The full path integral

After integrating over the field fluctuations

$$G^{>}(-i\tau, \boldsymbol{r}) = \int_{0}^{\boldsymbol{r}} \mathcal{D}\boldsymbol{z}\ \mathrm{e}^{-S[\boldsymbol{z},\tau]}$$

$$S[\boldsymbol{z},\tau] = S_0[\boldsymbol{z},\tau] - \bar{F}[\boldsymbol{z},\tau]$$

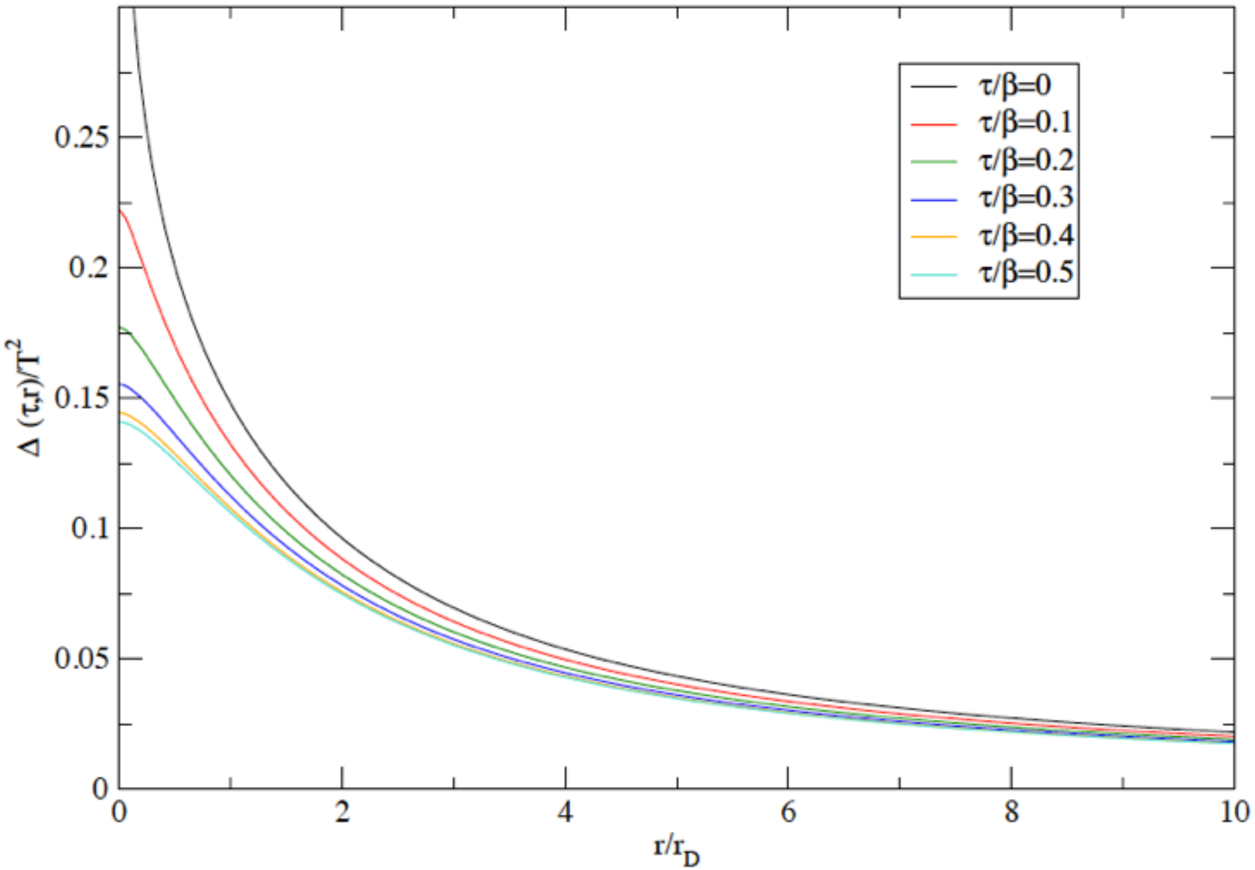
$$S_0[\boldsymbol{z},\tau] = \int_0^{\tau} d\tau' \frac{1}{2} M \dot{\boldsymbol{z}}^2$$

$$\bar{F}[\boldsymbol{z},\tau] = \frac{g^2}{2} \int_0^{\tau} d\tau' \int_0^{\tau} d\tau'' \Delta(\tau' - \tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau''))$$

All plasma information is contained in $\Delta(\tau, z)$ (~ density-density correlation function)

Subtract Coulomb interaction (density-density correlation fct)

$$\Delta(z, \boldsymbol{q}) = -\left(\frac{1}{\boldsymbol{q}^2 + \Pi(z, \boldsymbol{q})} - \frac{1}{\boldsymbol{q}^2}\right)$$



Logarithmic UV divergence at r=0 (for t=0)

Monte Carlo evaluation of the path integral

Strategy of MC calculation

$$\frac{G(\tau,\boldsymbol{r})}{G_0(\tau,\boldsymbol{r})} = \frac{\int_{\mathbf{0}}^{\boldsymbol{r}} \mathcal{D}\boldsymbol{z}\ \mathrm{e}^{-S_0[\boldsymbol{z}]}\ \mathrm{e}^{\bar{F}(\boldsymbol{z})}}{\int_{\mathbf{0}}^{\boldsymbol{r}} \mathcal{D}\boldsymbol{z}\ \mathrm{e}^{-S_0[\boldsymbol{z}]}} = \langle \mathrm{e}^{\bar{F}[\boldsymbol{z},\tau]} \rangle$$

Include interaction effects in the sampling of paths

$$S_\alpha[\boldsymbol{z},\tau] = S_0[\boldsymbol{z},\tau] - \alpha\bar{F}[\boldsymbol{z},\tau] \qquad (0 < \alpha < 1)$$

$$\frac{1}{G_\alpha(\tau,\boldsymbol{r})}\frac{\partial G_\alpha(\tau,\boldsymbol{r})}{\partial\alpha} = \frac{\int \mathcal{D}\mathbf{z}\ \bar{F}[\boldsymbol{z}]\ \exp\left[-S_\alpha[\mathbf{z}]\right]}{\int \mathcal{D}\mathbf{z}\ \exp\left[-S_\alpha[\mathbf{z}]\right]} = \langle \bar{F}[\boldsymbol{z}] \rangle_\alpha$$

Hence

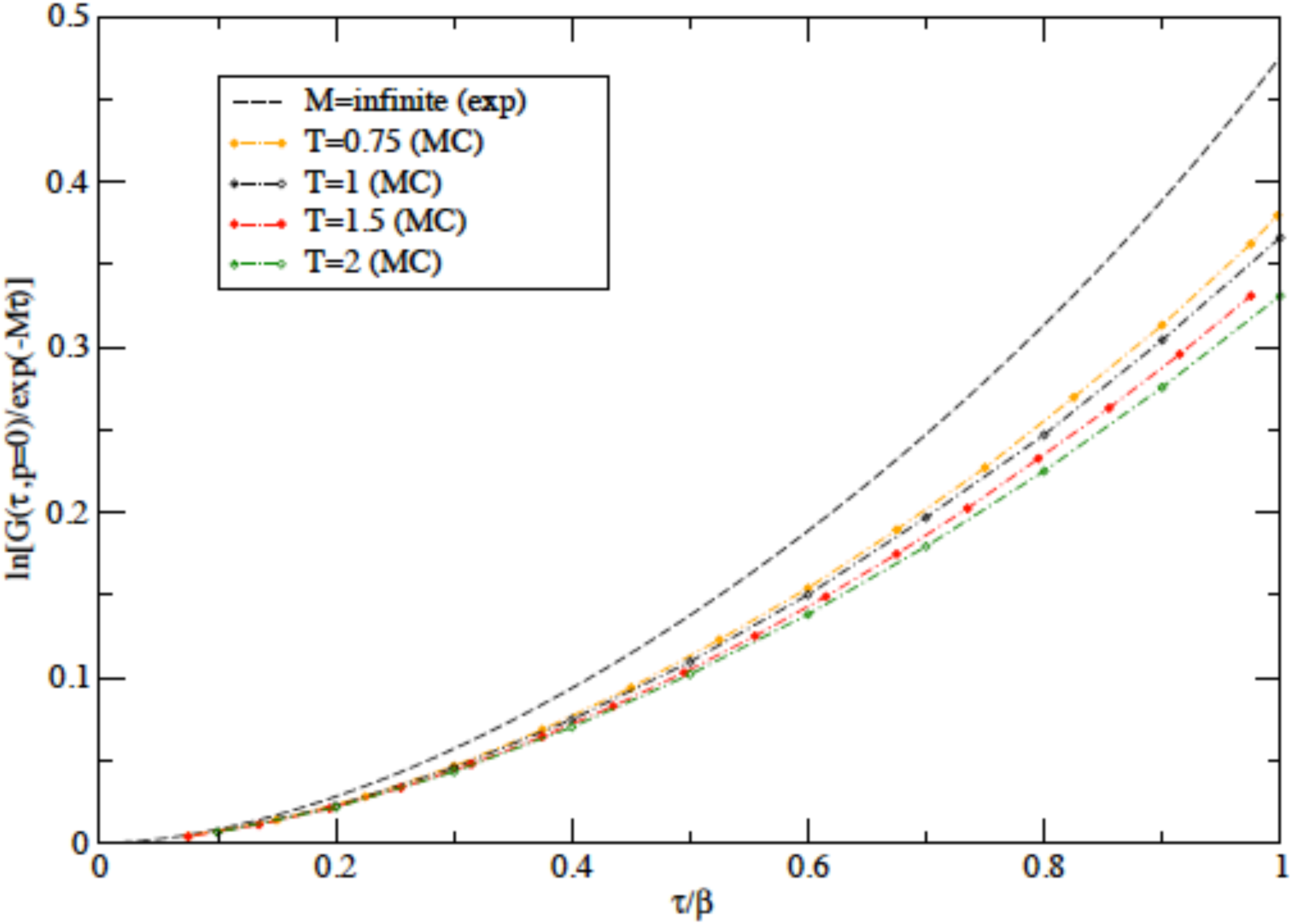
$$\ln\frac{G(\tau,\boldsymbol{r})}{G_0(\tau,\boldsymbol{r})} = \ln\left(\langle \mathrm{e}^{\bar{F}[\boldsymbol{z},\tau]} \rangle\right) = \int_0^1 d\alpha\ \frac{\partial \ln G_\alpha(\tau,\boldsymbol{r})}{\partial\alpha} = \int_0^1 d\alpha\ \langle \bar{F}[\boldsymbol{z}] \rangle_\alpha$$

Choice of parameters for the MC calculation

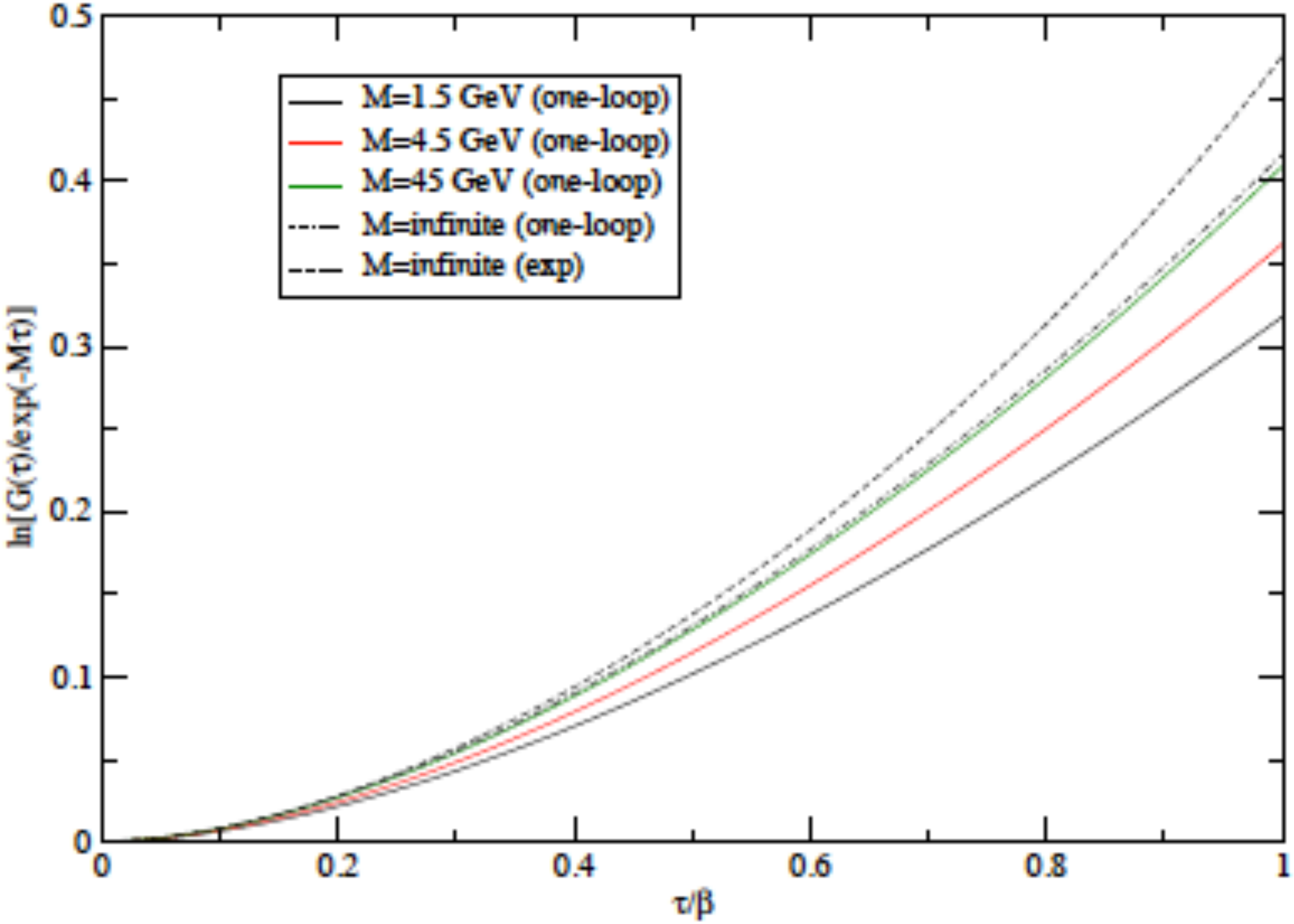
$$
\begin{aligned}
\frac{g^2}{4\pi} &\equiv C_F \alpha_s, \quad \textit{with} \quad \alpha_s = 0.3 \\
M &\approx 1.5\ \mathrm{GeV}\ \ (\mathrm{charm}) \\
T &\approx 200 - 400\ \mathrm{MeV}\ \ (T/M \ll 1)
\end{aligned}
$$

However, most results (appropriately scaled) depend only on T/M.

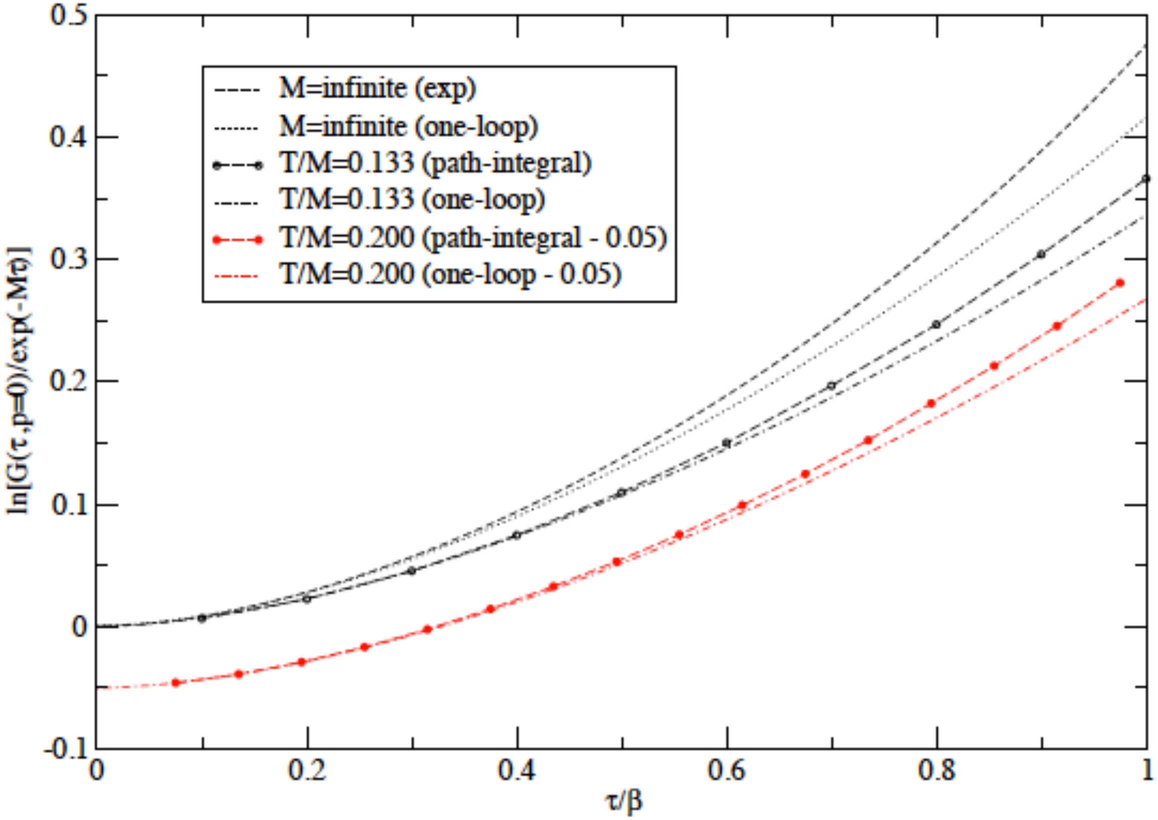
Euclidean correlator (p=0)



One-loop Euclidean correlator



Euclidean correlator
Comparison with one-loop



MEM reconstruction of the spectral density

Recovering the spectral function from the (numerical) correlator is a difficult problem

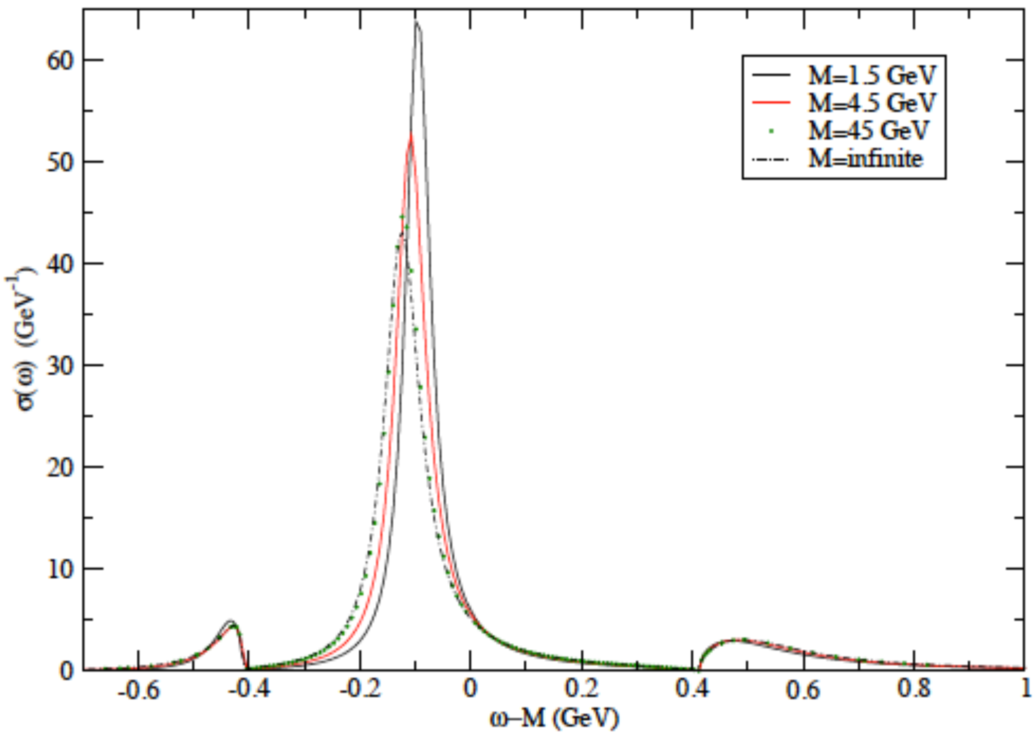
$$G^{>}(t=-i\tau)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\mathrm{e}^{-\omega\tau}\sigma(\omega)$$

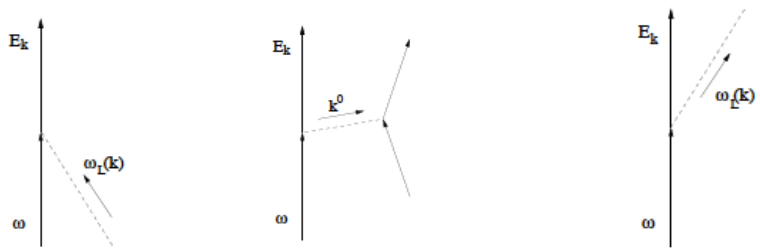
We have used the Maximum Entropy Method.
Results are sensitive to the choice of the default model

Sum rules imposed on the default model

$$\int\frac{d\omega}{2\pi}\,\sigma_{\rm def}(\omega)=1,\qquad \int\frac{d\omega}{2\pi}\,\omega\,\sigma_{\rm def}(\omega)=M$$

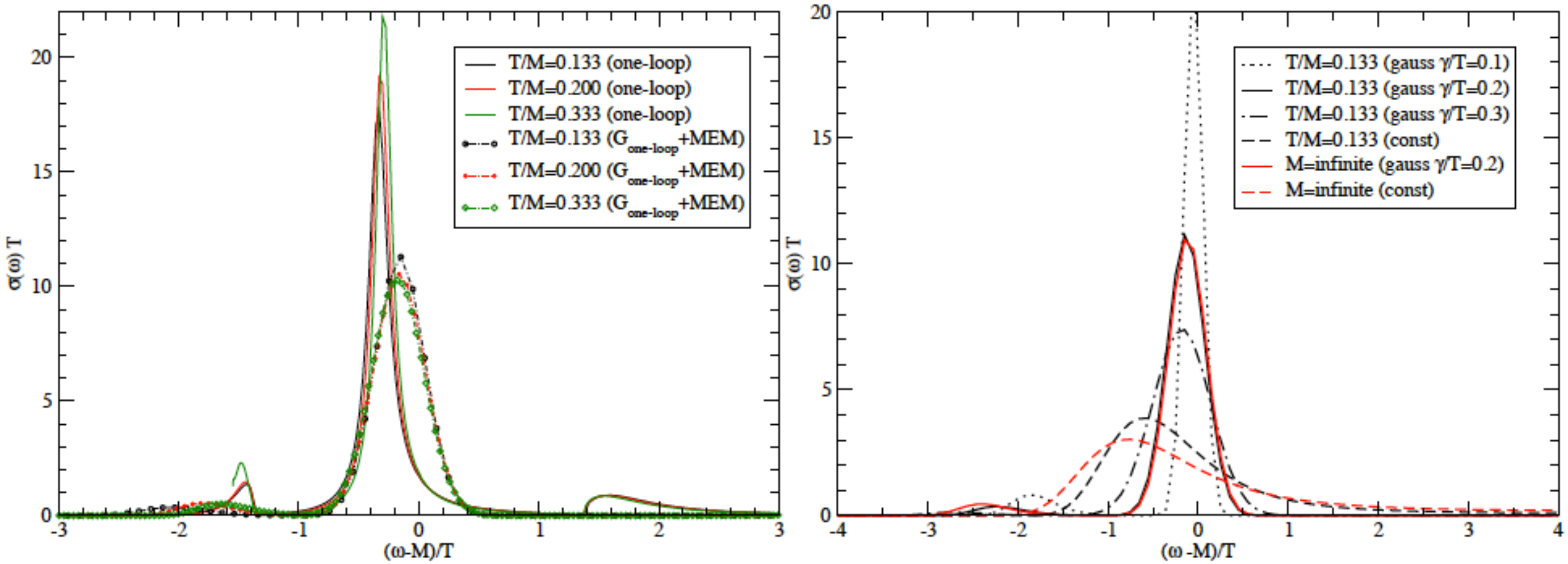
One-loop spectral function (exact)





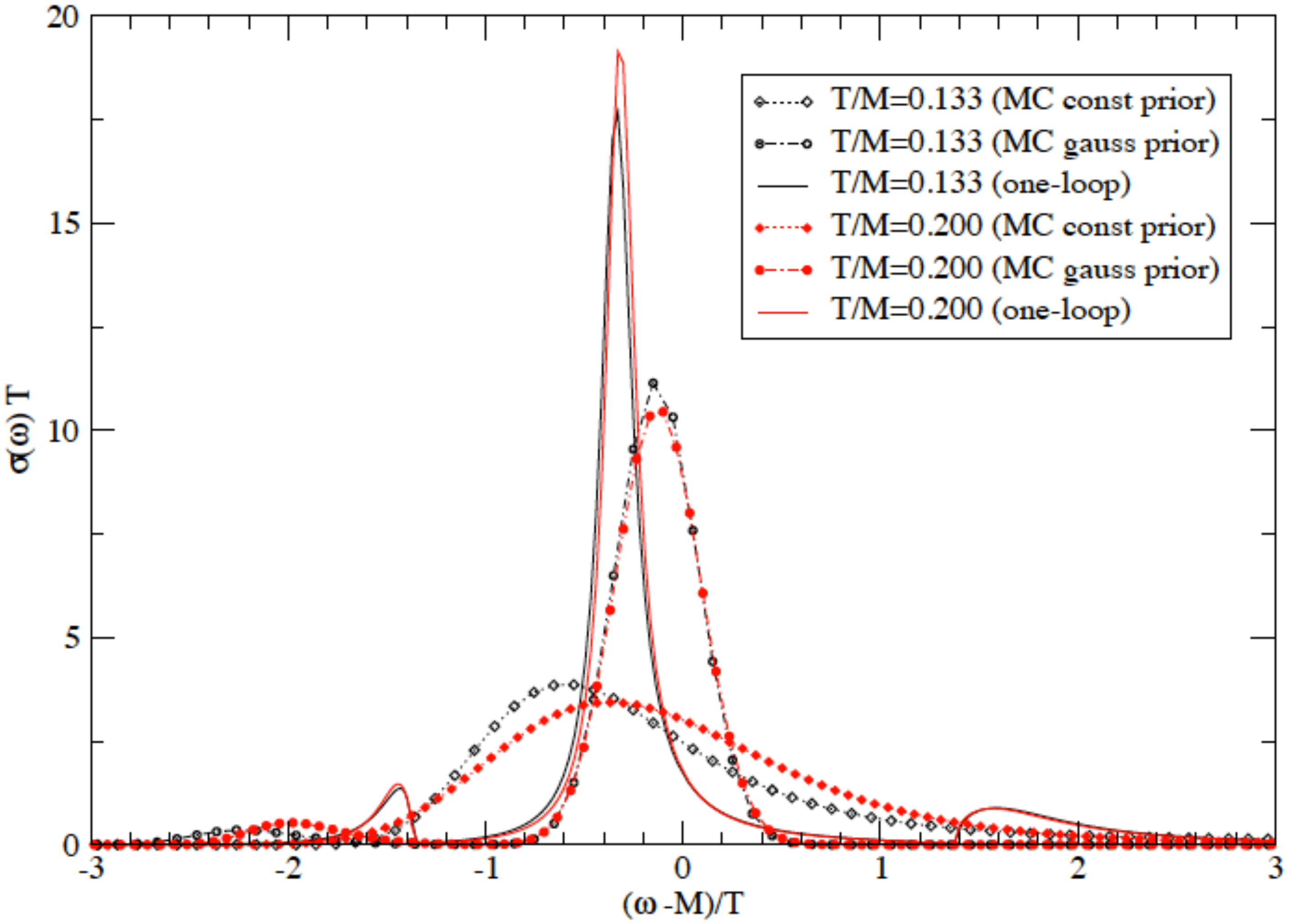
One-loop spectral function (reconstructed)

(sensitivity to the default model)



Spectral function

(MC vs one-loop)



A solvable toy model

A toy model

Fermion coupled to a single mode (harmonic oscillator)

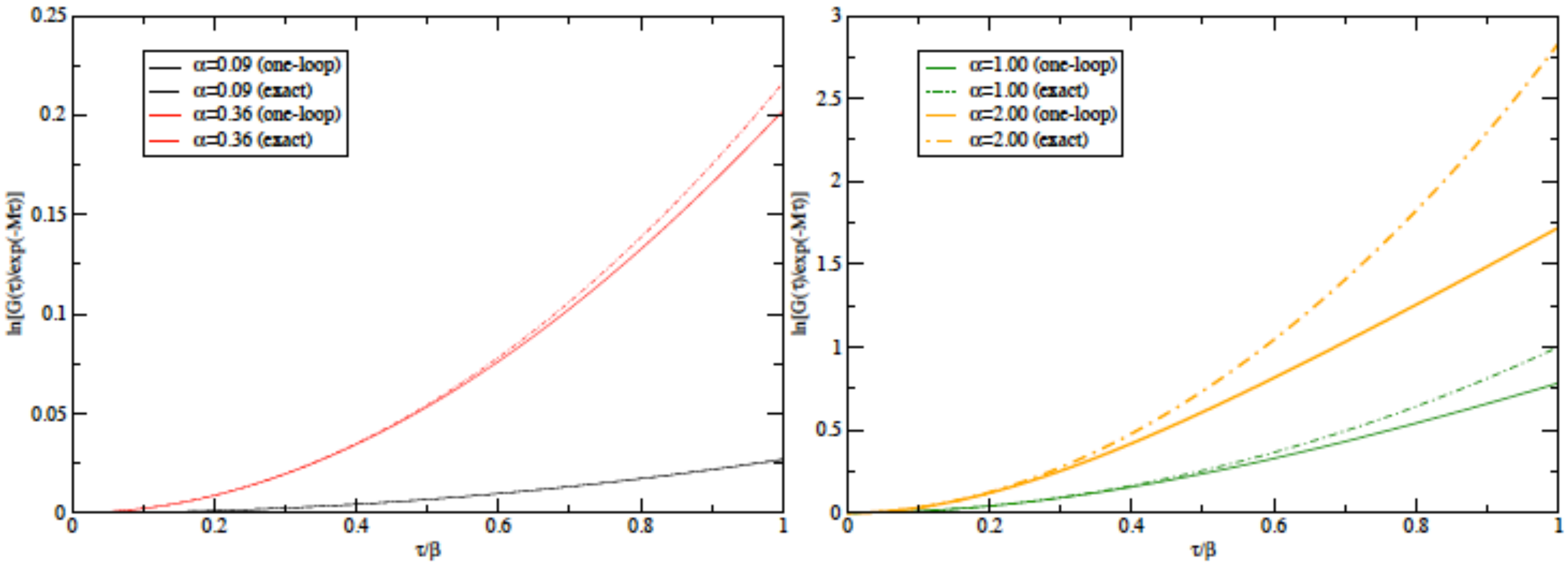
$$H = M\psi^\dagger\psi + \frac{1}{2}\left(\pi^2 + m_D^2\,\phi^2\right) + g\,\psi^\dagger\psi\,\phi, \qquad \phi \equiv \frac{a+a^\dagger}{\sqrt{2m_D}}$$

$[H, \psi^\dagger\psi] = 0$ and $\{\psi^\dagger, \psi^\dagger\} = 0 \;\Rightarrow$ only two cases are possible:

- N=0 $\Rightarrow \quad H_0 = m_D\left(a^\dagger a + 1/2\right);$
- N=1 $\Rightarrow \quad H_1 = (M - \alpha m_D) + m_D\left(b^\dagger b + 1/2\right)$, where

$$b \equiv a + \sqrt{\alpha}, \quad b^\dagger \equiv a^\dagger + \sqrt{\alpha} \quad \text{with} \quad \alpha \equiv \frac{g^2}{2m_D^3} \text{ (dimensionless)}$$

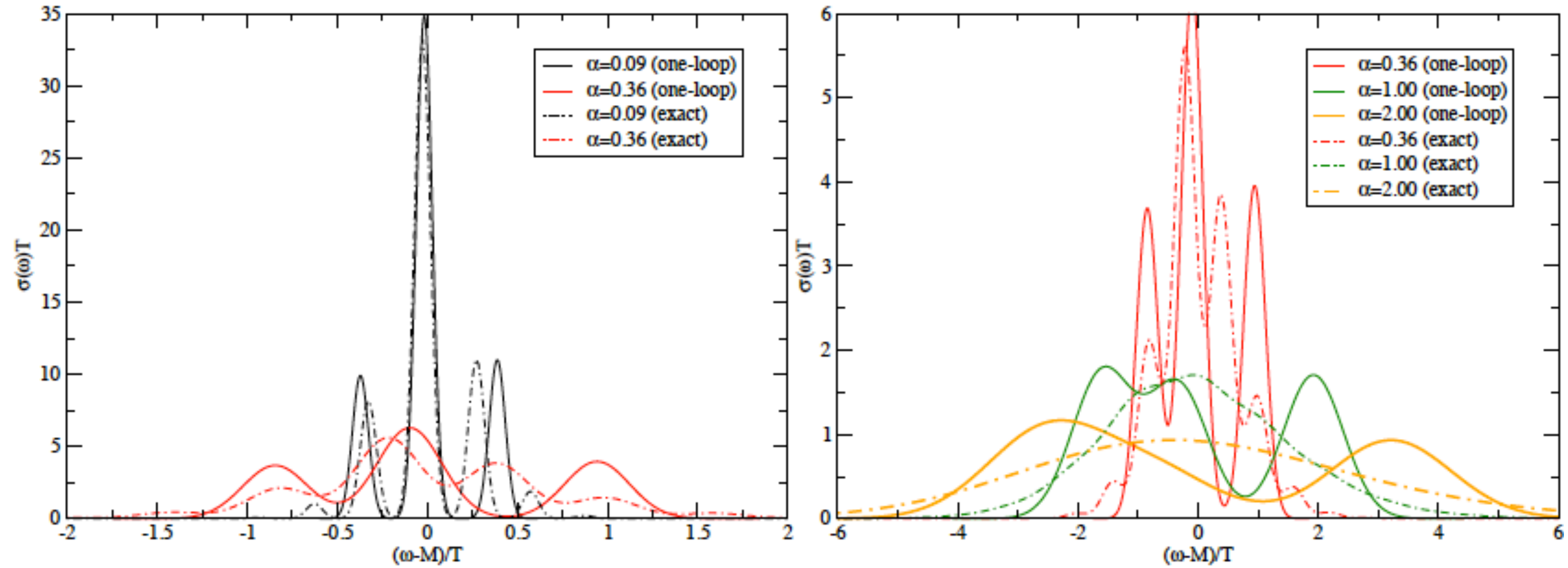
Euclidean correlator



$$\int_{-\infty}^{\infty} \frac{d\bar{\omega}}{2\pi}\, \sigma(\bar{\omega}) = 1, \qquad \int_{-\infty}^{\infty} \frac{d\bar{\omega}}{2\pi}\, \bar{\omega}\, \sigma(\bar{\omega}) = 0,$$

$$\int_{-\infty}^{\infty} \frac{d\bar{\omega}}{2\pi}\, \bar{\omega}^2\, \sigma(\bar{\omega}) = \alpha m_D^2 (1 + 2N), \quad \int_{-\infty}^{\infty} \frac{d\bar{\omega}}{2\pi}\, \bar{\omega}^3\, \sigma(\bar{\omega}) = \alpha m_D^3$$

Spectral function



$$\sigma(\bar{\omega}) = 2\pi\, \mathrm{e}^{-\alpha(1+2N)} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \sum_{p=0}^{n} \binom{n}{p} (N)^p (1+N)^{n-p} \delta\left(\bar{\omega} + \alpha m_D - (n-2p) m_D\right)$$

$\bar{\omega} \equiv \omega - M$ $\qquad$ $\gamma \sim \alpha T$ (artificial width)