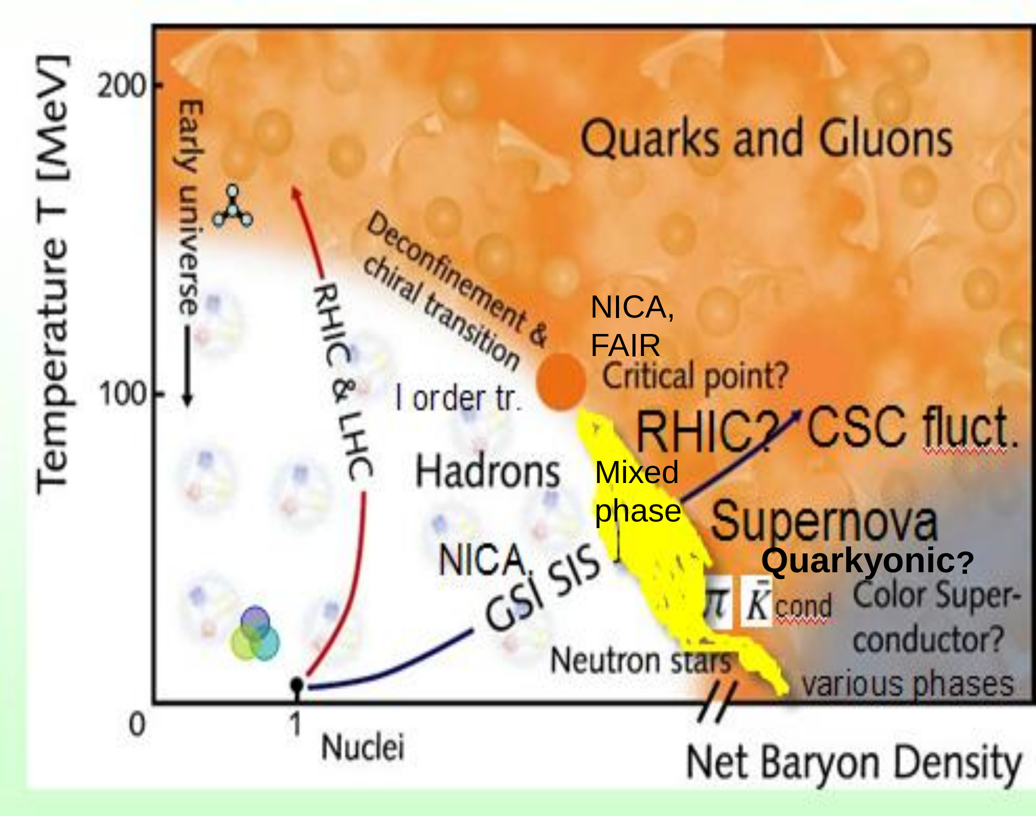
- Condensed matter
- Early Universe
- Supernovas
- Neutron stars
- Heavy ion collisions

Phase Diagrams
Water and Nuclear Matter

Variety of phases: 12 crystalline, 3 glass, liquid, vapor, CEP



Chapline et al. (2007)



Low density,low T: *HIC* (liquid-gas); *excited nuclei* (high spin, pairing); high density, low T: *SN,NS*: (NN-pairing, π,K,ρ-condensates; CSC, quarkyonic); high T *HIC*: (chiral restoration, deconfinement)

A general description of phase-tansition dynamics

Above critical point: Short-range excitations and soft collective modes.

Dyson equations for non-equilibrium Green functions should be considered with inclusion of effects of initial correlations – **should be used but not done.**

Dyson equations for non-equilibrium Green functions with suppression of effects of initial correlations (valid for $t \gg t_{cor} \sim$ fm)– formulated but not solved.

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Quasiclassical approximation for Dyson equations --

generalized Kadanoff-Baym (KB) kinetic equations for virtual particles with widths: (for $t \sim t_{rel} \gg t_{cor}, t_{micro} \sim 1/E_T$) --in use in some codes in an approximation by test particles.

Quasiparticle limit of KB kinetics –in use

Equations of non-ideal hydrodynamics (valid for $t \gg t_{rel}$) –in use

But most often simplified versions are used: Boltzmann-like (perturbative) kinetic equations, ideal hydro with phenomenologically introduced friction, etc.

Below critical point: Short-range excitations and soft collective modes**+ mean fields (order parameters) and their long-range fluctuations.**

Dyson equations for non-equilibrium Green functions **in presence of mean fields with inclusion of long-range fluctuations + equations for mean fields coupled with all fluctuations** with inclusion of effects of initial correlations – not done.

Dyson equations for non-equilibrium Green functions **in presence of mean fields with inclusion of long-range fluctuations + equations for mean fields coupled with all fluctuations** with suppression of effects of initial correlations (for $t \gg t_{cor} \sim fm$)– formulated but not solved.

Quasiclassical (Kadanoff-Baym) approximation for Dyson equations --

generalized KB kinetic equations for virtual particles **in presence of mean fields with inclusion of long-range fluctuations + equations for mean fields coupled with all fluctuations :** (for $t \sim t_{rel} \gg t_{cor}$) -- formulated but not solved.

Quasiparticle limit kinetic equations **in presence of mean fields with inclusion of long-range fluctuations + equations for mean fields coupled with all fluctuations :** (for $t \sim t_{rel} \gg t_{cor}$) -- formulated but not solved.

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Equations of non-ideal hydrodynamics **with mean fields** (for $t \gg t_{rel}$) –in use

Dynamics of mean fields with effects of long-range fluctuations –in use

Dynamics of mean fields without effects of long-range fluctuations –is most extensively used

Mean field description of the phase transition dynamics

II order phase transition for relativistic bosons:

π^0 in external scalar field deep well

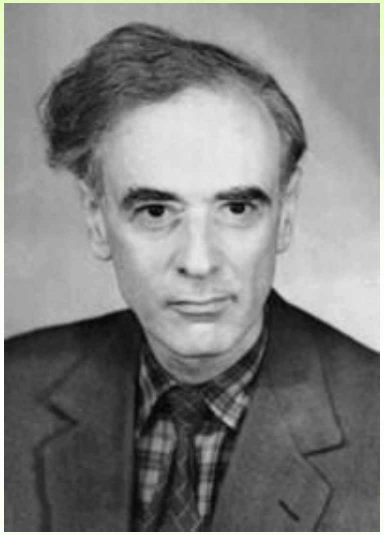
$-U>m^2$

I order phase transition for relativistic bosons:

π^0 in external scalar field U

in presence of a small scalar charge

Landau Phenomenological Description of Phase Transitions



Simplest case: one order parameter, homogeneous matter. Expand free energy in Φ and then coefficients, in $T-T_{cr}$ near critical point:

$$F = Const + \frac{a}{2}\phi^2 + \frac{b}{3}\phi^3 + \frac{c}{4}\phi^4 + h\phi$$

Either cubic or linear term can be eliminated by the shift of the order parameter.

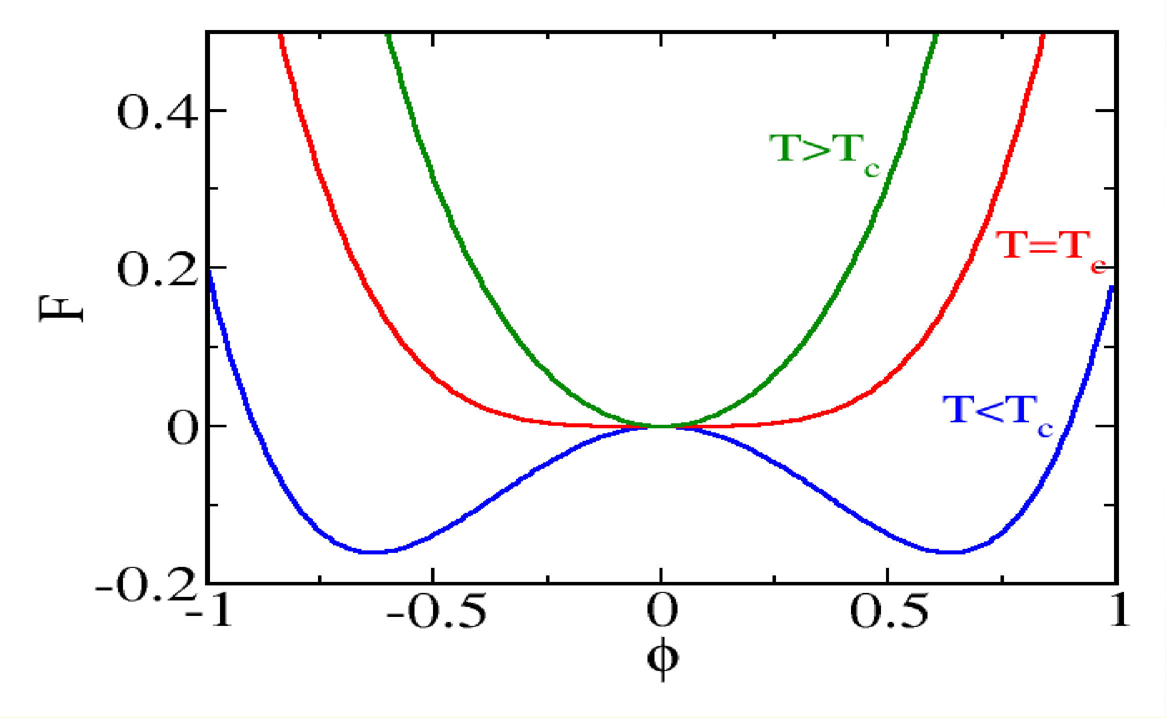
II order phase transition:

Specific heat C_v has finite value in crit. point ⟹ near critical point

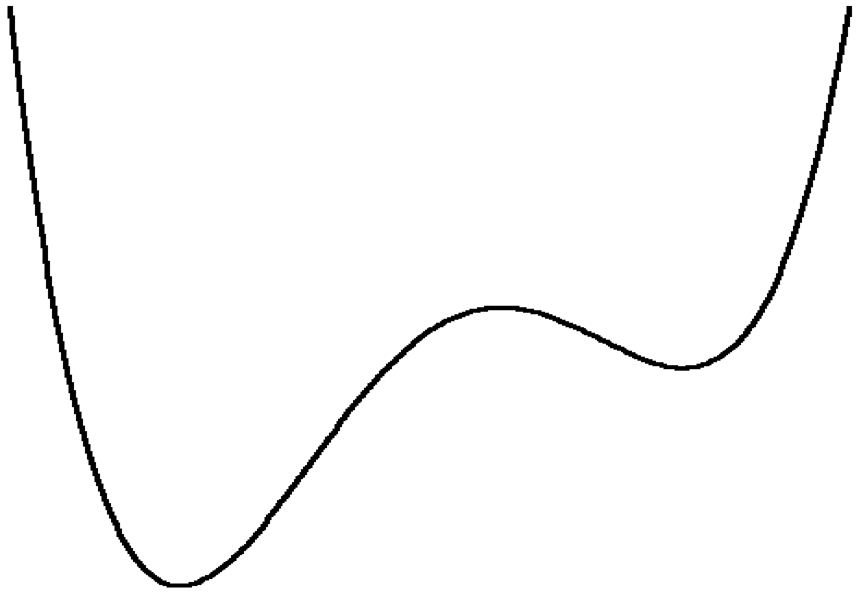
$F = -\alpha^2(T-T_{cr})^2/4c$, $\Phi \sim T-T_{cr}$, $a = \alpha\,(T-T_{cr})$, $b = h = 0$

I order phase transition: Φ has finite value, $h \neq 0$ (usually *b* is put zero)

II order phase transition



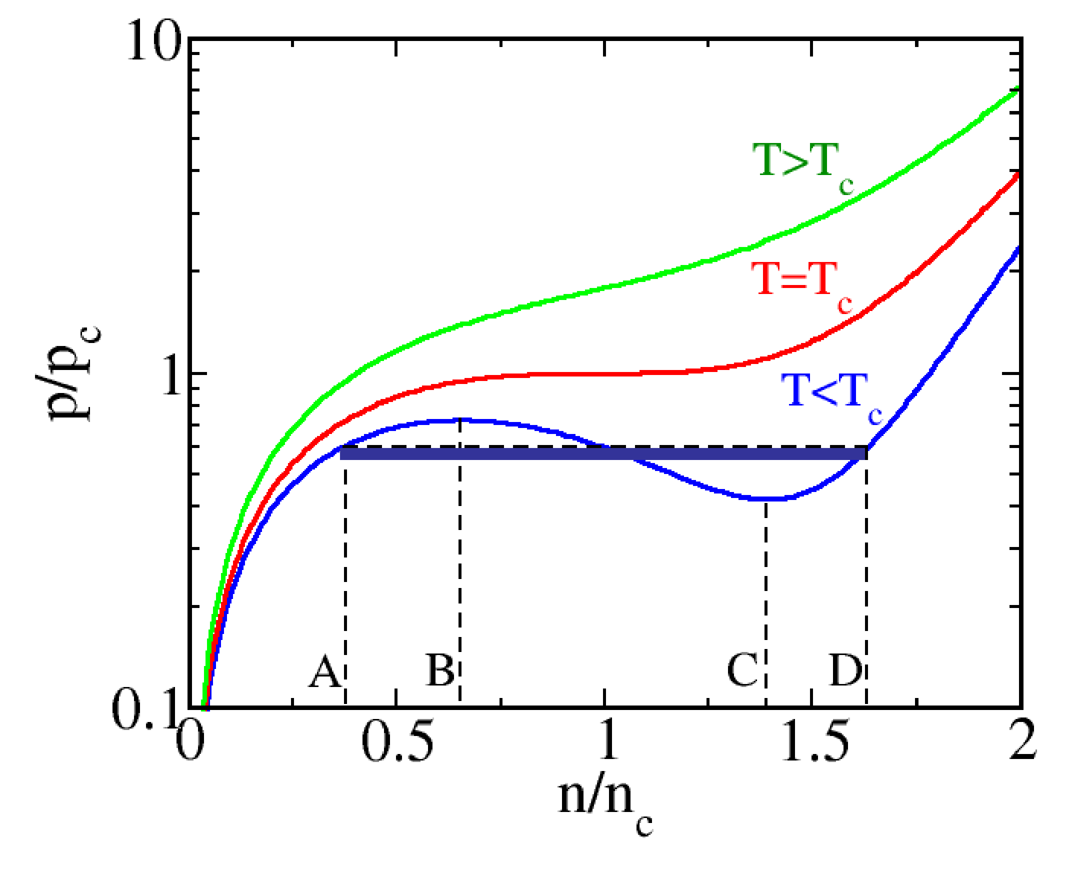
I order phase transition



$$a\phi + c\phi^3 + h = 0$$

,

I-order phase transition. Pressure isotherms



OA – homogeneous gas phase, dP/dn >0;
>D – homogeneous liquid phase, dP/dn >0;
BC – mechanically unstable, dP/dn <0;
AB(supercooled wapor),
CD (overheated liquid) – inhomogeneous, metastable, mechanically stable dP/dn >0, finite lifetime

Maxwell construction

$$\mu_{\text{gas}} = \mu_{\text{liquid}}$$

For slightly spatially inhomogeneous configurations

$$F = Const + \int d^3x \left(\frac{m}{2}(\nabla\phi)^2 + \frac{a}{2}\phi^2 + \frac{b}{3}\phi^3 + \frac{c}{4}\phi^4 + h\phi \right)$$

Dynamical description

in condensed matter there always exist slowly dissipating modes:

$$\frac{\partial \phi}{\partial t} = -\Gamma(\Delta)\frac{\delta F}{\delta \phi}$$

deviation from equilibrium is proportional to thermodynamical force

$$\Gamma(\Delta) = a_0 - a_1\Delta$$

is expanded in gradients

a_0=0 for description of conserved order parameter (like entropy)
a_1 =0 for non-conserved order parameter (like density)

$$\frac{\delta F}{\delta \phi} = -m\Delta\phi + a\phi + b\phi^2 + c\phi^3 + h$$

Dynamics on example of the non-conserved order parameter

II order phase transition:

$\Phi \sim exp(-i\omega t)$: $i\Gamma\omega = \alpha\,(T - T_{cr})$ for small Φ, so, amplitude of the order parameter grows with time in the whole system

I order phase transition:

Dynamics of the order parameter is more specific

$\Phi = \Phi_0\,\psi$, $\tau = t / t_0$, $\Phi \sim T_{cr} - T$, $t_0 \sim 1/(T_{cr} - T)$, $\xi = r/l_0$, $l_0 \sim 1/(T_{cr} - T)^{1/2}$

$$\frac{\partial \psi}{\partial \tau} = \Delta\psi + 2\psi(1 - \psi^2) + \boxed{\varepsilon}$$

$$\Delta = \frac{\partial}{\partial \xi^2} + \frac{d-1}{\xi}\frac{\partial}{\partial \xi} + \frac{1}{\xi^2}\hat{L}^2; \xi = |\vec{\xi}|$$

First time derivative (!) compared to example of relativistic bosons considered above

See A. Patashinsky, B. Shumilo, JETP 50 (1979) 712

Different solutions for ε=0:

d=1, ε=0: no curvature, surface tension σ=0,

$$\frac{\partial \psi}{\partial \tau} = \frac{\partial^2}{\partial \xi^2}\psi + 2\psi(1-\psi^2)$$

stationary kink

$$\psi = \pm \tanh(\xi - \xi_0), \xi_0 = const.$$

d>1, ε=0:

$$\frac{\partial \psi}{\partial \tau} = \frac{\partial^2}{\partial \xi^2}\psi + \frac{d-1}{\xi}\frac{\partial \psi}{\partial \xi} + 2\psi(1-\psi^2)$$

Curvature, surface tension

$$\psi = \pm \tanh(\xi - \xi_0), \xi_0 = \xi_0(\tau).$$

$$\frac{\partial}{\partial \tau}\xi_0(\tau) = -\frac{d-1}{\xi_0(\tau)}$$

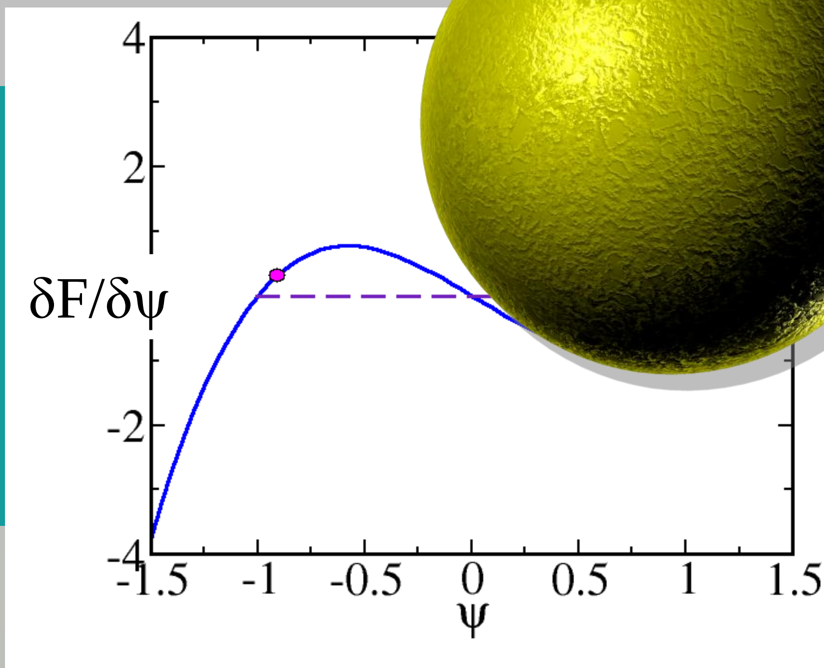
In case of II order phase transition (ε=0)

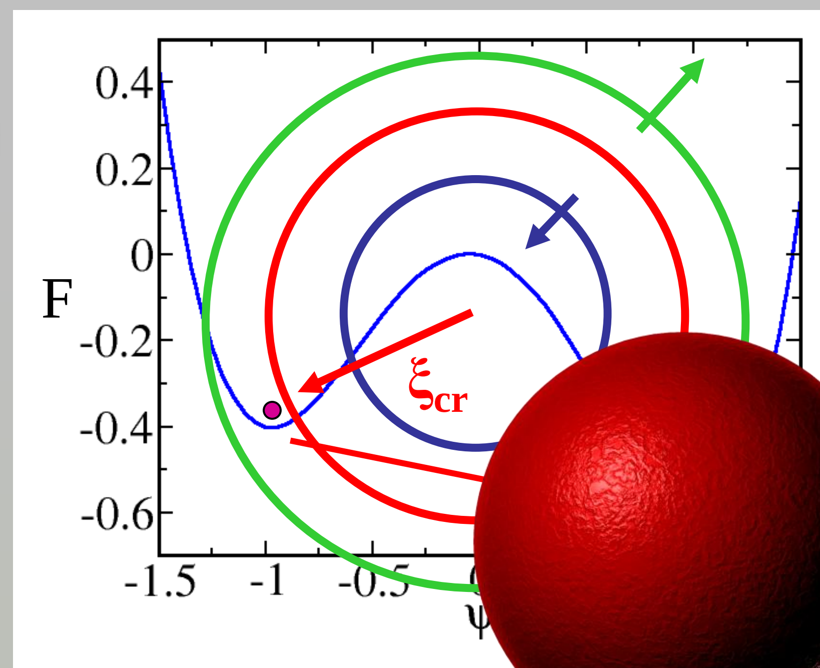
size of the seed diminishes with time owing to surface tension

Solution in the metastable region

d>1, ε«1, ε>0:

$$\psi = \mp\tanh(\xi - \xi_0) + \frac{\epsilon}{4};\ \frac{\partial}{\partial\tau}\xi_0(\tau) = -\frac{d-1}{\xi_0(\tau)} \pm$$





- slabs (d=1) have no critical size
- For d>1, rods (d=2) and droplets/bubbles (d=3) have critical sizes: *only seeds produced in metastable matter in fluctuations with overcritical size may grow to stable phase*

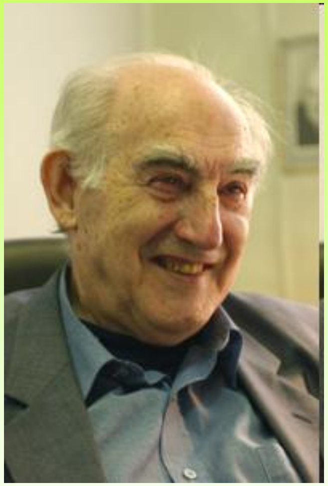
Boiling process

(close to T_c , when overcritical seeds are produced rapidly)

Beyond mean field: fluctuations

- **Long-range fluctuations of the order parameter** (very important at least near critical point)
- **Short-range fluctuations** (renormalize coefficients of Landau functional, and produce noise term in equation for the order parameter)

Fluctuation region near T_{cr}



Ginzburg criterion: $W \sim \exp(-\delta F(T)/T)$,

$\delta F \sim \alpha^2 (T - T_{cr})^2 / c$, $l_0 \sim 1/(T_{cr} - T)^{1/2}$

At $T = T_{fl}$ the fluctuation forming in a minimal volume $\sim l_0^3$ is probable ($W \sim 1$).

Then fluctuations dominate for T near T_{cr}

estimated by Ginzburg number $Gi = |T_{cr} - T_{fl}| / T_{cr} \sim 1$

Ginzburg–Levanyuk criterion: $C_V^{MF} \sim C_V^{fl}$

CSC fluctuations for $T>T_{csc}$

D.V. Phys. Rev C69 (2004) 06529

(.) In some models T_{csc}~100 MeV. At $T<T_{csc}$
I or/and II order CSC phase transitions are possible.

Fluctuation region, Gi ~1, can be very broad: $(0.5\text{-}1.5)T_{csc}$

A hope to observe CSC fluctuation effects at T above T_{CSC}!

(.) Coherence length $l_0 \sim 0.2\ \mathrm{fm}\ |(T_{cr}-T)/T_{cr}|^{-1/2}$ is short at T not too close to T_{csc} .Thus fluctuations of density with

$\rho(t)>\rho_{cr}$ and $T < T_{csc}(\rho(t))$ in $\sim l_0^3$ volume are rather probable and may result in **accumulation of CSC domains**

with $\rho(t) -\rho_{cr} << \rho_{cr}$ dissolving slowly in time since

$t_0 \sim 1/|T_{cr}-T| \sim 1/(\rho- \rho_{cr})$ is sufficiently large.

In such a way **a kind of mixed phase with inhomogeneous T-ρ profile** could be formed *if system lived sufficiently large time (!?)*

- Main role of the noise term is that it generates initial fluctuations which then either grow or diminish with time

Role of noise

$$\frac{\partial \psi}{\partial \tau} = \frac{\partial^2}{\partial \xi^2}\psi + 2\psi(1-\psi^2) + \epsilon + \boxed{\theta}$$

The noise term describes the short-distance fluctuations. The correlation radii both in space and time is negligible in comparison to correlation radii of order parameter. Thus the noise can be considered to be delta-correlated:

$$\langle \theta(\vec{x}_1, t_1)\theta(\vec{x}_2, t_2)\rangle = A\delta(\vec{x}_1 - \vec{x}_2)\delta(t_1 - t_2)$$

$$\psi = \mp \tanh(\xi - \xi_0) + \frac{\epsilon}{4} + \boxed{\chi(\xi, \tau)}$$ ←Amplitude

$$\frac{\partial}{\partial \tau}\xi_0(\tau) = -\frac{d-1}{\xi_0(\tau)} \pm 3/2\epsilon - 6\boxed{\chi(\xi, \tau)}$$ ←Radius

$$\boxed{\frac{\partial \chi}{\partial \tau} = \Delta_\xi \chi - 4\chi + \theta}$$ Response to the noise

Noise also affects seed shape

Conclusion to above sects.

- There are similarities and differences in description of the dynamics of the I order phase transitions for relativistic bosons in external fields and for the order parameter in non-relativistic systems.

  Main difference is connected with 2-time derivative of the field in eq. of motion in considered above relativistic case and with 1-time derivative for non-relativistic systems.

Manifestation of phase transition instabilities in solutions of kinetic equations

Consider appearance of instabilities in particle distributions

- Quasiparticle description
- Beyond quasiparticle approximation

Quasiparticle limit

The standard Landau qp. kin. eq. can be derived from KB kin. eq. integrating the latter in ϵ, in the limit for particle mass width $\Gamma \to 0$:

$$\partial_t f^{\rm qp}(X,\boldsymbol{p}) + \frac{\partial\varepsilon(X,\boldsymbol{p})}{\partial \boldsymbol{p}} \partial_{\mathbf{X}} f^{\rm qp}(X,\boldsymbol{p}) - \partial_{\mathbf{X}}\varepsilon(X,\boldsymbol{p})\, \frac{\partial f^{\rm qp}(X,\boldsymbol{p})}{\partial \boldsymbol{p}} = C^{\rm qp}(X,\boldsymbol{p}),$$

$C^{\rm qp}(X,\boldsymbol{p}) \equiv C(F = A^{qp} f^{qp}, ...)$.

The qp dispersion relation for the energies $\varepsilon(X,\boldsymbol{p})$ follows from the retarded eq.:

$$\varepsilon^2(X,\boldsymbol{p}) = m^2 + \boldsymbol{p}^2 + \mathrm{Re}\,\Pi^R\left(X, \varepsilon(X,\boldsymbol{p}), \boldsymbol{p}\right).$$

This dispersion eq. may have several solutions (branches) which contribute separately.

Note: Relativistic Boltzmann eq. is obtained **only** in perturbative limit: $\varepsilon(X,\boldsymbol{p}) \simeq \sqrt{m^2 + \boldsymbol{p}^2}$, with simplest $\propto ff(1-f)(1-f)$ collision term.

Kinetics of Bose-Einstein condensation

D.V., Phys.Atom. Nucl. **59** (1996) 2015

Consider evolution of initially non-equilibrium spatially homogeneous π^0 gas with only elastic collisions of quasiparticles, $L_{int} = -g\phi^4/4$,

$$\epsilon_p^2 = m^{*\,2} + \boldsymbol{p}^2, \quad m^{*\,2} = m^2 + g\int \frac{f_p}{2\epsilon_p}\frac{d^3p}{(2\pi)^3}$$

$$St[f_p] = g^2 \int F[f](2\pi)^4\delta^{(4)}(p + p_2 - p_3 - p_4)\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}$$

$$F[f] = (1 + f_p)(1 + f_{p2})f_{p3}f_{p4} - (1 + f_{p3})(1 + f_{p4})f_{p1}f_{p2}$$

Integrating in angles in ϵ variables we get

$$\partial_t f_\epsilon = \frac{g^2}{64\pi^3\epsilon}\int F[f]Dd\epsilon_3 d\epsilon_4, \quad \epsilon + \epsilon_2 = \epsilon_3 + \epsilon_4$$

$$D = \frac{1}{p}\min[p, p_1, p_2, p_3, p_4]$$

Kinetics of Bose-Einstein condensation

Let us characterize $f(\epsilon)$ by two parameters of initial distribution $f(t=0)=f_0$, the amplitude n_0 and the energy scale ϵ_0. Simplifying further put $m^* \simeq m$.

In dimensionless variables $\chi = f(\epsilon)/n_0$ and $\widetilde{\epsilon} = (\epsilon - m)/\epsilon_0$:

$$\frac{d\chi}{d\tau} = \int F[\chi] D d\widetilde{\epsilon}_3 d\widetilde{\epsilon}_4 \qquad (*)$$

with $\chi(\widetilde{\epsilon} \to 0) \to 1$,

$$t = \frac{64\pi^3\sqrt{m^2 + \boldsymbol{p}^2}}{g^2\epsilon_0^2 n_0(1+n_0)}\tau.$$

$$F[\chi] = \left(\chi_3\chi_4 - \chi\chi_2 + n_0[(\chi_1+\chi_2)\chi_3\chi_4 - (\chi_3+\chi_4)\chi\chi_2]\right)/(1+n_0)$$

depends only weakly on n_0.

In thermal equilibrium

$$f_p^{\rm eq} = \left[\exp\left((\sqrt{m^2+\boldsymbol{p}^2} - \mu)/T\right) - 1\right]^{-1}$$

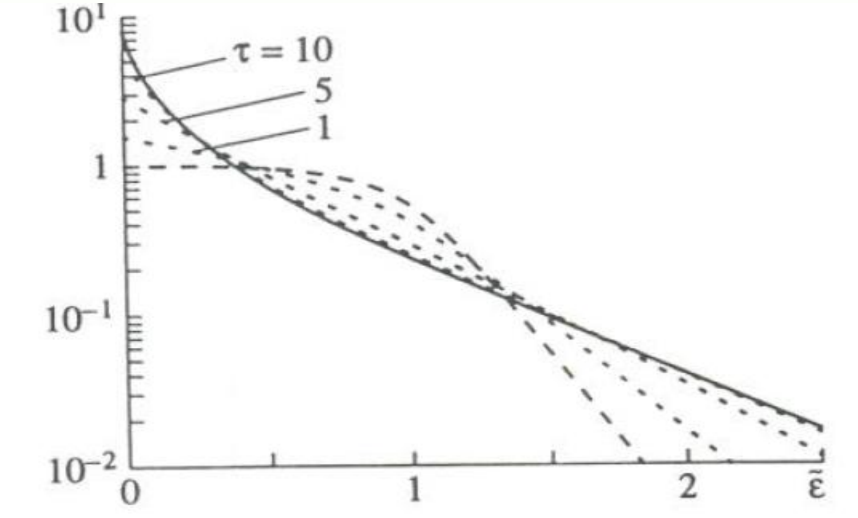
Kinetics of induced Bose-Einstein condensation

Equating number of particles and energy in initial and equilibrium configurations for $\mu = m$ we determine critical parameter n_0^{cr} for occurring of induced Bose-Einstein condensation in the course of equilibration process. Condensation occurs for $n_0 \geq n_0^{cr}$.

Take initial distribution in the form:

$$f_0 = \frac{2n_0}{\pi} \mathrm{arctg} e^{\gamma(1-\tilde{\epsilon}/\epsilon_0)}, \quad \gamma = 5.$$

Numerical solution $f(t)$ (see D. Semikoz, I. Tkachev, Phys.Rev. D55 (1977) 489) for $n_0 = 1 < n^{cr} \simeq 2.8$ is as follows:



Solid line is equilibrium distribution

Strong enhancement of the distribution at small momenta for τ~10

Kinetics of Bose-Einstein condensation

For $n_0 \gg 1$ there exists a self-similar solution of kinetic equation

$$f(\epsilon,\tau) = A^{-\beta}(\tau) f_s(\bar{\epsilon}), \quad \bar{\epsilon} = (\epsilon - m)/A(\tau), \quad \beta = const > 0$$

where

$f_s(\bar{\epsilon})$ obeys equation:

$$\beta f_s(\bar{\epsilon}) + \bar{\epsilon}\frac{df_s(\bar{\epsilon})}{d\bar{\epsilon}} = St[f_s(\bar{\epsilon})]/C_s$$

and

$$A(\tau) = [2C_s(\tau_c - \tau)(\beta - 1)]^{1/(2(\beta-1))}, \quad C_s = const > 0$$

For finite time τ_c the distribution function becomes singular due to condensation. For $\tau > \tau_c$ kinetic equation should be modified with taking into account of the accumulation of the condensate mean field.

Kinetics of Bose-Einstein condensation

Our kinetic equation (*) allows for **stationary solutions**

$$f(\epsilon') = (\epsilon')^{-\alpha}, \quad 0 < \epsilon' = \epsilon - m \ll \epsilon_0$$

with

$\alpha = 0$ (describes equilibrium state with $\mu < m$),

$\alpha = 1$ (describes equilibrium state with $\mu = m$),

and **for $f \gg 1$ there appear new two solutions** with

$\alpha = 7/6$ (describes turbulence of classical waves and corresponds to the constant flow of particles towards the region of small momenta)

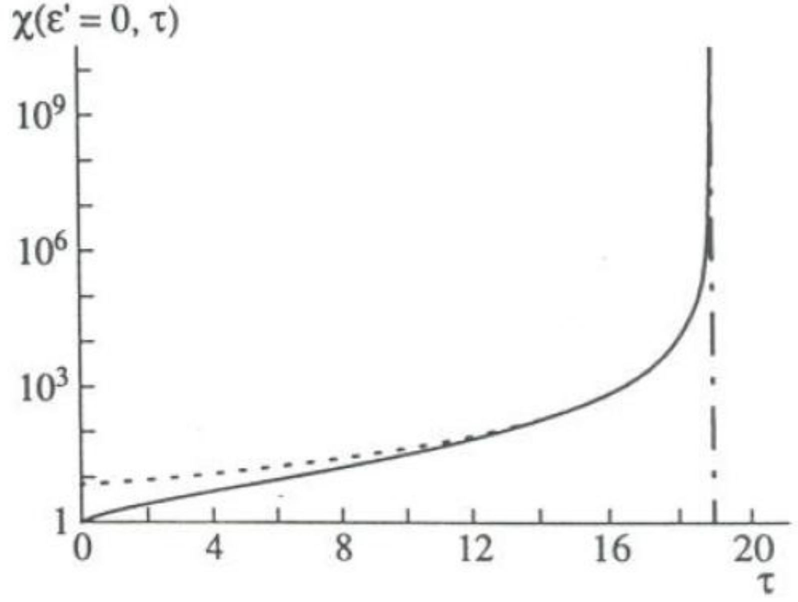
and

$\alpha = 3/2$.

In reality the later two stationary regimes may not be reached since for $\tau > \tau_c$ there appears Bose condensate. Numerical analysis shows that for $\epsilon' \ll 10^{-2}\epsilon_0$ the power law corresponds to $\alpha \simeq 1.24$ and solution is not yet stationary but self-similar.

Kinetics of Bose-Einstein condensation

Solid line is numerical solution, dash line is self-similar solution $f(\epsilon' = 0, \tau) \propto (\tau_c - \tau)^{-2.6}$ corresponding to $\alpha \simeq 1.24$



Typical values of parameters

For $\rho(t=0) \sim 6\rho_0$ and $\varepsilon_0 \sim 2m_\pi$ (Δ-isobar region for pion production), $n_0 \sim 7 >> n_{cr}$

t_{cond} ($\tau \sim 20$) $< t_{evol} \sim (10\text{-}20)$ fm (time of fireball evolution)

At $\varepsilon > 2m$ distribution reaches equilibrium for $\tau > 1$

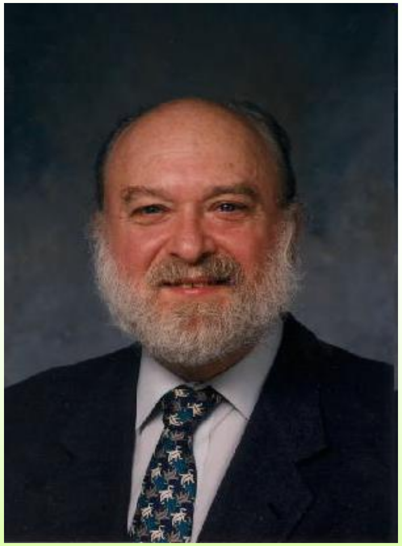
For 10^{-2} m $< \varepsilon - m << m$ turbulence regime

For $\varepsilon - m << 10^{-2}$ m for $\tau > 14$ self-similar solution

For τ = 19 there appears singularity at ε=m

signalizing start of Bose-Einstein condensation

- **Kinetics of the pion condensation phase transition, solution of KB equation**

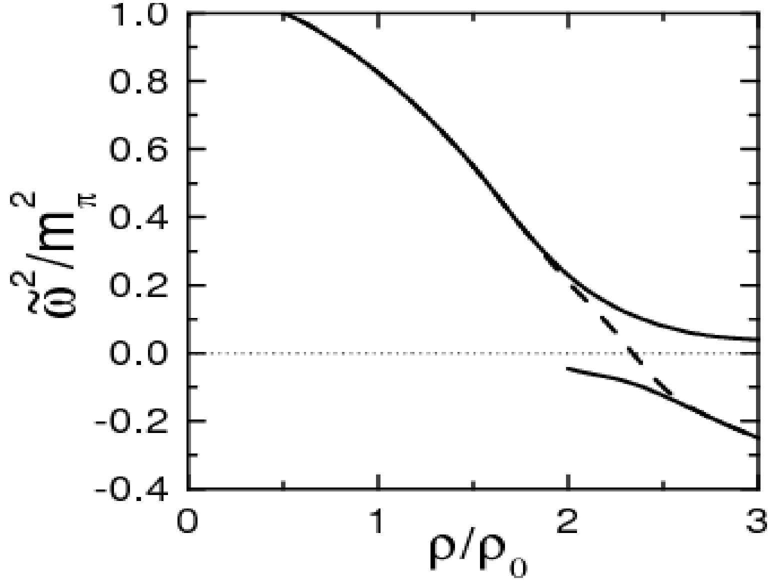


Density dependence of the effective pion gap in symmetric nuclear matter

Pion has attractive p wave interaction to nucleon. $\mathrm{Re}\Pi^R_\pi \propto -\boldsymbol{p}^2 p_F^N(\rho)$ and

$$\widetilde{\omega}^2 \equiv -\min_{|\boldsymbol{p}|}\mathrm{Re}G^R_\pi(\epsilon=0,|\boldsymbol{p}|) = m_\pi^2 + \boldsymbol{p}_m^2 + \mathrm{Re}\Pi^R_\pi(\epsilon=0,p_m)$$

changes sign at some critical density $\rho = \rho_{cr}$ and becomes negative that signalizes pion condensation phase transition.



ρ_0 is nuclear saturation density

$p_m \sim p_F$ **pion condensation is example of condensation to inhomogeneous state**

For $\rho > \rho_{cr}$ upper branch corresponds to metastable state, lower, to stable one.

I order phase transition due to pion fluctuations

See D.V., I. Mishustin, Sov.J.Nucl.Phys.35 (1982) 667;
A.B.Migdal, E.Saperstein, M.Troitsky, D.V. Phys.Rept.192 (1990) 179.

With taking into account fluctuations always (!) I order phase transition

Note that quarkyonic phase has much similar with the pion condensate one

Transport scheme. Wigner transformation

Separate in Dyson equations slow space-time macro-motion $X = \frac{1}{2}(x_1 + x_2)$ and rapid relative motion $\xi = x_1 - x_2$ related to micro- processes, e.g. for $t_{cor} \ll t_{micro} \sim 1/\epsilon_F \ll 1/\Gamma$. For any two-point function

$$F^{ij}(X;p) = \int d\xi e^{\mathrm{i}p\xi} F^{ij}\left(X + \xi/2, X - \xi/2\right), \qquad i,j \in \{-+\}$$

Gradient approximation (for $\Delta p_\mu \Delta X^\mu \gg \bar{h}$) converts

$$\int d\xi e^{\mathrm{i}p\xi} \left(\int dz f(x,z)\varphi(z,y) \right) = \left(\exp\left[\frac{\mathrm{i}\bar{h}}{2} \left(\partial_p \partial_{X'} - \partial_X \partial_{p'} \right) \right] f(X,p)\varphi(X',p') \right)_{p'=p, X'=X}$$

$$\simeq f(X,p)\varphi(X,p) + \frac{\mathrm{i}\bar{h}}{2} \left\{ f(X,p), \varphi(X,p) \right\},$$

where the first order terms are given by Poisson brackets

$$\{f(X,p), \varphi(X,p)\} = \frac{\partial f}{\partial p^\mu} \frac{\partial \varphi}{\partial X_\mu} - \frac{\partial f}{\partial X^\mu} \frac{\partial \varphi}{\partial p_\mu}$$

In such a way one obtains Kadanoff-Baym kinetic equation.

Generalized kinetic eq. in physical notation

Within the first-order gradient approximation, the KB eq. is

$$DF(X,p) - \left\{\Gamma_{in}, \mathrm{Re}G^R\right\} = C(X,p),$$

Here the differential drift operator

$$D = \left(v_\mu - \frac{\partial \mathrm{Re}\Pi^R}{\partial p^\mu}\right)\partial_X^\mu + \frac{\partial \mathrm{Re}\Pi^R}{\partial X^\mu}\frac{\partial}{\partial p_\mu}$$ describes drag flow in a mean field

The Poisson bracketed term describes back flow due to fluctuations.

Cf. a toy-ship moving in a bath: drag flow near ship and back flow at edges.

Case $C = 0$: Vlasov collision-less dynamics. Also $C = 0$ is fulfilled for thermal equilibrium when $\partial_X F = 0$. The collision term:

$$C(X,p) = \Gamma_{in}(X,p)\widetilde{F}(X,p) - \Gamma_{out}(X,p)F(X,p) = A\Gamma[\gamma - f].$$

$$\begin{aligned} F(X,p) &= A(X,p)f(X,p) = (\mp)\mathrm{i}G^{-+}(X,p), \\ \widetilde{F}(X,p) &= A(X,p)[1 \mp f(X,p)] = \mathrm{i}G^{+-}(X,p), \end{aligned}$$

F is 4-phase-space probability – a generalized virtual particle distribution function.

$$A(X,p) \equiv -2\mathrm{Im}G^R(X,p) = \widetilde{F} \pm F = \mathrm{i}\left(G^{+-} - G^{-+}\right)$$

Clearly we deal with a generalized kinetic equation!

Pion cond. phase transition in dense nucleon system

Consider relaxation of a pion distribution in (quasi)equilibrium nucleon environment. Assume $\rho_\pi \ll \rho_N$. Using also that $m_\pi/m_N \sim 1/7 \ll 1$ we can neglect a feedback of pions onto nucleons.

Thus we may drop pion distrib. dependence in all self-energy terms.

Distribution of virtual pions (ϵ is not connected with $\boldsymbol{p}$) is found from the Kadanoff-Baym kinetic equation:

$$\frac{\Gamma_\pi}{2} B^\pi_\mu \partial^\mu_x f_\pi(\epsilon, \boldsymbol{p}, t, \boldsymbol{r}) = \Gamma^\pi_{in} - \Gamma^\pi f_\pi,$$

$B^\mu_\pi = A_\pi \left[\left(2p^\mu - \frac{\partial \mathrm{Re}\Pi^R_\pi}{\partial p_\mu} \right) - M_\pi \Gamma^{-1}_\pi \frac{\partial \Gamma_\pi}{\partial p_\mu} \right]$ is the normalized spectral function, Γ_π is the width, A_π is ordinary spectral function, Γ^π_{in} is the gain term which does not depend on δf_π in our approximation, $M_\pi = \epsilon^2 - m^2_\pi - \boldsymbol{p}^2 - \mathrm{Re}\Pi^R_\pi$.

Assuming $f_\pi(\epsilon, \boldsymbol{p}, t, \boldsymbol{r}) = f^{\mathrm{eq}}_\pi(\epsilon) + \delta f_\pi(\epsilon, \boldsymbol{p}, t, \boldsymbol{r})$, $f^{\mathrm{eq}}_\pi(\epsilon)$ is equilibrium distribution, we find

$$\frac{1}{2} B^\mu_\pi \partial^\mu_x \delta f_\pi + \delta f_\pi = 0,$$

whereas for space-homogeneous case it follows the solution:

$$\delta f_\pi(\epsilon, \boldsymbol{p}) = \delta f_0(\epsilon, \boldsymbol{p}) e^{-2t/B_0(\epsilon, \boldsymbol{p})}$$

See Yu. Ivanov, J. Knoll, H. van Hees, D.V. Phys.Atom.Nucl.64 (2001) 652.

Pion cond. phase transition in a dense nucleon system

For small ϵ, and $|\boldsymbol{p}| \simeq p_m$ the pion width $\Gamma_\pi = \beta(p_m)\epsilon$, with $\beta(p_m) = const > 0$, and the second term proves to be dominant in B_0^π.

This case is not described by qp. approx. (!).

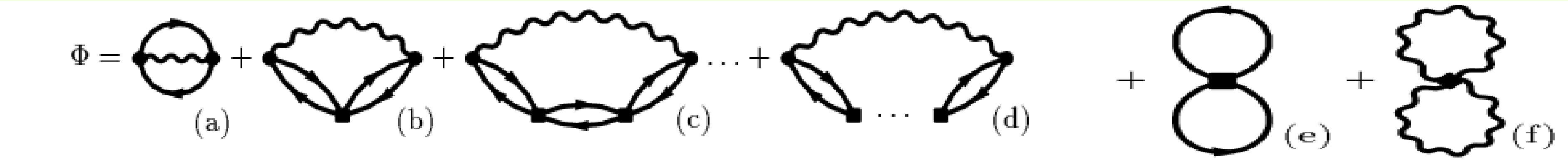
Thus $B_\pi^0 = \frac{2\beta}{\widetilde{\omega}^2}$ and

$$\delta f_\pi(\epsilon, \boldsymbol{p}) \simeq \delta f_0 \exp(-\widetilde{\omega}^2 t/\beta).$$

Initial virtual pion fluctuation (for small ϵ and $\boldsymbol{p} \neq 0$) is damped for $\rho < \rho_{cr}$ (when $\widetilde{\omega}^2 > 0$) and it grows in time for $\rho > \rho_{cr}$ (when $\widetilde{\omega}^2 < 0$).

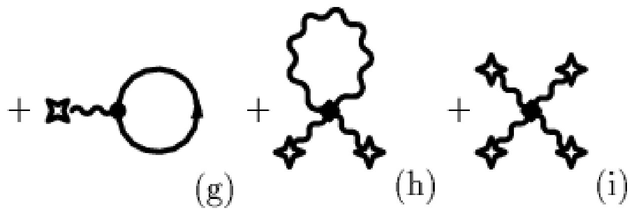
- ✔ Initial correlations are disregarded in the KB kin.eq. Therefore in case $\widetilde{\omega}^2 < 0$, to generate initial pion distribution one needs to add a noise term in the kinetic eq. The corresponding term is added in phenomenological treatment of the dynamics of phase transitions.
- ✔ In this particular example the pion **width drives the phase transition.**

We demonstrated only onset of instability for $\rho>\rho_{cr}$ whereas solution of the problem requires to incorporate interaction with the mean field, e.g. to the diagrams



one should add

diagrams with mean field



Conclusion to “Kinetic description” sect.

- Dynamics of Bose-Einstein condensation in case of elastic collisions

and of condensation in presence of dissipative processes are very different.

In the latter case we have shown solution of the KB equation beyond the quasiparticle approximation.

Back to dynamics of mean field but now for phase transition to the state with finite momentum

A.B.Migdal, E.Saperstein, M.Troitsky, D.V. Phys.Rept.192 (1990) 179.

The mean field free energy density for the phase transition to inhomogeneous state (II order phase transition)

Consider charged pion condensation

$$\delta F = -[\Delta_0 + \alpha_4(k_0^2 - k^2)^2]|\psi|^2 + \frac{\Lambda(k_0)}{2}|\psi|^4, \quad \alpha_4 < 0$$

Similar to A-phase of smectic liquid crystals

Different structures with the same volume energy but different surface energies:

the plane layers

$$\psi = a e^{ik_0 x},$$

the cylindrical layers

$$\psi = a e^{ik_0 \rho},$$

the spherical layers

$$\psi = a e^{ik_0 r},$$

the disordered phase

$$\psi = \frac{1}{\sqrt{N}}\left\{\sum_{i=1}^{N_1} a_i \exp\left[ik_0\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}\right] + \sum_{i=N_1+1}^{N} a_i \exp\left[ik_0\sqrt{(x-x_i)^2 + (y-y_i)^2}\right] + a_0 e^{ik_0 x}\right\}, \quad N \gg 1, \quad N^{-1}\sum_{i=0}^{N} a_i^2 = a^2,$$

Finite size effects

For simplest model, the running wave $\psi = a\xi(r)\exp[ik_0 r]$ corresponds to the minimum of the surface energy

$$\alpha_4\Delta^2\xi - 4(k_0\nabla)^2\alpha_4\xi + \Delta_0\xi - \Delta_0\xi^*\xi^2 = 0, \quad a^2 = \Delta_0/\Lambda$$

In half-space medium z<0 there are solutions of two types:

$$\psi = a\tanh\left[\frac{z - z_0}{\sqrt{2}\,l_\parallel}\right]e^{ik_0 z}, \quad l_\parallel = (-4\alpha_4 k_0^2/|\Delta_0|)^{1/2},$$

corresponds to $k_0 \parallel z$, and the second solution

$$\psi \simeq a\left\{1 - C_1 e^{z/l_\perp}\cos\left(\frac{z}{l_\perp} + C_2\right) - \frac{C_1^2}{10}e^{2z/l_\perp} \times\left[1 + \sin^2\left(\frac{z}{l_\perp} + C_2\right)\right] - \cdots\right\}e^{ik_0 y}$$

This solution corresponds to smaller surface energy

relates to $k_0 \perp z$. Here, $l_\perp = (-2\alpha_4)^{1/4}\Delta_0^{-1/4}$, $C_1 \simeq \sqrt{2}$ and $C_2 \simeq \pi/4$.

Finite size effects

For spherical system of the radius R to the minimum of the surface energy corresponds solution:

$$\psi = a \tanh\left[\frac{|z| - \sqrt{R^2 - x^2 - y^2}}{\sqrt{2}\, l_{\parallel}}\right] e^{ik_0 z}$$

An elongation of the nucleus along k_0 direction is energetically favorable

Dynamics of the I order phase transition to inhomogeneous state

D.V. Phys.Scripta 47 (1993) 333

With taking into account fluctuations – always I order phase transition → add h-term to the Free energy density:

$$\delta F = \hat{L}|\psi|^2 + \frac{\Lambda|\psi|^4}{2} - h\psi\sqrt{\frac{\psi^*}{\psi}} - h\psi^*\sqrt{\frac{\psi}{\psi^*}}$$

$$\hat{L} = -\Delta_0 - \alpha_4(\Delta + k_0^2)^2,$$

$$\partial_t \xi = 2\xi(1-\xi^2) + \tilde{h} + \tilde{\hat{L}}\xi$$

Here, the new dimensionless variables are introduced:

$$\tilde{r} = r/(-8k_0^2\alpha_4/\Delta_0)^{1/2}, \quad \tilde{h} = 2h/\Delta_0\psi_0,$$

$$\psi_0 = \sqrt{\Delta_0/\Lambda}, \quad \tilde{t} = t\Gamma\,\Delta_0/2,$$

$$\tilde{\hat{L}} = (\boldsymbol{n}\tilde{\nabla})^2 - \alpha\,\tilde{\Delta}^2, \quad \boldsymbol{n} = \boldsymbol{k}_0/k_0, \quad \alpha = \Delta_0/4k_0^2$$

Dynamics of the I order phase transition to inhomogeneous state

For initially spherical seed of radius $R_0 > R_{cr}$, $k_0 || x$

$$\xi = -\tanh\left(|\tilde{x}| - \sqrt{|\tilde{R}_0^2 - \tilde{\rho}^2|}\,\nu - \tfrac{3}{2}\tilde{h}\tilde{t}\right) + \frac{\tilde{h}}{4},$$

$$\nu = \mathrm{sgn}\,(\tilde{R}_0 - \tilde{\rho}),$$

$$\xi(r, 0) = -\tanh\left(|\tilde{x}| - \sqrt{|\tilde{R}_0^2 - \tilde{\rho}^2|}\,\nu\right) + \frac{\tilde{h}}{4},$$

$$\xi(\tilde{R}_0, 0) \simeq 0$$

For $t \to 0$ we have $v_y \to 0$, while for $t \to \infty$ we obtain $v_y \to v_x$. So, an initially spherical germ is elongated in an ellipsoid of the eccentricity

$$\varepsilon = [(\tfrac{3}{2}\tilde{h}\tilde{t})^2 + \tilde{R}_0^2]^{1/2}/(\tfrac{3}{2}\tilde{h}\tilde{t} + \tilde{R}_0)$$

The maximum of eccentricity, $\varepsilon_{max} = 1/\sqrt{2}$, is achieved at the characteristic time $\tilde{t} \simeq 2\tilde{R}_0/3\tilde{h}$. Then $\varepsilon \to 1$ for $t \to \infty$.

Stick-like structures are observed in A-smectics near critical point

Dynamics of initially non-spherical seeds

For parallelepiped-like shape

$$\psi = e^{ik_0 y}\psi_0 \Big\langle \tanh[\tilde{y}_0 - |\tilde{y}| + \tfrac{3}{2}\tilde{h}\tilde{t}]$$

$$+ \frac{\tilde{h}}{4} - \exp\{[(|\tilde{x}| - \tilde{x}_0)/\tilde{l}_\perp - \tfrac{3}{2}\tilde{h}\tilde{t}]v_1\}$$

$$\times\ v_1 \cos\left(\frac{|\tilde{x}| - x_0}{\tilde{l}_\perp} - \tfrac{3}{2}\tilde{h}\tilde{t}\right)$$

$$- \exp\{[(|\tilde{z}| - \tilde{z}_0)/\tilde{l}_\perp - \tfrac{3}{2}\tilde{h}\tilde{t}]v_2\}$$

$$\times\ v_2 \cos\left(\frac{|\tilde{z}| - \tilde{z}_0}{\tilde{l}_\perp} + \tfrac{3}{2}\tilde{h}\tilde{t}\right)\Big\rangle,$$

$$v_1 = \operatorname{sgn}\left(\frac{\tilde{x}_0 - |\tilde{x}|}{\tilde{l}_\perp} + \tfrac{3}{2}\tilde{h}\tilde{t}\right),$$

$$v_2 = \operatorname{sgn}\left(\frac{\tilde{z}_0 - |\tilde{z}|}{\tilde{l}_\perp} + \tfrac{3}{2}\tilde{h}\tilde{t}\right), \quad l_\perp = \alpha^{1/4} \ll 1.$$

The velocity is $\tilde{v}_{\tilde{y}} = \frac{3}{2}\tilde{h}$, while $\tilde{v}_{\tilde{x}} = \tilde{v}_{\tilde{z}} = \frac{3}{2}\tilde{h}\tilde{l}_\perp \ll \tilde{v}_{\tilde{y}}$

Dynamics of initially non-spherical seeds

For cylindrical-like shape

$$\psi = e^{ik_{0}z}\psi_0 \Big\langle \tanh\,[\tilde{z}_0 - |\tilde{z}| + \tfrac{3}{2}\tilde{h}\tilde{t}] + \frac{\tilde{h}}{4}$$
$$- \exp\{[(\tilde{\rho} - \tilde{\rho}_0)/\tilde{l}_\perp - \tfrac{3}{2}\tilde{h}\tilde{t}]\nu_3\}$$
$$\times\, \nu_3 \cos\left(\frac{\tilde{\rho} - \rho_0}{\tilde{l}_\perp} - \tfrac{3}{2}\tilde{h}\tilde{t}\right)\Big\rangle,$$
$$\nu_3 = \mathrm{sgn}\left(\frac{\tilde{\rho}_0 - \tilde{\rho}}{\tilde{l}_\perp} + \tfrac{3}{2}\tilde{h}\tilde{t}\right).$$

This solution describes a cylinder which rapidly elongates and slowly expands in the perpendicular direction. One can also find the dynamics of many other configurations.

Conclusion to sect. “Phase transition to inhomogeneous state”

- There are many different structures with the same volume energy and different surface energies
- Even initially spherical overcritical seeds grow anisotropically.

Hydrodynamical description

Hydrodynamical eqs. are derived from kinetical eq. for $t \gg t_{kin}$

Consider

I-order phase transition of the liquid-gas type in condensed matter and in nuclear systems e.g., liquid-gas and hadron-quark I order phase transitions in HIC

Hydrodynamics of the first order phase transition:

V.Skokov, D.V. , arXiv 0811.3868, JETP Lett. 90 (2009) 223;
Nucl. Phys. A828 (2009) 401; A846 (2010).

We solve the system of non-ideal hydro equations describing non-trivial fluctuations (droplets/bubbles, aerosol) in d=2 space +1 time dimensions numerically for van der Waals-like EoS, and for arbitrary d in the vicinity of the critical point analytically.

Non-ideal non-relativistic hydrodynamics

$$mn\left[\partial_t u_i + (\mathbf{u}\nabla)u_i\right] = -\nabla_i P$$

$$+\nabla_k\left[\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right) + \zeta\delta_{ik}\mathrm{div}\mathbf{u}\right], \quad (8)$$

$$\partial_t n + \mathrm{div}(n\mathbf{u}) = 0, \quad (9)$$

$$T\left[\frac{\partial s}{\partial t} + \mathrm{div}(s\mathbf{u})\right] = \mathrm{div}(\kappa\nabla T)$$

$$+\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right)^2 + \zeta(\mathrm{div}\mathbf{u})^2. \quad (10)$$

Here η and ζ are shear and bulk viscosities; $\mathbf{u}$ is the velocity of the element of the fluid; s is the entropy density; κ is the thermal conductivity; d is the dimensionality of space.

Dynamics of the phase transition is controlled by the slowest mode

In collective processes u is usually small,

therefore in analytical treatment we neglect u^2 terms

Qualitative analysis and rough estimates

typical time for density fluctuation: $t_{\rho} \sim R$ (constant velocity)

R (t) is the size of evolving seed

typical time for heat transport $t_T \sim R^2 \; c_V / \kappa$, c_V is specific heat density

We introduce R_{fog} -- typical seed size at which $t_{\rho} = t_T$

$t_{\rho} > t_T$ for $R(t) < R_{fog}$: **Density evolution stage**

(isothermal)

$t_T > t_{\rho}$ for $R(t) > R_{fog}$: **Heat transport stage**

Seeds with $R \sim R_{fog}$ are accumulated with passage of time: ⇨ **fog stage**

for H-QGP phase transition $R_{fog} \sim$ 0.1-1 fm, for liquid-gas ~1-10 fm, fireball evolution time $t_{evol} \sim 10$ fm

Thermal conductivity effects should be incorporated in hydro simulations of HIC

Next is coalescence stage (occurs at still larger time scale),
see Lifshiz, Pitaevsky, Physical Kinetics. v. X

Supercooled gas; overheated liquid; aerosol-like mixture in spinodal region

Expand the Landau free energy in $\delta\rho = \rho - \rho_{\rm r}$ and δ T near the reference point, close to ρ_{cr}, T_{cr}

$$\delta F = \int \frac{d^3x}{\rho_{\rm r}} \left[\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda(\delta\rho)^4}{4} - \frac{\lambda v^2(\delta\rho)^2}{2} - \epsilon\delta\rho \right]$$

Surface term

$v^2 \sim (T_{cr} - T)$ in mean field treatment

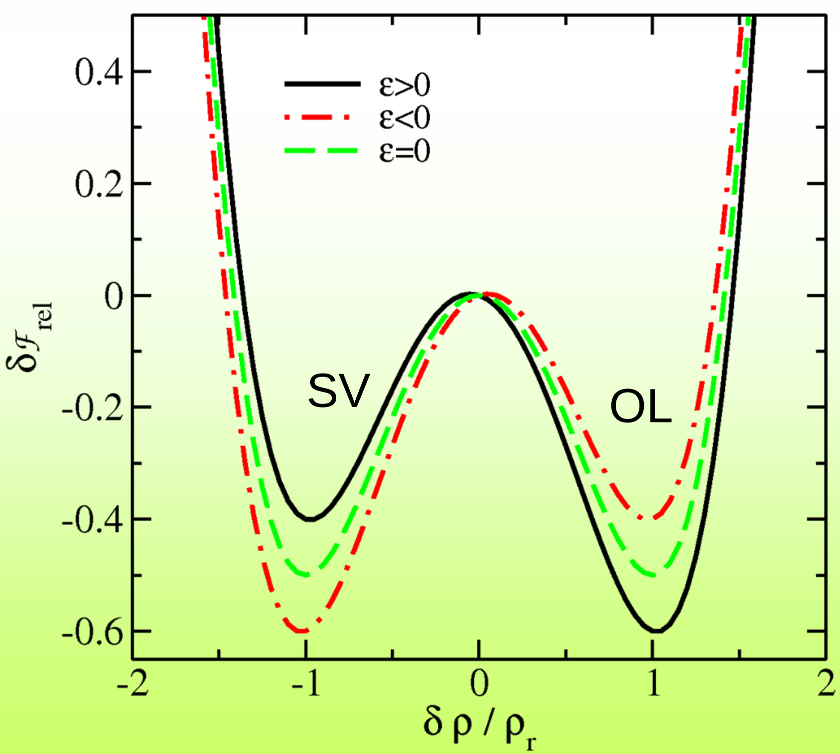
$$\delta P = \rho \frac{\delta[F_L(T,\delta\rho)]}{\delta(\delta\rho)}\Big|_T$$

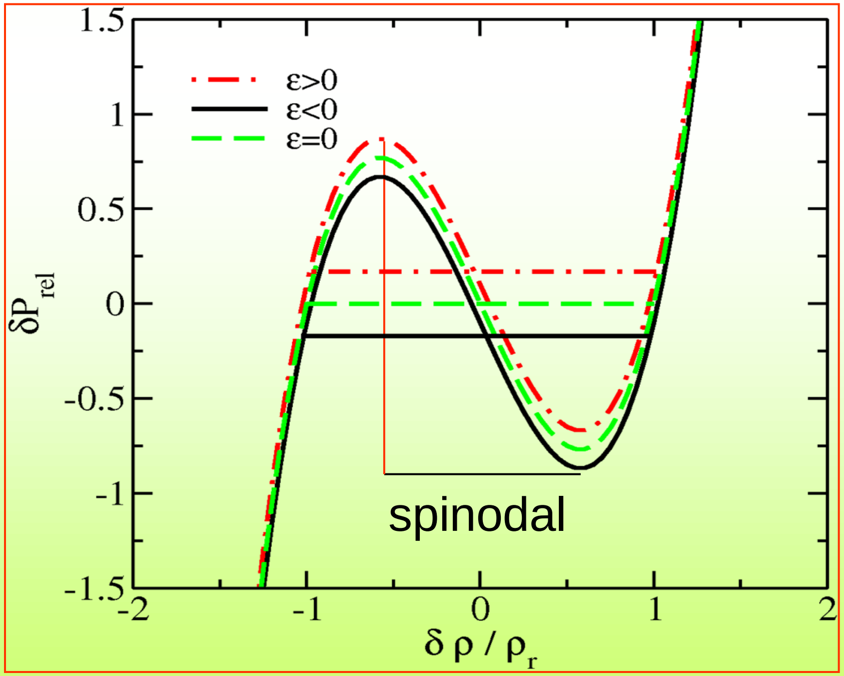
ρ=mn

$$\delta\mathcal{F}_{rel} = \delta\mathcal{F}_L / \mathcal{F}_L(T_{cr}, \rho_{cr}) \qquad \delta P_{rel} = \delta P / P(T_{cr}, \rho_{cr})$$

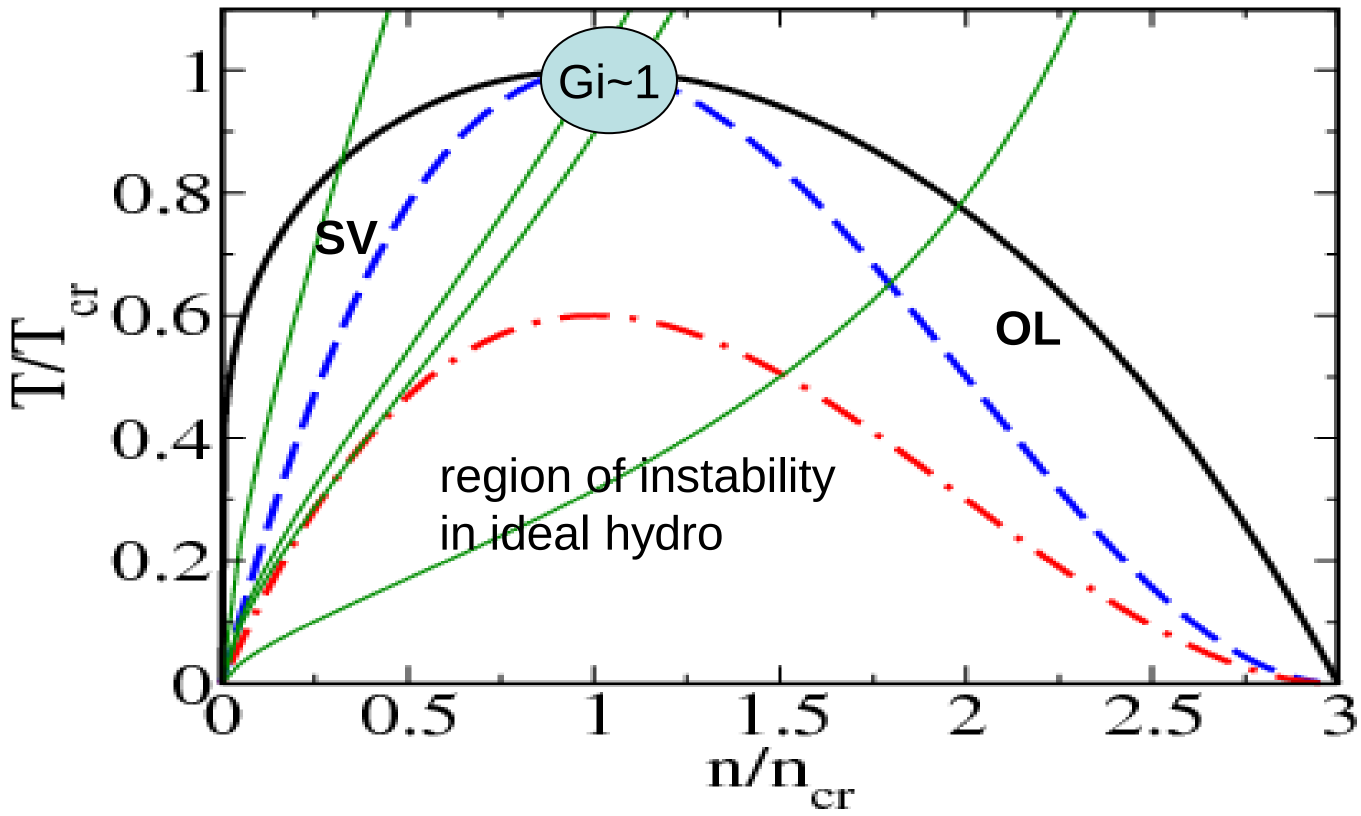
at critical point

isotherms





Constant entropy trajectories



- - - isothermal spinodal, - . - . - isoentropic spinodal, ▬ Maxwell construction

T_{max} =0.6 T_{cr} for van der Waals EoS

Is fluctuation region broad or narrow?

For the hadron quark phase transition we estimate

$$\mathrm{Gi} \gtrsim 1.4(100\ \mathrm{MeV\ fm}^{-2}/\sigma_0)^6$$

σ_0 is surface tension

For the liquid-gas transition

$$\mathrm{Gi} \sim 10(T_{cr}/18.6\ \mathrm{MeV})^6$$

in both cases fluctuation region might be very broad

In thermodynamical description fluctuation effects should be incorporated in EoS.

Mean field vs. fluctuations

For $\mathrm{Gi} \gtrsim 1$

stationary system is not uniform due to permanently creating and decaying fluctuations (it looks like a sup right before boiling)

For **dynamical system** (*like fireball in HIC*)

since typical time for developing of critical fluctuations is large, $t_0 \sim |T-T_{cr}|^{-1}$ (at least near critical point), fluctuations may have not sufficient time to appear ⇨

One can consider **mean field EoS** provided fireball evolution time $t_{evol} < t_0$

(argument by Zeldovich, Mikhailov UFN (1987) in description of explosion phenomena)

Dynamics of I order phase transition near critical point

viscosities

From Navier-Stokes and continuity equations neglecting u^2 terms:

$$-\frac{\partial^2\delta\rho}{\partial t^2} = \Delta\left[c\Delta\delta\rho + \lambda v^2\delta\rho - \lambda(\delta\rho)^3 + \epsilon - \rho_{\rm r}^{-1}\left(\frac{4}{3}\eta_{\rm r} + \zeta_{\rm r}\right)\frac{\partial\delta\rho}{\partial t}\right]$$

See D.V. Phys.Scripta 47 (1993) 333 $\quad \delta\rho = \rho - \rho_{\rm r}$

In dimensionless variables

$$-\beta\frac{\partial^2\psi}{\partial\tau^2} = \Delta_\xi\left(\Delta_\xi\psi + 2\psi(1-\psi^2) + \widetilde{\epsilon} - \frac{\partial\psi}{\partial\tau}\right)$$

$$\delta\rho = v\,\psi, \quad \xi_i = x_i/l, \quad \tau = t/t_0$$

$$l = \left(\frac{2c}{\lambda v^2}\right)^{1/2}, \quad t_0 = \frac{2(\frac{4}{3}\eta_{\rm r} + \zeta_{\rm r})}{\lambda v^2\rho_{\rm r}}, \quad \widetilde{\epsilon} = \frac{2\epsilon}{\lambda v^3} \quad \beta = \frac{c\rho_{\rm r}^2}{(\frac{4}{3}\eta_{\rm r} + \zeta_{\rm r})^2}$$

$$v \propto |T - T_{\rm cr}|^{1/2} \longrightarrow t_0 \propto |T - T_{\rm cr}|^{-1}$$

processes in the vicinity of the critical point prove to be very slow

Peculiarities of hydro- description

Eq. is the 2-order in time derivatives -- beyond the standard Ginzburg-Landau description where:

$$- \rho_{\mathrm{r}}^{-2}\left(\tilde{d}\eta_{\mathrm{r}}+\zeta_{\mathrm{r}}\right)\frac{\partial\delta\rho}{\partial t}=\frac{\delta[F(T,\delta\rho)]}{\delta(\delta\rho)}|_{T}.$$

thermodynamical force

However for a produced fluctuation two initial conditions should be fulfilled

$$\delta\rho(t=0,\vec{r})=\delta\rho(0,\vec{r}),\qquad \frac{\partial\delta\rho(t,\vec{r})}{\partial t}|_{t=0}\simeq 0$$

initial stage of fluctuation dynamics is not described in GL approximation; at large t one can use the GL description

Flow-experiments at RHIC indicate on very low viscosity

Conformal theories show minimum η/s ~1/4π:

η/s ratio is under extensive discussion in the literature

However η/s does not appear in equations of motion for fluctuations

Dynamics of the density mode is controlled by another parameter β, which enters together with the second derivative in time. This parameter is expressed in terms of the **surface tension** and the **viscosity**

$$\beta = \frac{\sigma_0^2\, m}{32\, T_{\rm cr}\, [\frac{4}{3}\,\eta_{\rm r} + \zeta_{\rm r}]^2}$$

$$\sigma_0^2 = 32\, m\, \rho_{\rm cr}^2 T_{\rm cr}\, c$$

surface tension

The **larger viscosity** and the **smaller surface tension**,

the effectively **more viscous** is the fluidity of seeds.

β<<1 is the regime of effectively viscous fluidity

β >>1 is the regime of perfect fluidity

for liquid-gas phase transition β~0.01;

for H-QGP phase transition: β~0.02-0.2 , even for η/s ~1/4π:

Effectively very viscous fluidity of density fluctuations in the course of the phase transition!

Equation for the density fluctuation is supplemented by

the heat transport equation

for the variations of the entropy and temperature

For small u:

$$T_r\left[\partial_t \delta s - s_r (n_r)^{-1} \partial_t \delta n\right] = \kappa_r \Delta \delta T.$$

The variation of the temperature is related to the variation of the entropy density $s[n, T]$ by

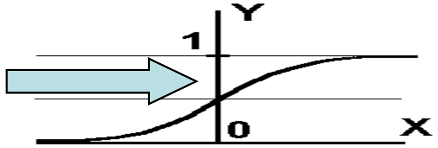
$$\delta T \simeq T_r (c_{V,r})^{-1} \left(\delta s - (\partial s / \partial n)_{T,r} \delta n\right),$$

Stage $t_\rho >> t_T$, limit of a large thermal conductivity, seeds evolve at almost constant T

$$\delta n(t,r) \simeq \frac{v(T)}{m}\left[\pm \mathrm{th}\,\frac{r-R_n(t)}{l} + \frac{\epsilon}{2\lambda_{cr} v^3(T)}\right] + (\delta n)_{cor}$$

$(\delta n)_{cor}$ is a small correction responsible for the baryon number conservation

stable seed interior



metastable exterior

$$\frac{\beta t_0^2}{2}\frac{d^2 R_n}{dt^2} = \frac{3\epsilon}{2\lambda_{cr} v^3(T)} - \frac{2l}{R_n} - \frac{t_0}{l}\frac{dR_n}{dt}$$

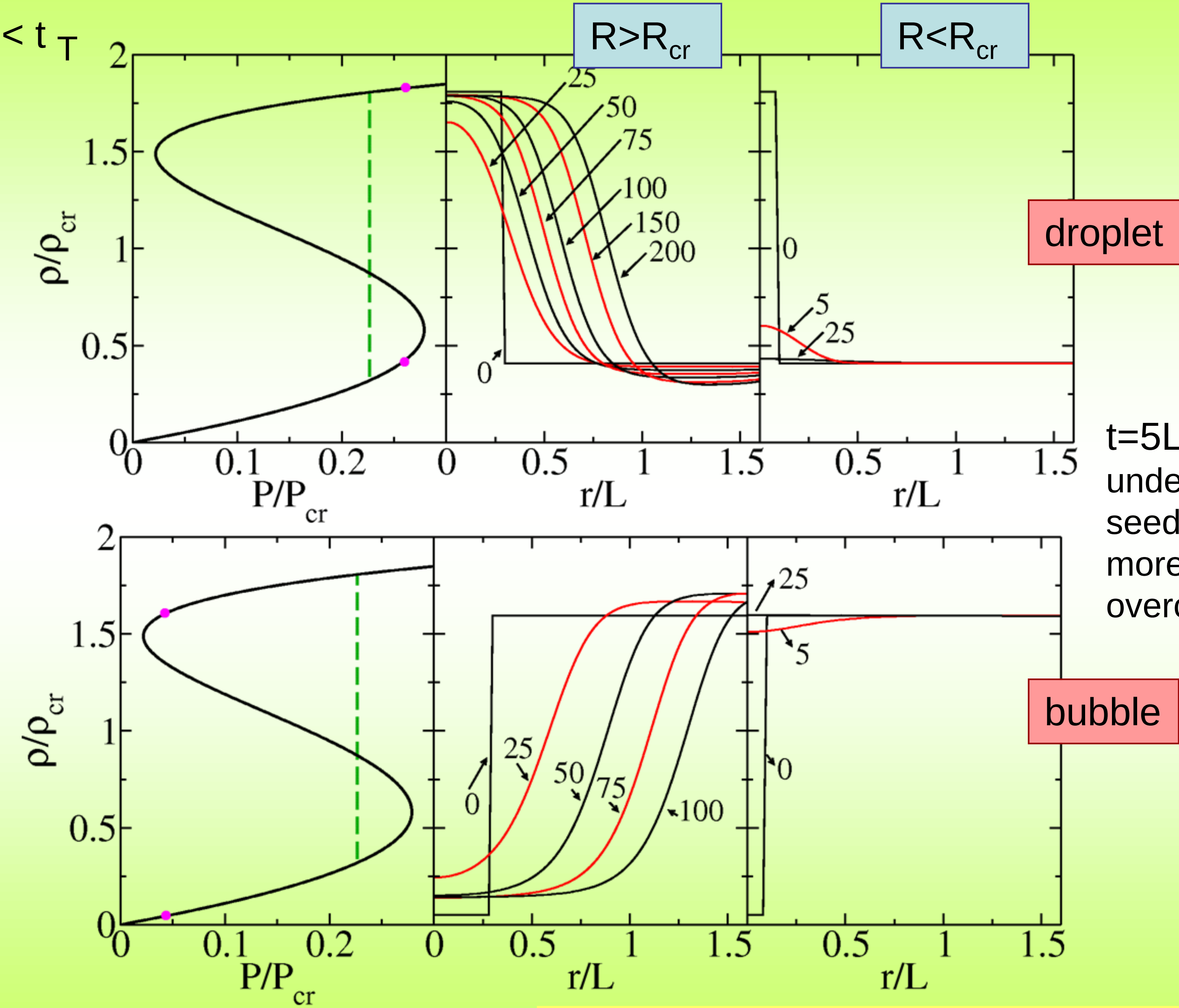
$R_{cr} = 4l\lambda_{cr} v^3(T)/(3\epsilon)$. First the bubble/droplet size $R_n(t) > R_{cr}$ grows with an acceleration and then it reaches a steady grow regime with a constant velocity $u_{as} = \frac{3\epsilon l}{\lambda_{cr} v^3(T) t_0} \propto \gamma_\epsilon |T_{cr} - T|^{1/2}$. (for $t >> t_0 \beta$)

$$\delta s = \left(\frac{\partial s}{\partial n}\right)_T \left\{\frac{v(T)}{m}\left[\pm \mathrm{th}\frac{r-R_n(t)}{l} + \frac{\epsilon}{2\lambda_{cr} v^3(T)}\right] + (\delta n)_{cor}\right\}$$

seeds with $R<R_{cr}$ disolve

Hadron-QGP phase transition: droplet/bubble evolution from metastable phases

For $t_\rho < t_T$

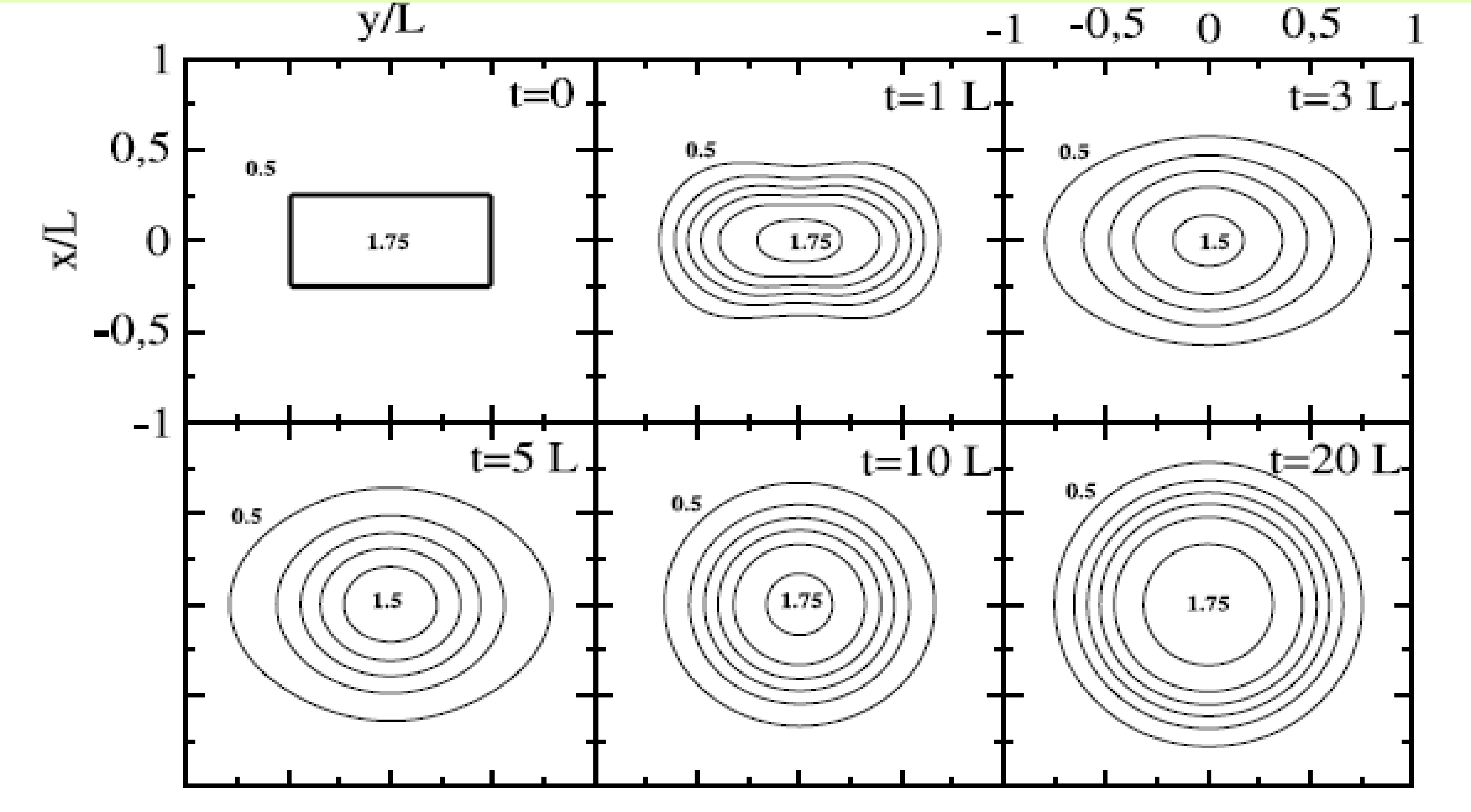


$t=5L=25$ fm, undercritical seeds dissolve more rapidly, overcritical-slowly

$(T_{cr} - T)/T_{cr} = 0.15$; $T_{cr}=162$ MeV; $L=5$ fm; $\beta = 0.2$

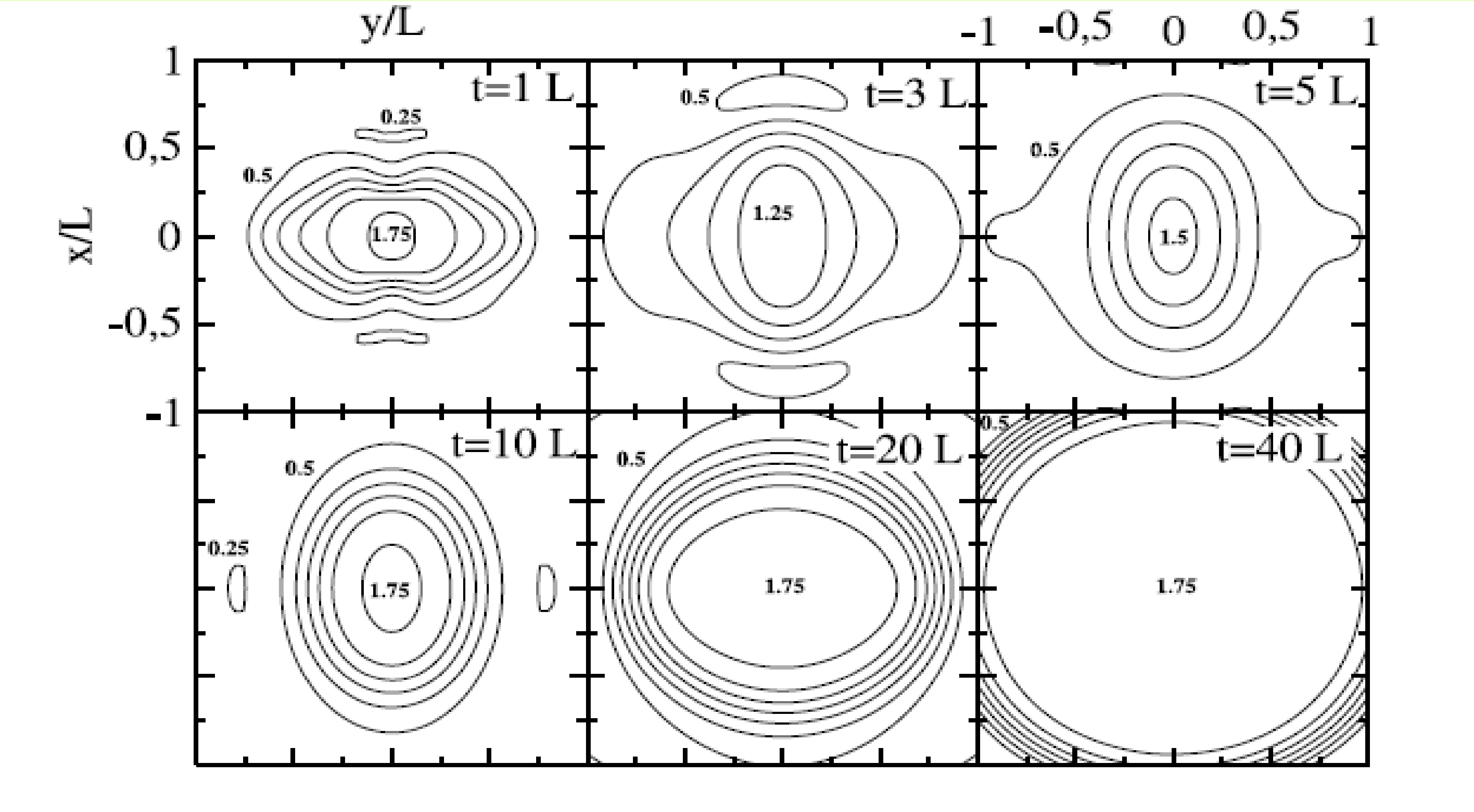
Change of the seed shape with time

Iso-lines of the density n/n_{cr} with increment 0.25



Initially anisotropic droplet slowly acquires spherical form $\beta = 0.1 \ll 1$

Change of the seed shape with time



For almost perfect fluid the process is more peculiar and still more slow $\beta=1000>>1$

Limit of zero thermal conductivity

$$\delta n(t,r) \simeq \frac{v(\tilde{s})}{m}\left[\pm \mathrm{th}\,\frac{r-R_n(t)}{l} + \frac{\epsilon}{2\lambda_{P,max} v^3(\tilde{s})}\right] + (\delta n)_{cor},$$

but now at fixed entropy per baryon rather than at fixed T

$$\delta\tilde{s} = 0 = (\delta s n_{P,max} - s_{P,max}\delta n)/n^2_{P,max}$$

An illustration: a metastable state with growing droplets of overcritical size

(occurs provided $t_{evol} >> t_0$)

inhomogeneous matter

An illustration: a metastable state

(at $\mathbf{t}_{\text{evol}} \ll \mathbf{t}_{0}$)

overcritical droplets/bubbles have no time to be prepared and to grow

almost homogeneous matter

Our calculations show that most probably namely this case is realized in actual HIC when trajectory passes metastable OL or SV regions

Instabilities in spinodal region

aerosol-like mixture of bubbles and droplets (**mixed phase**)

$$\delta n = \delta n_0 \exp[\gamma t + i\,\mathbf{pr}],$$

$$\delta s = \delta s_0 \exp[\gamma t + i\,\mathbf{pr}],$$

$$T = T_> + \delta T_0 \exp[\gamma t + i\,\mathbf{pr}]$$

$T_>$ is the temperature of the uniform matter

From equations of non-ideal hydro:

$$\gamma^2 = -p^2\left[u_T^2 + \frac{(\tilde{d}\eta + \zeta)\gamma}{mn} + cp^2 + \frac{u_{\tilde{s}}^2 - u_T^2}{1 + \kappa p^2/(c_V \gamma)}\right]$$

$u_{\tilde{s}}^2 = m^{-1}(\partial P/\partial n)_{\tilde{s}}$ and $u_T^2 = m^{-1}(\partial P/\partial n)_T$ are speeds of sound

V.Skokov, D.V. , arXiv 0811.3868, JETP Lett. 90 (2009) 223; Nucl. Phys. A828 (2009) 401; A846 (2010); J. Randrup, PRC79 (2009) 024601, arXiv arXiv:1007.1448

Three solutions

For small momenta:

$$\gamma_{1,2} = \pm i u_{\tilde{s}} p + \left[\frac{\kappa}{c_V}\left(\frac{u_T^2}{u_{\tilde{s}}^2} - 1\right) - \frac{\tilde{d}\eta + \zeta}{mn}\right]\frac{p^2}{2},$$

Density mode

$$\gamma_3 = -\frac{\kappa u_T^2 p^2}{u_{\tilde{s}}^2 c_V}\left[1 - \frac{u_T^2 - u_{\tilde{s}}^2}{u_{\tilde{s}}^2 u_T^2}\left(c + \frac{\kappa u_T^4}{u_{\tilde{s}}^2 c_V^2} - \frac{(\tilde{d}\eta + \zeta)\kappa u_T^2}{mn c_V u_{\tilde{s}}^2}\right)p^2\right]$$

Thermal mode

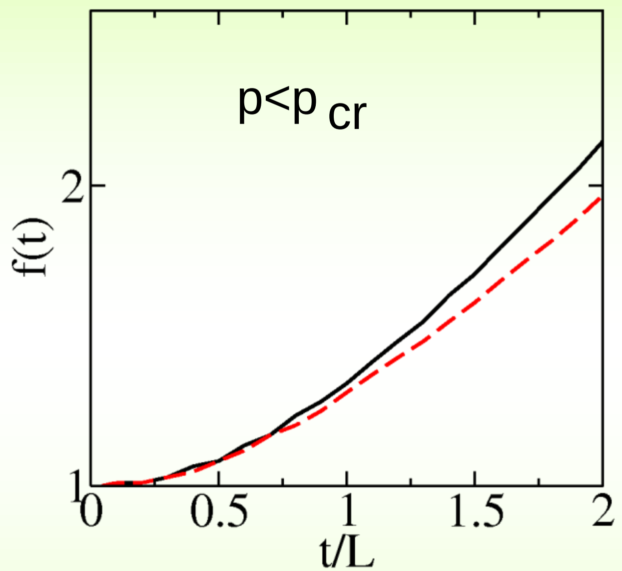
Does instability arise after the trajectory crosses the isothermal spinodal line or adiabatic one?

Limit of large thermal conductivity

$\kappa \gg \nu c_V \sqrt{c}$, $\nu = (u_{\tilde{s}}^2 - u_T^2)/(-u_T^2)$

instability arises for the density mode, when trajectory crosses isothermal spinodal line

amplitude of the growing modes



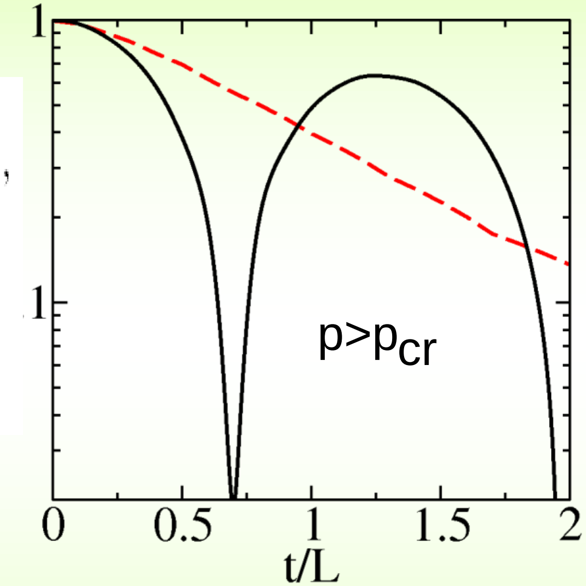
for most rapidly growing modes:

$$\gamma_m = \frac{(-u_T^2) m n_{cr}}{(2\sqrt{\beta}+1)(\tilde{d}\eta + \zeta)},$$

$$p_m^2 = \frac{(-u_T^2)\sqrt{\beta}}{(2\sqrt{\beta}+1)c}.$$

⇨ $R_m \sim 1/p_m$

oscillating modes for β>>1



β=0.1 dash line, β=10 solid line

t=2L=10 fm

$$\delta T_0 = \delta n_0 \frac{Ts[1 - n(\partial s/\partial n)_T/s]}{c_V n[1 + \kappa/(\sqrt{c}\, c_V)]}$$

Far from critical point time evolution is rapid –effect of warm Champagne

Limit of small thermal conductivity $\kappa \ll \nu c_V \sqrt{c}$

Instability arises when trajectory crosses isothermal spinodal line, but now for the thermal mode

$$p_m^2 \simeq -u_T^2/(2c), \qquad \gamma_{3m} = \gamma_3(p_m) \simeq \frac{\kappa u_T^4}{4 c c_V u_{\tilde{s}}^2}.$$

Limit of $\kappa = 0$ (like in ideal hydro. calculations) **is special: no thermal mode**

Instability arises for the density mode far below T_{cr}, only when trajectory crosses adiabatic spinodal line

$$\gamma^2 = -p^2\left[u_{\tilde{s}}^2 + \frac{(\tilde{d}\eta + \zeta)\gamma}{mn} + cp^2\right].$$

Solution is similar to that for the density modes at large κ, but now the entropy per baryon is fixed rather than the temperature.

ideal hydro *(at least without taking of special care)* **cannot correctly describe dynamics of the first-order phase transition.**

Values of viscosities and thermal conductivity

There exist many (although very different) estimates of viscosities in hadron and quark matter and almost no appropriate estimates of the heat conductivity

Viscosities in SHMC model: hadron phase

A.Khvorostukhin, V.Toneev, D.V. Nucl.Phys. A845:106 (2010)

(From V.Toneev presentation)

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm MF} + \mathcal{L}_{\rm ex}$$

$$\mathcal{L}_{\rm bar} = \sum_{b\in\{\rm bar\}} \left[i\bar{\Psi}_b \left(\partial_\mu + i\, g_{\omega b}\, \chi_\omega\, \omega_\mu \right) \gamma^\mu\, \Psi_b - m_b^*\, \bar{\Psi}_b\, \Psi_b \right].$$

$\{b\} = N(938),\ \Delta(1232),\ \Lambda(1116),\ \Sigma(1193),$ $\Xi(1318),\ \Sigma^*(1385),\ \Xi^*(1530),$ and $\Omega(1672),$

$$m_b^*/m_b = \Phi_b(\chi_\sigma \sigma) = 1 - g_{\sigma b}\, \chi_\sigma\, \sigma / m_b\,,\ b \in \{b\}$$

$$\mathcal{L}_{\rm MF} = \frac{\partial^\mu \sigma\ \partial_\mu \sigma}{2} - \frac{m_\sigma^{*2}\, \sigma^2}{2} - U(\sigma) - \frac{\omega_{\mu\nu}\, \omega^{\mu\nu}}{4} + \frac{m_\omega^{*2}\, \omega_\mu \omega^\mu}{2}$$

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu\ .$$

isospin-symmetric hadronic matter

$$m_m^*/m_m = |\Phi_m(\chi_\sigma \sigma)|\,, \quad \{m\} = \sigma, \omega\,.$$

Brown-Rho scaling ansatz

$$\Phi = \Phi_N = \Phi_\sigma = \Phi_\omega = \Phi_\rho = 1 - f, \quad f = g_{\sigma N}\ \chi_\sigma\ \sigma / m_N$$

$$U = m_N^4 \left(\frac{b}{3} f^3 + \frac{c}{4} f^4 \right).$$

$$\mathcal{L}_{\rm ex} = \sum_{\rm bos \in \{ex\}} \mathcal{L}_{\rm bos}$$

$$\{\rm ex\} = \pi; K, \bar{K}; \eta(547); \sigma', \omega', \rho'; K^{*\pm,0}(892), \eta'(958), \phi(1020)$$

Two phase model

Quark-gluon phase, HQB model: the IG of the massive quarks, antiquarks and gluons

$$\varepsilon^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) = \sum_{a\in\{q\}} \varepsilon_a^{\mathrm{IG}}(T,\mu_a) + B$$

$$P^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) = \sum_{a\in\{q\}} P_a^{\mathrm{IG}}(T,\mu_a) - B$$

$$n_{\mathrm{bar}}^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) = \sum_{a\in\{q\}} b_a\, n_a^{\mathrm{IG}}(T,\mu_a)$$

$$n_{\mathrm{str}}^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) = \sum_{a\in\{q\}} s_a\, n_a^{\mathrm{IG}}(T,\mu_a)$$

$\{q\} = q, \bar{q}, g$

Gibbs conditions:

$$P^{\mathrm{SHMC}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) = P^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) \ ,$$

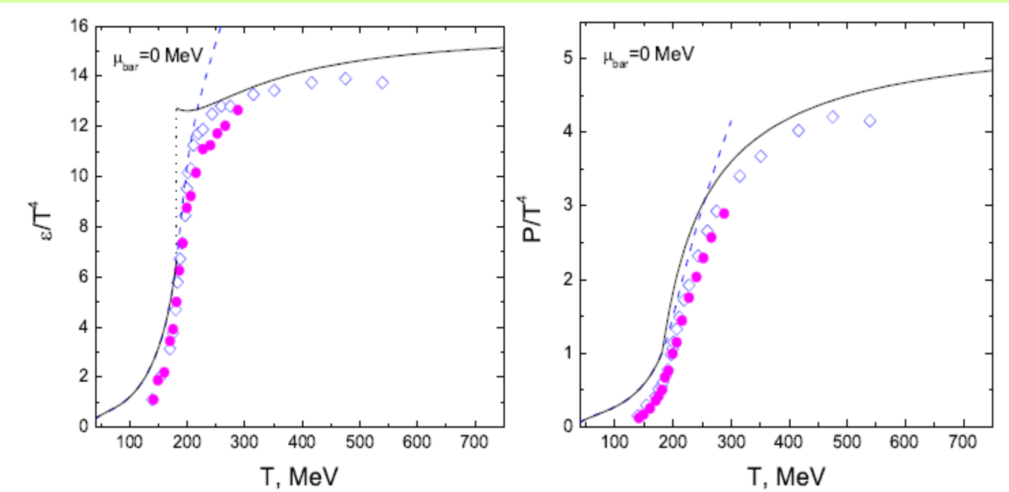
$$n_{\mathrm{bar}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) = \alpha\, n_{\mathrm{bar}}^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) + (1-\alpha)\, n_{\mathrm{bar}}^{\mathrm{SHMC}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}})$$

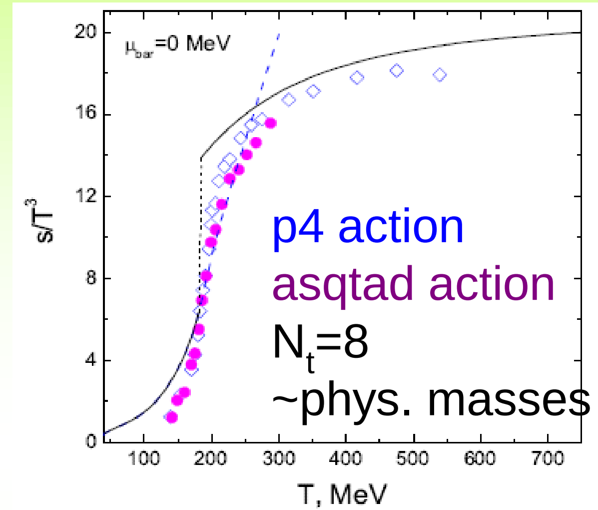
$$0 = \alpha\, n_{\mathrm{str}}^{\mathrm{HQB}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) + (1-\alpha)\, n_{\mathrm{str}}^{\mathrm{SHMC}}(T,\mu_{\mathrm{bar}},\mu_{\mathrm{str}}) \ ,$$

$$\alpha = V^{\mathrm{HQB}}/V$$

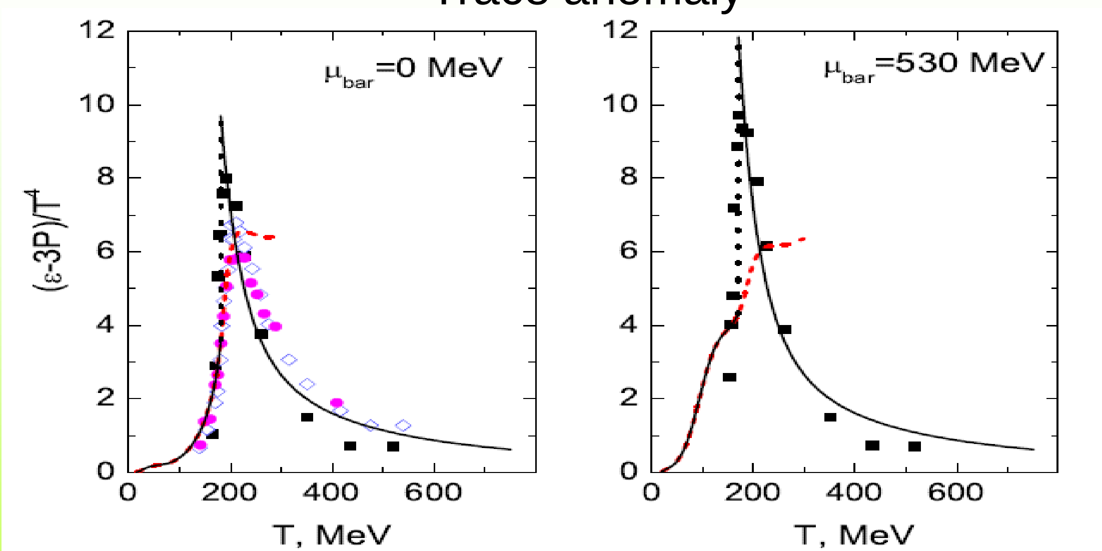
Comparison of EoS with lattice data

energy density pressure entropy





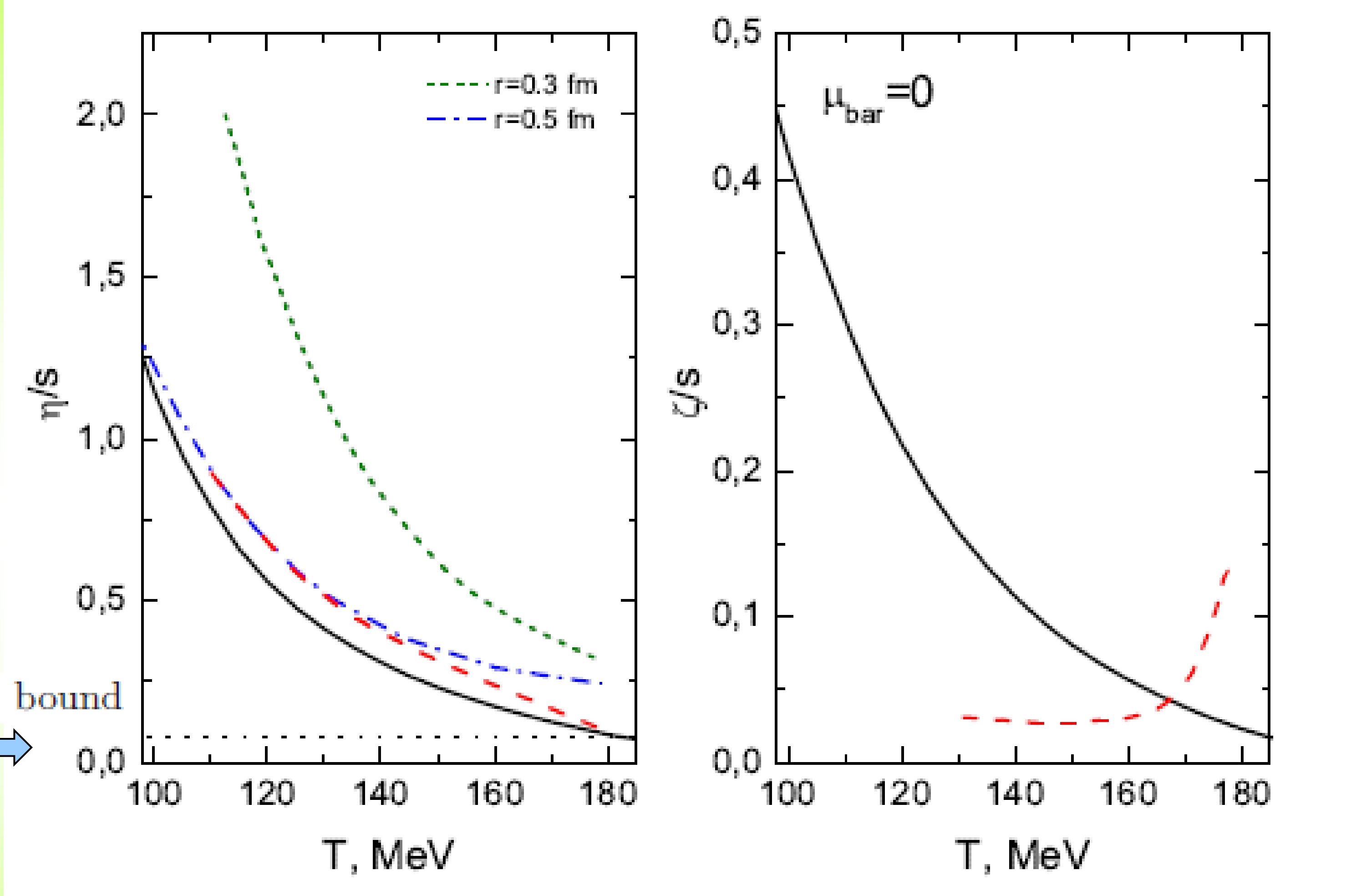
Trace anomaly



A.Bazavov et al., Phys. Rev. **D80,** 014504 (2009)

Z.Fodor et al., Phys. Lett. **B568**, 73 (2003)

Viscosity behavior for μ_{bar}=0



lower AdS/CFT bound $\eta/s = 1/4\pi$

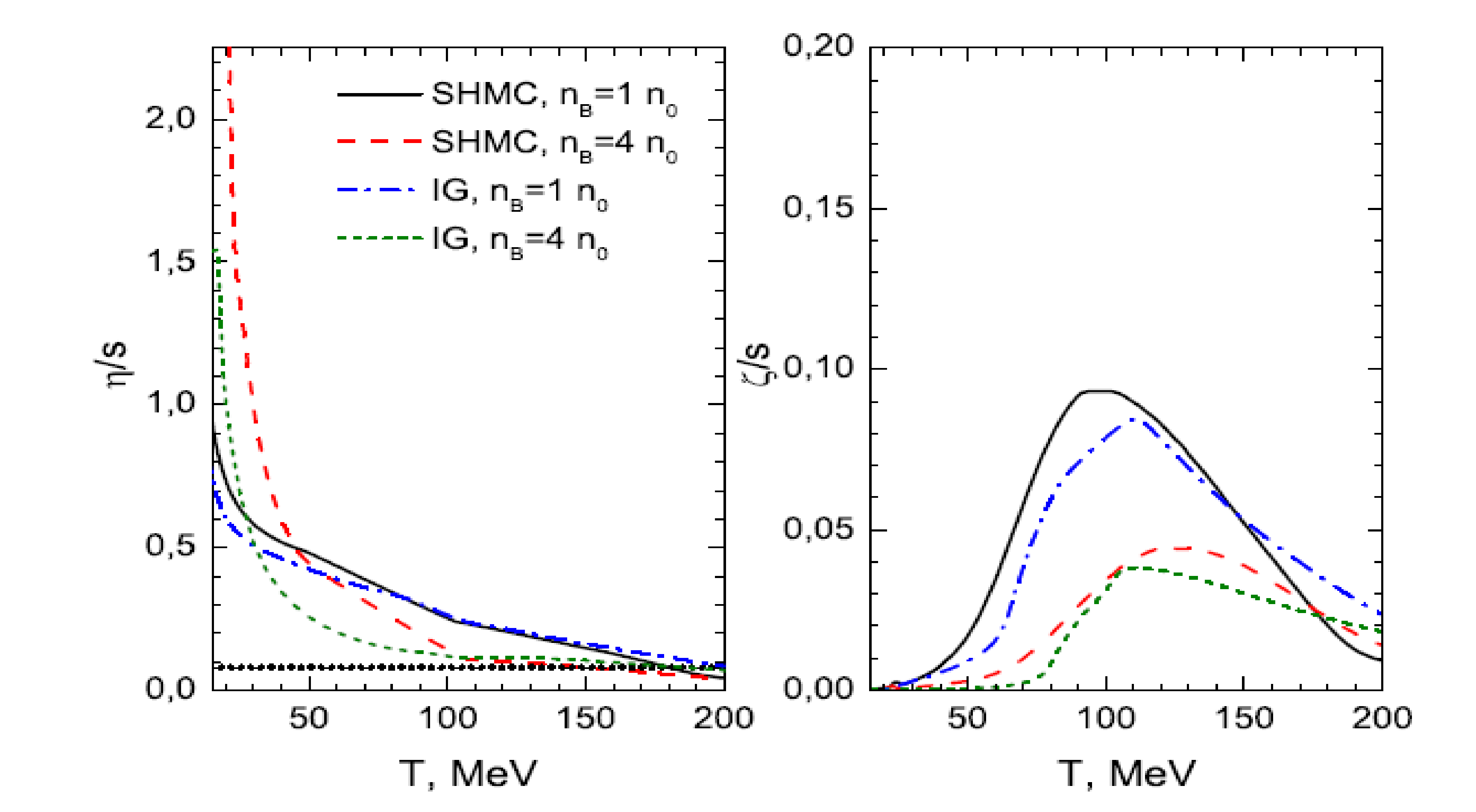
Excluded-volume hadron gas model: M. Gorenstein et al., Phys. Rev. **C77**, 024911 (2008)

$$\eta = \frac{5}{64\sqrt{\pi}} \frac{\sqrt{mT}}{r^2}$$

Resonance gas with Hagedorn states: J.Naronha-Hostler et al., Phys. Rev. Lett. **103**, 172302 (2009)

$$\rho(m) = m^{-a} \ \exp(m/T_{\rm H})$$

Viscosities in SHMC model for baryon enriched matter



Conclusions to sect. Hydro

The larger viscosity and the smaller surface tension the effectively more viscous is the fluidity

Anomalies in thermal fluctuations near CEP (which are under extensive discussion) may have not sufficient time to develop

→ argument in favor of mean field EoS

Thus T_{cr} calculated in thermal models might be significantly higher than the value which may manifest in fluctuations in HIC

Heat transport effects play important role

Effects of spinodal decomposition can be easier observed since they require a shorter time to develop

Since in reality κ is not zero, spinodal instabilities start to develop when the trajectory crosses the isothermal spinodal line rather than the adiabatic one as it were in ideal hydro, i.e. at much higher T. This favors observation of manifestation of spinodal decomposition in the H-QGP phase transition in HIC

Concluding:

- One may hope to observe non-monotonous behavior of different observables in HIC due to manifestation of non-trivial fluctuation effects (especially of **spinodal decomposition** at I order hadron-quark phase transition) at monotonous increase of collision energies:

collision energy increase with a certain energy step will be possible at FAIR and NICA

What could be a final state in stationary system?

Mixed phase vs. Maxwell construction

- One conserved charge – Maxwell construction.
- Otherwise a possibility of mixed phase.

Baryon charge conservation, strangeness in strong interactions, lepton charge in weak interactions

Consider example of stationary **pasta phases** in cold neutron stars

Bulk calculations: Gibbs equilibrium conditions

- In Maxwell construction $P_I = P_{II}$, $\mu_{bar,I} = \mu_{bar,II}$,

But from local charge neutrality condition on Maxwell construction

$\mu_{Q,I}^{loc} \neq \mu_{Q,II}^{loc}$ since charge densities in phases I and II are different

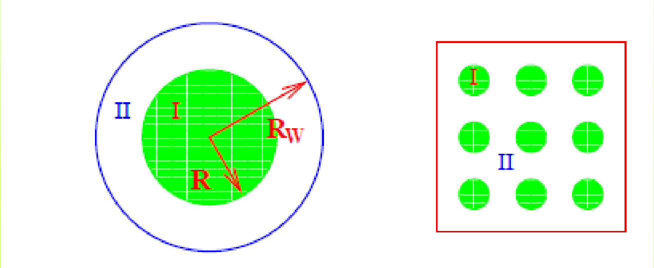
N. Glendenning, Phys Rev. D46 (1992) 1274: **must be** $\mu_{Q,I} = \mu_{Q,II}$

Charge can be conserved only globally

that allows to fulfill all Gibbs conditions. *He concluded:*

Always should exist a structured mixed phase consisted of neutral Wigner-Seitz cells (predicted by D.Rawenhall, Ch.Pethick, J. Wilson, Phys.Rev.Lett . 50 (1983) 2066)

Picture for d=3: droplets:



Finite size Coulomb+surface effects were disregarded. However they should be properly incorporated

Finite size effects
on example of droplets (D)

For a given volume fraction factor $f = (R/R_W)^3$, the total energy E may be written as the sum of the volume energy E_V, the Coulomb energy E_C and the surface energy E_S,

$$E = E_V + E_C + E_S. \tag{1}$$

We further assume, for simplicity, that baryon number (ρ_B^α) and charge (ρ_Q^α) densities are uniform in each phase α, $\alpha = I, II$. Then, E_V can be written as $E_V/V_W = f\epsilon^I(\rho_B^I) + (1-f)\epsilon^{II}(\rho_B^{II})$ in terms of the energy densities ϵ^α, $\alpha = I, II$. The surface energy E_S may be represented as $E_S/V_W = f \times 4\pi\sigma/R$ in terms of the surface tension σ. The Coulomb energy E_C is given by

$$E_C/V_W = f \times \frac{16\pi^2}{15}\left(\rho_Q^I - \rho_Q^{II}\right)^2 R^2. \tag{2}$$

The optimal value of R_D is determined by the minimum condition,

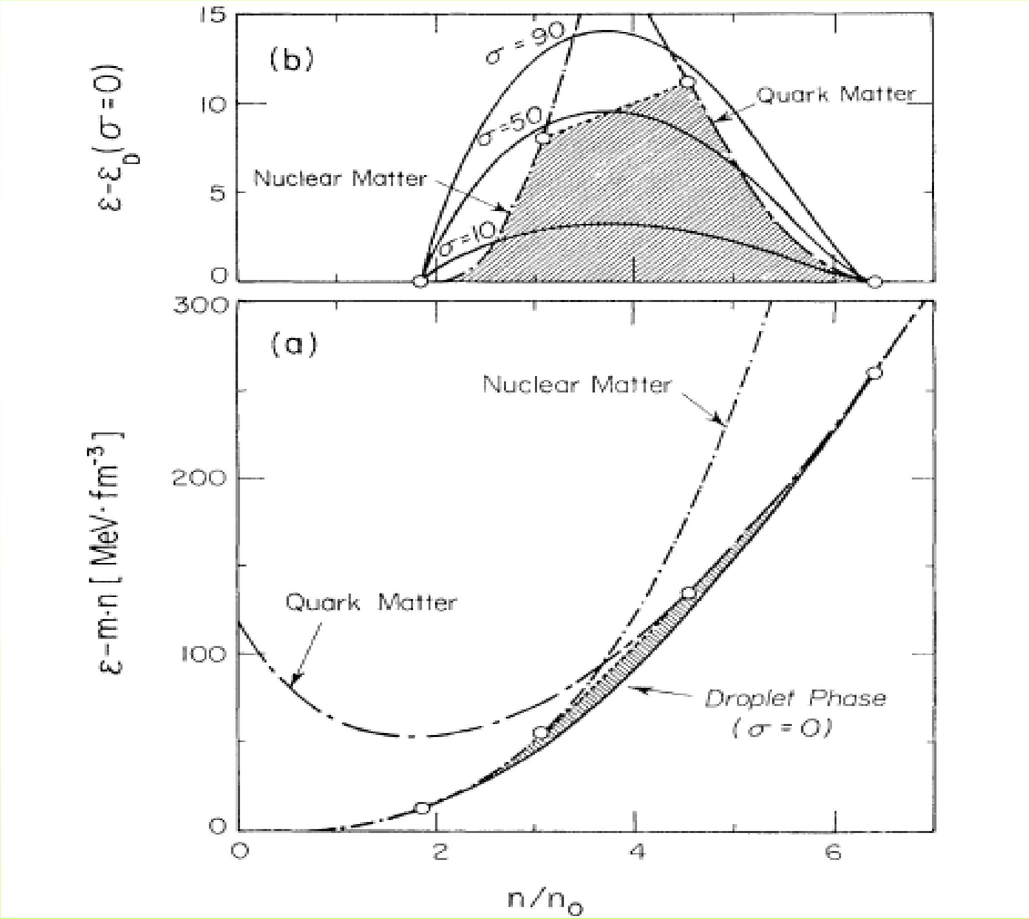
$$\left.\frac{\partial(E/V_W)}{\partial R}\right|_f = 0, \tag{3}$$

$R_D \sim \sigma^{1/3}/|\delta\rho|^{2/3}$ grows with σ

Finite size effects

on example of hadron-quark phase transition

- H. Heiselberg, Ch. Pethick, E. Staubo, Phys.Rev.Lett. 70 (1993) 1355.



(b) Energy density of droplet phase relative to values for different σ.

Only in hatched area (for σ< 70 MeV/fm^2) **droplet phase is energetically favorable,**

in disagreement with above statement that mixed phase should always exit.

Solution of puzzle: additional equation for the electric potential

D.V., M. Yasuhira, T.Tatsumi Nucl.Phys.A723 (2003)291

The thermodynamic potential enjoys the invariance under a gauge transformation, $V(\vec{r}) \to V(\vec{r}) - V^0$ and $\mu_i^\alpha \to \mu_i^\alpha + N_i^{\mathrm{ch},\alpha} V^0$, with an arbitrary constant V^0. Hence the chemical potential μ_i^α acquires physical meaning only *after gauge fixing* [c] .

V must fulfill Poisson eq. disregarded in bulk treatment of the mixed phase

$$\Delta V^\alpha(\vec{r}) = 4\pi e^2 \rho^{\mathrm{ch},\alpha}(\vec{r}) \qquad (8)$$

when we say $\mu_e^I \neq \mu_e^{II}$ within the Maxwell construction, it means nothing but the difference in the electron number density n_e in two phases, $n_e^I \neq n_e^{II}$; this is because $n_e = \mu_e^3/(3\pi^2)$, if the Coulomb potential is *absent*. Once the Coulomb potential is taken into account, using eq. (8), n_e can be written as

$$n_e^\alpha = \frac{(\mu_e^\alpha - V^\alpha)^3}{3\pi^2}. \qquad (10)$$

Screening effect

For $R_D \gg \lambda$ (Debye size) the Coulomb energy is reduced to the surface one:

The full surface tension $\sigma_{\rm tot}^{\rm spher}$ then renders

$$\sigma_{\rm tot}^{\rm spher} = \sigma + \sigma_V = \sigma - \lambda_D^{\rm I} \frac{\beta_0 \alpha_0 [\alpha_0 + 4/3]}{3(1+\alpha_0)^2}.$$

For $\sigma+\sigma_V>0$ – **Maxwell construction instead of mixed phase**

Have we mixed phase or Maxwell construction depends on the value of surface tension.

Nuclear pasta *(in RMF model)*

structure of the inner crust of neutron stars

Thermodynamic potential

$$\Omega = \Omega_B + \Omega_M + \Omega_e,$$

$$\Omega_B = \int d^3r \left[\sum_{i=p,n} \left(\frac{2}{(2\pi)^3} \int_0^{k_{Fi}} d^3k \sqrt{m_B^{*2} + k^2} - \rho_i \nu_i \right) \right],$$

$$\Omega_M = \int d^3r \left[\frac{(\nabla\sigma)^2}{2} + \frac{m_\sigma^2 \sigma^2}{2} + U(\sigma) - \frac{(\nabla\omega_0)^2}{2} - \frac{m_\omega^2 \omega_0^2}{2} - \frac{(\nabla\rho_0)^2}{2} - \frac{m_\rho^2 \rho_0^2}{2} \right],$$

$$\Omega_e = \int d^3r \left[-\frac{1}{8\pi e^2} (\nabla V_{\rm Coul})^2 - \frac{(V_{\rm Coul} - \mu_e)^4}{12\pi^2} \right],$$

$$\nu_p = \mu_B - \mu_e + V_{\rm Coul} - g_{\omega N}\omega_0 - g_{\rho N}\rho_0, \qquad \nu_n = \mu_B - g_{\omega N}\omega_0 + g_{\rho N}\rho_0,$$

$$m_B^* = m_B - g_{\sigma N}\sigma,$$

The parameter set is chosen to reproduce nuclear matter saturation properties.

Nuclear pasta *in RMF model*

Toshiki Maruyama, T.Tatsumi, D.V., T.Tanigawa, S. Chiba, Ph.Rev.C72(2005) 015802.

Equations of motion From $\frac{\delta\Omega}{\delta\phi_i(\mathbf{r})} = 0\ (\phi_i = \sigma, \rho_0, \omega_0, V_{\mathrm{Coul}}, \rho_n, \rho_p, \rho_e,)$, we get

$$
\begin{aligned}
-\nabla^2\sigma + m_\sigma^2\sigma &= -\frac{dU}{d\sigma} + g_{\sigma N}(\rho_n^{(s)} + \rho_p^{(s)}) \\
-\nabla^2\omega_0 + m_\omega^2\omega_0 &= g_{\omega N}(\rho_p + \rho_n) \\
-\nabla^2\rho_0 + m_\rho^2\rho_0 &= g_{\rho N}(\rho_p - \rho_n) \\
\nabla^2 V_{\mathrm{Coul}} &= 4\pi e^2 \rho_{\mathrm{ch}} \\
\mu_n = \mu_B &= \sqrt{k_{Fn}^2 + {m_B^*}^2} + g_{\omega N}\omega_0 - g_{\rho N}\rho_0 \\
\mu_p = \mu_B - \mu_e &= \sqrt{{k_{Fp}}^2 + {m_B^*}^2} + g_{\omega N}\omega_0 + g_{\rho N}\rho_0 - V_{\mathrm{Coul}} \\
\rho_e &= -(\mu_e - V_{\mathrm{Coul}})^3/3\pi^2
\end{aligned}
$$

(charge density $\rho_{\mathrm{ch}} = \rho_p + \rho_e$)

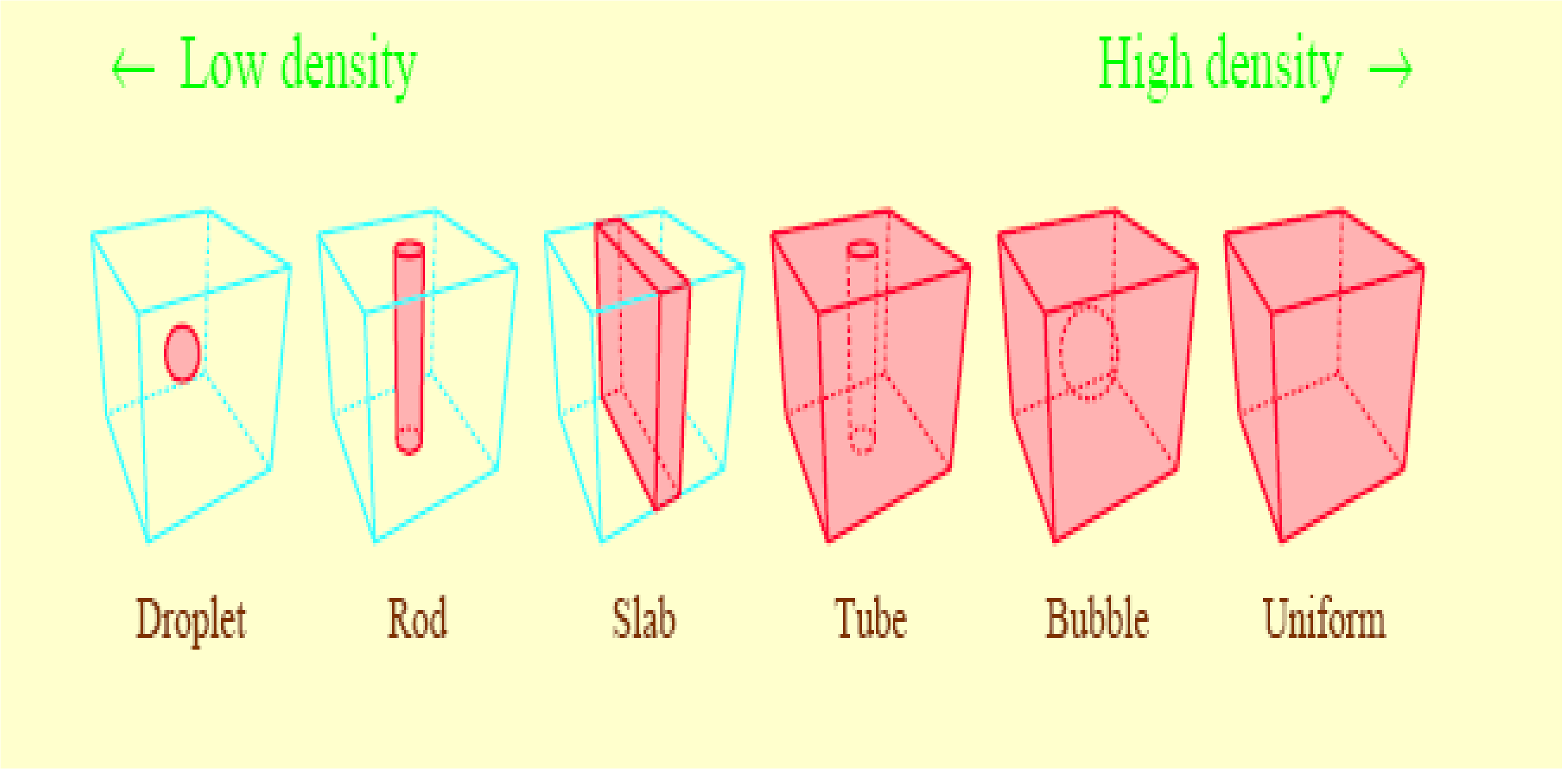
Poisson eq. is non linear.

Wigner-Seitz cell approximation and numerical solution

We fitted parameters to describe finite nuclei properties:
No external surface tension parameter (!)

Nuclear pasta structures

from T. Maruyama presentation

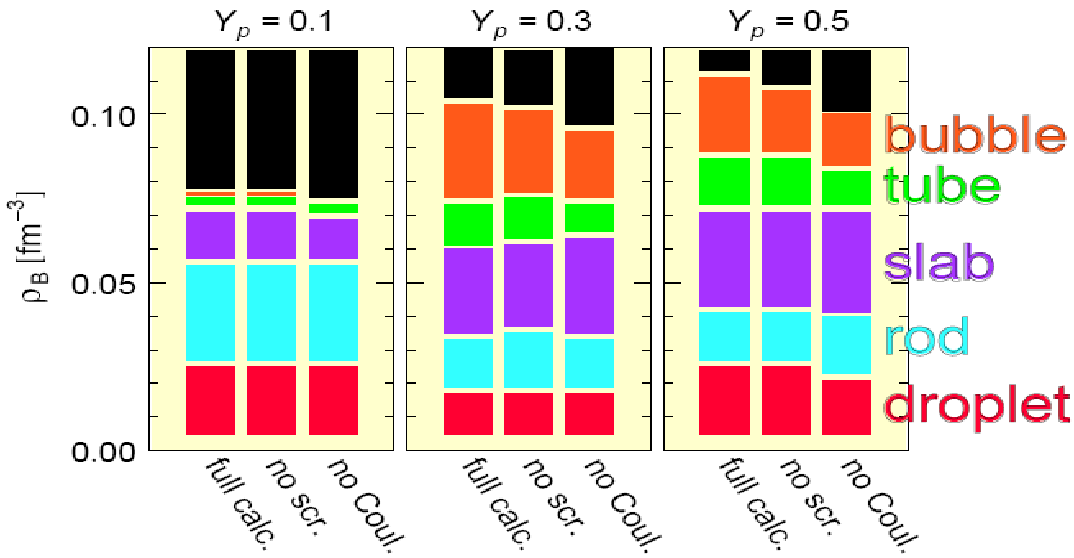


Nuclear pasta *in the inner crust of NS*

Coulomb screening effects

We compare 3 calculations:
(1) full calculation,
(2) no electron screening (uniform electron),
(3) no Coulomb interaction (corresponds to bulk calc).

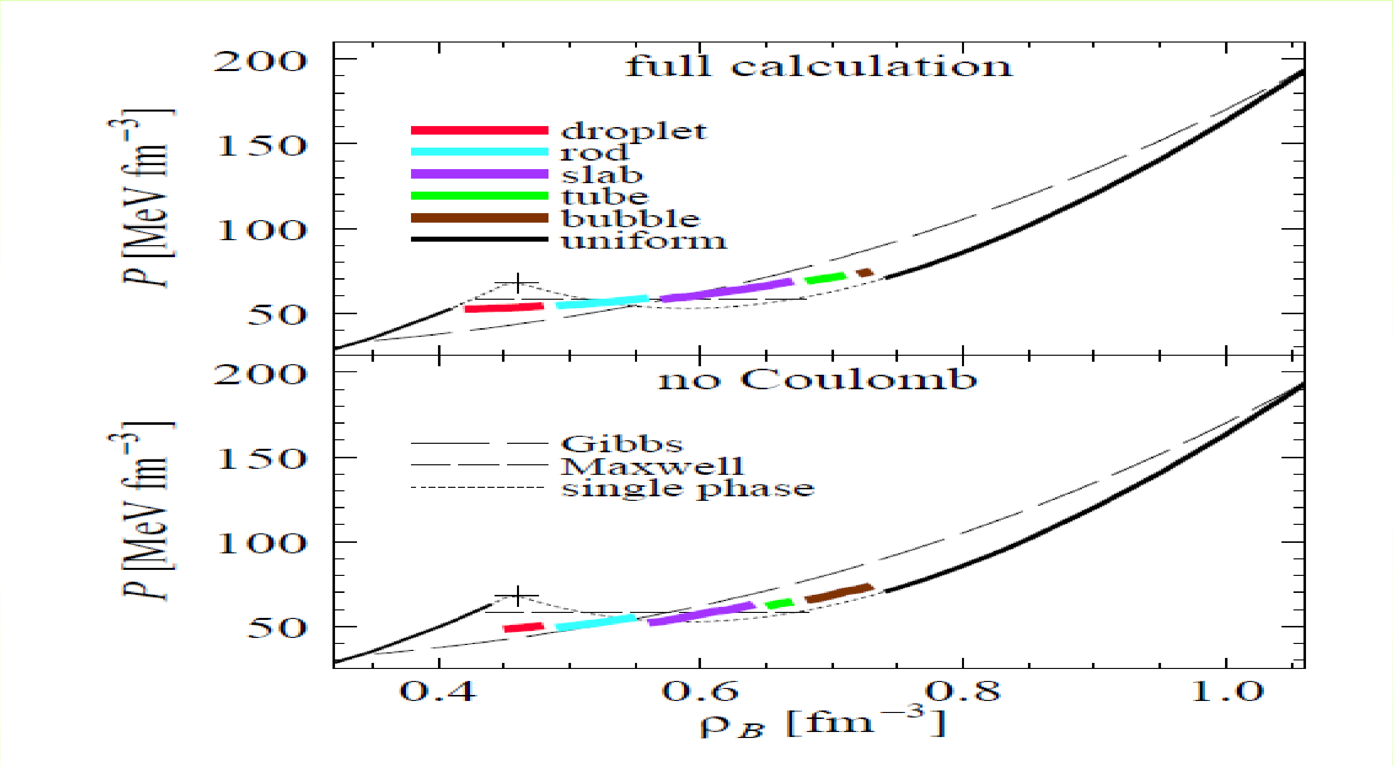
⋆ "No Coulomb interaction" result includes Coulomb interaction only in the total energy.
Neglect $V_{\rm Coul}$ to determine ρ_e and μ_p.



- The region of Pasta structures are significantly affected by the Coulomb screening effects.
- Especially the "bubble" structure is much affected.

Kaon condensation pasta *in RMF model*

Toshiki Maruyama, T.Tatsumi, D.V., T.Tanigawa, T.Endo,S. Chiba,
Phys.Rev.C73 (2006) 035802



“No Coulomb” means perturbative treatment of the Coulomb effect

without screening

Resulting EOS is much closer to that given by Maxwell construction than to that of mixed phase with bulk calculations

Conclusions to sect. Pasta

- With inclusion of Coulomb screening effects paradox: *“Gibbs conditions vs Maxwell construction”* is resolved.
- Peculiar structures of “Pasta” affect transport properties of neutron stars.
- Resulting EoS is closer to that given by Maxwell construction.