Dense QCD Phases in Heavy Ion Collisions

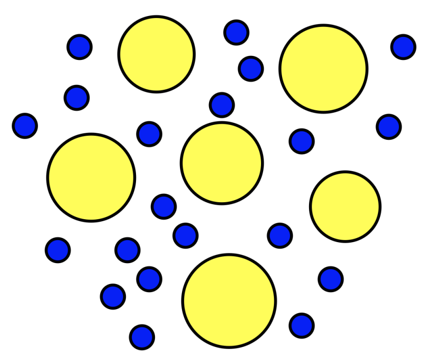
JINR, Dubna, August 21 – September 4, 2010

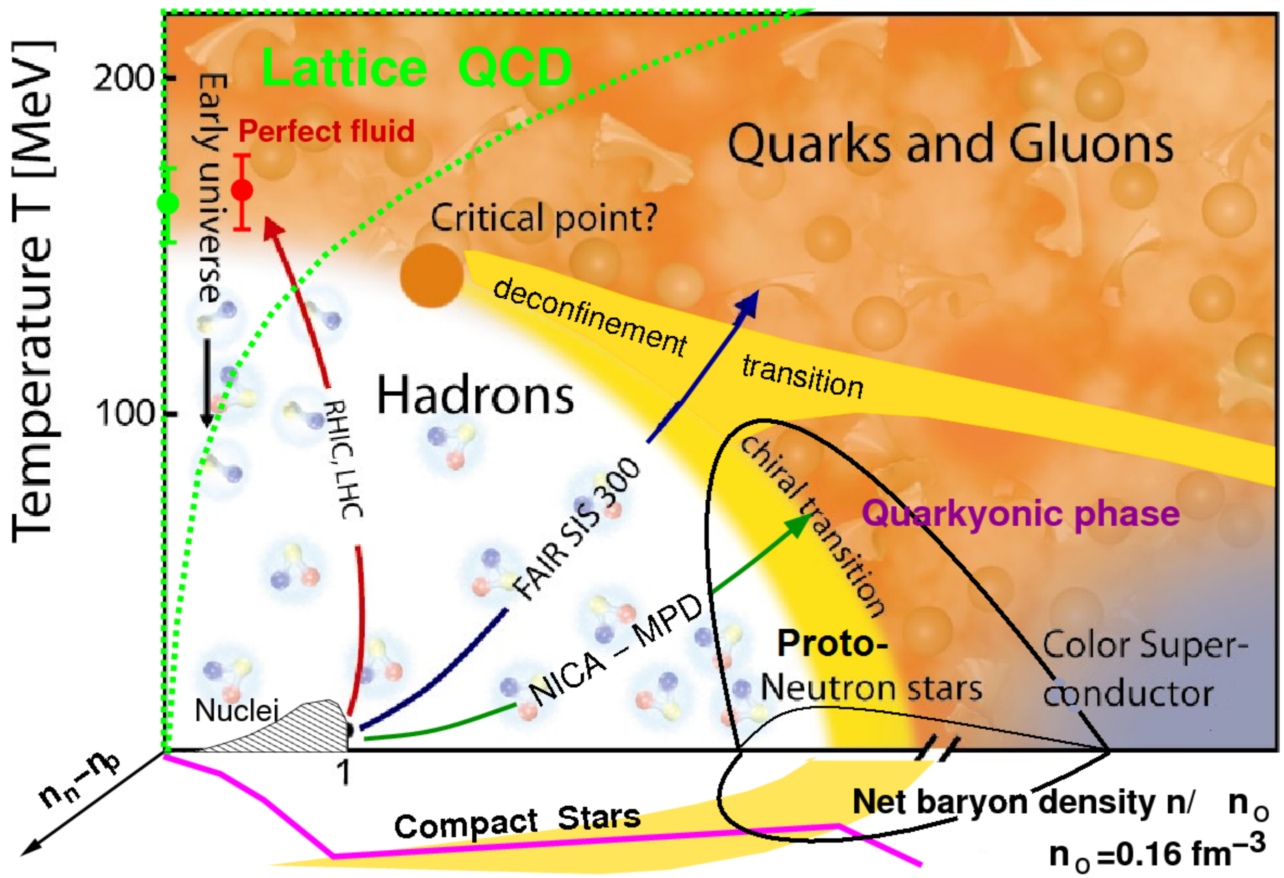
Phase Transitions & Instabilities

Jørgen Randrup (LBNL)

Lecture #1: Phase coexistence

Lecture #2: Phase separation





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Phase Transitions & Instabilities

Static

Phase coexistence

Illustrative examples

Finite range effects

Dynamic

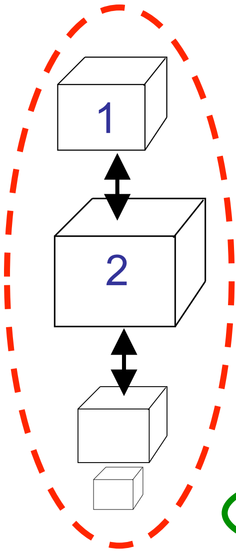
Phase crossing

Mean field instabilities

Instabilities in fluid dynamics

Instabilities in chiral dynamics

Basic thermodynamics



$\boldsymbol{X}_1 = \{E_1, N_1, V_1, \ldots\} \Rightarrow S_1(\boldsymbol{X}_1)$

$\boldsymbol{X}_2 = \{E_2, N_2, V_2, \ldots\} \Rightarrow S_2(\boldsymbol{X}_2)$

$\boldsymbol{X} = \{E, N, V, \ldots\} = \boldsymbol{X}_1 + \boldsymbol{X}_2 + \ldots$

$S = S_1 + S_2 + \ldots$

$$\lambda_i^\ell \equiv \frac{\partial S_i}{\partial X_i^\ell} \quad \begin{cases} \lambda_i^E = \dfrac{\partial S_i}{\partial E_i} = \beta_i = \dfrac{1}{T_i} \\ \lambda_i^N = \dfrac{\partial S_i}{\partial N_i} = \alpha_i = -\dfrac{\mu_i}{T_i} \\ \lambda_i^V = \dfrac{\partial S_i}{\partial V_i} = \pi_i = \dfrac{p_i}{T_i} \end{cases}$$

The combined system is in equilibrium provided S has a local *maximum* - which requires $\delta S = 0$ and $\delta^2 S < 0$:

$$\delta X^\ell = \sum_i \delta X_i^\ell \doteq 0 \quad \begin{cases} \delta E = 0 \\ \delta N = 0 \\ \delta V = 0 \end{cases}$$

δS:
$$0 \doteq \delta S([\boldsymbol{X}_i]) = \sum_i \delta S_i(\boldsymbol{X}_i = \bar{\boldsymbol{X}}_i) = \sum_{i\ell} \left(\frac{\partial S_i}{\partial X_i^\ell}\right)_{\boldsymbol{X}_i = \bar{\boldsymbol{X}}_i} \delta X_i^\ell = \sum_\ell \left(\sum_i \lambda_i^\ell \, \delta X_i^\ell\right)$$

$$\Rightarrow \quad \lambda_1^\ell \doteq \lambda_2^\ell \doteq \ldots \quad \begin{cases} T_1 = T_2 = \ldots \\ \mu_1 = \mu_2 = \ldots \\ p_1 = p_2 = \ldots \end{cases}$$

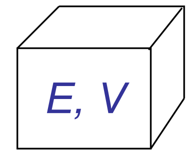
$\delta^2 S$:
$$0 \dot{>} \sum_i \delta^2 S_i(\boldsymbol{X}_i = \bar{\boldsymbol{X}}_i) = \sum_{i\ell_1\ell_2} \left(\frac{\partial^2 S_i}{\partial X_i^{\ell_1} \partial X_i^{\ell_2}}\right)_{\boldsymbol{X}_i = \bar{\boldsymbol{X}}_i} \delta X_i^{\ell_1} \, \delta X_i^{\ell_2}$$

=> Only negative eigenvalues of the entropy curvature matrices:
$$\frac{\partial^2 S_i}{\partial X_i^{\ell_1} \partial X_i^{\ell_2}}$$

Thermodynamics with no conserved charge:

Statistical equilibrium in bulk matter

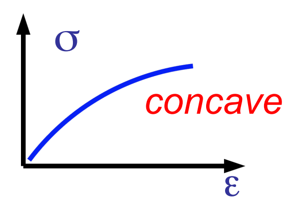
Control parameter(s) $\{X\}$:
- Energy $E = V\varepsilon$ $\varepsilon = E/V$
- Volume $V \to \infty$

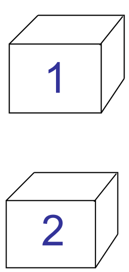


Entropy function $S\{X\}$: $S(E,V) = V\sigma(\varepsilon)$

Derivative(s) $\lambda_X = \partial_X S$:
- $\beta = 1/T \;=\; \partial_E S(E,V) \;=\; \partial_\varepsilon \sigma(\varepsilon)$ *temperature*
- $\pi = p/T \;=\; \partial_V S(E,V) \;=\; \sigma - \beta\varepsilon$ *pressure*

Thermodynamic (local) ***stability***: $\delta^2 S_{tot} < 0$
=> Entropy curvature $\partial_\varepsilon^2 \sigma$ must be *negative*





Thermodynamic ***coexistence***:
=> $T_1 = T_2$ & $p_1 = p_2$

<=> $\sigma(\varepsilon)$ *has common tangent!*

?!

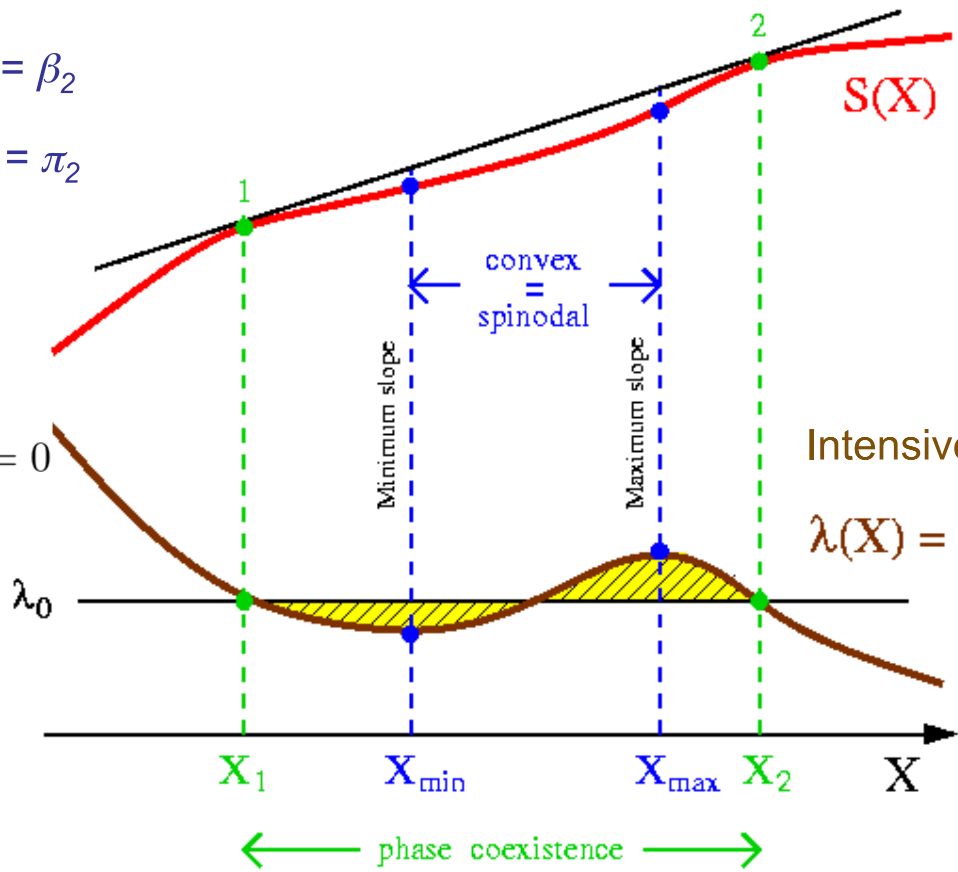
First order <=> Phase coexistence <=> Spinodal instability

Extensive variable *X*

Entropy function *S(X)*

… occur when S(X) is locally convex:

$$X=E: \begin{cases} \partial_\varepsilon \sigma(\varepsilon) = \beta: \ \beta_1 = \beta_2 \\ \pi = \sigma - \beta\varepsilon: \ \pi_1 = \pi_2 \end{cases}$$



$$\int_{X_1}^{X_2} dX\,(\lambda(X) - \lambda_0) = 0$$

Maxwell construction:

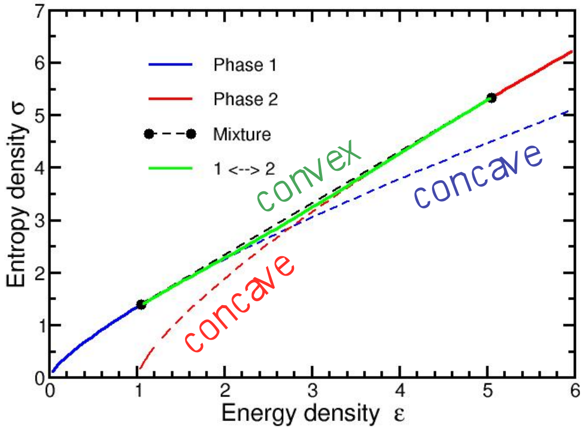
Intensive variable:

$\lambda(X) = -dS/dX$

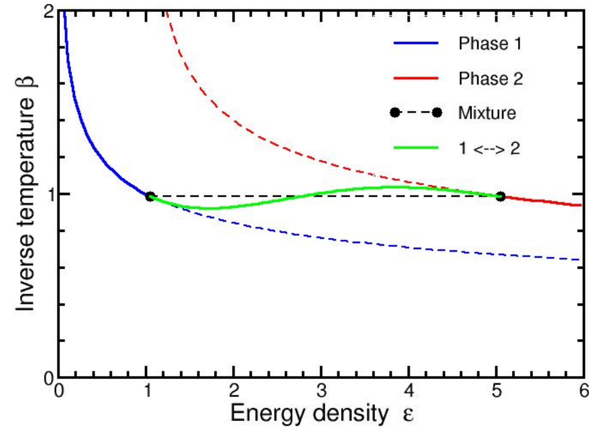
$X=E \Rightarrow \lambda=\beta$

Phase transformation with no conserved charge:

Entropy density: $\sigma(\varepsilon)$

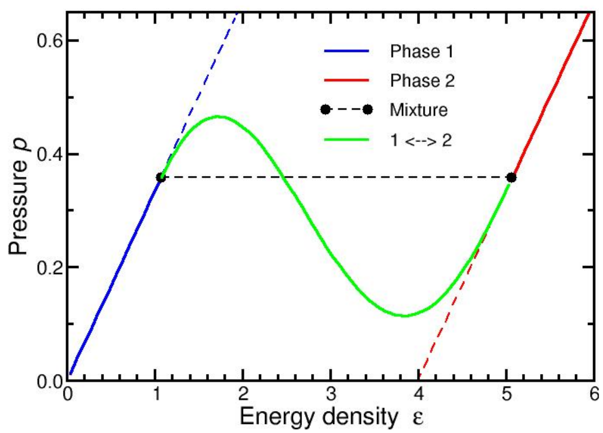


Inverse temperature: $\beta(\varepsilon) = \partial\sigma/\partial\varepsilon$

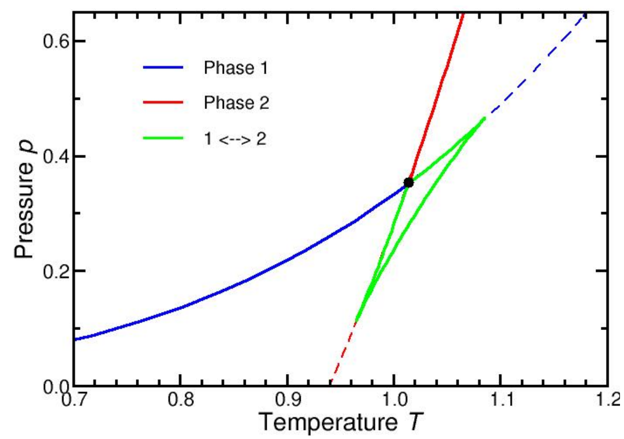


Equation of State

Pressure: $p(\varepsilon) = T\sigma - \varepsilon$



Pressure: $p(T)$



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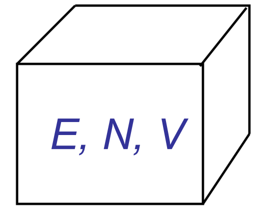
Thermodynamics with one conserved charge:

Statistical equilibrium in bulk matter

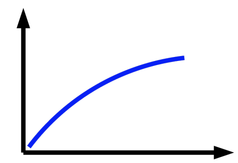
Control parameter(s) $\{X\}$:

$$\begin{cases} \text{Energy } E = V\varepsilon & \varepsilon = E/V \\ \text{Charge } N = V\rho & \rho = N/V \\ \text{Volume } V \to \infty & \end{cases}$$



Entropy function $S\{X\}$:

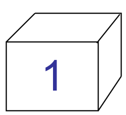
$$S(E,N,V) = V\sigma(\varepsilon,\rho)$$

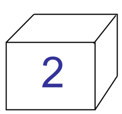


Derivative(s) $\lambda_X = \partial_X S$:

$$\begin{cases} \beta = 1/T = \partial_E S(E,N,V) = \partial_\varepsilon \sigma(\varepsilon,\rho) \\ \alpha = -\mu/T = \partial_N S(E,N,V) = \partial_\rho \sigma(\varepsilon,\rho) \\ \pi = p/T = \partial_V S(E,N,V) = \sigma - \beta\varepsilon - \alpha\rho \end{cases}$$

Thermodynamic (local) ***stability***: $\delta^2 S_{tot} < 0$
=> Curvature matrix $\{\partial_\chi \partial_{\chi'} \sigma(\varepsilon,\rho)\}$ has only *negative* eigenvalues





Thermodynamic ***coexistence***: $\delta S_{tot} = 0$
=> $T_1 = T_2$ & $\mu_1 = \mu_2$ & $p_1 = p_2$

<=> $\sigma(\varepsilon,\rho)$ *has common tangent!*

?!

Microcanonical -> Canonical:

entropy density $\sigma(\varepsilon,\rho)$ $\Rightarrow$

$$\begin{cases} \beta(\varepsilon,\rho) = \partial_\varepsilon \sigma(\varepsilon,\rho) = 1/T(\varepsilon,\rho) \\ \alpha(\varepsilon,\rho) = \partial_\rho \sigma(\varepsilon,\rho) = -\mu(\varepsilon,\rho)/T(\varepsilon,\rho) \end{cases}$$

temperature

chemical potential

$$\Rightarrow \begin{cases} p(\varepsilon,\rho) = \sigma T - \varepsilon + \mu\rho \\ h(\varepsilon,\rho) = p + \varepsilon \end{cases}$$

pressure

enthalpy density

<u>Canonical representation:</u> only <E> is specified

Then replace S by $S' = S - \beta E$ and require $\delta S'=0$ & $\delta^2 S'<0$

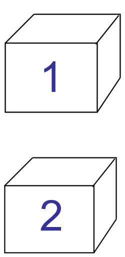
Or, equivalently, consider $F = -TS' = E - TS$ and require $\delta F=0$ & $\delta^2 F>0$

free energy density

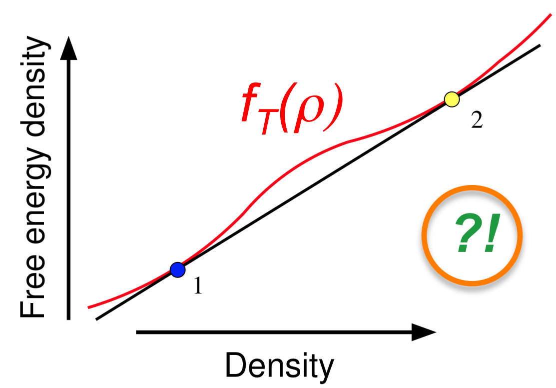
$$f_T(\rho) \equiv \varepsilon_T(\rho) - T\sigma_T(\rho) = \mu_T(\rho)\rho - p_T(\rho)$$

$$\mu_T(\rho) = \partial_\rho f_T(\rho)$$

$$\sigma_T(\rho) = -\partial_T f_T(\rho)$$



Phase coexistence $\Leftrightarrow$ *$f_T(\rho)$ has common tangent!*



Phase Transitions & Instabilities

Phase coexistence

Illustrative examples

Finite range effects

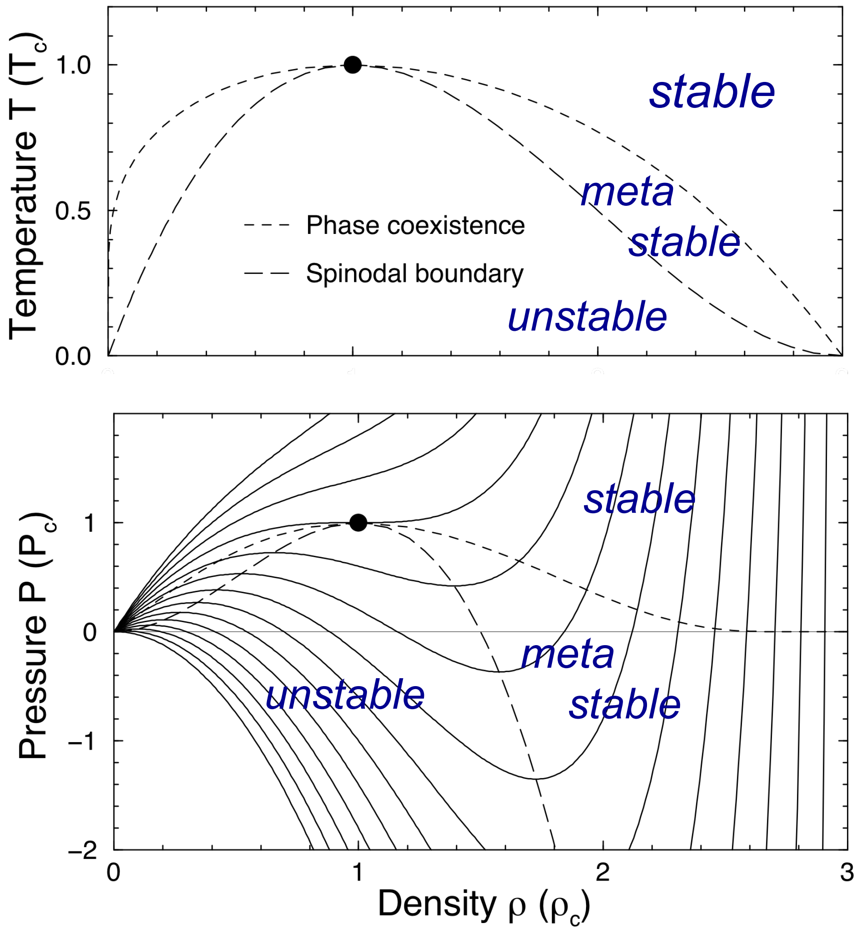
Phase crossing

Mean field instabilities

Instabilities in fluid dynamics

Instabilities in chiral dynamics

Van der Waals fluid



Phase Diagram

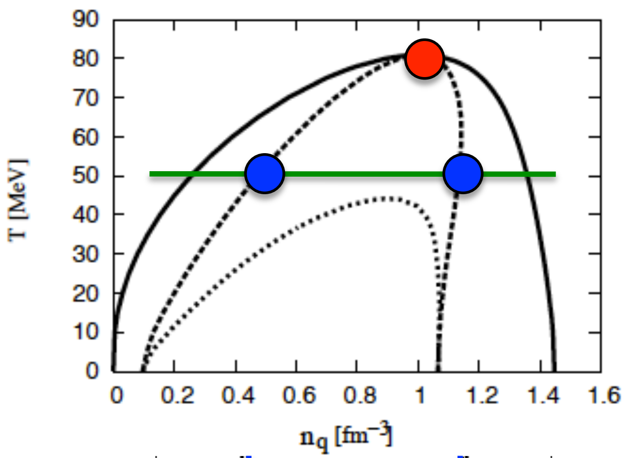
Equation of State:

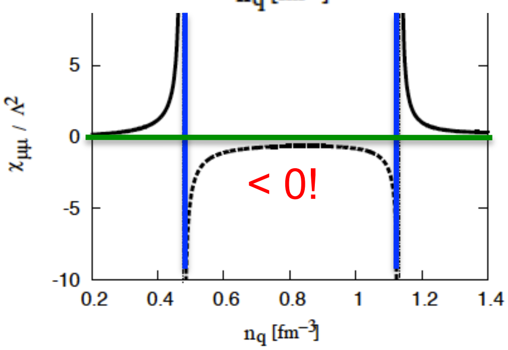
$$p = \frac{\rho T}{1 - b\rho} - a\rho^2$$

?!

Density fluctuations in the presence of spinodal instabilities

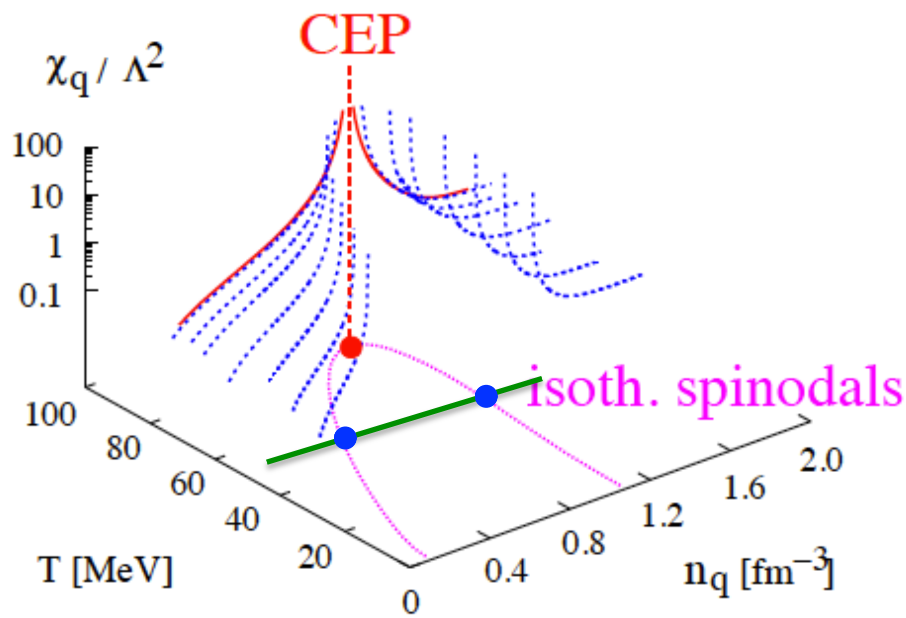
C. Sasaki, B. Friman, K. Redlich, Phys. Rev. Lett. 99, 232301 (2007)





Nambu – Jona-Lasino model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + \mu\gamma_0)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\vec{\tau}\gamma_5\psi\right)^2\right]$$



Net quark number susceptibility at T=50 MeV as a function of the quark number density across the first-order phase transition

The net quark number susceptibility in the stable and meta-stable regions

Inclusion of interaction energy

Entropy density of non-interacting system: $\sigma_{\text{free}}(\varepsilon,\rho)$

Interaction-energy density: $w(\rho)$

Entropy density with interaction included: $\sigma(\varepsilon,\rho) \doteq \sigma_{\text{free}}(\varepsilon - w(\rho),\rho)$

=>
$$\begin{cases} \beta(\varepsilon,\rho) = \dfrac{\partial\sigma(\varepsilon,\rho)}{\partial\varepsilon} = \dfrac{\partial\sigma(\varepsilon-w(\rho),\rho)}{\partial\varepsilon} = \beta_{\text{free}}(\varepsilon-w(\rho),\rho) \\ \alpha(\varepsilon,\rho) = \dfrac{\partial\sigma(\varepsilon,\rho)}{\partial\rho} = \dfrac{\partial\sigma(\varepsilon-w(\rho),\rho)}{\partial\rho} = \alpha_{\text{free}}(\varepsilon-w(\rho),\rho)-\beta_{\text{free}}(\varepsilon-w(\rho),\rho)\partial_\rho w(\rho) \end{cases}$$

Chemical potential is shifted: $\mu(T;\rho) = \mu_{\text{free}}(T;\rho) + \partial_\rho w(\rho)$

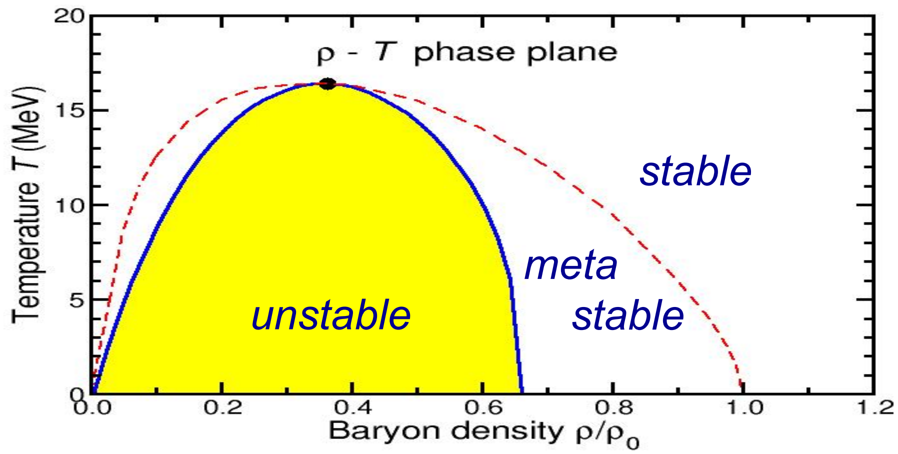
Free energy density is shifted: $f = \varepsilon - T\sigma = f_{\text{free}} + w$

Note: $\partial_\rho f(T;\rho) = \partial_\rho f_{\text{free}}(T;\rho) + \partial_\rho w(\rho) = \mu_{\text{free}}(T;\rho) + \partial_\rho w(\rho) = \mu(T;\rho)$

Pressure is augmented by $p_{\text{int}}(\rho) = \rho\dfrac{\partial w}{\partial\rho} - w(\rho) = \rho^2\dfrac{\partial}{\partial\rho}\dfrac{w}{\rho}$ since

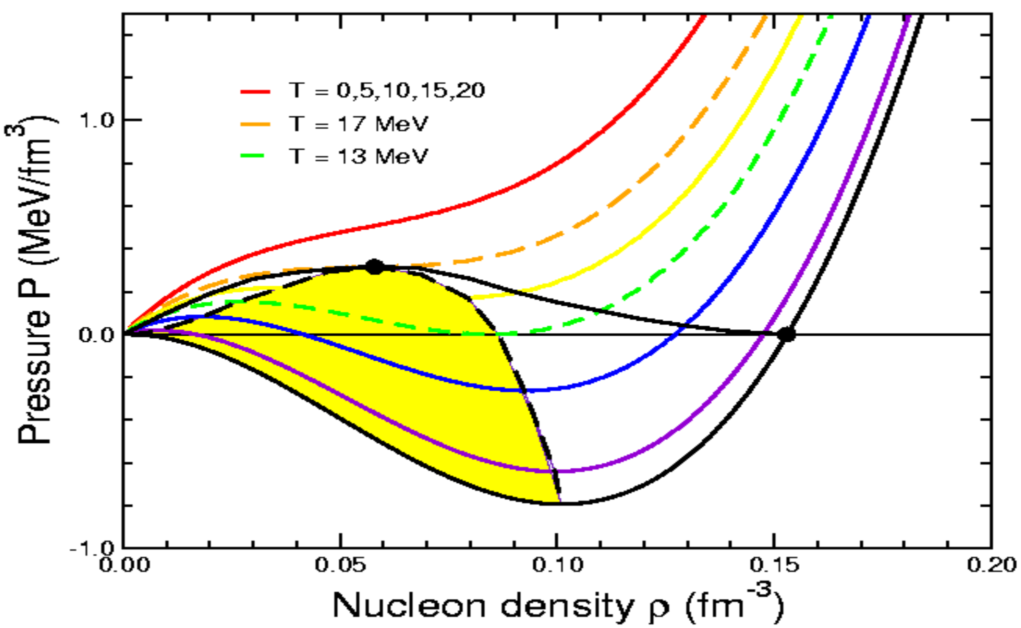
$$p = \rho\mu - f = \rho(\mu_{\text{free}}+\partial_\rho w)-(f_{\text{free}}+w) = (\rho\mu_{\text{free}}-f_{\text{free}})+(\rho\partial_\rho w-w) = p_{\text{free}}+p_{\text{int}}$$

Nuclear matter



Phase diagram

$$\varepsilon(T;\rho) = \varepsilon_{\rm FG}(T;\rho) + w(\rho)$$



Equation of state: $p_T(\rho)$

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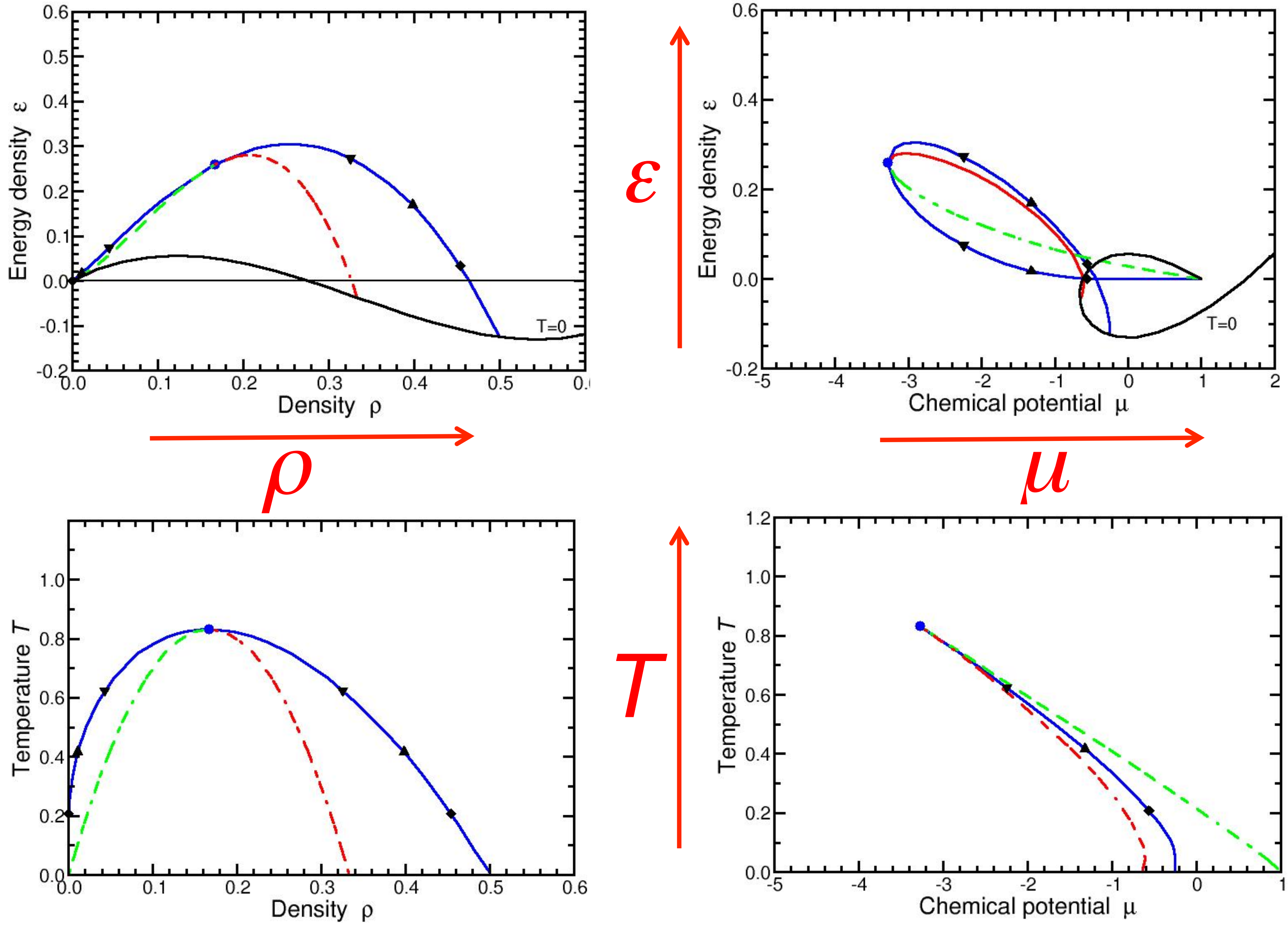
Nuclear phase diagram in different representations



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Isentropic changes

Entropy density: $\sigma(\varepsilon,\rho)$

Energy density: ε

Net baryon density: ρ

Temperature: $T(\varepsilon,\rho) = 1/\sigma_\varepsilon$

Chemical potential: $\mu(\varepsilon,\rho) = -T\sigma_\rho$

Pressure: $p(\varepsilon,\rho) = T\sigma - \varepsilon + \mu\rho$

Enthalpy density: $h(\varepsilon,\rho) = p + \varepsilon$

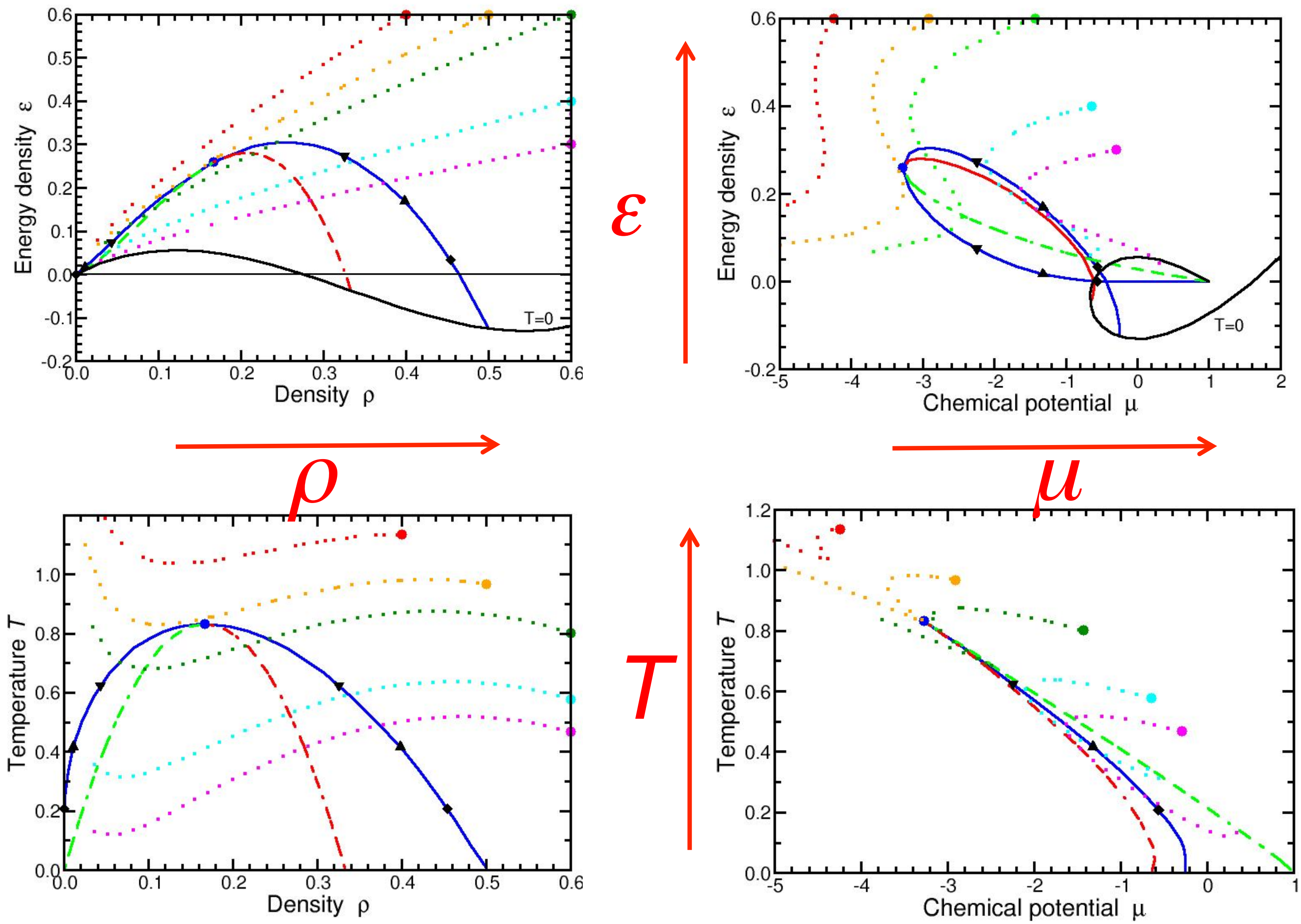
Entropy per (net) baryon: $s(\varepsilon,\rho) = \sigma/\rho$

Changes: $(\delta\varepsilon,\delta\rho)$ => δs :

$$\rho^2 T\delta s = \rho^2 T\delta(\sigma/\rho) = \rho T\delta\sigma - T\sigma\delta\rho = \rho\delta\varepsilon - \mu\rho\delta\rho - [h-\mu\rho]\delta\rho = \rho\delta\varepsilon - h\delta\rho$$

$$\delta s = 0 \quad \Rightarrow \quad \rho\delta\varepsilon = h\delta\rho$$

Isentropic phase trajectories in different representations



Phase Transitions & Instabilities

Phase coexistence

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Phase crossing

Mean field instabilities

Instabilities in fluid dynamics

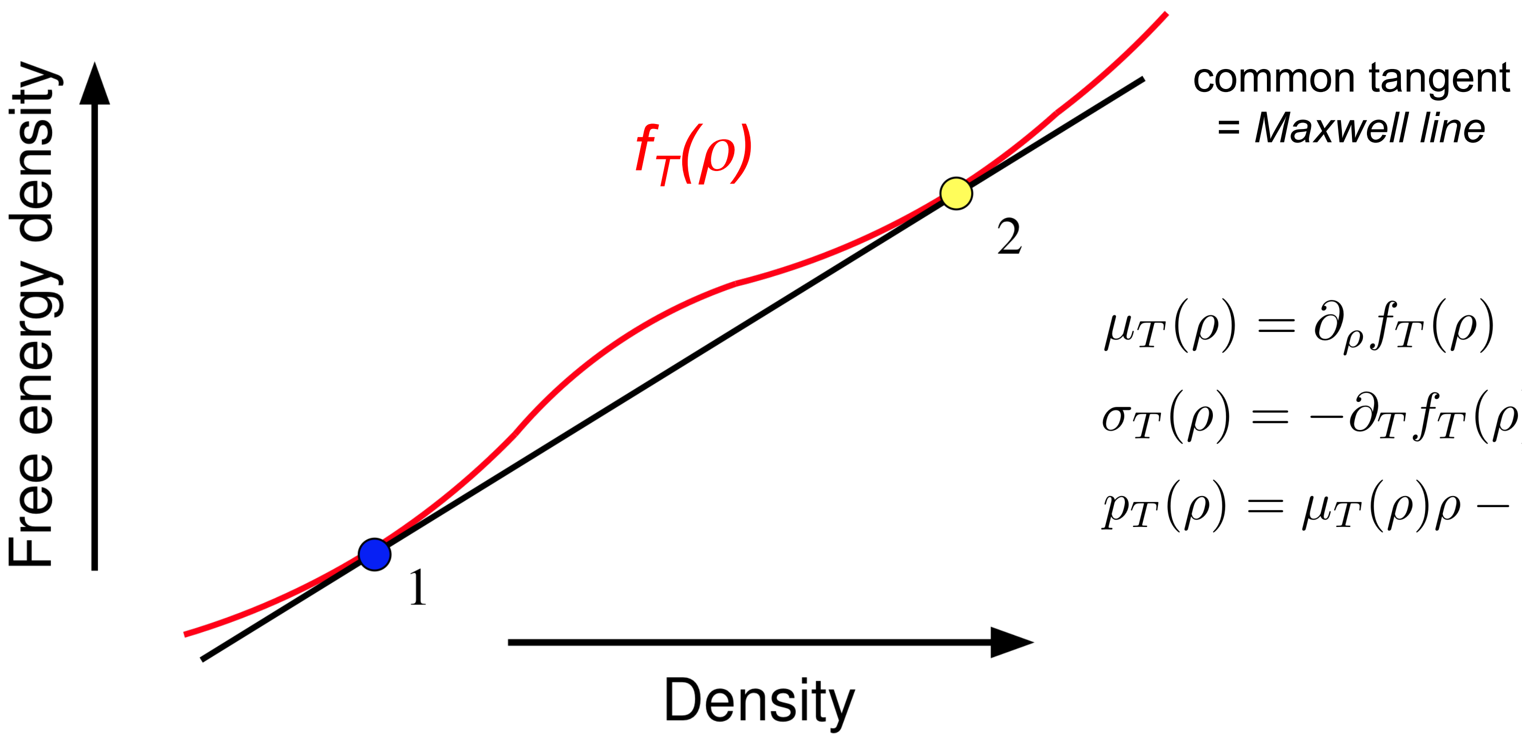
Instabilities in chiral dynamics

Canonical description: T specified

Free energy density

$$f_T(\rho) \equiv \varepsilon_T(\rho) - T\sigma_T(\rho) = \mu_T(\rho)\rho - p_T(\rho)$$

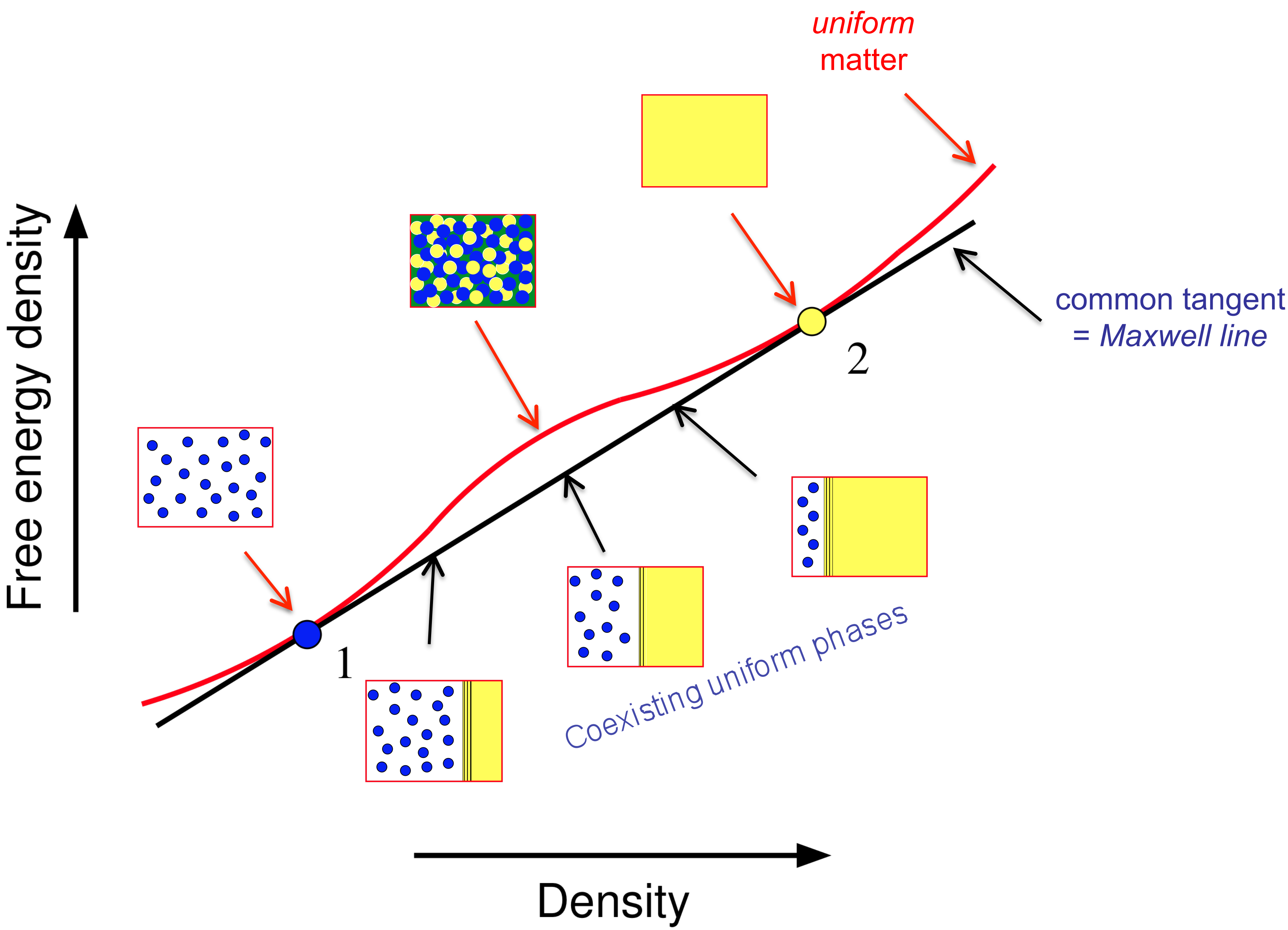
Phase coexistence ⇔ common tangent:



$$\mu_T(\rho) = \partial_\rho f_T(\rho)$$

$$\sigma_T(\rho) = -\partial_T f_T(\rho)$$

$$p_T(\rho) = \mu_T(\rho)\rho - f_T(\rho)$$



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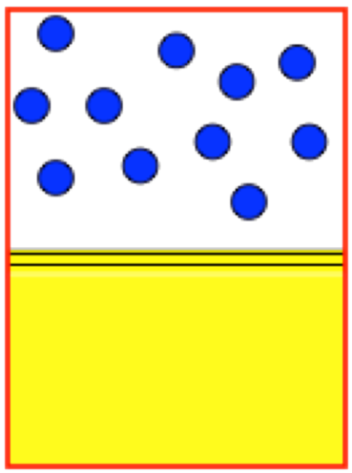
Nuclear liquid-gas phase coexistence

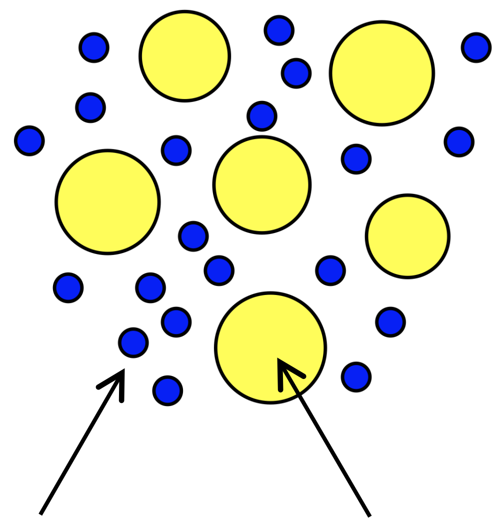
nucleon gas phase



nuclear liquid phase
(nuclear matter)

can *coexist* in mutual equilibrium

≠



nucleons fragments

phase mixture

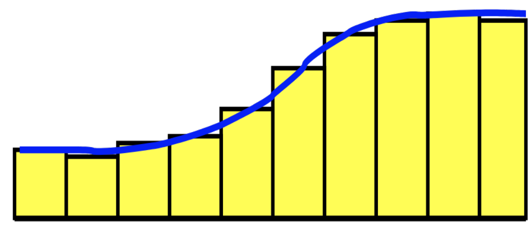
Equation of state: Finite range

Free energy density for uniform matter: $f_0(\rho, T)$

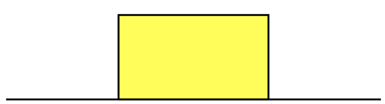
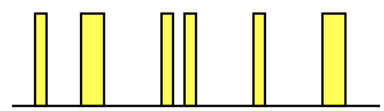
But we need to treat non-uniform systems: $\rho(\boldsymbol{r}),\ T(\boldsymbol{r})$

Local density approximation:

$$f[\rho(\cdot), T(\cdot)](\boldsymbol{r}) \ \doteq\ f_0(\rho(\boldsymbol{r}), T(\boldsymbol{r}))$$



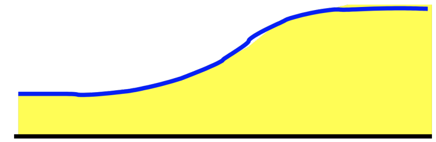
… implies:

F() = F()

No good!

=> Finite range *must* be taken into account

Non-uniform density $\tilde{\rho}(\boldsymbol{r})$



gradient contribution $\quad \tilde{w}(\boldsymbol{r}) \equiv w_0(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$

local entropy density: $\quad \tilde{\sigma}(\boldsymbol{r}) \equiv \sigma(\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r}))$

$\Rightarrow$ total entropy: $S[\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})] \equiv \int \tilde{\sigma}(\boldsymbol{r}) d\boldsymbol{r}$

$$\Rightarrow \begin{cases} \tilde{\beta}(\boldsymbol{r}) \equiv \dfrac{\delta S}{\delta \tilde{\varepsilon}(\boldsymbol{r})} \Rightarrow \tilde{T}(\boldsymbol{r}) \\ \tilde{\alpha}(\boldsymbol{r}) \equiv \dfrac{\delta S}{\delta \tilde{\rho}(\boldsymbol{r})} \Rightarrow \tilde{\mu}(\boldsymbol{r}) \end{cases}$$

?!

$\Rightarrow$ local pressure $\tilde{p}(\boldsymbol{r})$ & local enthalpy density $\tilde{h}(\boldsymbol{r})$ & ...

Note: Constant $T(\boldsymbol{r})$ and $\mu(\boldsymbol{r})$ => constant $p(\boldsymbol{r})$

Canonical scenario: constant temperature T

$\Rightarrow$ Free energy density: $\tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$

J. Randrup, Phys. Rev. C 79, 054911 (2009)

The gradient term modifies the local pressure

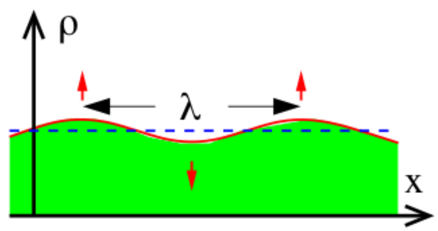
Small deviations from uniformity:

$$p(r) \approx p_0(\varepsilon(r), \rho(r)) - C\rho_0 \nabla^2 \rho(r)$$

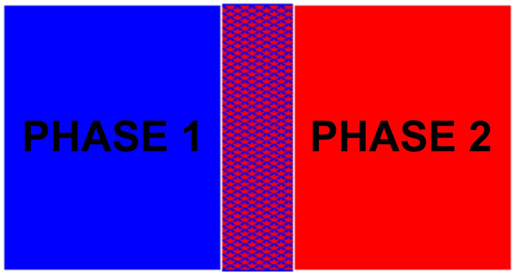
Small harmonic density undulations:

$$\rho(r,t) = \rho_0 + \delta\rho(x,t) \doteq \rho_0 + \rho_k \mathrm{e}^{ikx - i\omega t}$$

$$\boxed{p_k \rightarrow p_k + C\rho_0 k^2 \rho_k}$$



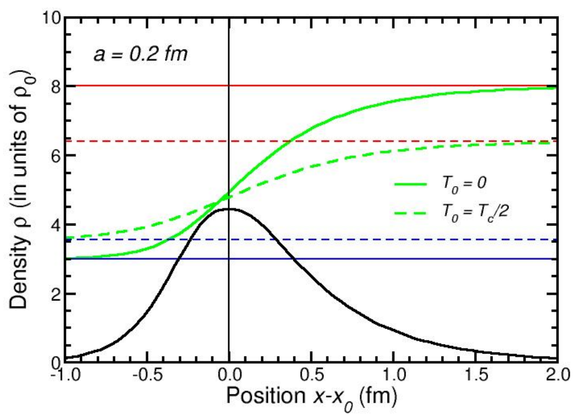
The gradient term generates a phase boundary



Global equilibrium requires constant *T, μ, p*:

$$0 \doteq \delta S - \beta_0 \delta E - \alpha_0 \delta N = \int dx \left\{ [\tilde{\beta}(x) - \beta_0]\delta\tilde{\varepsilon}(x) + [\tilde{\alpha}(x) - \alpha_0]\delta\tilde{\rho}(x) \right\}$$

$$\Rightarrow \quad \tilde{\beta}(x) = \beta_0 \ \ \& \ \ \tilde{\alpha}(x) = \alpha_0 \quad \Rightarrow \quad \tilde{p}(x) = p_0$$

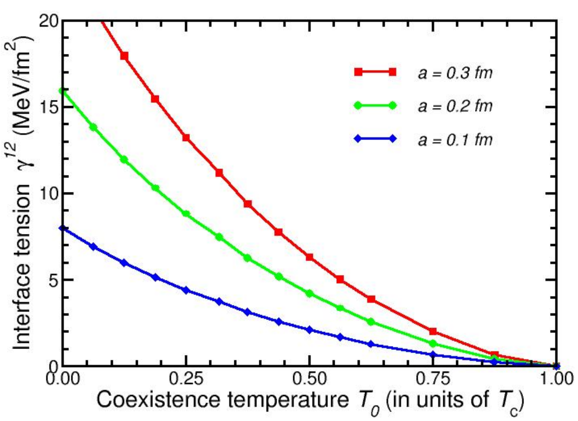


The interface density profile is determined by

$$C\partial_x^2 \rho(x) \doteq \mu_T(\rho(x)) - \mu_0 = \partial_\rho \Delta f_T(\rho(x))$$

where $\quad \Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho)$

$$f_T^M(\rho) \equiv f_T(\rho_i) + \mu_0(\rho - \rho_i) \leq f_T(\rho)$$



The interface tension is given by

$$\gamma_T^{12} = \int_{\rho_1}^{\rho_2} d\rho \, [2C\Delta f_T(\rho)]^{\frac{1}{2}}$$

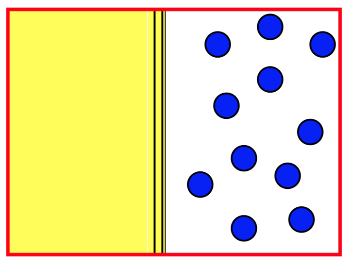
ρ(x) not Needed!

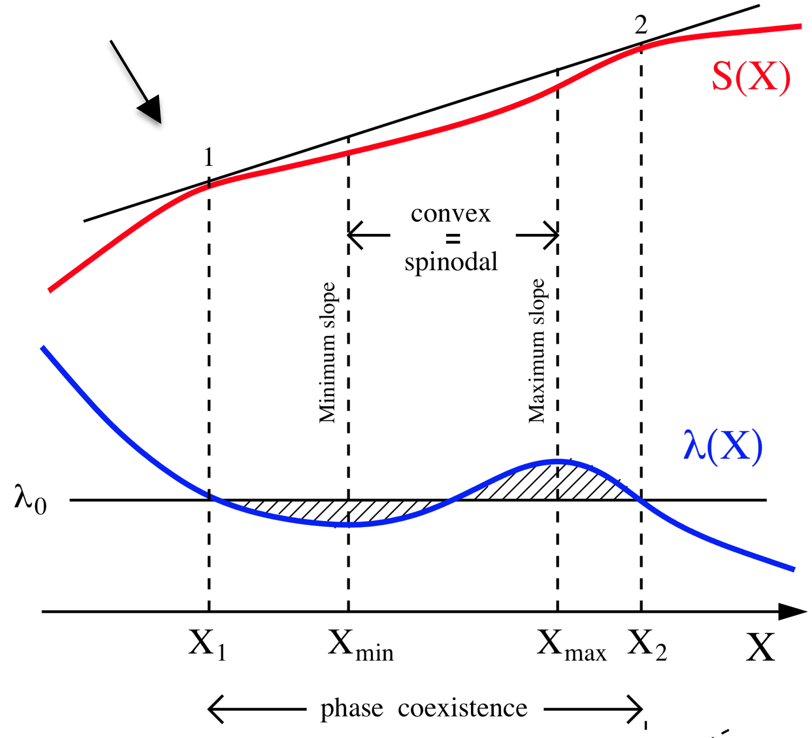
?!

JRandrup: Dubna School, 2010

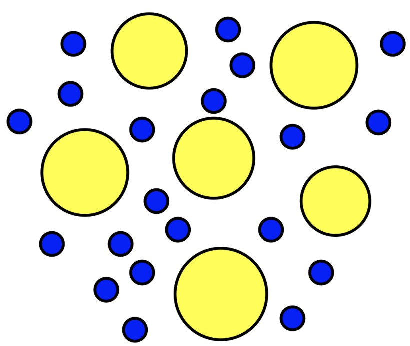
J. Randrup, Phys. Rev. C 79, 054911 (2009)

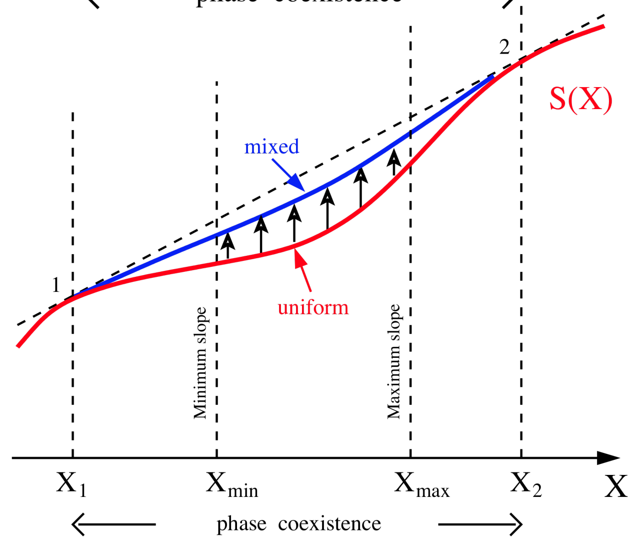
Separated phases:

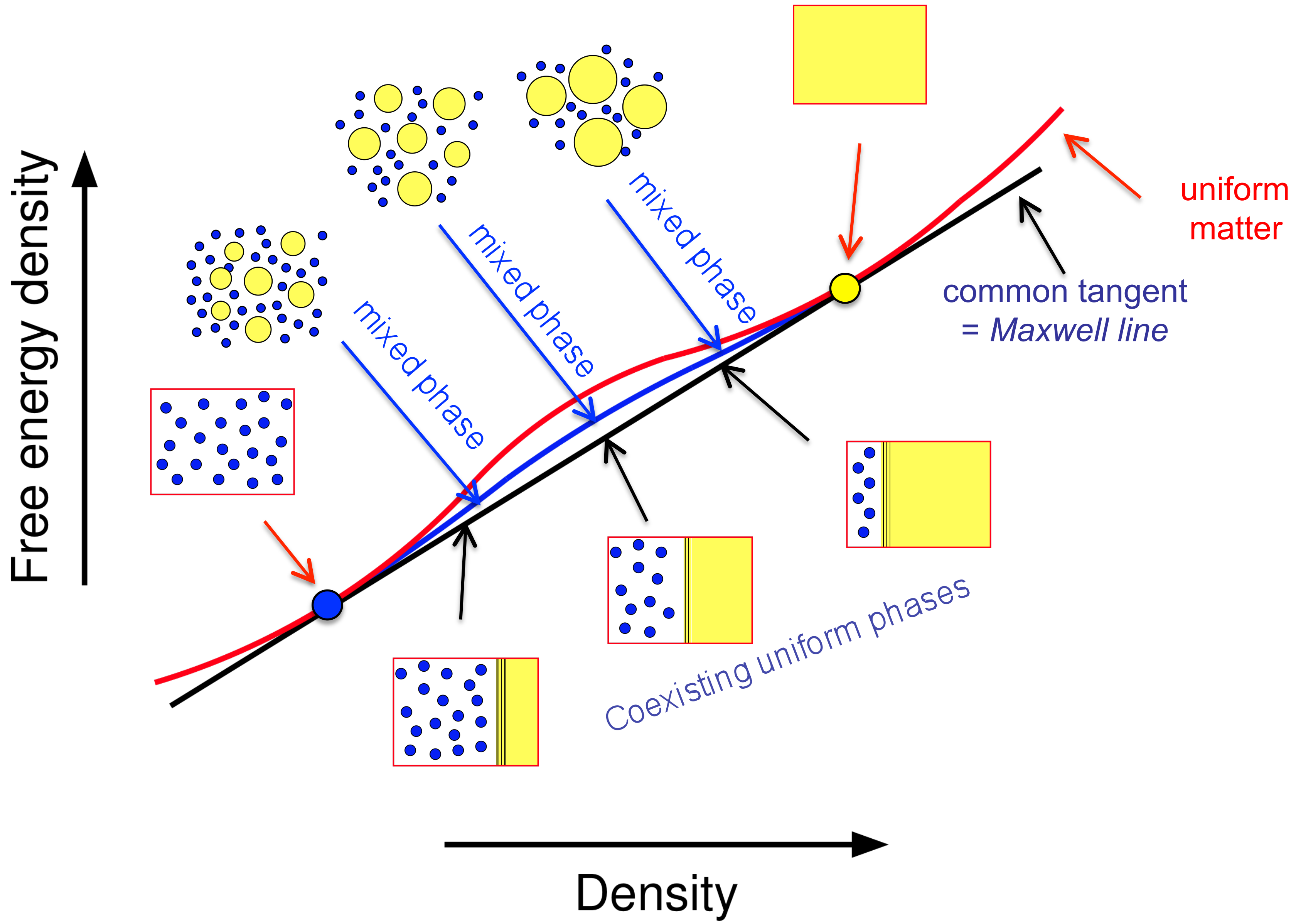




Mixed phase:







JRandrup: Dubna School, 2010

Phase Transitions & Instabilities

Phase coexistence

Illustrative examples

Finite range effects

Phase crossing

Mean field instabilities

Instabilities in fluid dynamics

Instabilities in chiral dynamics

Hadron Gas versus Quark-Gluon Plasma

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

$$p_\pi(T) = -g_\pi \int_{m_\pi}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 - \mathrm{e}^{-\beta\epsilon}]$$

$$p_N(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon-\mu_0)}]$$

$$p_{\bar{N}}(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon+\mu_0)}]$$

$$p_w(\rho) = \rho\partial_\rho w(\rho) - w(\rho)$$

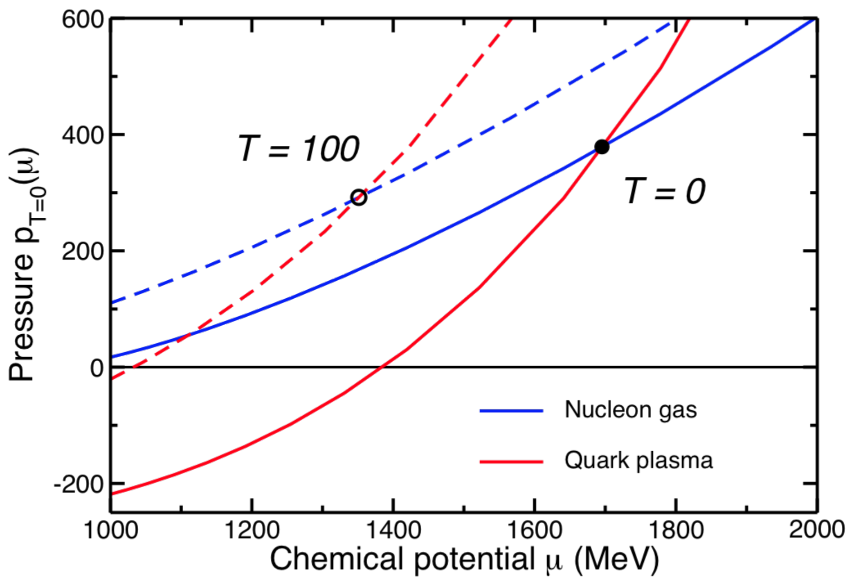
$$w(\rho) = \left[-A\left(\frac{\rho}{\rho_s}\right)^\alpha + B\left(\frac{\rho}{\rho_s}\right)^\beta\right]\rho$$

$$\mu = \mu_0 + \partial_\rho w = 3\mu_q$$

$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

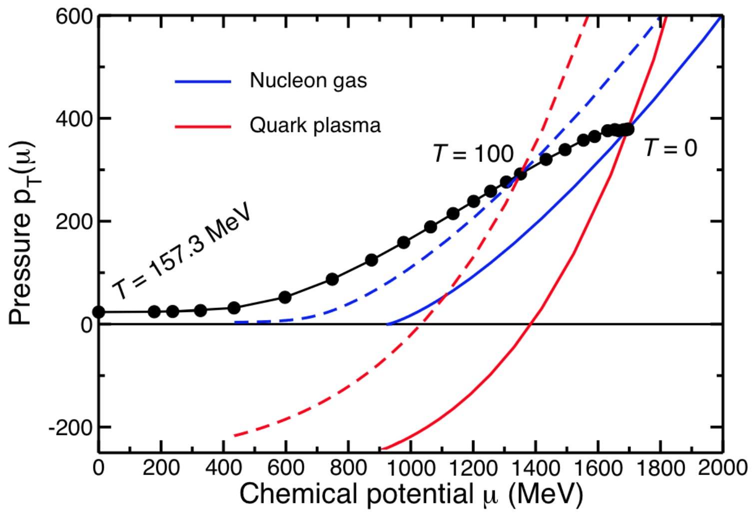
$$p_g = g_g \frac{\pi^2}{90} T^4$$

$$p_q + p_{\bar{q}} = g_q \left[\frac{7\pi^2}{360}T^4 + \frac{1}{12}\mu_q^2 T^2 + \frac{1}{24\pi^2}\mu_q^4\right]$$

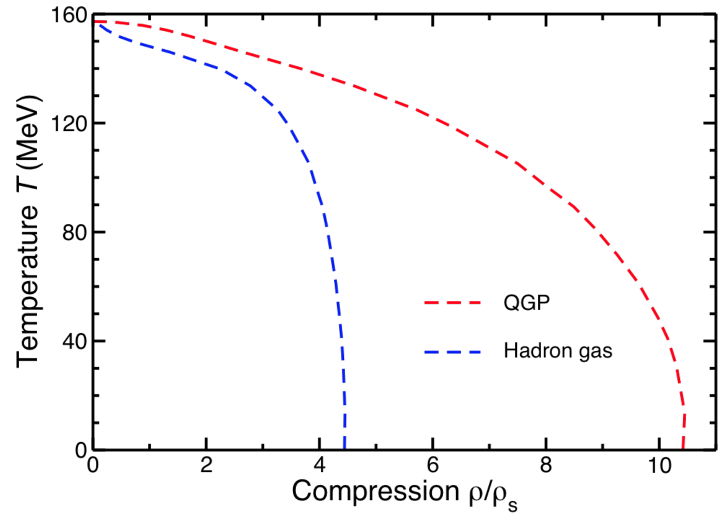


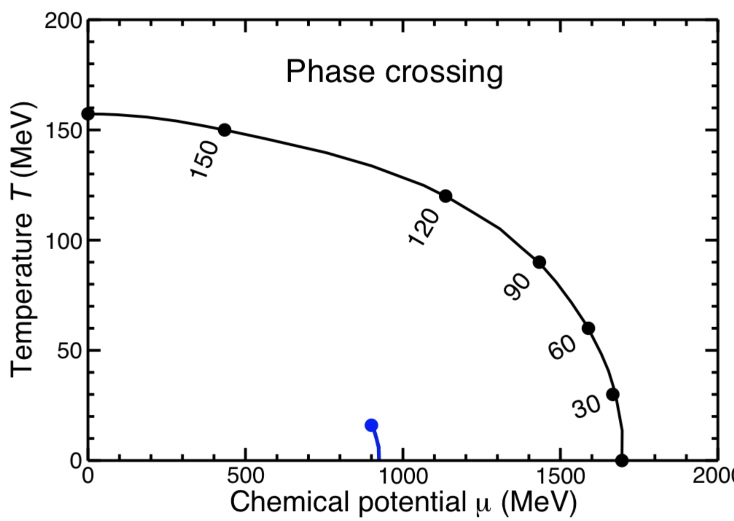
Hadron Gas versus Quark-Gluon Plasma

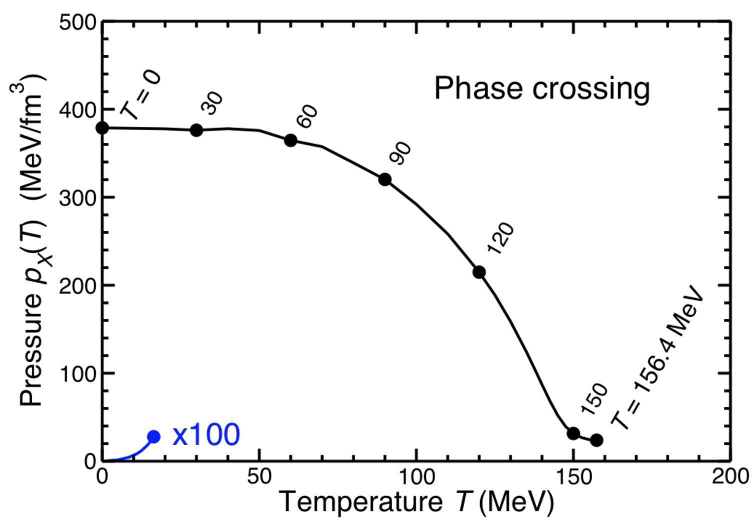
Phase crossing



Phase "boundaries"







JRandrup: Dubna School, 2010

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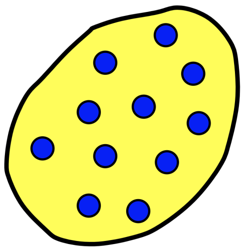
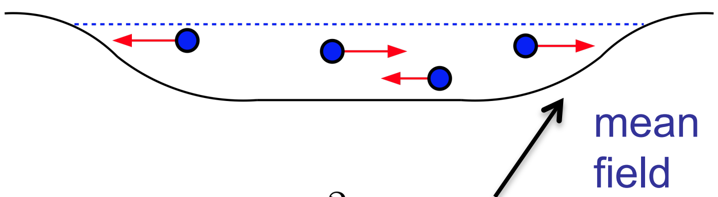
Nuclear dynamics at $E_{coll} \approx E_{Fermi}$



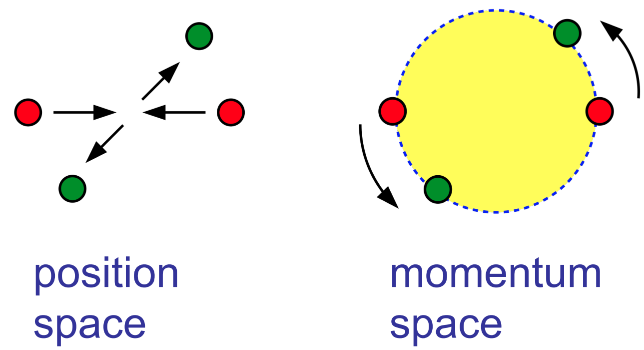
Individual nucleons move in common one-body field while occasionally experiencing Pauli-suppressed binary collisions

One-particle Hamiltonian



$$h[f](\boldsymbol{r},\boldsymbol{p}) = \frac{p^2}{2m^*} + U[\rho](\boldsymbol{r})$$

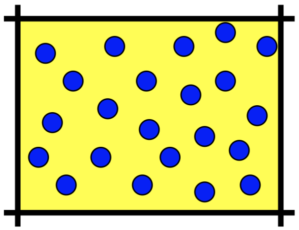
Two-body collisions



The state of the system is characterized by its reduced one-particle phase-space density:

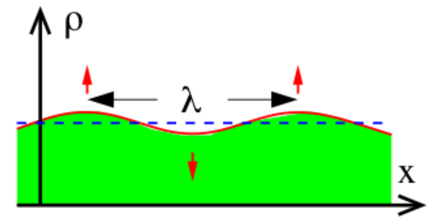
$$f(r,p)$$

Instabilities in Fermi liquids: Nuclear matter



$$\delta f(\boldsymbol{r},\boldsymbol{p},t) = f(\boldsymbol{r},\boldsymbol{p},t) - f_0(\boldsymbol{p})$$

$$\delta f(\boldsymbol{r},\boldsymbol{p},t) = \sum_{\boldsymbol{k}} f_{\boldsymbol{k}}(\boldsymbol{p},t)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$



$$h[f](\boldsymbol{r},\boldsymbol{p}) = \frac{p^2}{2m^*} + U(\rho(\boldsymbol{r})) \qquad \frac{\partial}{\partial t}\delta f + \boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{r}}\delta f - \frac{\partial f_0}{\partial \boldsymbol{p}}\cdot\left(\frac{\partial U}{\partial \rho}\frac{\partial}{\partial \boldsymbol{r}}\delta\rho\right) = 0 \qquad \boldsymbol{v} = \frac{\partial h}{\partial \boldsymbol{p}} = \frac{\boldsymbol{p}}{m^*}$$

$$\delta\rho(\boldsymbol{r},t) = g\int\frac{d^3\boldsymbol{p}}{h^3}\,\delta f(\boldsymbol{r},\boldsymbol{p},t) \quad => \quad \rho_{\boldsymbol{k}}(t) = g\int\frac{d^3\boldsymbol{p}}{h^3}\,f_{\boldsymbol{k}}(\boldsymbol{p},t)$$

$$f_{\boldsymbol{k}}(\boldsymbol{p},t) = f_{\boldsymbol{k}}(\boldsymbol{p})\,\mathrm{e}^{-i\omega_k t} \quad => \quad (-\omega_k + \boldsymbol{v}\cdot\boldsymbol{k})f_{\boldsymbol{k}}(\boldsymbol{p}) = \boldsymbol{v}\cdot\boldsymbol{k}\frac{\partial f_0}{\partial \epsilon}\frac{\partial U}{\partial \rho}\rho_{\boldsymbol{k}}$$

$$1 \doteq \frac{\partial U}{\partial \rho}g\int\frac{d^3\boldsymbol{p}}{h^3}\frac{\boldsymbol{v}\cdot\boldsymbol{k}}{\boldsymbol{v}\cdot\boldsymbol{k}-\omega_k}\frac{\partial f_0}{\partial \epsilon} = \frac{\partial U}{\partial \rho}g\int\frac{d^3\boldsymbol{p}}{h^3}\frac{(\boldsymbol{v}\cdot\boldsymbol{k})^2}{(\boldsymbol{v}\cdot\boldsymbol{k})^2-\omega_k^2}\frac{\partial f_0}{\partial \epsilon}$$

Finite range: $\tilde{g}(r_{12}) : \ \tilde{U} = \tilde{g} * U : \ \partial_\rho U \ \rightarrow \ \tilde{g}_k \partial_\rho U$

?!

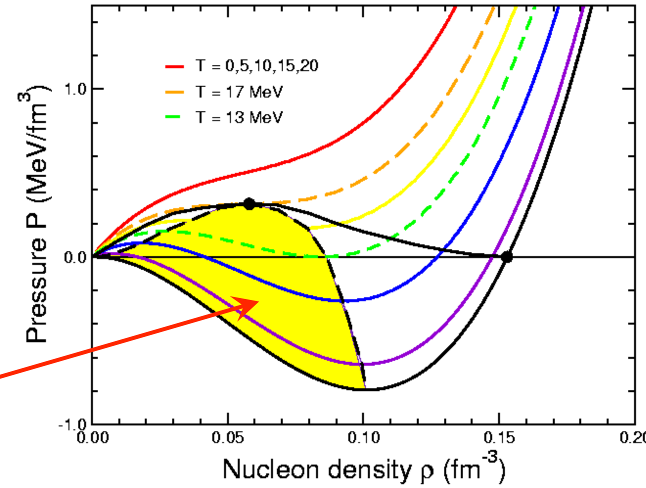
Nuclear spinodal instabilities

Spinodal region: F_0 < -1
Matter is thermodynamically and mechanically unstable

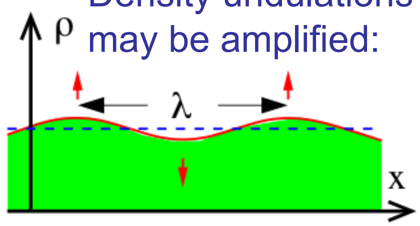
?!

$$F_0 = \frac{\partial h}{\partial \epsilon_F}$$

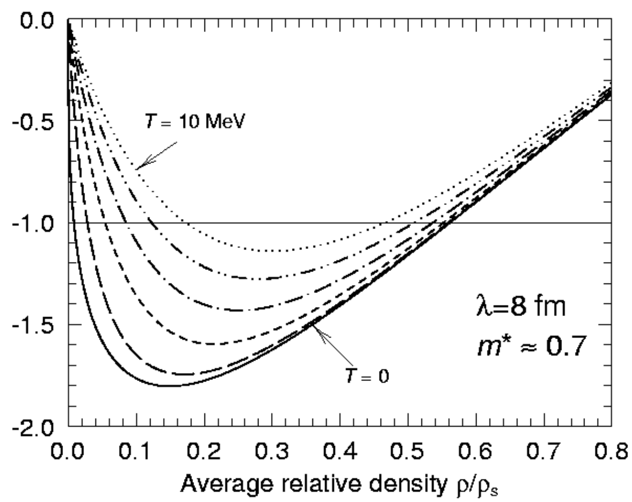
Nuclear Matter Equation of State:

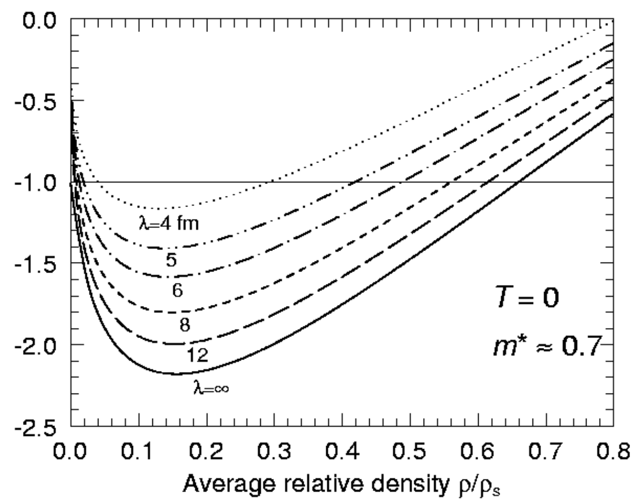


Density undulations may be amplified:



The Landau parameter F_0 depends on ρ, *T*, λ:

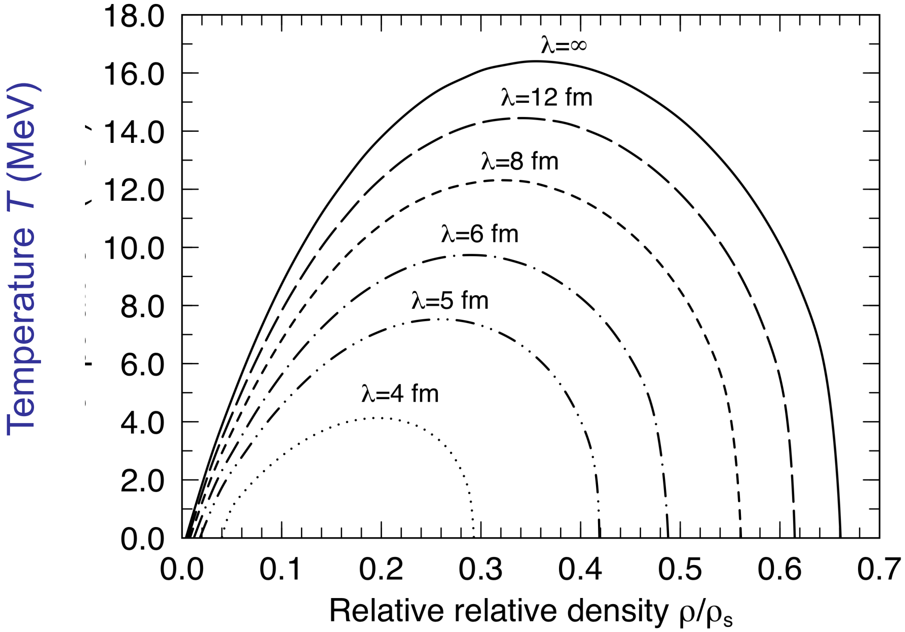




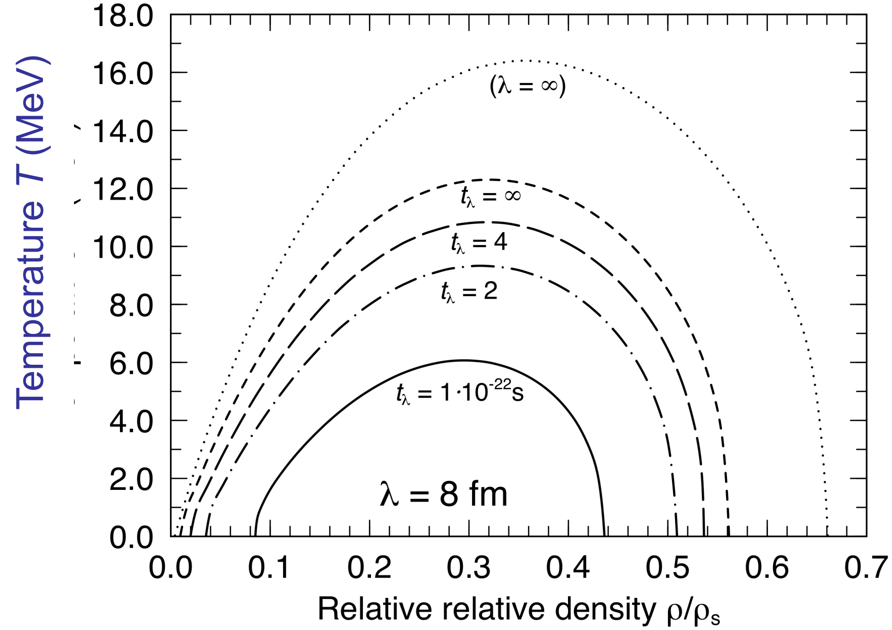
JRandrup: Dubna School, 2010

Spinodal boundaries in the (ρ,T) phase plane:

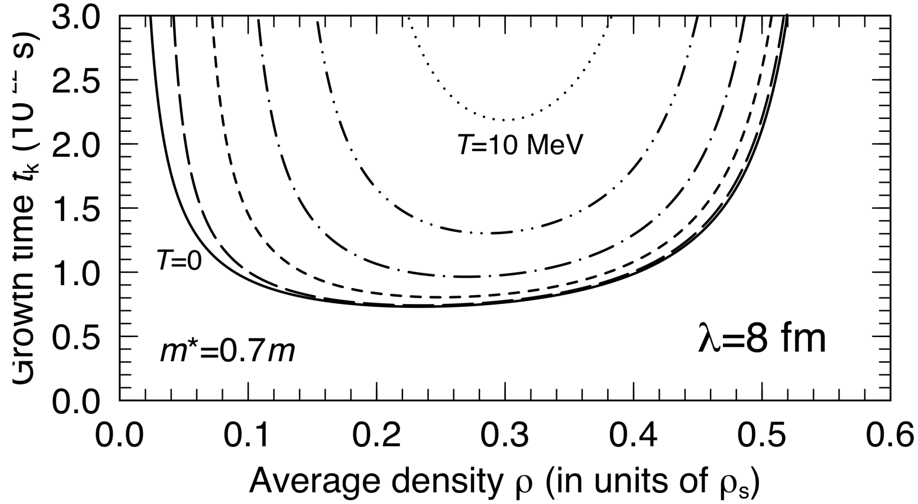
Spinodal boundary
for given wave length λ

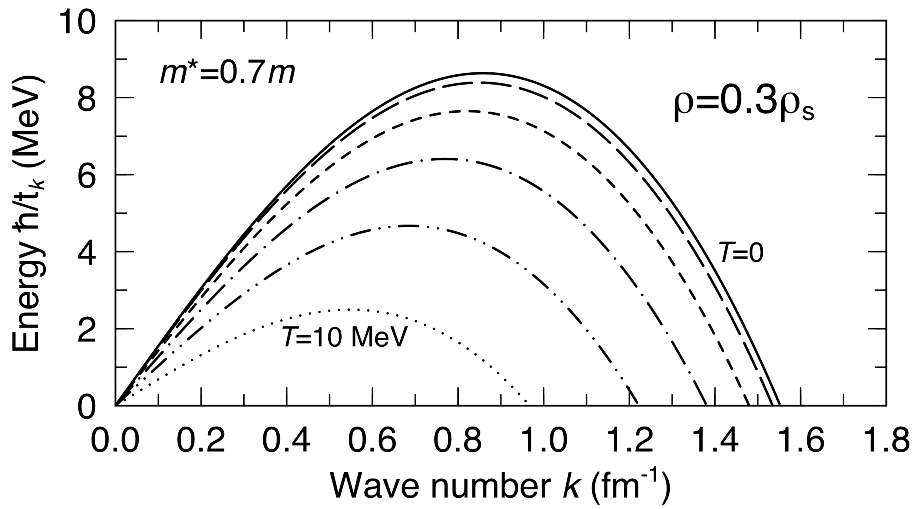


Growth times t_λ for λ = 8 fm

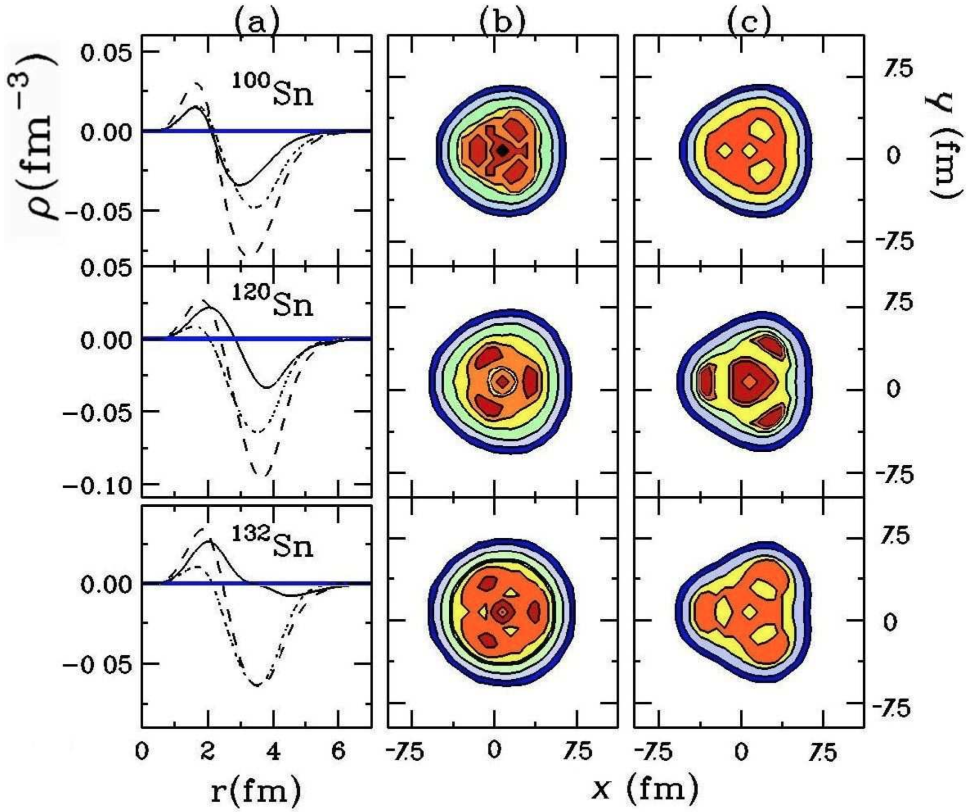


Dependence of growth rates on density, temperature and wave length:





Spinodal instabilities in finite nuclear systems



RPA calculations for unstable octupole modes in Sn isotopes:

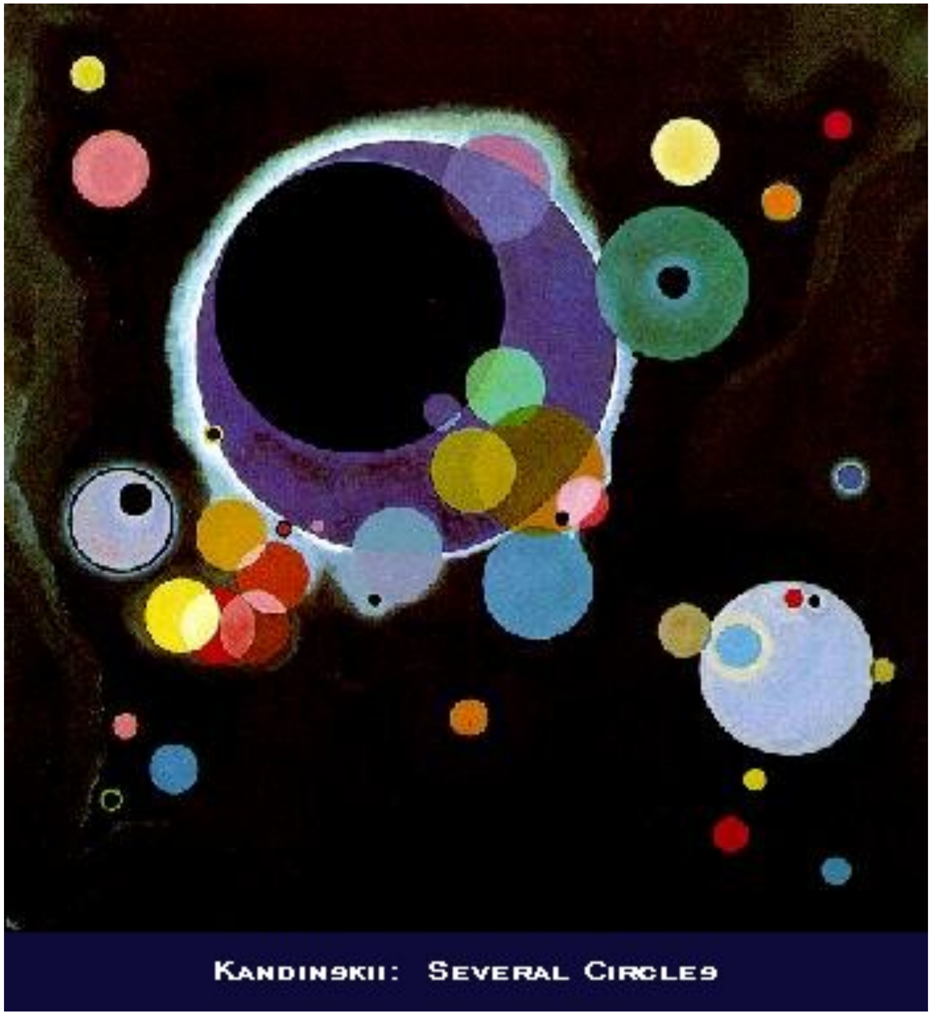
(a) radial dependence of the form factor at the dilution D = 1:5 for neutrons (solid), protons (dotted), and nucleons (dashed);

(b) contour plots of the perturbed neutron density;

(c) contour plots of the perturbed proton density.

M. Colonna, Ph. Chomaz, S. Ayik, Phys. Rev. Lett. 88 (2002) 122701

Statistical multifragmentation:



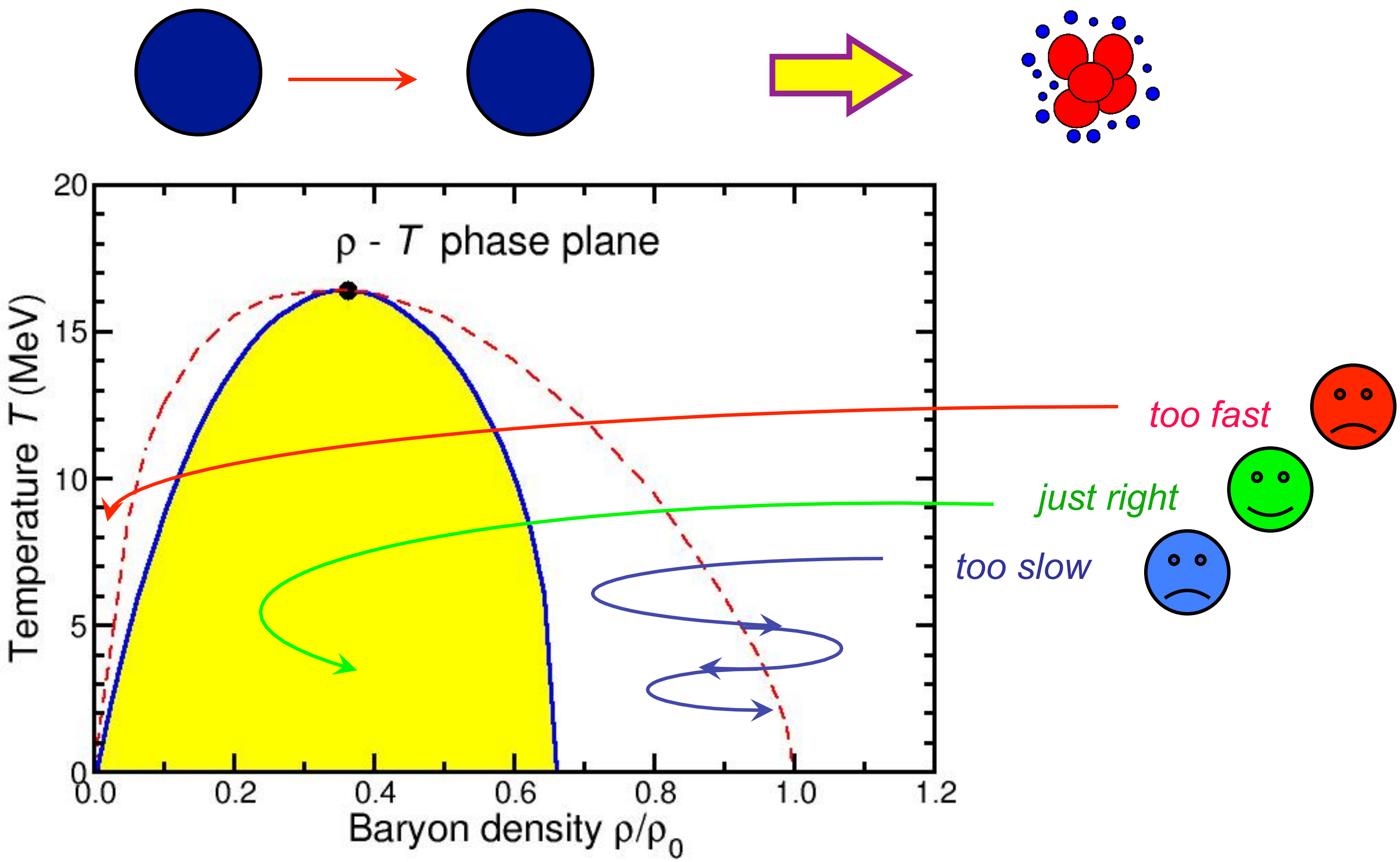
=> *Different* fragment sizes

(Igor Mishustin, 2003)

Spinodal fragmentation:

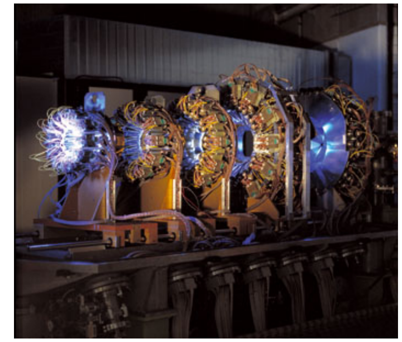
=> *Equal* sizes

Optimal collision energy



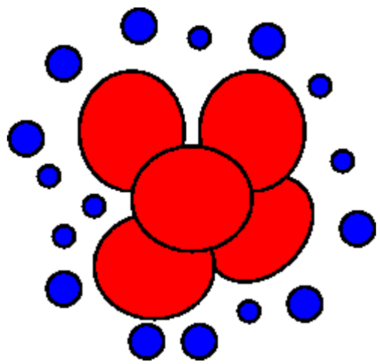
Experiment: *INDRA @ GANIL*

B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252



INDRA

32 MeV/A Xe + Sn (b=0)

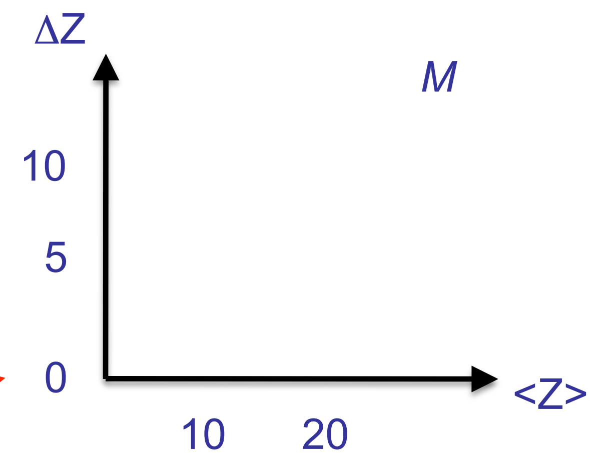


Analysis:

For each event having *M* IMFs, calculate mean IMF charge <Z> and IMF charge dispersion ΔZ.

(L.G. Moretto)

Make LEGO plot of (<Z>, ΔZ):



For events with ΔZ=0, all *M* IMFs have the same charge

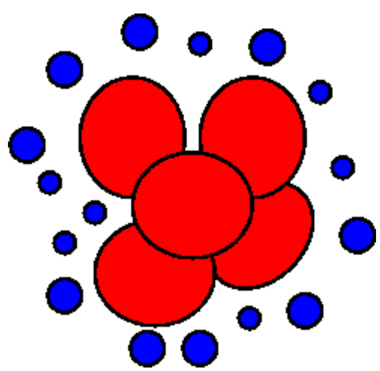
Transport calculations

… suggest a visible spinodal signal:

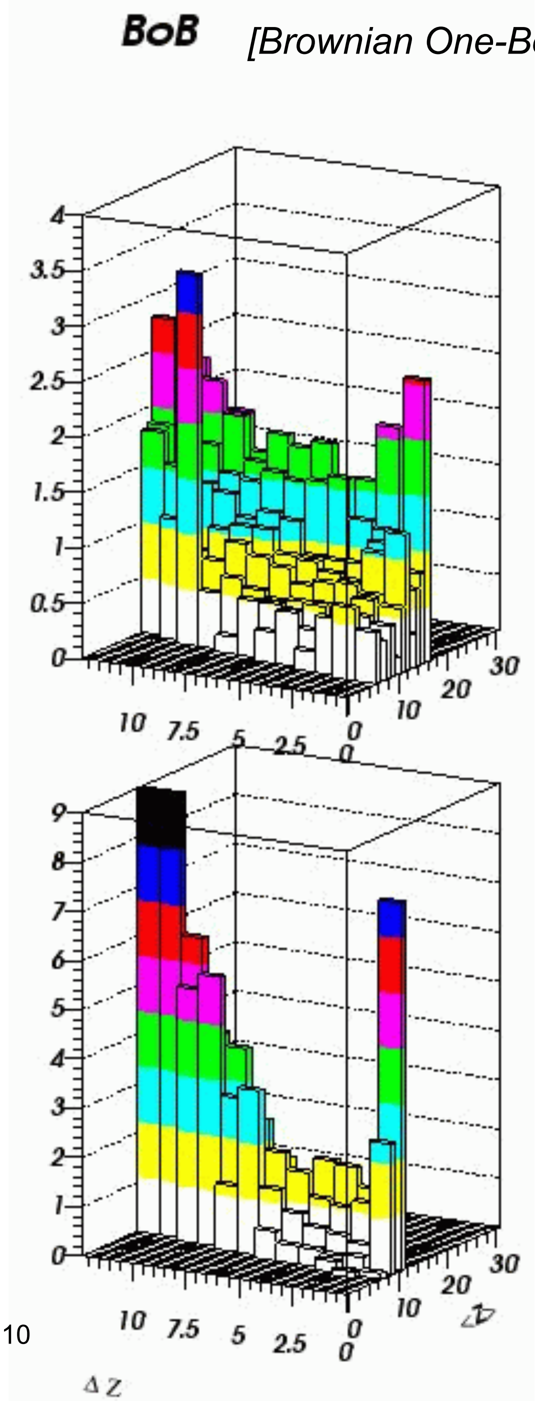
Brownian One-Body dynamics *)
≈ Boltzmann-Langevin

$$\delta K[f] \;\rightarrow\; -\delta \boldsymbol{F}\cdot\frac{\partial f}{\partial \boldsymbol{p}}$$

32 MeV/A Xe + Sn (b=0):



BoB *[Brownian One-Body model]*



M = 4

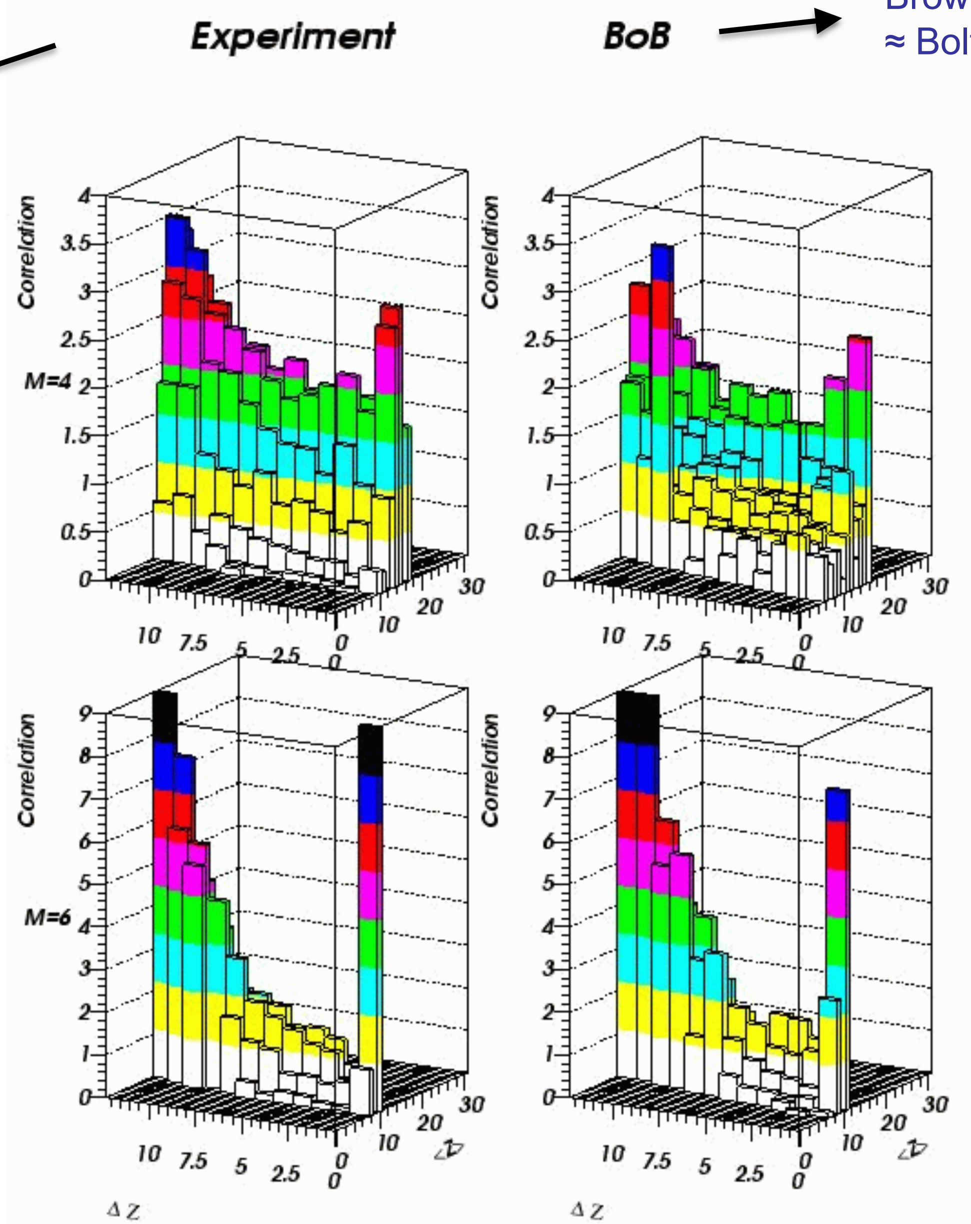
M = 6

*) Ph. Chomaz, M. Colonna, A. Guarnera, J. Randrup,
Physical Review Letters 73 (1994) 3512

Experiment: *INDRA @ GANIL*

$$\delta K[f] \;\rightarrow\; -\delta \boldsymbol{F}\cdot\frac{\partial f}{\partial \boldsymbol{p}}$$

Brownian One-Body dynamics
≈ Boltzmann-Langevin



B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252

Ph. Chomaz *et al*, Phys. Rev. Lett. 73 (1994) 3512