With the Functional Renormalization Group

towards

the QCD phase diagram

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Dense Matter In Heavy Ion Collisions and Astrophysics

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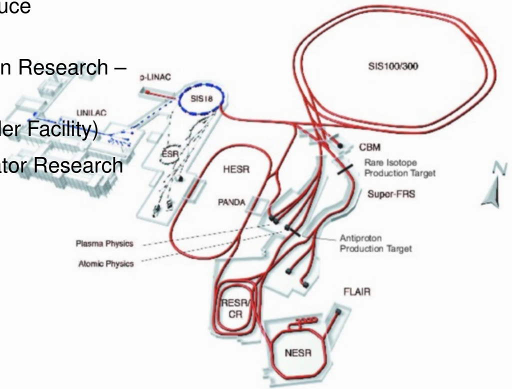
Heavy-Ion Collision Experiments

aim: create hot and dense QCD matter → elucidate its properties

QCD under extrem conditions: very active field (August 2010)

- RHIC @ BNL (Au-Au collisions $\sqrt{s_{NN}} \sim 200$ GeV)
- LHC @ CERN (higher energies)
- LeRHIC @ BNL (low-energy scan to produce $n_B >> n_0 \sim 0.17$ fm^{-3})
- FAIR @ GSI (Facility for Antiproton and Ion Research – hopefully SIS-300)
- NICA @ JINR (Nuclotron-based Ion Collider Facility)
- J-PARC @ JAERI (Japan Proton Accelerator Research Complex)



Outline

- **QCD phase diagram**
  - ▷ Landau-Ginzburg functional
  - ▷ Size of the critical region

- **Functional Renormalization Group (FRG)**
  - ▷ properties of the FRG
  - ▷ truncation schemes

- **Applications to the QCD phase diagram**
  - ▷ Mean-field approximation
  - ▷ $N_f = 2$ and $N_f = 2 + 1$ chiral models
  - ▷ Polyakov loop dynamics
  - ▷ Beyond Mean-Field
  - ▷ ...with the FRG

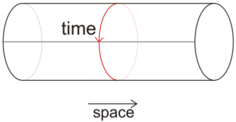
QCD Phase Transitions

QCD has (at least) three order parameters:

1. quark confinement: Polyakov loop Φ (and $\bar{\Phi}$)

   deconfinement transition in Euclidean spacetime

   - $(N_c \times N_c)$-matrix: $L(\vec{x}) = \mathcal{P} \exp \left\{ -ig \int_0^{\beta \equiv 1/T} dx_4 A_4(x_4, \vec{x}) \right\}$; $\mathcal{P}$: path ordering



   - traced Polyakov loop:

$$l = \frac{1}{N_c} \mathrm{tr}_c L$$

   - under (non-periodic) gauge transformation: $l \to z_k l$

     but gauge action still symmetric ($A_4 \to A_4 + const$) $\rightarrow$ **center symmetry**

     - center Z_{N_c} of $SU(N_c)$: elements of the center commute with all $SU(N_c)$ elements

       $\rightarrow \quad z_k = \exp(2\pi i k/N_c)\mathbf{1} \qquad k = 0, \ldots, N_c - 1$

QCD Phase Transitions

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1. quark confinement: Polyakov loop Φ (and $\bar{\Phi}$)

   deconfinement transition in Euclidean spacetime

- quark fields break center symmetry explicitly

  $\longrightarrow$ **center symmetry exact only in pure gluonic theory**

  (quarks absent or infinitely heavy $m_q \to \infty$)

- expectation value of traced Polyakov loop:

$$\Phi = \langle l(\vec{x}) \rangle = \exp(-\beta F_q) \qquad ; \qquad \bar{\Phi} = \langle l^\dagger(\vec{x}) \rangle = \exp(-\beta F_{\bar{q}})$$

- $F_q (F_{\bar{q}})$ free energy of a **static** quark (antiquark) in hot gluonic medium
- correlations

$$\langle l^\dagger(\vec{x}) l(\vec{y}) \rangle = \exp(-\beta F_{\bar{q}q}(x-y))$$

  $F_{\bar{q}q}$ excess free energy for an antiquark at x and a quark at y

QCD Phase Transitions

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   deconfinement transition in Euclidean spacetime

- in **confining** phase: free energy of single quark **diverges** ($F_q \to \infty$)

$$\Phi \to 0$$

- potential between a quark and antiquark increases linearly at long distances

  ($F_{\bar{q}q}(r \to \infty) \to \sigma r$)

  correlations **vanish**: $\langle l^\dagger(r \to \infty) l(0) \rangle \to 0$

- Expected behavior of **Polyakov loop in pure Yang-Mills**

Confined (disordered) phase

- free energy $F_q \to \infty$
- Polyakov loop $\Phi = 0$
- correlations:

$$\langle l^\dagger(r \to \infty) l(0) \rangle \to 0$$

Deconfined (ordered) phase

- free energy $F_q < \infty$
- Polyakov loop $\Phi \neq 0$
- correlations:

$$\langle l^\dagger(r \to \infty) l(0) \rangle \to |\langle l \rangle|^2 \neq 0$$

$\to$ behavior similar to magnetization in **classical spin systems**

deconfinement phase transition in pure Yang-Mills has similar features with a phase transition of **three-dimensional Z_{N_c}-spin models**

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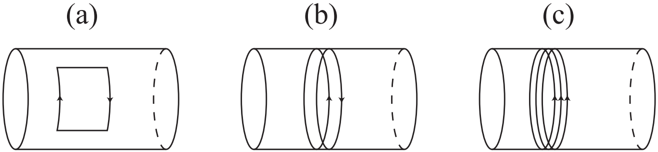
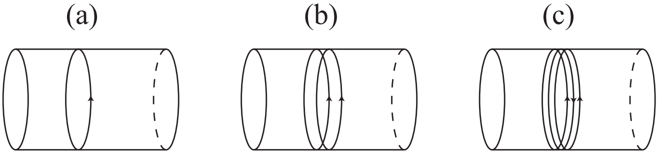


center symmetry respecting



center symmetry breaking

[K. Fukushima, Annals Phys. **304** 72 (2003)]

QCD Phase Transitions

QCD has (at least) three order parameters:

1. quark confinement: Polyakov loop Φ (and $\bar{\Phi}$)

   deconfinement transition in Euclidean spacetime

2. chiral symmetry restoration: chiral condensate $\langle \bar{q} q \rangle$

   - chiral symmetry in vacuum **spontaneously** broken (this is the source of hadron masses)
   - classical QCD symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A$ in **chiral limit**
   - axial anomaly: $U(1)_A$ broken explicitly to Z_{2N_f} by quantum effects

     $U(1)_A$ current not conserved anymore: $\partial_\mu j_5^\mu \sim \tilde{F}F$ (RHS: related to topological charge density)

     gauge configurations with **non-trivial topology** are microscopically responsible for the **axial anomaly**

     if gauge configurations are dominated by topologically **trivial** sectors $\rightarrow$ axial current could be conserved

   $\rightarrow$ effective restoration of axial symmetry in the medium

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   - chiral symmetry in vacuum **spontaneously** broken (this is the source of hadron masses)
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   - spontaneous chiral symmetry breaking

$$SU(N_f)_{L+R \equiv V} \times U(1)_B$$

$\rightarrow$ $N_f^2 - 1$ massless Nambu-Goldstone bosons ($N_f > 1$)

   - chiral condensate

$$\langle \bar{q}q \rangle = \langle \bar{q}_R q_L + \bar{q}_L q_R \rangle \qquad q_{R/L} = \frac{1}{2}(1 \pm \gamma_5) q \quad \text{right/left projected fields}$$

   - Expected behavior of **chiral condensate**

Broken (ordered) phase	Symmetric (disordered) phase
■ condensate $\langle \bar{q}q \rangle \neq 0$	■ condensate $\langle \bar{q}q \rangle \neq 0$

QCD Phase Transitions

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1. quark confinement: Polyakov loop Φ (and $\bar{\Phi}$)

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3. color superconductivity: diquark condensate $\langle qq \rangle$

   - QCD at high baryon density: one-gluon exchange $\rightarrow$ formation of Cooper pairs

   $\rightarrow$ normal quark matter becomes color superconducting (CSC) phase with diquark condensates at **asymptotic high density and sufficiently low temperature**

   - since quarks carry not only spin but also color and flavor

     various pairing patterns are possible

   - If all gaps $\Delta_{ud}, \Delta_{us}, \Delta_{ds}$ are non-vanishing $\rightarrow$ Color and flavor d.o.f. are entangled

     $\rightarrow$ color-flavor-locked (CFL) phase

     see lecture by M. Buballa

QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

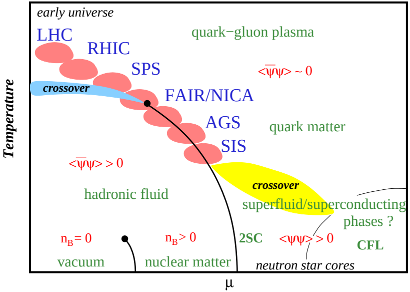
1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$

   order parameter:

   $$\langle \bar{q} q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken}, T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase}, T > T_c \end{cases}$$

   associate limit: $m_q \longrightarrow 0$



chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high T

QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$

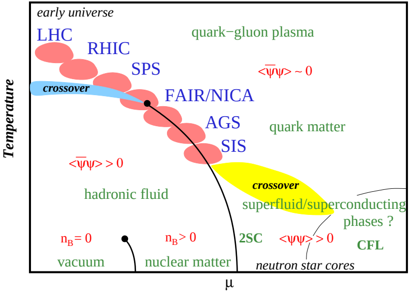
   order parameter:

   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

2. de/confinement

   order parameter: Polyakov loop variable

   $$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase, } T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase, } T > T_c \end{cases}$$



At densities/temperatures of interest only model calculations available

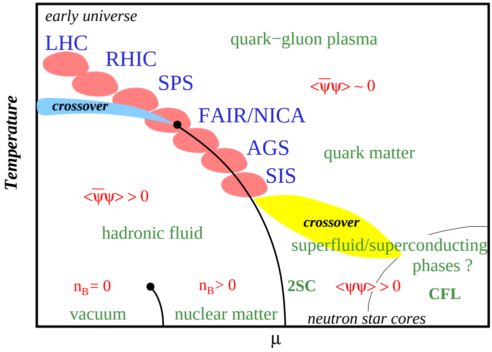
alternative: ➜ dressed Polyakov loop (or dual condensate)

it relates chiral and deconfinement transition to spectral properties of Dirac operator

effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

The conjectured QCD Phase Diagram



At densities/temperatures of interest
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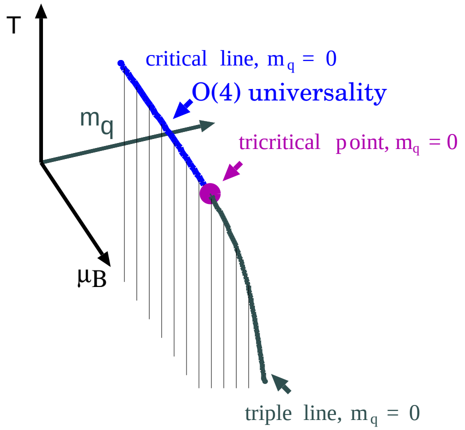
Open issues:

related to chiral & deconfinement transition

- ▷ existence of CEP?
- ▷ its location?
- ▷ additional CEPs? How many?
- ▷ coincidence of both transitions at $\mu = 0$?
- ▷ quarkyonic phase at $\mu > 0$?
- ▷ chiral CEP/ deconfinement CEP?
- ▷ so far only MFA results effect of fluctuations (e.g. size of crit. reg.)?
- ▷ ...

Phase diagram in (T, μ_B, m_q)-space

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$



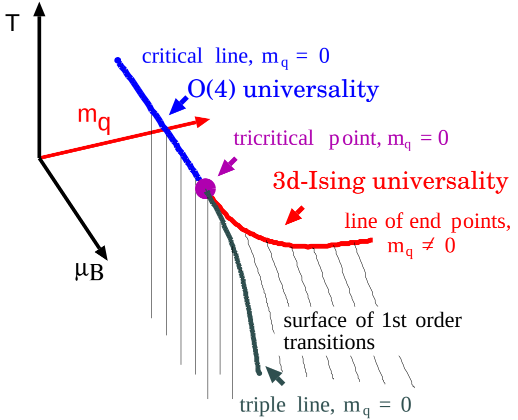
General properties

- chiral limit
  tricritical point
  (Gaussian fixed
  point)

Phase diagram in (T, μ_B, m_q)-space

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$

$m_q \neq 0$: no symmetry remains $\longrightarrow$ only one critical mode σ (Ising) ($\vec{\pi}$ massive)



General properties

- chiral limit
  tricritical point
  (Gaussian fixed
  point)
- finite m_q
  critical endpoints
  (3D-Ising class)

Landau-Ginzburg approach

see lecture by D.N. Voskresensky and talk by P. Büscher

Landau-Ginzburg potential: expansion in order parameter $\vec{\phi} = (\sigma, \vec{\pi})$

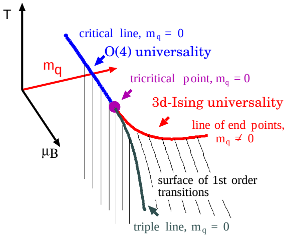
$$\Omega(T,\mu;\phi) \sim a(T,\mu)\vec{\phi}^2 + b(T,\mu)\vec{\phi}^4 + c\vec{\phi}^6 + m\sigma \quad ; \quad c > 0$$

$\underline{m = 0}$:

- $\underline{2^{\text{nd}}\text{ order line:}}$ $a = 0,\ b > 0$
  4 fields massless $\rightarrow O(4)$ universality
- $\underline{\text{tricritical point:}}$ $b = 0$
  $a = b = 0 \Rightarrow$ mean-field exponent
- $\underline{1^{\text{st}}\text{ order line:}}$ $b < 0$

$\underline{m \neq 0}$:

- 2^{nd} order line $\longrightarrow$ crossover
- tricritical point $\longrightarrow$ critical point
  end point of a 1^{st} order line
  σ massless, $\vec{\pi}$ massive $\rightarrow$ Ising class
- 1^{st} order line $\longrightarrow$ 1^{st} order line



What are the sizes of the critical regions?

$\rightarrow$ Ginzburg criterion

Ginzburg criterion

Ginzburg criterion: size of crit. region$\leftrightarrow$ break down of mean-field theory

Landau-Ginzburg potential for 2$^{\text{nd}}$ order phase transition

$$\Omega(T,\mu;\phi) \sim d(\vec{\nabla}\phi)^2 + a' t\phi^2 + b\phi^4 \qquad ; \qquad t = (T - T_c)/T_c$$

$\Rightarrow$ **Ginzburg-Levanyuk temperature** τ_{GL}

For $t < \tau_{GL}$ fluctuations are important

$$|t| \sim \frac{T_c^2}{a' d^3} b^2 \equiv \tau_{GL} \sim m_q^{4/5} \sim m_\pi^2$$

but this **criterion is useless here**

- size depends on microscopic dynamics
- even universality arguments not applicable

example for O(2) class

He^4 λ-transition: $\tau_{GL} \sim 10^{-15}$

$O(2)$ spin model: $\tau_{GL} \sim 0.3$

both systems in $O(2)$ class

but τ_{GL} differs!

expectation $\Rightarrow$ size of crit. region shrinks as $m_q \to 0$ ($\tau_{GL} \sim b^2$)

Non-trivial critical region suppression

Suppression of size of crit. region,
where non-trivial critical behavior sets in
also observed in other models

Critical region suppression ($\mu = 0$)

- Yukawa theory with spon. χ SB — Rosenstein et al. 1994
- Gross-Neveu model (large-N) — Kocic, Kogut 1995
- MC simulations confirm these results

Outline

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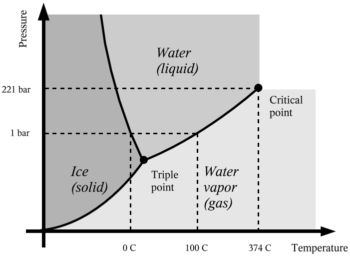
  - ▷ properties of the FRG
  - ▷ truncation schemes
- ○ **Applications to the QCD phase diagram**
  - ▷ Mean-field approximation
  - ▷ $N_f = 2$ and $N_f = 2 + 1$ chiral models
  - ▷ Polyakov loop dynamics
  - ▷ Beyond Mean-Field
  - ▷ ...with the FRG

Why non-perturbative Renormalization Group?

example: phase diagram of water



- allows to describe physics across different length scales

  2^{nd}-order phase transition $\rightarrow$ long-wavelength fluctuations ($\xi \rightarrow \infty$)

  critical opalescence: light is strongly scattered

  dissimilar systems exhibit **same** critical exponents $\rightarrow$ universality

  assign each system to a universality class

bridge the gap

microscopic theory $\longrightarrow$ macroscopic (effective) theory

loose irrelevant details of the microscopic theory

QCD

- chiral fermions, implementation of quarks w/ & w/o quark masses
- with **standard perturbation theory**

  $\rightarrow$ **not possible** to describe spontaneous symmetry breaking

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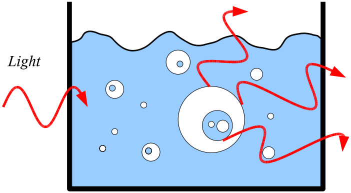
critical opalescence



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On the History of Renormalization Group

- RG is a systematic theory of crit. phenomena

  $\rightarrow$ qualitative & quantitative

- Name historically, nowadays:

  scale dependence of physics

  strategy to solve problems with many scales

▷ pioneered by A. Petermann & E.C.G. Stückelberg (1953)

▷ Gell-Mann & Low (1954) $\rightarrow$ asymptotic behavior of Green's functions in QED

▷ Bogoliubov & Shirkov (1959)

▷ Kadanoff (1966)

▷ **K.G. Wilson** (1970)

▷ C.G. Callan and K. Symanzik (1970)

▷ F. Wegner and A. Houghton (1973)

▷ ...

Kenneth Geddes Wilson

- born June 8th, 1936 in Waltham, Massachusetts
- Ph.D., California Institute of Technology, 1961
- long time at Cornell University, NY
- since August 1988 at Ohio State University (Columbus,OH)

Nobel prize 1982

theory for critical phenomena in connection with phase transitions

Idea of the Renormalization Group

- Quantum field theory: generating functional

$$\mathcal{Z}[J] = \frac{1}{\mathcal{N}} \int \mathcal{D}\phi e^{-S[\phi,J]} \quad ; \qquad \text{“ill-defined”}$$

- path integral $\Longleftrightarrow$ functional diffential equation (FDE)
- FDE well-defined since original divergences are relegated to the boundary values of its solution

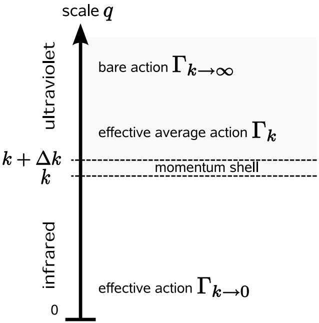
(Wilsonian) RG's

describe very efficiently universal and non-universal aspects of phase diagrams

Exact RG ≡ FRG ≡ Wetterich Equations ≡ . . .

average effective action Γ_k:

- Γ_k contains **only** fluctuations with $q^2 \geq k^2$
- $\Gamma_k \sim$ coarse-grained free energy with length scale $\sim 1/k$
  $\rightarrow$ effective action for **field averages over volume** $\sim 1/k^d$
- implement IR cutoff $R_k(q)$
- k large: Γ_k close to microscopic action
- lowering k: successive inclusion of fluctuations
- $k = 0$: IR cutoff is absent
  $\rightarrow \Gamma_0 \equiv \Gamma$, i.e. **all fluctuations** included.



Γ_k interpolates between S_{class} and Γ

$$\Gamma_\Lambda = S_{\text{class}} \qquad ; \qquad \lim_{k\to 0} \Gamma_k = \Gamma$$

$\Rightarrow$ ability to follow $k \rightarrow 0$ evolution ≡ ability to solve the theory

Exact RG ≡ FRG ≡ Wetterich Equations ≡ . . .

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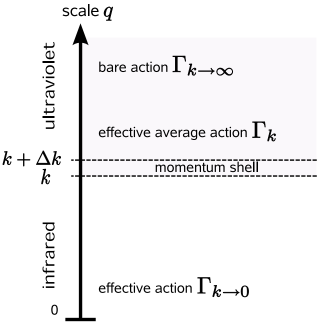
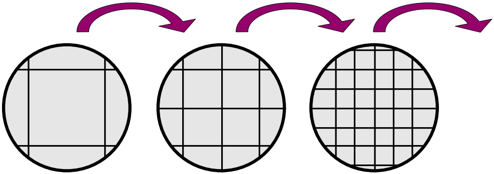


▷ procedure: step-by-step magnification of the smallest scale up to larger scales.

microscopical $\longrightarrow$ **macroscopical**

▷ look at physics with a **microscope** with varying resolution



Exact RG $\equiv$ FRG $\equiv$ Wetterich Equations $\equiv \ldots$

QFT in $d = 1 + 3$: generating functional

$$Z[j] = \int \mathcal{D}\phi \, \exp\left[-S[\phi] + \int j\,\phi\right]$$

addition of an IR cutoff term

$$Z_k[j] = \int \mathcal{D}\phi \, \exp\left[-S[\phi] - \Delta S_k[\phi] + \int j\,\phi\right]$$

Exact RG $\equiv$ FRG $\equiv$ Wetterich Equations $\equiv \ldots$

addition of an IR cutoff term

$$Z_k[j] = \int \mathcal{D}\phi \, \exp\left[-S[\phi] - \Delta S_k[\phi] + \int j\,\phi\right]$$

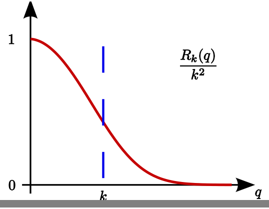
- choice (quadratic in the fields)

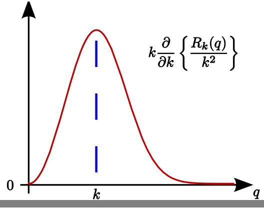
$$\Delta S_k[\phi] = \frac{1}{2}\int_q \phi(-q) R_k(q) \phi(q)$$

with:

$\lim_{q^2/k^2 \to \infty} R_k(q) = 0$: remove cutoff for $k \to 0$ and UV not suppressed

$\lim_{k \to \infty (\Lambda)} R_k(q) \to \infty$: no modes are integrated out
$\to$ acts like a functional $\delta(\phi)$





Exact RG $\equiv$ FRG $\equiv$ Wetterich Equations $\equiv \ldots$

addition of an IR cutoff term

$$Z_k[j] = \int \mathcal{D}\phi \, \exp\left[-S[\phi] - \Delta S_k[\phi] + \int j\,\phi\right]$$

- modified Legendre transform

$$\begin{aligned}\Gamma_k[\phi] &= -\ln Z_k[j] + j\phi - \Delta S_k[\phi] \\ &= -\ln \int \mathcal{D}\tilde{\chi} e^{-S[\tilde{\chi}+\phi] - \Delta S_k[\tilde{\chi}] + \frac{\delta \Gamma_k[\phi]}{\delta \phi}\tilde{\chi}}\end{aligned}$$

- 1st term: $S[\phi]$ classical contribution
- 2nd term: $\tilde{\chi}$ fluctuations with background field ϕ

$$\lim_{k\to\Lambda} \Delta S_k[\phi] \to \infty \quad : \quad \Gamma_\Lambda[\phi] = S[\phi]$$

Exact RG $\equiv$ FRG $\equiv$ Wetterich Equations $\equiv \dots$

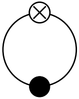
addition of an IR cutoff term

$$Z_k[j] = \int \mathcal{D}\phi \, \exp\left[-S[\phi] - \Delta S_k[\phi] + \int j\,\phi\right]$$

flow equation for average effective action **[Wetterich ’93]**

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\; k\partial_k R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k}\right) \quad ; \quad \Gamma_k^{(2)}[\phi] = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



RG Approaches

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$ $\longrightarrow$ $k\partial_k \equiv \partial_t$

1 Exact RG

ERG (average effective action) **[Wetterich]**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

2 Proper-time RG

PTRG **[Liao]**

$$\partial_t \Gamma_k[\phi] = -\frac{1}{2} \int_0^\infty \frac{d\tau}{\tau} \left[\partial_t f_k(\Lambda^2 \tau) \right] \mathrm{Tr} \exp\left(-\tau \Gamma_k^{(2)} \right)$$

3 other approximations

Truncations

exact RG **impossible** to solve $\rightarrow$ **systematic** approximations needed

$\Rightarrow$ projection onto *sub-theory* space

- derivative expansion

$$\Gamma_k[\phi] = \int d^d x \left\{ U_k(\phi) + \frac{1}{2} Z_k \partial_\mu \phi \partial_\mu \phi + \ldots + \mathcal{O}(\partial^4) \right\}$$

- expansion in powers of the fields

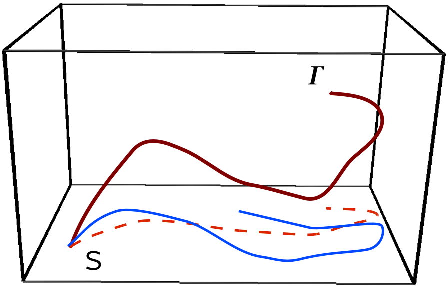
$$\Gamma_k[\phi] = \sum_n \frac{1}{n!} \int \left(\prod_i^n d^d x_i \phi(x_i) \right) \Gamma_k^{(n)}(x_1, \ldots, x_n)$$

- ... (some more expansion schemes)

Truncations

consider a 3-dim. subset of operators

here: RG flow in a 3-dim. space of all action functionals:



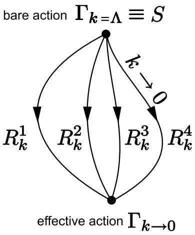
▷ projection of exact flow on subspace of truncation (dashed)

→ does **not** coincide with approximate flow (blue)

(omission of operator in 3rd direction)

▷ enlarge subspace
(of relevant operators)

→ improve approximation



▷ → choose "optimized" IR regulator [Litim, Pawlowski]

Truncations

example: scalar theory with Z_2-symmetry

$$S_{\text{eff}} = \int d^d x \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi^2) \right\}$$

$\rightarrow$ lowest order of derivative expansion (LPA)

Truncations

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- further reduction: potential expansion: $V(\phi^2) = \frac{a_2}{2!}\phi^2 + \frac{a_4}{4!}\phi^4 + \frac{a_6}{6!}\phi^6 + \dots$

Truncations

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- β-functions for coefficients a_i:

$$\begin{aligned}
\partial_t a_2 &= 2a_2 - \frac{\zeta}{2}\frac{a_4}{1+a_2} \\
\partial_t a_4 &= (4-d)a_4 - \zeta\left[\frac{2}{5}\frac{a_6}{1+a_2} - \frac{a_4^2}{(1+a_2)^2}\right] \\
\partial_t a_6 &= \ldots \\
&\vdots
\end{aligned}$$

Truncations

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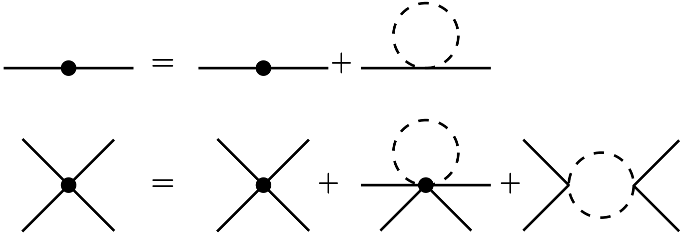
- "Feynman diagram" representation of the β-functions:



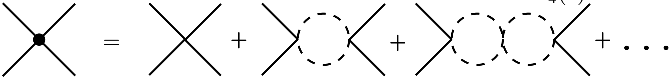
Integrating the β-functions

- consider quartic coupling a_4 in $d = 4$:

ignore a_6 contribution and use $a_2 \ll 1$ at cutoff scale:

$$\partial_t a_4 = \zeta a_4^2$$

$$\to a_4(t) = \frac{a_4(0)}{1 - \zeta a_4(0) t} = \frac{a_4(0)}{1 - \zeta a_4(0) \ln(k/\Lambda)} = \frac{a_4(0)}{1 - a_4(0)\,\cdot}$$



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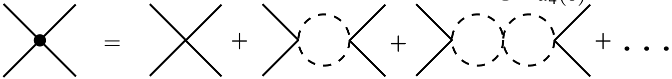
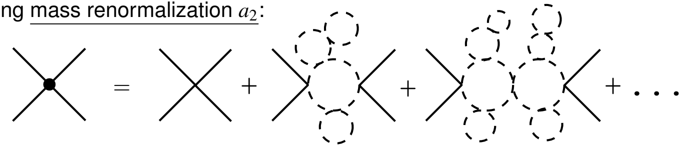


- including mass renormalization a_2:



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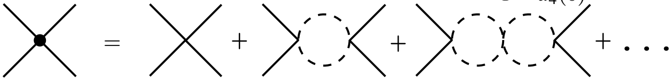

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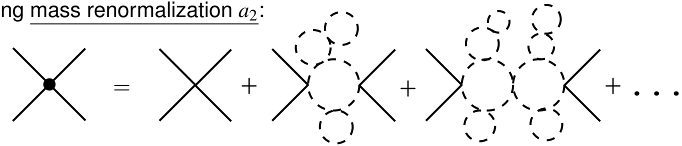
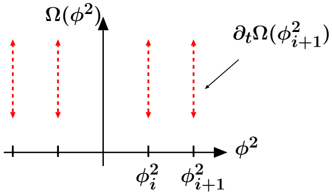


- **lowest level approximation** in NPRG contains even **improved ladder Schwinger-Dyson results**

Solving Flow Equations

in general two possibilities

1.) Solve coupled flow eqs on ϕ^2 **grid**:



2.) **Taylor expansion** around ϕ_0^2:

$$\Omega(\phi^2) = \sum_{n=0}^{N} a_n(\phi^2 - \phi_0^2)^n$$

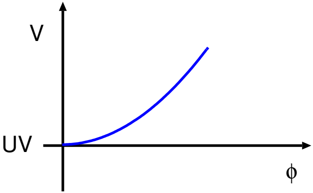
- Initial condition at high UV cutoff e.g. $\Lambda = 1000$ MeV

$$V_\Lambda = \frac{1}{4}\lambda_\Lambda(\phi^2)^2 \qquad \text{symmetric potential}$$

- Fixed UV parameterization (e.g. λ_Λ) such to reproduce physics in the IR (e.g. $\phi_0 \equiv f_\pi \sim 93$ MeV)

skip evolution

Scale Evolution of the Potential (Vacuum)



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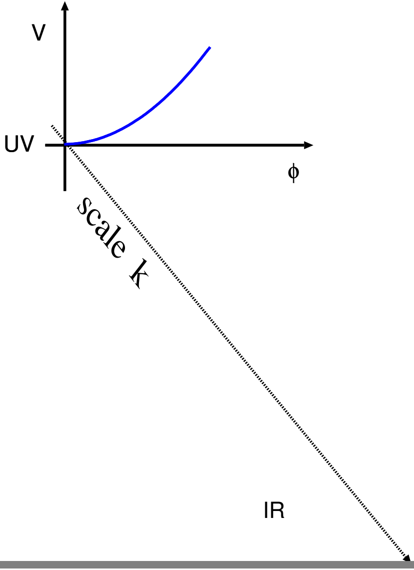

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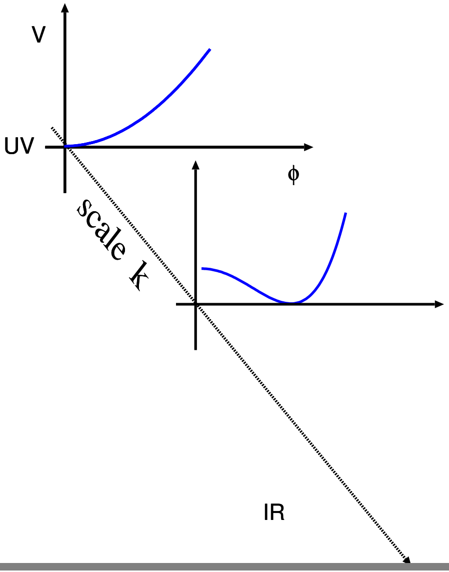

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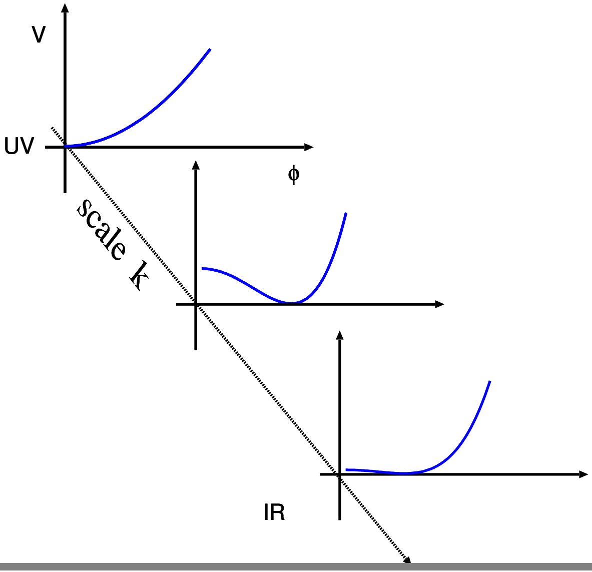

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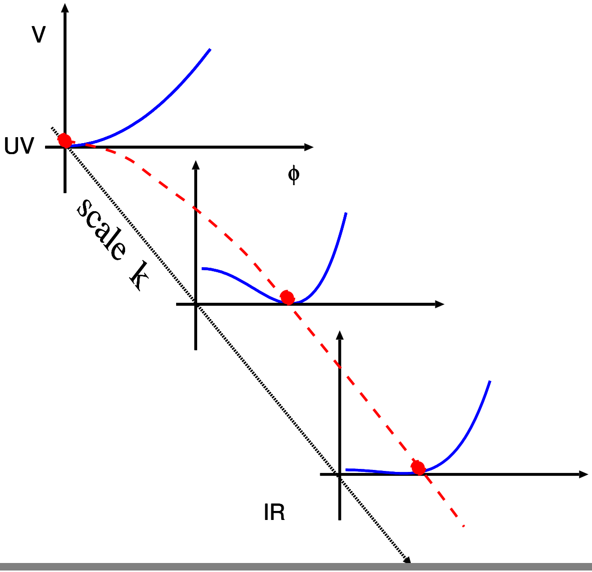

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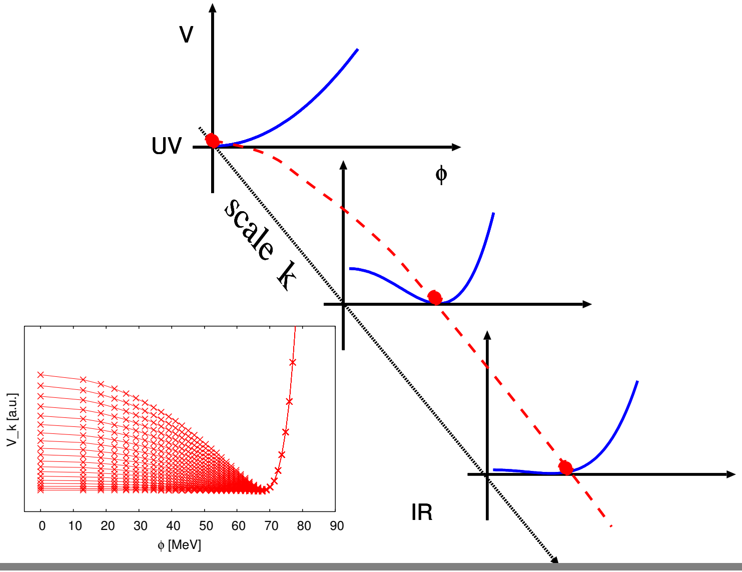

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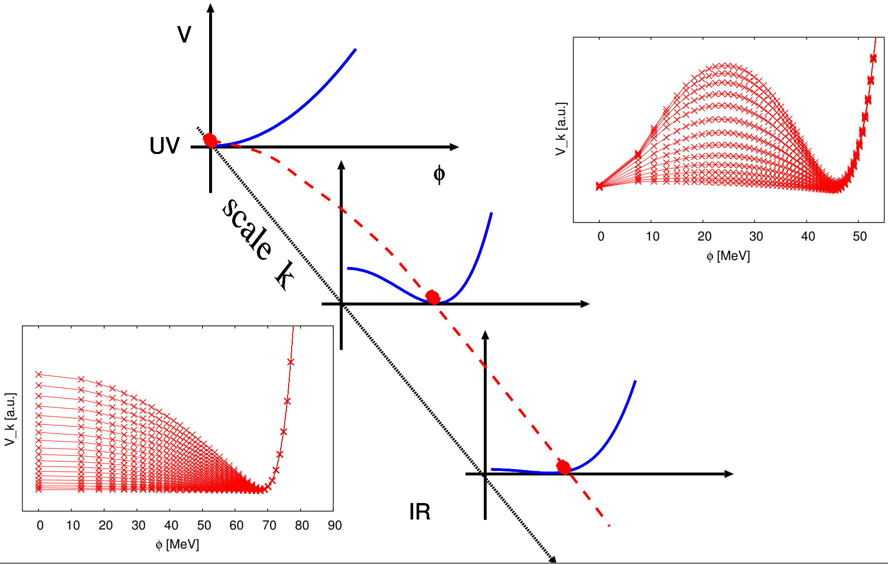
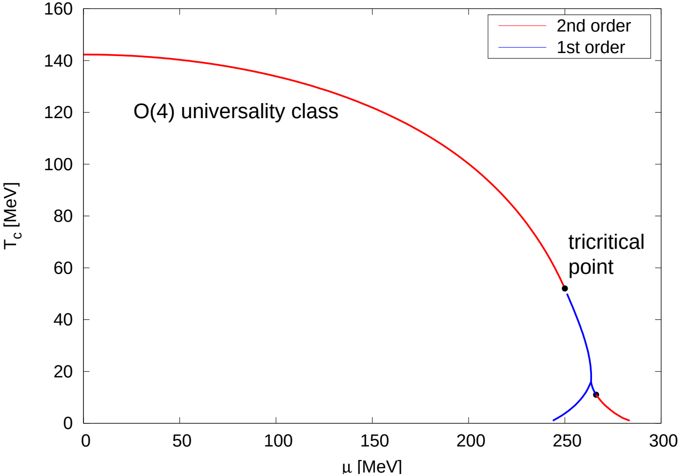


Chiral Phase Diagram $N_f = 2$ and $m_q \sim 280$ MeV

[BJS, J. Wambach, '05 & '06]

FRG analysis:

$O(4) \sim SU(2) \times SU(2)$ chiral limit



Chiral Phase Diagram $N_f = 2$ and $m_q \sim 280$ MeV

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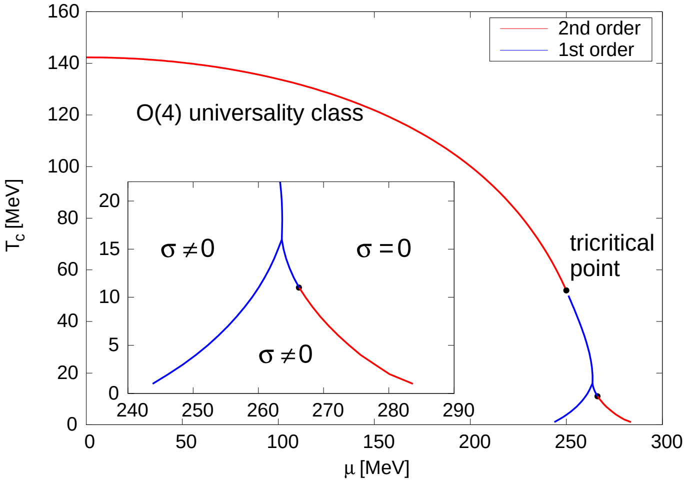
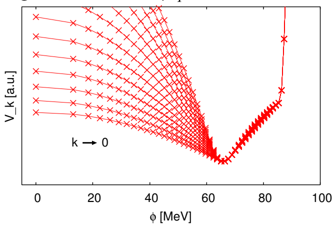
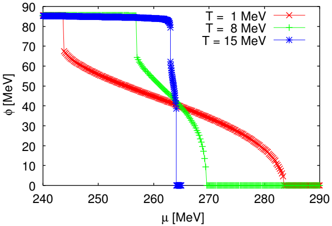
$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!



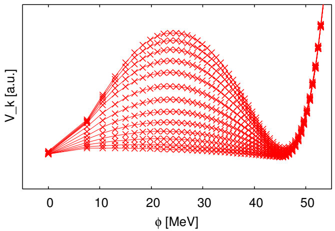
A Second (new) Phase Transition

e.g. $T = 6$ MeV, $\mu_q = 254$ MeV

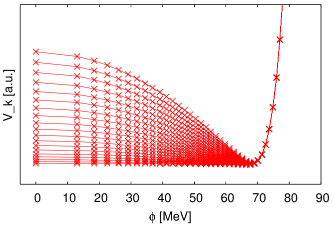




first-order phase transition



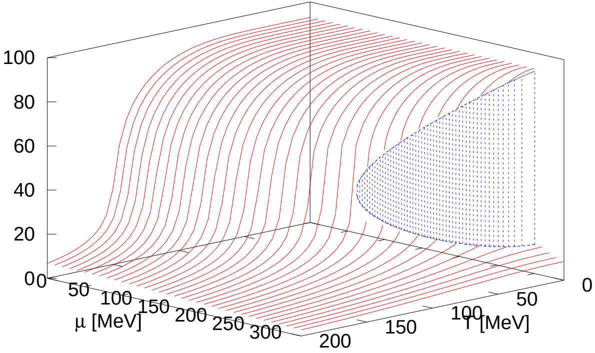
second-order phase transition



Finite Pion Masses

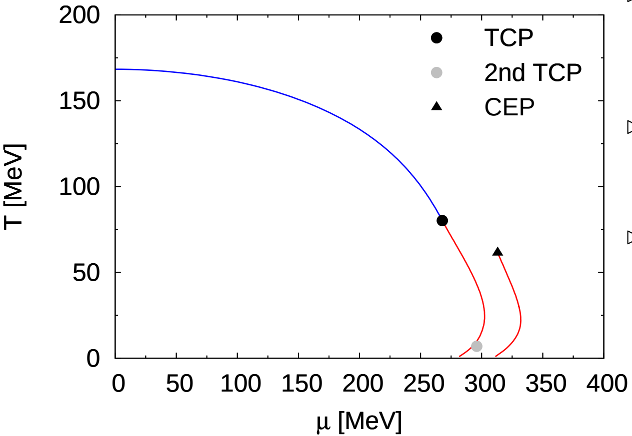
order parameter: $\phi(T,\mu)$

- 2nd-order transition $\rightarrow$ crossover
- shift of "T_c"
- shift tricritical point $\rightarrow$ critical



Phase Diagram with FRG

$m_\pi \sim 138$ MeV



▷ bending usual for RG

Clausius-Clapeyron relation ok

▷ 2nd tricritical point

in chiral limit

▷ features

parameter independent

but locations TCP/CEP

parameter dependent

[BJS,Wambach '05 & '06]

Schladming Winter School

49. Internationale Universitätswochen für Theoretische Physik

Physics at all scales:
The Renormalization Group

Schladming, Styria, Austria, February 26 - March 5, 2011

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Sebastian Diehl (University of Innsbruck)	**Ultracold Quantum Gases and the Functional Renormalization Group**
Richard J. Furnstahl (Ohio State University)	**The Renormalization Group in Nuclear Physics**
Anna Hasenfratz (University of Colorado)	**Exploring the Conformal Window**
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(University of Sussex)	
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Lorenz von Smekal (TU Darmstadt)	**Universal Aspects of QCD-like Theories**
Uwe C. Täuber (Virginia Tech Blacksburg)	**Renormalization Group: Applications in Statistical Physics**

If you wish to apply, please access the web page and complete the registration form as soon as possible, but not later than **February 18, 2011**. More Information about the school can be found on the web page as well.

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	Reinhard Alkofer	Universitätsplatz 5, A-8010 Graz, Austria
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	Heimo Latal	Fax: +43 316 380 9820
	Leopold Mathelitsch	E-mail: theor.physik@uni-graz.at
		http://physik.uni-graz.at/schladming2011/

Das Land Steiermark → Wissenschaft und Forschung

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KONICA MINOLTA

Majer Computer Software

With the Functional Renormalization Group

towards

the QCD phase diagram

Bernd-Jochen Schaefer

University of Graz, Austria

second part

Helmholtz International Summer School

Dense Matter In Heavy Ion Collisions and Astrophysics

24th Aug. - 4th Sept, 2010

Dubna, Russia

Outline

- **QCD phase diagram**
  - ▷ Landau-Ginzburg functional
  - ▷ Size of the critical region

- **Functional Renormalization Group (FRG)**
  - ▷ properties of the FRG
  - ▷ truncation schemes

- **Applications to the QCD phase diagram**
  - ▷ Mean-field approximation
  - ▷ $N_f = 2$ and $N_f = 2 + 1$ chiral models
  - ▷ Polyakov loop dynamics
  - ▷ Beyond Mean-Field
  - ▷ ...with the FRG

$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling h:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar σ_a and pseudoscalar π_a nonet

fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2 \text{tr}[M^\dagger M] - \lambda_1 (\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)] + \text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

Mean field approximation

- partition function:

$$\mathcal{Z}(T,\mu) = \int \mathcal{D}\bar{q}\mathcal{D}q \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \exp\left\{ i\int_0^{1/T} dt d^3x \left(\mathcal{L}_{N_f=3} + \sum_f \mu_f \bar{q}_f \gamma_0 q_f \right) \right\}$$

- two chiral condensates: non-strange σ_x and strange σ_y ($N_f = 2+1$)
- integrate fermions (→ determinant) , drop meson integration

Grand potential

$$\Omega(T,\mu) = -\frac{T \ln \mathcal{Z}}{V} = U_{\text{meson}}(\sigma_x,\sigma_y) + \Omega_{\bar{q}q}(T,\mu;\sigma_x,\sigma_y)$$

with mesonic potential $U(\sigma_x,\sigma_y)$ and

Quark contribution:

$$\Omega_{\bar{q}q}(T,\mu) = -2N_c T \sum_{flavor} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln(1+e^{-(E_{q,f}-\mu_f)/T}) + \ln(1+e^{-(E_{q,f}+\mu_f)/T}) \right\}$$

➜ **divergent vacuum contribution neglected** ⇒ influences phase diagram

Non-strange $\sigma_x(T,\mu)$
strange $\sigma_y(T,\mu)$ } via $\frac{\partial\Omega}{\partial\sigma_0} = \frac{\partial\Omega}{\partial\sigma_8}\Big|_{\sigma_0=\sigma_x,\sigma_8=\sigma_y} = 0$

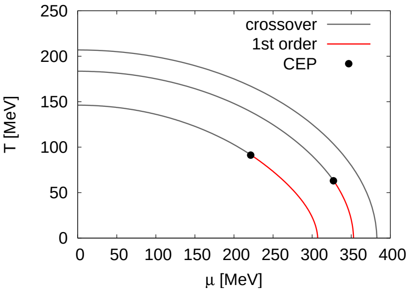
Phase diagram for $N_f = 2+1 \quad (\mu \equiv \mu_q = \mu_s)$

- Model parameter fitted to (pseudo)scalar meson spectrum:
- PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV $\rightarrow$ broad resonance

➜ existence of CEP depends on m_σ!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here mean-field approximation)

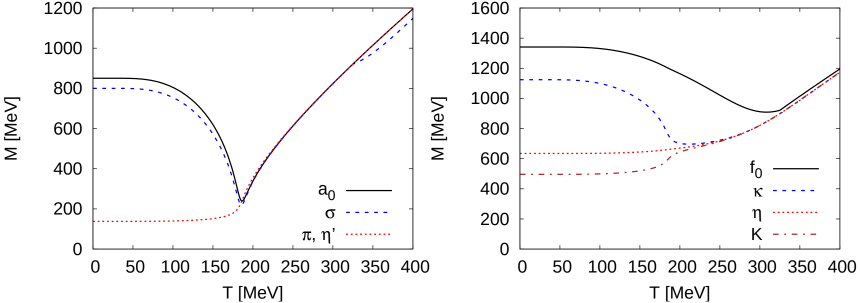
with $U(1)_A$ **[BJS, M. Wagner '09]**



In-medium meson masses

Finite temperature axis: $\mu = 0$

masses **without** $U(1)_A$ anomaly



- At low temperatures: mesons dominate
- At high temperatures: quarks dominate

In-medium meson masses

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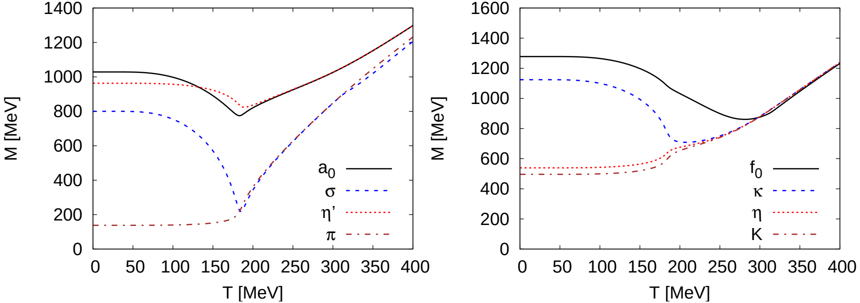
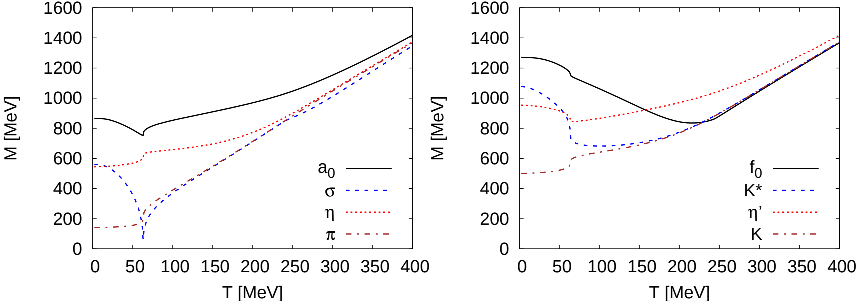


- At low temperatures: mesons dominate
- At high temperatures: quarks dominate

In-medium meson masses

slide through CEP: $\mu = \mu_c$

masses **with** $U(1)_A$ anomaly



- At low temperatures: mesons dominate
- At high temperatures: quarks dominate

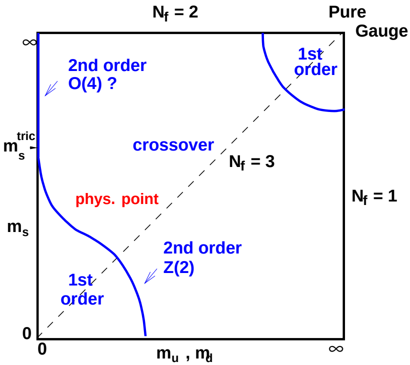
Mass sensitivity

Chiral limit: RG arguments $\rightarrow$ for $N_f \geq 3$ first-order [Pisarski, Wilczek '84]

Columbia plot: **[Brown et al. '90]**

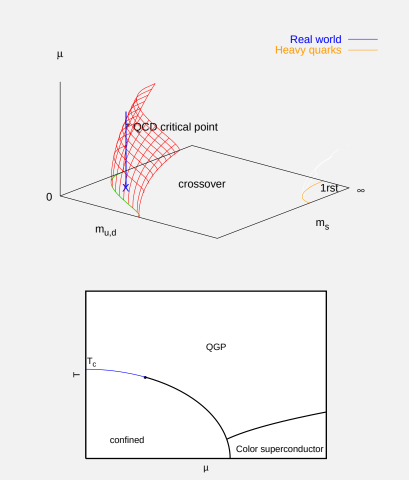
$T_d^{N_c=3} \sim 270$ MeV

$T_\chi \sim 150 \ldots 190$ MeV

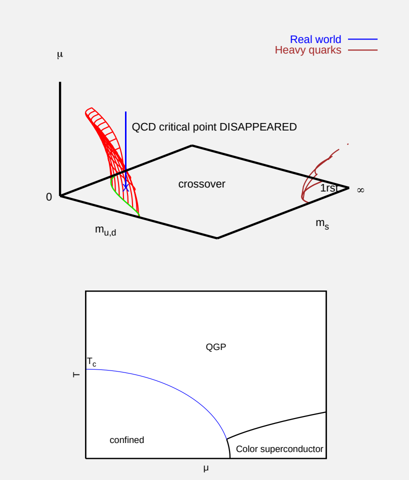


Mass sensitivity (lattice, $N_f = 3, \mu_B \neq 0$)

Standard scenario: $m_c(\mu)$ **increasing**



Nonstandard scenario: $m_c(\mu)$ **decreasing**

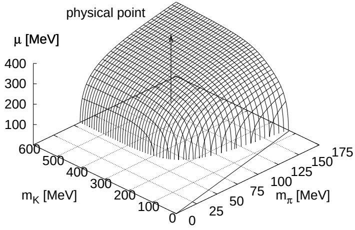


[de Forcrand, Philipsen: hep-lat/0611027]

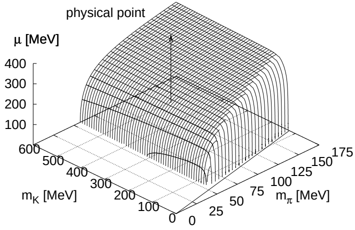
Chiral critical surface ($m_\sigma = 800$ MeV)

➜ standard scenario for $m_\sigma = 800$ MeV (as expected)

with $U(1)_A$



without $U(1)_A$



[BJS, M. Wagner, '09]

Note: 't Hooft coupling μ-independent

PNJL with (unrealistic) large vector int. $\longrightarrow$ bending of surface

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Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

- **logarithmic potential:** **Rößner et al. 2007**

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3$$

- **Fukushima** **Fukushima 2008**

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

a controls deconfinement $\quad$ b strength of mixing chiral & deconfinement

Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

in presence of dynamical quarks: $T_0 = T_0(N_f, \mu)$ **BJS, Pawlowski, Wambach, 2007**

N_f	0	1	2	$2+1$	3
T_0 [MeV]	270	240	208	187	178

$\mu \neq 0 :$ $\bar{\phi} > \phi$

since $\bar{\phi}$ is related to free energy gain of antiquarks

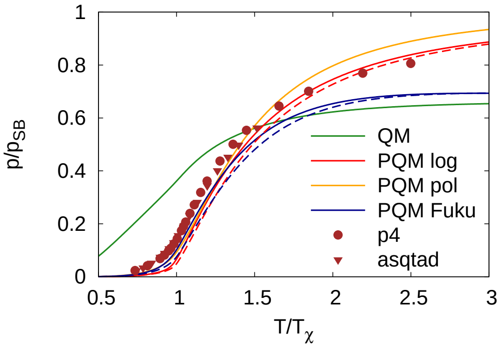
in medium with more quarks → antiquarks are more easily screened.

QCD Thermodynamics $N_f = 2 + 1$

[BJS, M. Wagner, J. Wambach '10]

SB limit: $\frac{p_{\rm SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations



- ▷ solid lines:
  PQM with lattice masses (HotQCD)
  $m_\pi \sim 220, m_K \sim 503$ MeV
- ▷ dashed lines:
  (P)QM with realistic masses

lattice data: [Bazavov et al. '09]

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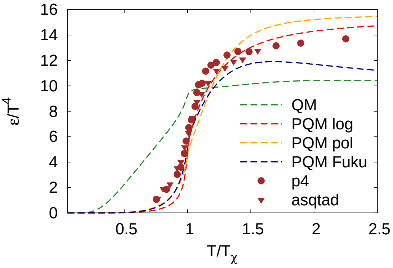


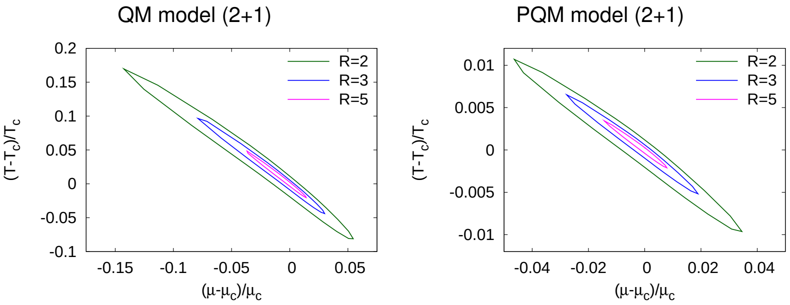
solid lines: $m_\pi \sim 220, m_K \sim 503$ MeV (HotQCD)
[Bazavov et al. '09]

Critical region

contour plot of **size of the critical region** around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop



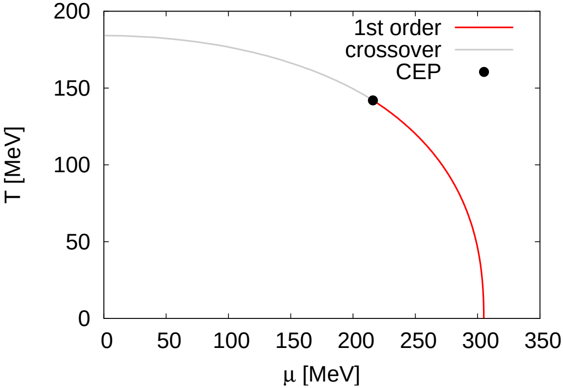
[BJS, M. Wagner; in preparation]

$N_f = 2$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide

■ for PQM model



[BJS, Pawlowski, Wambach '07]

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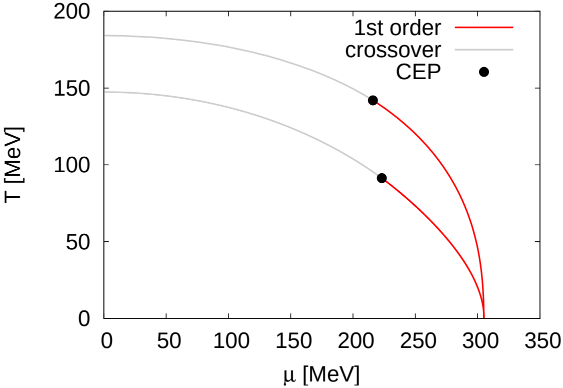

- for PQM model
- for QM model (lower lines)

[BJS, Pawlowski, Wambach '07]

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Summary of QM and PQM models in mean field approximation

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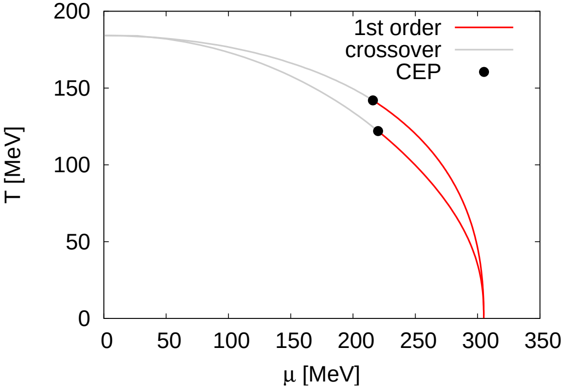

- for PQM model
- for PQM model **with** μ-modification in Polyakov loop potential (lower lines)

[BJS, Pawlowski, Wambach '07]

Outline

- **QCD phase diagram**
  - ▷ Landau-Ginzburg functional
  - ▷ Size of the critical region
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  - ▷ properties of the FRG
  - ▷ truncation schemes
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  - ▷ ...with the FRG

Importance of Dirac term

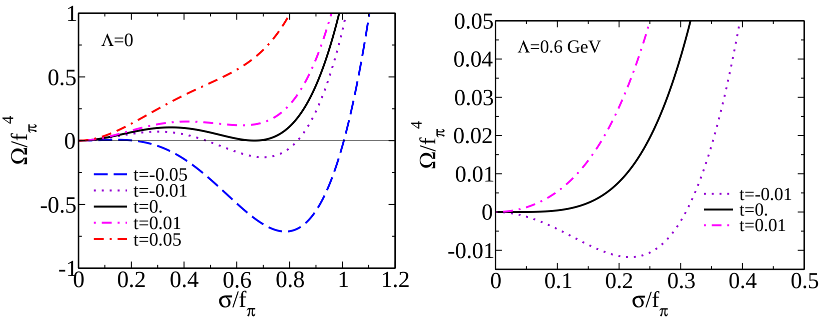
[V. Skokov, B. Friman, K.Redlich, BJS; arXiv:1005.3166]

Thermodynamic potential (numerical results for $\mu = 0$)

$$\Omega = U_{\text{Pol}} + U_{\text{meson}} + \Omega_{q\bar{q}} \qquad \text{with}$$

$$\Omega_{q\bar{q}} = -2N_f \int \frac{d^3p}{(2\pi)^3} \left\{ N_c E_q \theta(p^2 - \Lambda^2) + T \ln N_q + T \ln N_{\bar{q}} \right\}$$

$$N_q = 1 + 3\Phi e^{-\beta(E_q-\mu)} + 3\bar{\Phi} e^{-2\beta(E_q-\mu)} + e^{-3\beta(E_q-\mu)}$$



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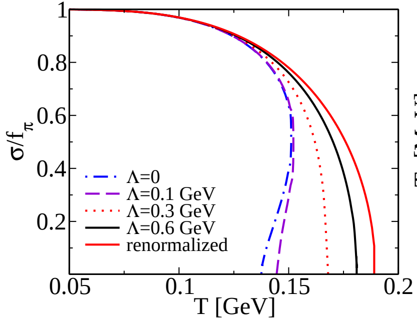

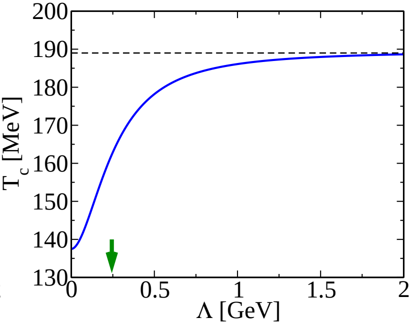

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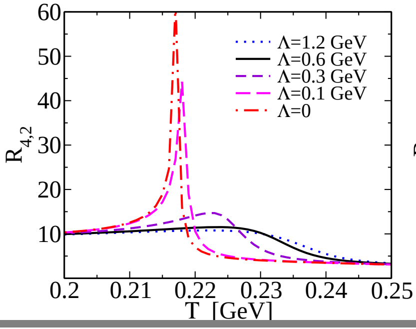
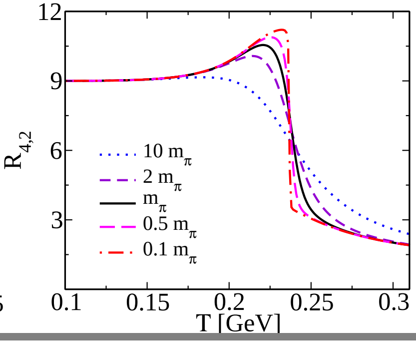




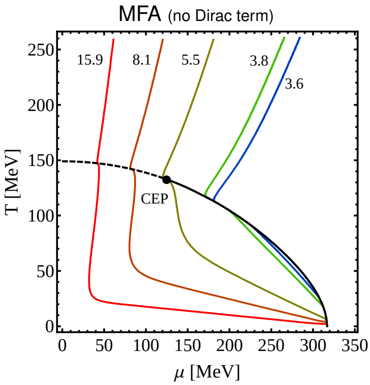
Isentropes $s/n = \mathrm{const}$ and Focussing

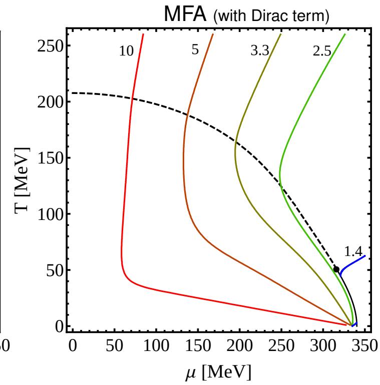
[E. Nakano, BJS, B.Stokic, B.Friman, K.Redlich '10]

here: $N_f = 2$ QM model: kink in MFA are washed out in FRG

$\rightarrow$ no focussing if fluctuations taken into account

a) influence of Dirac term   b) smallnest of critical region





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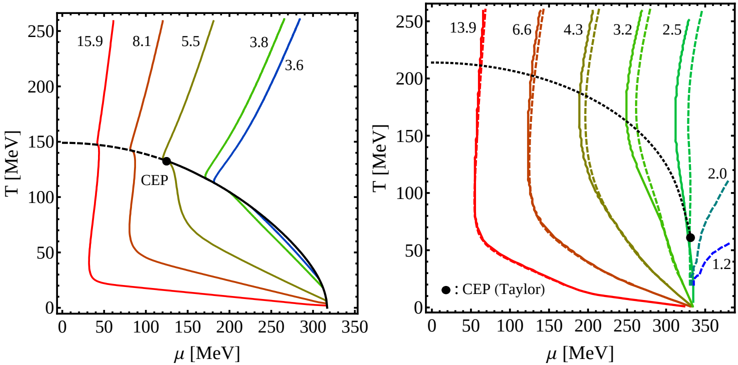
MFA (no Dirac term)   FRG (full grid and Taylor expansion)



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kink structure at boundary in mean field approximation

$\Rightarrow$ remnant of first-order transition in chiral limit

if Dirac term neglected

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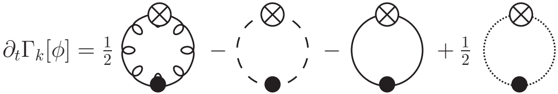
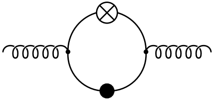
$\mathbf{T}_0(N_f, \mu)$ modification

full QCD FRG flow: gluon , ghosts, quark and meson (via hadronization) fluctuations
[J. Braun, H. Gies, L.M. Haas, F. Marhauser, J.M. Pawlowski et al.]



in presence of dynamical quarks
gluonic contribution modified:



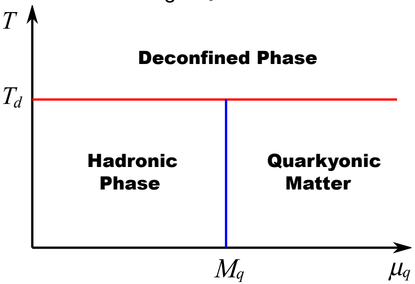
pure YM flow
($\rightarrow$ Polyakov loop potential):

$$T_0 \leftrightarrow \Lambda_{QCD} \qquad : \qquad T_0 \rightarrow T_0(N_f, \mu)$$

[BJS, Pawlowski, Wambach, 2007]
[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Quarkyonic Phase

large N_c world



[McLerran, Pisarski '07]

- if $\mu < M_q \sim M_B/N_c \sim O(1)$
  ➜
  hadronic phase with zero baryon density
- if $T > T_d \sim \Lambda_{\text{QCD}}$
  ➜
  # d.o.f. jump from $O(1)$ to $O(N_c^2)$ (gluons)
  deconfined phase
- since quark loops are suppressed by $1/N_c$
  T_d is **μ-independent**
- if $\mu > M_q$ ➜
  non-zero baryon density

quarkyonic phase is confining but chirally restored (➜parity-doubled hadrons)

What happens at $N_c = 3$?

Phase diagram $N_f = 2 + 1$

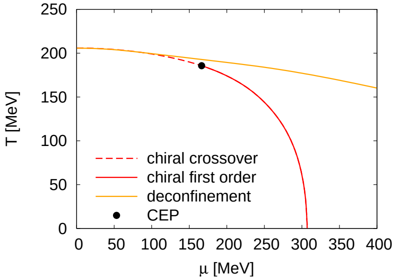
[BJS, M. Wagner; in preparation]

influence of Polyakov loop

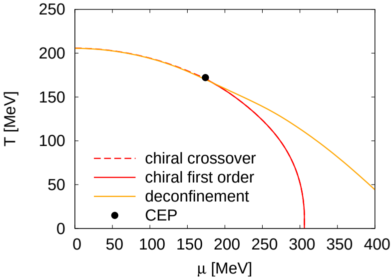
Logarithmic Polyakov loop potential

Mean-field approximation

$T_0 = 270$ MeV (constant)



$T_0(\mu)$ (i.e. with μ corrections)



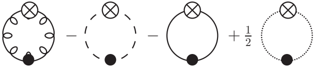
shrinking of possible quarkyonic phase

Functional Renormalization Group

similar conclusion if **fluctuations** are included

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ 

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

PQM truncation $\mathrm{N_f} = 2$ **[Herbst, Pawlowski, BJS, arXiv:1008.0081]**

$$\Gamma_k = \int d^4x \left\{ \bar{\psi} \left(\not{D} + \mu\gamma_0 + ih(\sigma + i\gamma_5 \vec{\tau}\vec{\pi}) \right) \psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \Omega_k[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] \right\}$$

Initial action at UV scale Λ:

$$\Omega_\Lambda[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] = U(\sigma, \vec{\pi}) + \mathcal{U}(\Phi, \bar{\Phi}) + \Omega_\Lambda^\infty[\sigma, \vec{\pi}, \Phi, \bar{\Phi}]$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

Functional Renormalization Group

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

Flow equation for PQM $\mathbf{N_f = 2}$ **[Herbst, Pawlowski, BJS; arXiv:1008.0081]**

$$\partial_t \Omega_k = \frac{k^5}{12\pi^2} \left[-\frac{2N_f N_c}{E_q} \left\{ 1 - N_q(T,\mu;\Phi,\bar{\Phi}) + N_{\bar{q}}(T,\mu;\Phi,\bar{\Phi}) \right\} + \frac{1}{E_\sigma} \mathrm{coth}\left(\frac{E_\sigma}{2T}\right) + \frac{3}{E_\pi} \mathrm{coth}\left(\frac{E_\pi}{2T}\right) \right]$$

with $E_{\sigma,\pi,q} = \sqrt{k^2 + m^2_{\sigma,\pi,q}}\,,\ m_\sigma^2 = 2\Omega_k' + 4\sigma^2\Omega_k''\,,\ m_\pi^2 = 2\Omega_k'\,,\ m_q^2 = g^2\sigma^2$

and

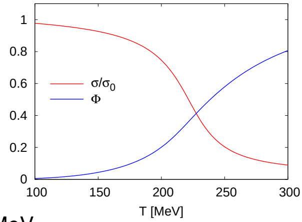
$$N_q(T,\mu;\Phi,\bar{\Phi}) = \frac{1 + 2\bar{\Phi} e^{\beta(E_q-\mu)} + \Phi e^{2\beta(E_q-\mu)}}{1 + 3\bar{\Phi} e^{\beta(E_q-\mu)} + 3\Phi e^{2\beta(E_q-\mu)} + e^{3\beta(E_q-\mu)}}$$

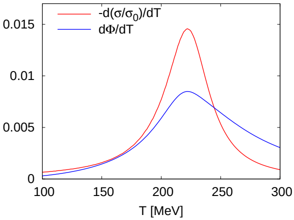
$$N_{\bar{q}}(T,\mu;\Phi,\bar{\Phi}) = N_q(T,-\mu;\Phi,\bar{\Phi})|_{\mu\to-\mu}$$

cf. [Skokov et al. arXiv:1004.2665]

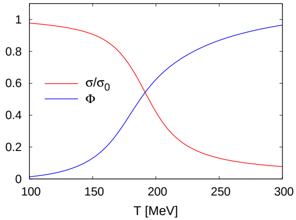
$\mu = 0$: order parameters and T-derivatives

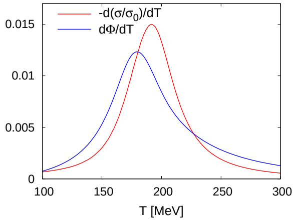
$T_0 = 270$ MeV



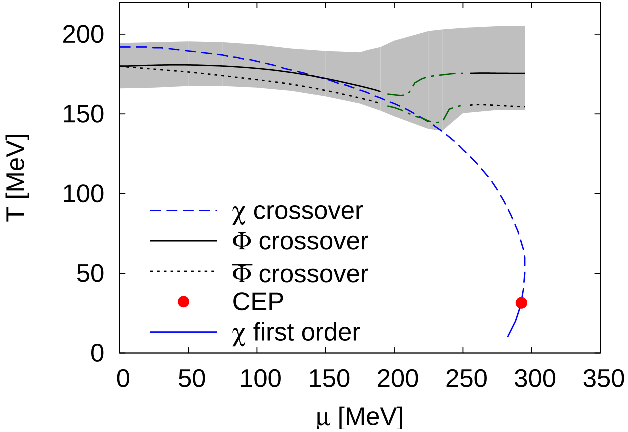


$T_0 = 208$ MeV



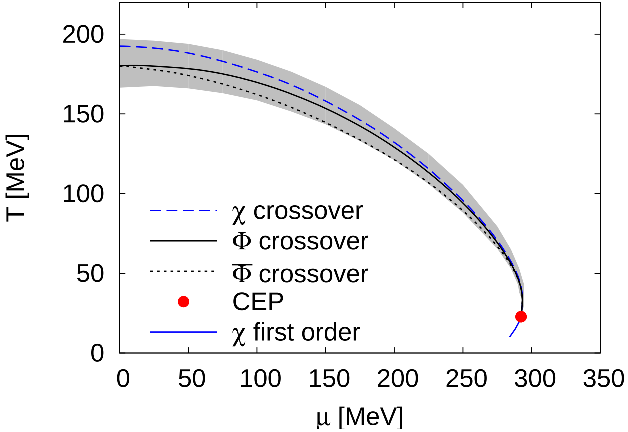


Phase diagram $T_0 = 208$ MeV



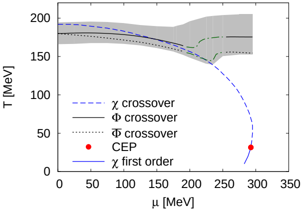
[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Phase diagram $T_0(\mu), T_0(0) = 208$ MeV



[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Thermodynamics



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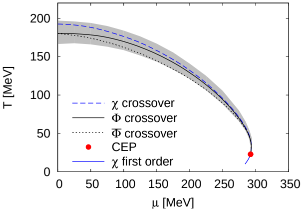
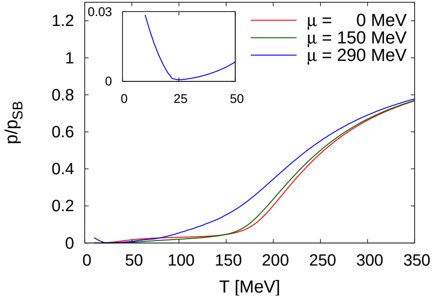
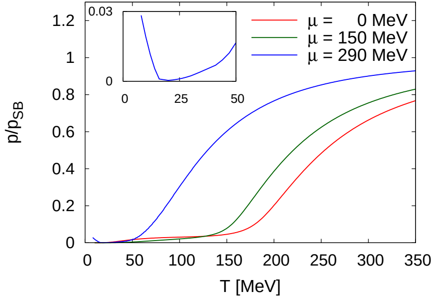


$T_0(\mu)$ MeV





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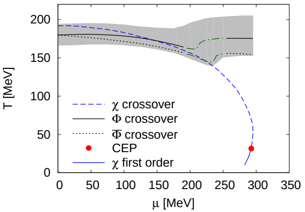

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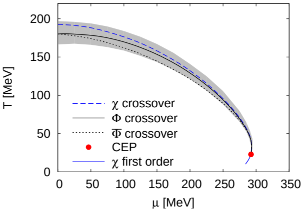

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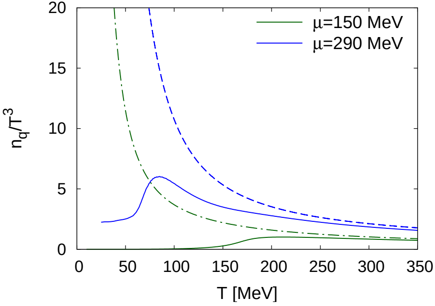
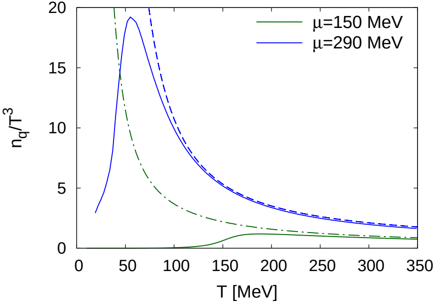
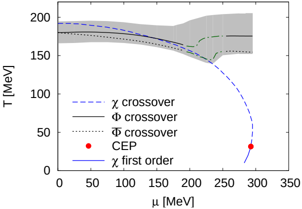




Thermodynamics



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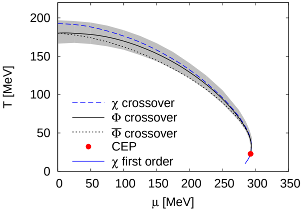
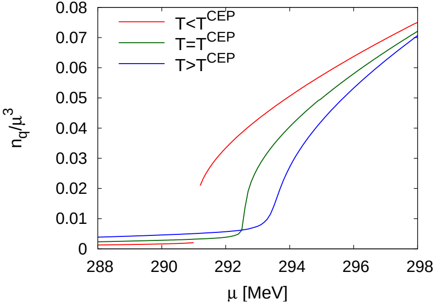
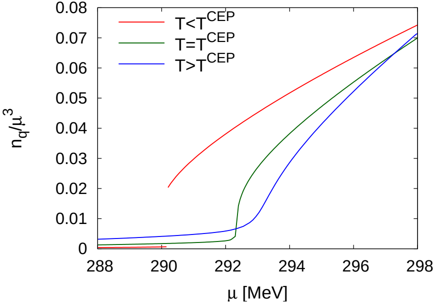


$T_0(\mu)$ MeV





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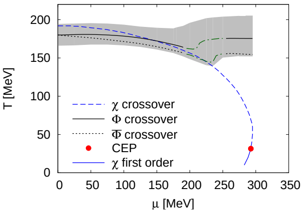

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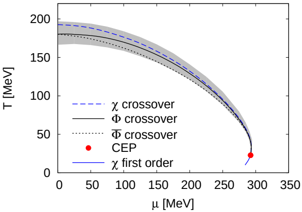
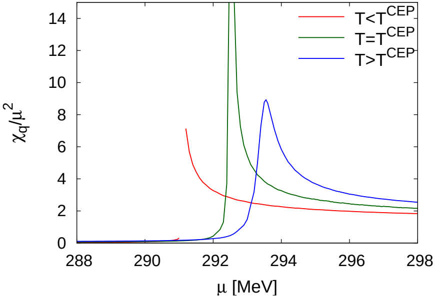
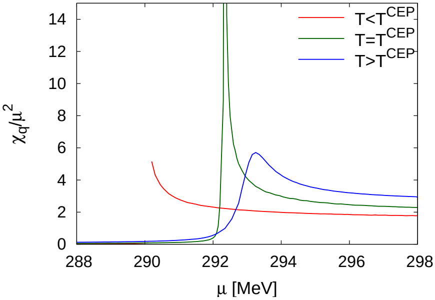


$T_0(\mu)$ MeV





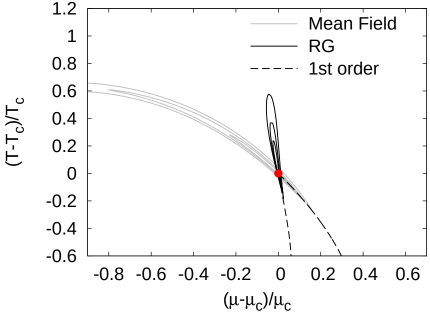
Critical region

similar conclusion if **fluctuations** are included

fluctuations via Functional Renormalization Group

comparison: $N_f = 2$ **QM model**

Mean Field $\leftrightarrow$ RG analysis

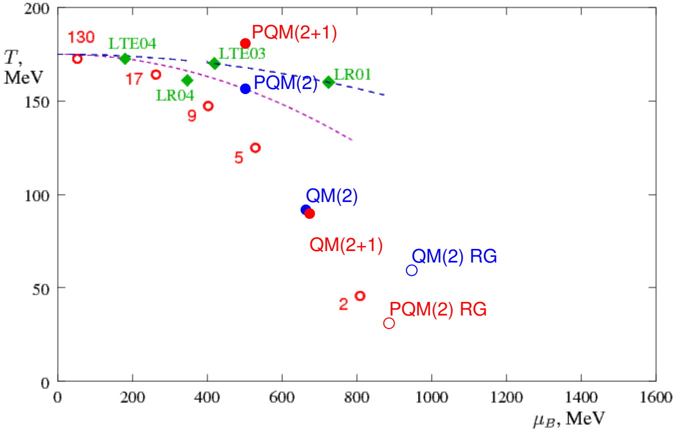


[BJS, J. Wambach ’06]

Critical Endpoints

model studies vs. lattice simulations

Blue points: models   Lines & green points: lattice   Red circles: Freezeout points for HIC



lattice methods:

- reweighting
- imaginary μ_B
- Taylor expansion around $\mu_B = 0$

Summary

- $N_f = 2$ and $N_f = 2 + 1$ chiral (Polyakov)-quark-meson model study

➜ Mean-field approximation and FRG

➜ **fluctuations are important**

functional approaches (such as the presented FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

Findings:

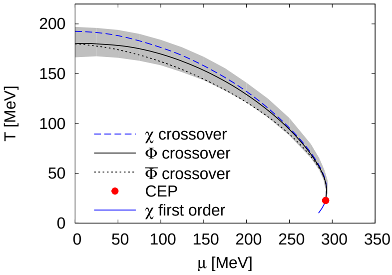
▷ matter **back-reaction to YM sector**: $T_0 \Rightarrow T_0(N_f, \mu)$

▷ **FRG with PQM truncation**: Chiral & deconfinement transition **coincide** for $N_f = 2$ with $T_0(\mu)$-corrections

▷ same conclusion for $N_f = 2 + 1$?

▷ **role of quantum fluctuations**

effects of Dirac term in a mean-field approximation



Outlook:

▷ include **glue dynamics** with FRG

→ towards full QCD

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49. Internationale Universitätswochen für Theoretische Physik

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Schladming, Styria, Austria, February 26 - March 5, 2011

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