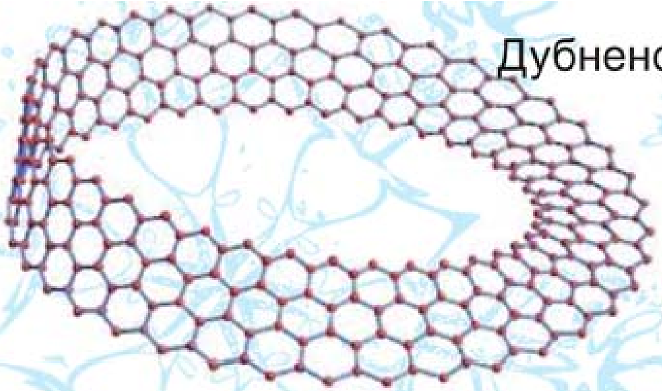

Дубненская международная школа по теоретической физике / DIAS-TH

XI Зимняя школа по теоретической физике

28 января - 3 февраля 2013 г.

НАНОСТРУКТУРЫ

И НАНОМАСШТАБНЫЕ ЯВЛЕНИЯ

Квантовая оптика с атомными поляритонами

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**) *Российский квантовый центр, Сколково*

"I Did It My Way"

- ✓**1985-1990, Education:**

Erevan State University, Physics Department , Armenia

- ✓**1990-1993, PhD:**

M.V. Lomonosov Moscow State University, Physics Department
Supervisor is A.S.Chirkin

- ✓**1993 -** Candidate of Physics and Mathematics, MSU
- ✓**1993-1995**, International Laser Center, MSU, Post..Doc .
- ✓**1995-2010**, Assoc. Professor at Dept. of Physics and Applied Math. VISU
- ✓**2009**, Doctor of Science, Institute of Laser Physics, SBRAS, Nonsibirsk
- ✓**1999-2000**, Graduation at Innsbruck University, Austria, Group of Prof. P.Zoller
- ✓**2010-...**   Professor at VISU
- ✓**2013** Russian Quantum Center, Skolkovo

I have more than 50 papers in Refereed journals. They are Physical Review, JETP, JETP Letts, Applied Physics, IOP etc.

Содержание курса

Лекция 1. Термализация связанных атомно-оптических состояний.

- ✓Введение: Критические явления и фазовые переходы в связанных состояниях среды и поля;
- ✓Новые явления в атомных газах сверхвысокого давления;

Лекция 2. Высокотемпературные фазовые переходы в атомно-оптических системах.

- ✓Сверхизлучательный фазовый переход фотонов при ОС;
- ✓1D БЭК атомных поляритонов в микротрубках ;
- ✓2D БЭК фотонов.

Лекция 3. Квантовые технологии со связанными состояниями среды и поля.

- ✓Современные технологии для удержания и манипулирования атомами;
- ✓Поляритонные кристаллы - новые квантовые материалы для хранения и и обработки информации;
- ✓Поляритонные солитоны- новые объекты для передачи информации.

Общая информация

Ultra-High Density Atomic Gases

Old question: *What happens to radiation when many atoms interact "collectively" with light and how we can implement that?*

Dilute atomic gases

n is atomic gas density,

$\lambda \simeq 1\mu m$ is light wavelength .

$$n\lambda^3 << 1$$

$$n << 10^{12}\,cm^{-3}$$

Such densities are typical for atomic BEC and EIT observation, for various schemes of quantum optical memory

Ultra-high density atomic medium

$$n\lambda^3 > 1,$$

$$n \simeq 10^{15} \div 10^{16}\,cm^{-3}$$

Plan of Lecture 1

1. Bose-Einstein Condensation (BEC) phenomena.

2. Outlook of phase transitions and coherent effects for various coupled matter-field states in condensed matter physics.

3. Thermalization of coupled atom-field (dressed) states in the presence of optical collisions.

   - *atom-field interaction in dressed state picture;*
   - *the effect of optical collisions.*

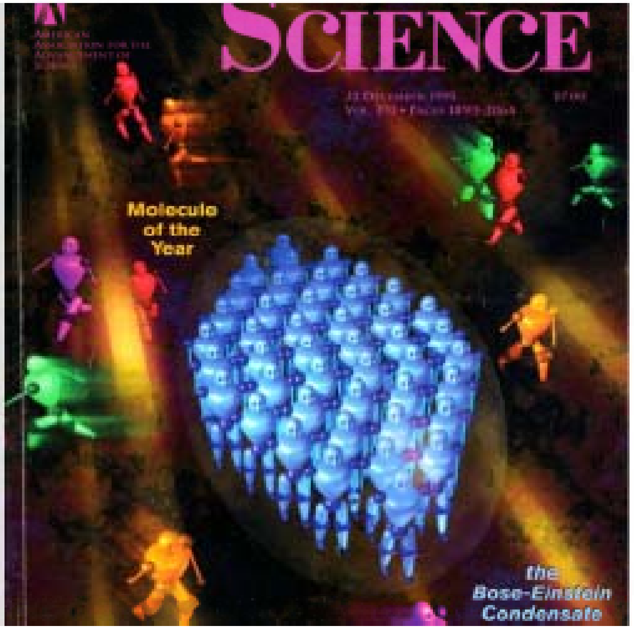
4. Outlook

Бозе-эйнштейновский конденсат (БЭК) – новое состояние материи

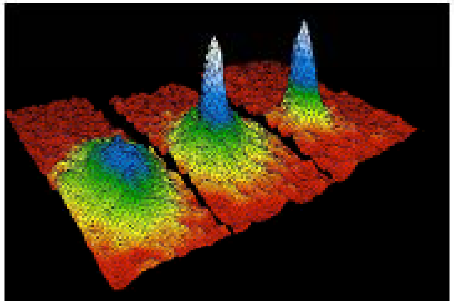
С 1995г. начинается новая эра в исследовании вещества

Eric A. Cornell **Wolfgang Ketterle** **Carl E. Wieman**

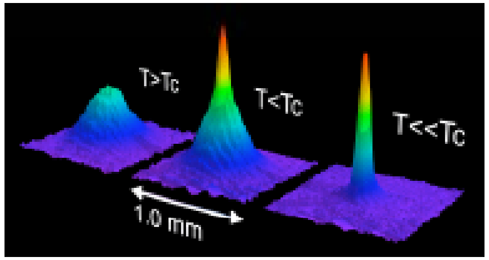
Все атомы в одном и том же состоянии



Атомы формируют одну большую материальную волну де Бройля



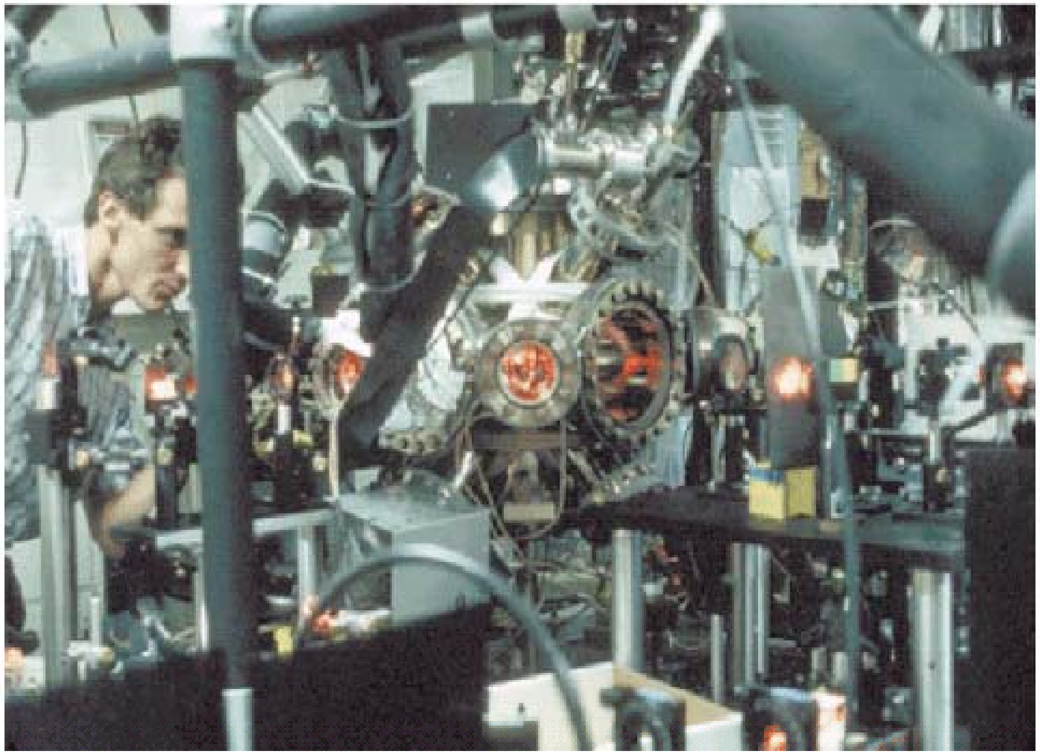
JILA ^{87}Rb

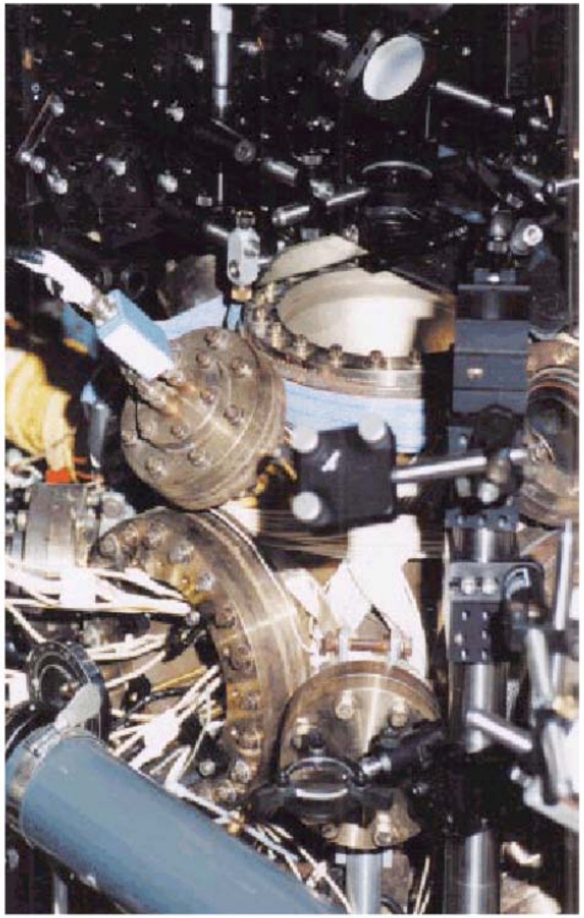


MIT ^{23}Na

Установка по получению атомных конденсатов

В.Кеттерле, УФН, т. 173, с. 1339 (2003)

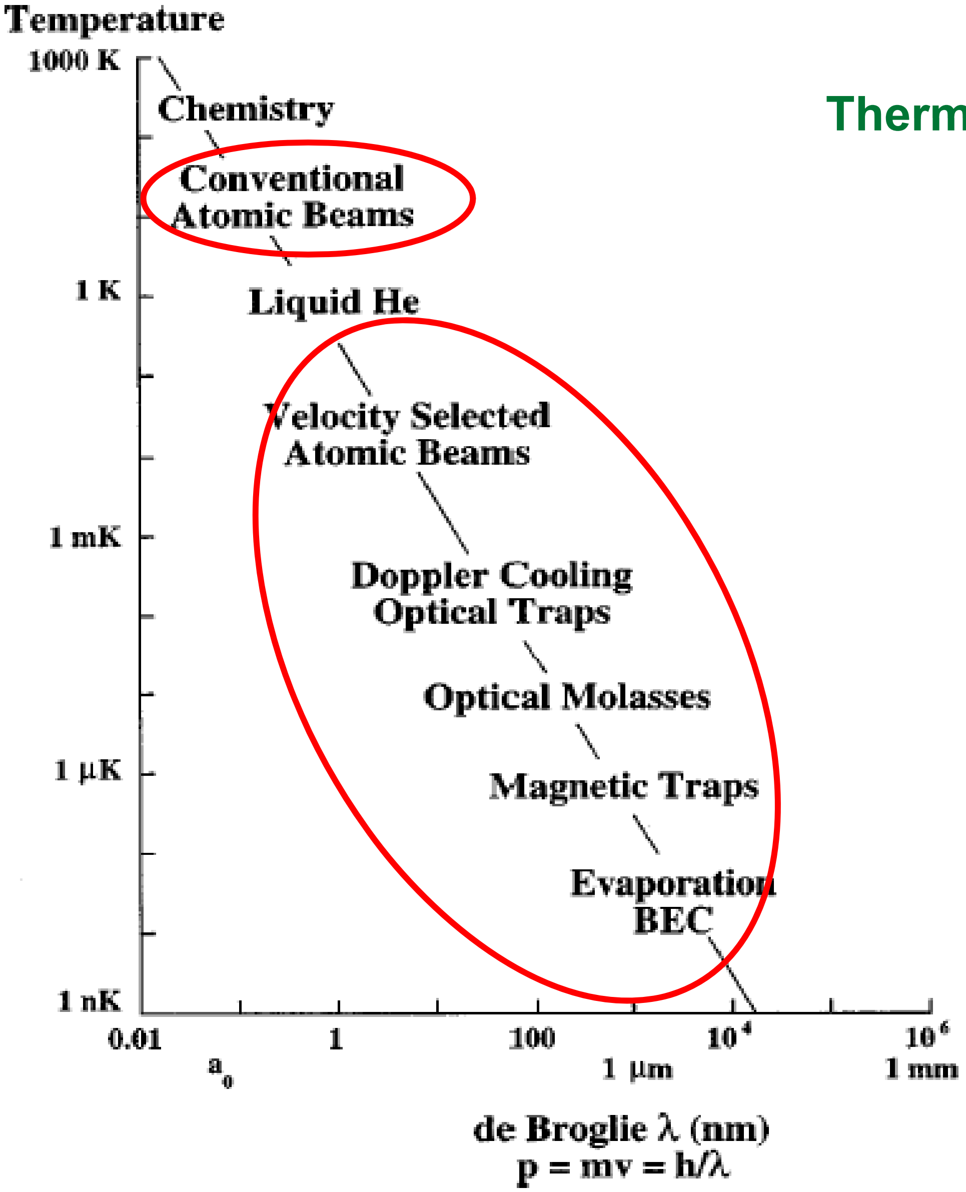




1. Глубокое лазерное охлаждение
2. Охлаждение испарением

Магнито-оптическая ловушка

Понижение температуры – главный способ получения БЭК атомов



Thermal De Broglie wavelength

$$\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$

Illustrative plot of various phenomena along a scale of temperature plotted against the de Broglie Wavelength

Может ли Газ Атомов Быть Квантовым?

Magneto-optical trap

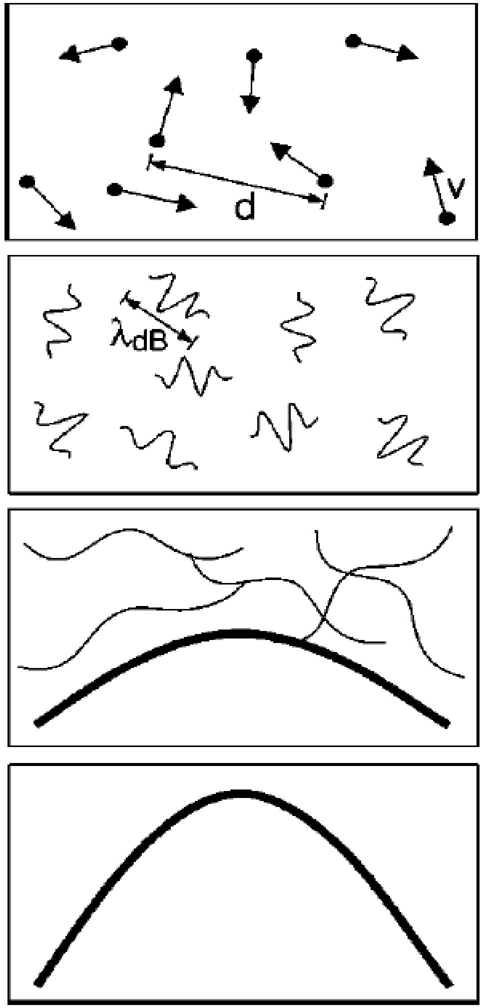
From room temperature to
100 μK

Molasses

$100\ \mu K \longrightarrow 10\ \mu K$

$n\lambda^3 = 10^{-7}$

Intrisically limited because
of the dissipative character of
the MOT.



High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"

$T=T_C$:
BEC

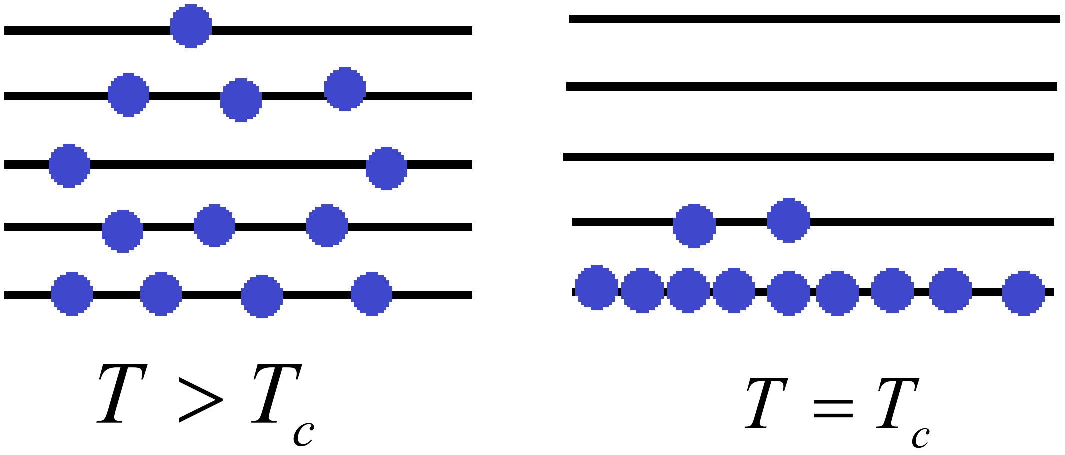
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0:
Pure Bose condensate
"Giant matter wave"

Конденсация Бозе-Эйнштейна

В 1924г. индийский физик С. Бозе опубликовал статью по статистике газов, в которой показал, что при определенных условиях частицы газа могут занимать основное состояние.

Это происходит в условиях фазового перехода в конденсированное состояние.



Критическую температуру T_c можно определить из условия

$$\lambda_{dB} \simeq r$$

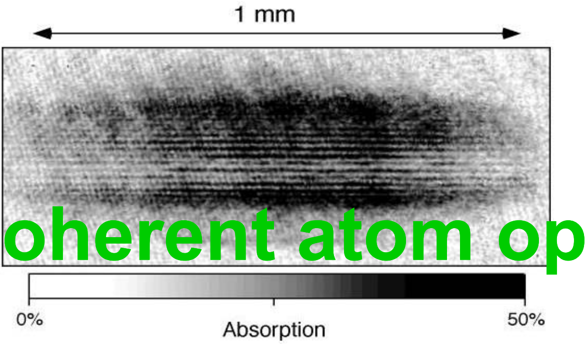
$$T_c \simeq \frac{h^2 n^{2/3}}{3k_Б m}$$

В эксперименте получают до 10^9 конденсированных атомов

Для атомов критическая температура лежит в области десятков наноКельвин

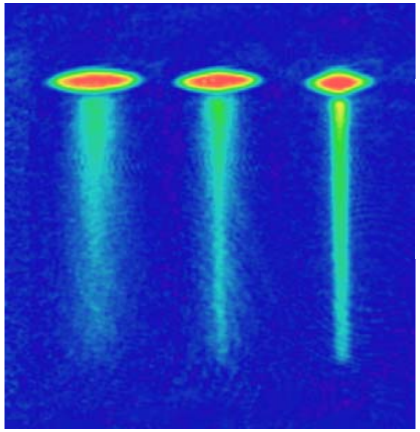
Bose-Einstein Condensate properties

matter-wave coherence



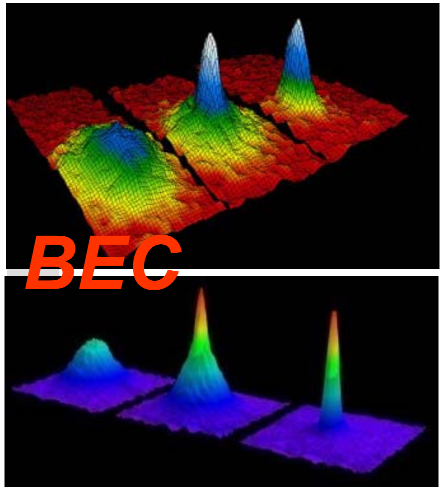
coherent atom optics

atom laser

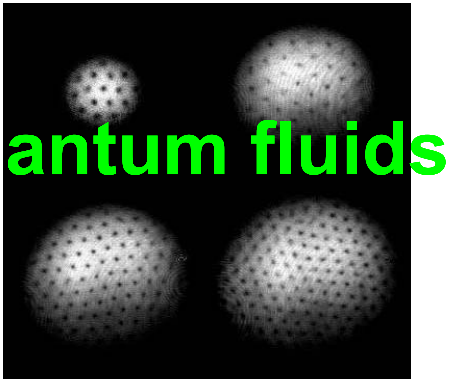


Основные свойства

1) Термодинамически равновесное состояние,
2) Когерентное состояние, характеризуемое определенной фазой и описываемое макроскопической волновой функцией.

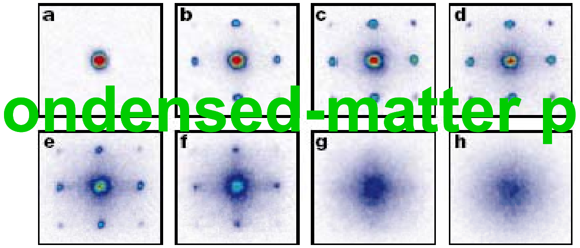


BEC



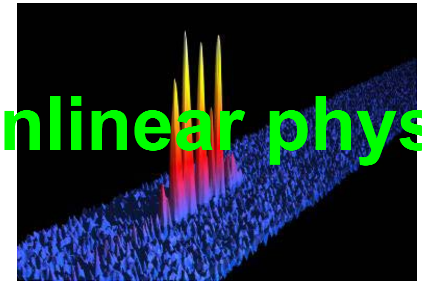
quantum fluids

vortices



condensed-matter physics

Mott insulator



nonlinear physics

solitons

Theoretical Description of the Condensate

The Hamiltonian:

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + V(\vec{r}_i) + \sum_{i<j} W(\vec{r}_i - \vec{r}_j)$$

Confining potential

Interactions between atoms

- At low temperature, we can replace the real potential $W(\vec{r}_i - \vec{r}_j)$

$$W(\vec{r}_i - \vec{r}_j) \longrightarrow g\,\delta(\vec{r}_i - \vec{r}_j) \qquad g = \frac{4\pi\hbar^2 a}{m},$$

a : scattering length

- Hartree approximation: $\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) \approx \psi(\vec{r}_1)\psi(\vec{r}_2)\ldots\ \psi(\vec{r}_N)$

Treatment valid in the dilute regime:

Gross-Pitaevski equation (or non-linear Schrödinger's equation) :

$$\left(-\frac{\hbar^2}{2m}\Delta + V(\vec{r}) + Ng\,\left|\psi(\vec{r})\right|^2\right)\psi(\vec{r}) = \mu\,\psi(\vec{r})$$

Complete analogy with laser beam propagation in optics!

Elementary Excitation Spectrum

Weakly interacting gas: Bogoliubov theory ,

Bogoliubov, N., 1947. *J. Phys. USSR* **11**, 23.

$$\hat{H} = \sum_p E_p \hat{a}_p^\dagger \hat{a}_p + \frac{1}{2} \sum_{p_1 p_2 p_3 p_4} U_{p_1 p_2 p_3 p_4} \hat{a}_{p_1}^\dagger \hat{a}_{p_2}^\dagger \hat{a}_{p_3} \hat{a}_{p_4}$$

- Hamiltoniam in momentum space

Diagonalization by using Bogoliubov transformation

$$\hat{a}_p = c_p \hat{A}_p + s_p \hat{A}^\dagger_{-p} \qquad \hat{a}_p^\dagger = c_p \hat{A}_p^\dagger + s_p \hat{A}_{-p}$$

with c_p, s_p real numbers satisfying $c_p^2 - s_p^2 = 1$.

Now Hamiltonian is $\hat{H} = E_0 + \sum_{p \neq 0} \mathscr{E}_p \hat{A}_p^\dagger \hat{A}_p$ where

$$E_0 = \frac{U_0 N(N-1)}{V} + \frac{1}{2} \sum_{p \neq 0} \left(\mathscr{E}_p - \frac{p^2}{2m} - \frac{U_0 N}{V} \right), \quad \mathscr{E}_p = \sqrt{\left(\frac{p^2}{2m} + \frac{U_0 N}{V} \right)^2 - \left(\frac{U_0 N}{V} \right)^2}.$$

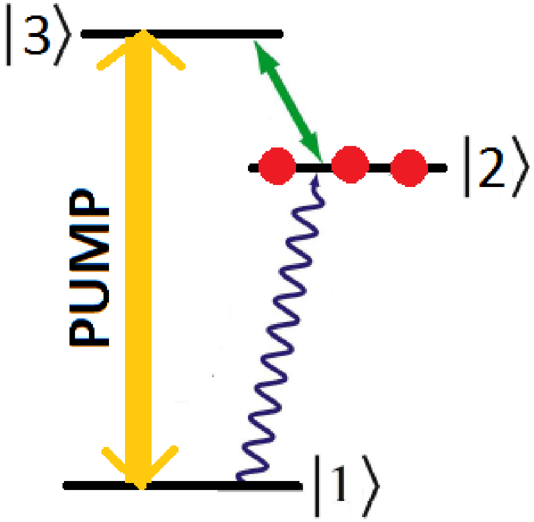
Elementary exitation spectrum

Or, $$\mathscr{E}_k \simeq \begin{cases} \sqrt{4\pi a \rho \hbar^2 k/m}, & \text{for } k \to 0, \\ \hbar^2 k^2/2m, & \text{for } k \to \infty \end{cases}$$

Sound velocity is $c = \sqrt{4\pi a \rho \hbar / m}$

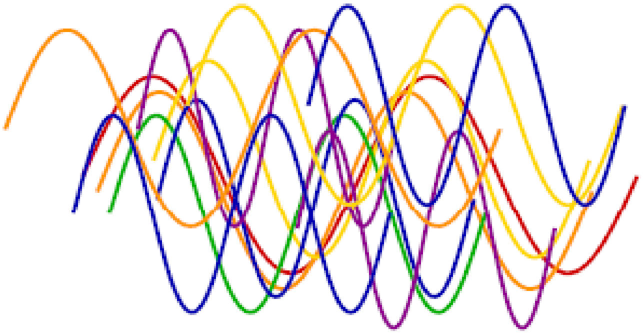
Light Amplification by Stimulated Emission of Radiation - LASERs

А.М. Прохоров, Ч.Х. Таунс,
Н.Г. Басов - 1954г.



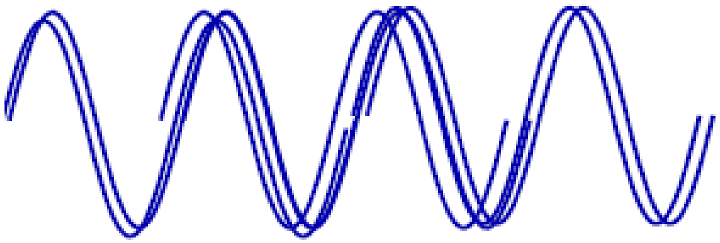
Белый свет (некогерентный)

Состоит из фотонов разных частот; амплитуда и фаза НЕ определены



лазер

амплитуда и фаза определены



Необходима инверсия населенностей!

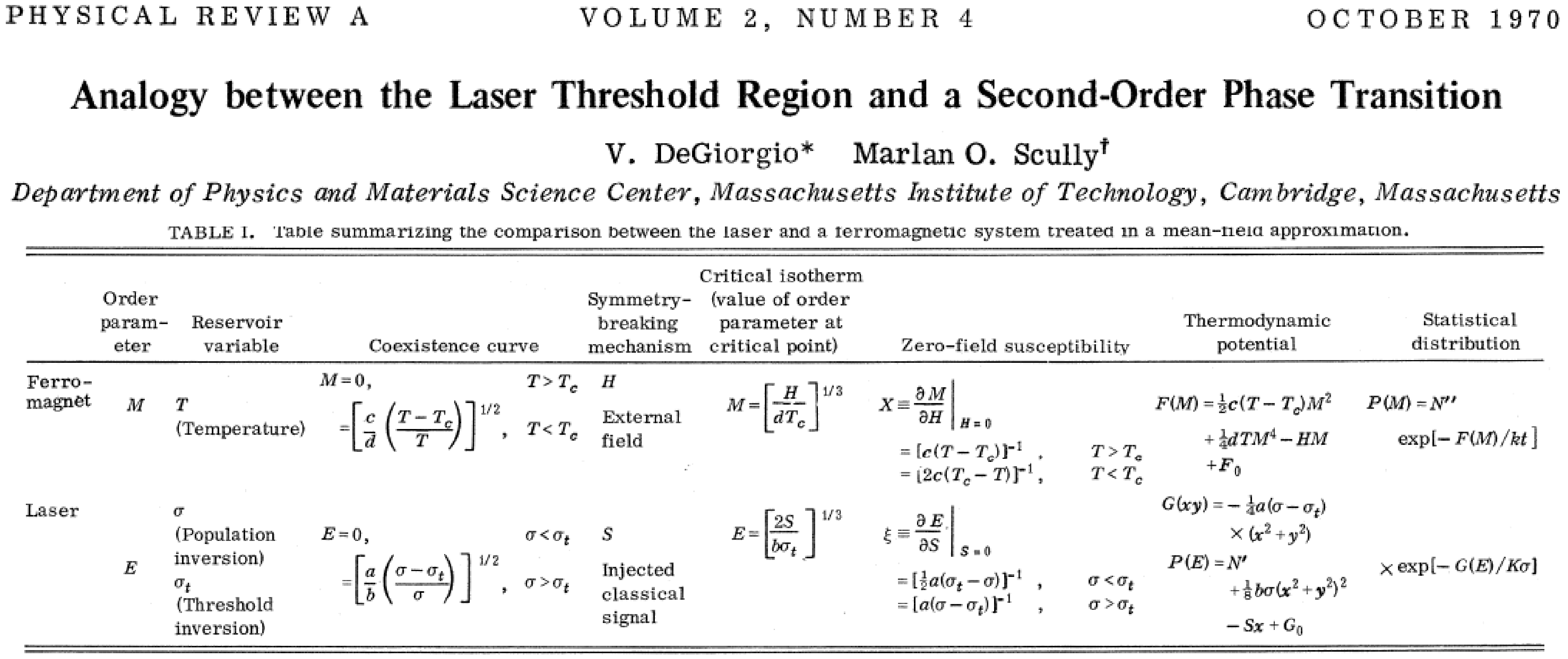
Phase Transitions in C.M. Systems

Phase transition to superfluid state with superconductors;

Bardeen-Cooper-Schrieffer (BCS) states, 1957

.**Second order phase transitions in ferromagnets –** ↑↓↑↓↓↑↑
↑↓↑↓↓↑↑

“spin up” , “spin down” **- *L.P. Kadanoff, M.D. Fisher, 1967***



PHYSICAL REVIEW A VOLUME 2, NUMBER 4 OCTOBER 1970

Analogy between the Laser Threshold Region and a Second-Order Phase Transition

V. DeGiorgio* Marlan O. Scully†

Department of Physics and Materials Science Center, Massachusetts Institute of Technology, Cambridge, Massachusetts

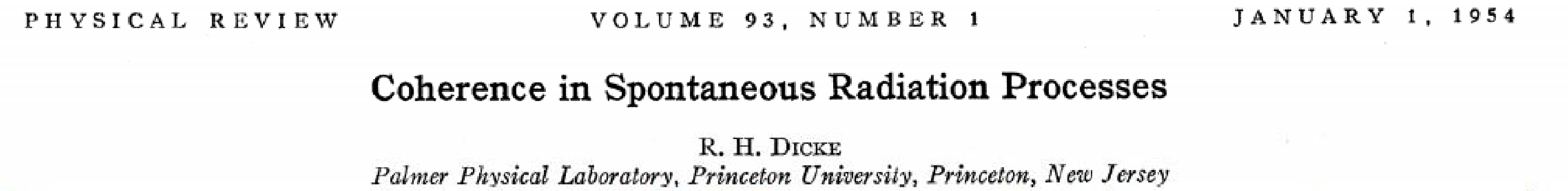
TABLE I. Table summarizing the comparison between the laser and a ferromagnetic system treated in a mean-field approximation.

	Order parameter	Reservoir variable	Coexistence curve	Symmetry-breaking mechanism	Critical isotherm (value of order parameter at critical point)	Zero-field susceptibility	Thermodynamic potential	Statistical distribution
Ferromagnet	M	T (Temperature)	$M=0$, $T>T_c$ $=\left[\frac{c}{d}\left(\frac{T-T_c}{T}\right)\right]^{1/2}$, $T<T_c$	H External field	$M=\left[\frac{H}{dT_c}\right]^{1/3}$	$X\equiv\left.\frac{\partial M}{\partial H}\right\vert_{H=0}$ $=[c(T-T_c)]^{-1}$, $T>T_c$ $=[2c(T_c-T)]^{-1}$, $T<T_c$	$F(M)=\frac{1}{2}c(T-T_c)M^2$ $+\frac{1}{4}dTM^4-HM$ $+F_0$	$P(M)=N''$ $\exp[-F(M)/kt]$
Laser	E	σ (Population inversion) σ_t (Threshold inversion)	$E=0$, $\sigma<\sigma_t$ $=\left[\frac{a}{b}\left(\frac{\sigma-\sigma_t}{\sigma}\right)\right]^{1/2}$, $\sigma>\sigma_t$	S Injected classical signal	$E=\left[\frac{2S}{b\sigma_t}\right]^{1/3}$	$\xi\equiv\left.\frac{\partial E}{\partial S}\right\vert_{S=0}$ $=[\frac{1}{2}a(\sigma_t-\sigma)]^{-1}$, $\sigma<\sigma_t$ $=[a(\sigma-\sigma_t)]^{-1}$, $\sigma>\sigma_t$	$G(xy)=-\frac{1}{4}a(\sigma-\sigma_t)$ $\times(x^2+y^2)$ $+\frac{1}{8}b\sigma(x^2+y^2)^2$ $-Sx+G_0$	$P(E)=N'$ $\times\exp[-G(E)/K\sigma]$

Lasers are thermodynamically non-equilibrium systems!

Coherent Effects and Phase Transitions in Coupled Atom-light Systems

Dicke effect: Enhanced emission



PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

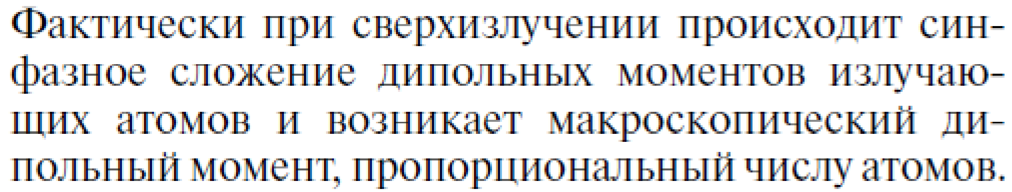
R. H. Dicke

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

$$H_{\text{int}} = \sum_{k,i} g_k \left(\psi_k^{\dagger} S_i^{-} e^{-i\mathbf{k}\cdot\mathbf{r}_i} + \text{H.c.} \right)$$

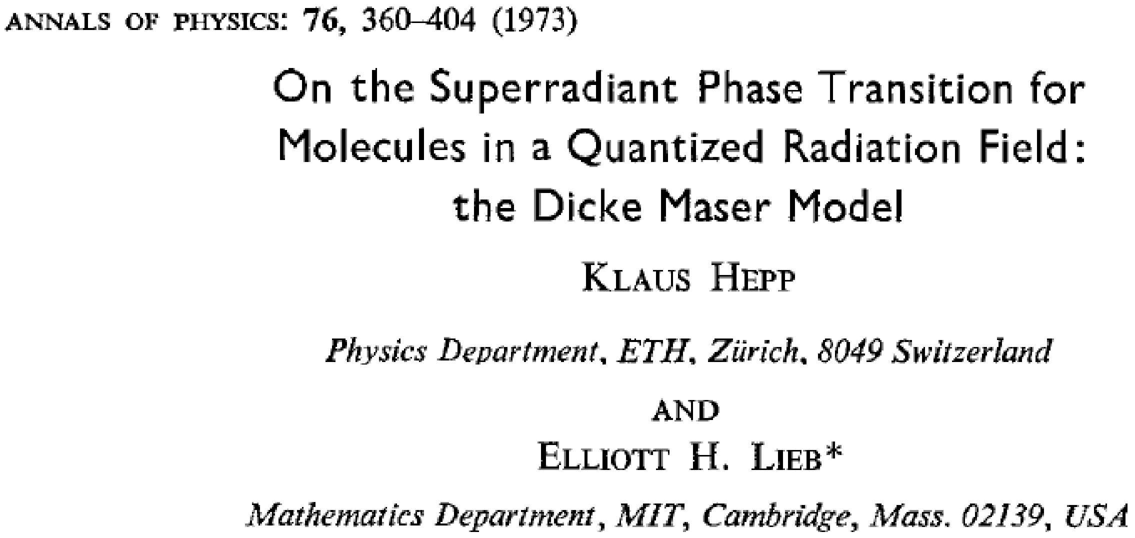
Intensity of radiation is $I \propto N^2$;

N is number of atoms



Фактически при сверхизлучении происходит синфазное сложение дипольных моментов излучающих атомов и возникает макроскопический дипольный момент, пропорциональный числу атомов.

“Superradiant” phase transition (BEC of photons)



ANNALS OF PHYSICS: 76, 360–404 (1973)

On the Superradiant Phase Transition for Molecules in a Quantized Radiation Field: the Dicke Maser Model

Klaus Hepp

Physics Department, ETH, Zürich, 8049 Switzerland

AND

Elliott H. Lieb*

Mathematics Department, MIT, Cambridge, Mass. 02139, USA

Main pecularities

Some early publications on superradiant PT

✓Y.K. Wang, F.T. Hioe, Phys. Rev. A, 7, 831 (1973);
✓K. Rzazewski, K.Wodkiewicz, W. Zakowicz, Phys. Rev.Lett., 35, 432 (1975),
✓J.Brankov, V.A. Zagrebnov, N.S. Tonchev, Theor. And Math.Phys. 22, 13 (1975)
✓V.I. Emeljanov, Yu. Klimontovich, Phys. Lett. A., 59 (1976),
✓В.Ф. Елесин, Ю.В. Копаев, Письма в ЖЭТФ, 24, 78 (1976),
✓J.Knight, Y.Aharonov, G.Hseih, Phys.Rev.A, 17, 1454 (1978)

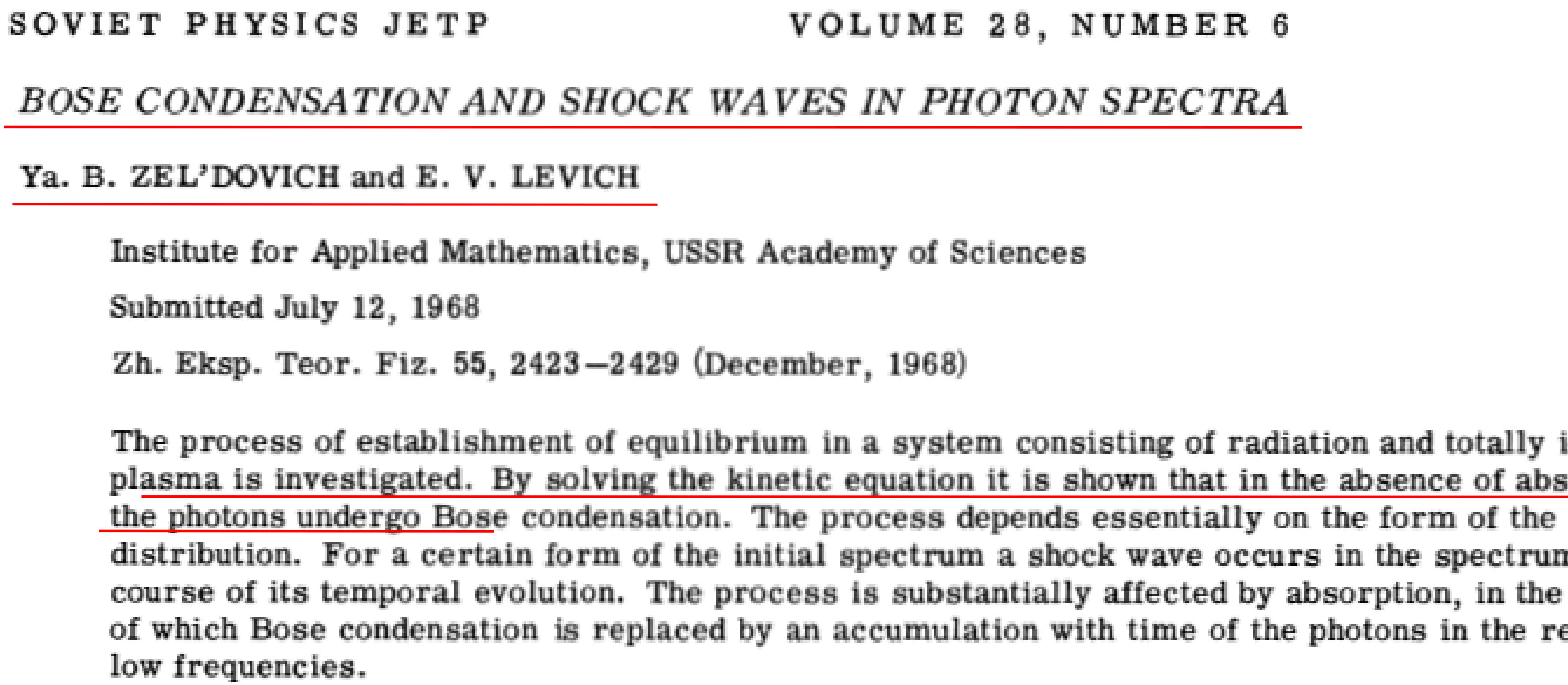
Основные свойства

✓Возникновение макроскопической спонтанной поляризации на нулевой частоте – БЭК фотонов;
✓Хим. потенциал =0.

Сложности модели

Противоречит правилу сумм, фиксирующее значение суммы *матричных элементов,* характеризующих переходы между состояниями рассматриваемой системы, Thomas-Kuhn-Reich, 1925

Ранние схемы получения БЭК фотонов



SOVIET PHYSICS JETP VOLUME 28, NUMBER 6 JUNE, 1969

BOSE CONDENSATION AND SHOCK WAVES IN PHOTON SPECTRA

Ya. B. ZEL'DOVICH and E. V. LEVICH

Institute for Applied Mathematics, USSR Academy of Sciences

Submitted July 12, 1968

Zh. Eksp. Teor. Fiz. 55, 2423–2429 (December, 1968)

The process of establishment of equilibrium in a system consisting of radiation and totally ionized plasma is investigated. By solving the kinetic equation it is shown that in the absence of absorption the photons undergo Bose condensation. The process depends essentially on the form of the initial distribution. For a certain form of the initial spectrum a shock wave occurs in the spectrum in the course of its temporal evolution. The process is substantially affected by absorption, in the presence of which Bose condensation is replaced by an accumulation with time of the photons in the region of low frequencies.

BEC of photons achieved by Compton scattering off a thermal electron gas

Vital Questions

1. How it is possible to achieve thermodynamically equilibrium state for coupled atom-light system?
2. Is it possible to achieve high temperature PT?

Centaur - Half-Human and Half-Animal Composition

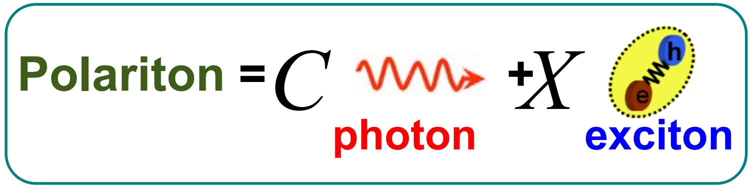
In Greek mythology, a Centaur (from Greek: *Κένταυροι) is a member of a* composite race of creatures, part human and part horse.

A bronze statue of a Centaur, after the Giuseppe Alessandro Furietti *Centaurs.*

Achilles educated by Chiron.
Jean-Baptiste Regnault , 1782

Polariton - Half-Matter and Half-Photon Composition in Physics

Polaritons are quantum Bosonic quasi-particles occurring at matter-field interface (*J.J. Hopfield*, 1959, *V.M. Agranovich, et al, 1960)*



Polariton = C **photon** $+X$ **exciton**

C **and** X **are Hopfield coefficients**

High temperature phase transitions is possible with low branch polaritons due to their small effective mass that is about $10^{-35}\,kg$

Theory on polariton BEC, nonlinear properties and superfluidity

- P. R. Eastham and P. B. Littlewood, *Phys.Rev.B* 64, 235101(2001);
- A. Kavokin, G. Malpuech, F.P. Laussy, *Phys. Lett. A* 306, 187 (2003);
- I. Carusotto, C. Ciuti, *Phys. Rev. Lett*. 93, 166401 (2004);
- O.L. Berman, Yu.E. Lozovik, D. W. Snoke, Phys.Rev.B, 77, 155317 (2008)

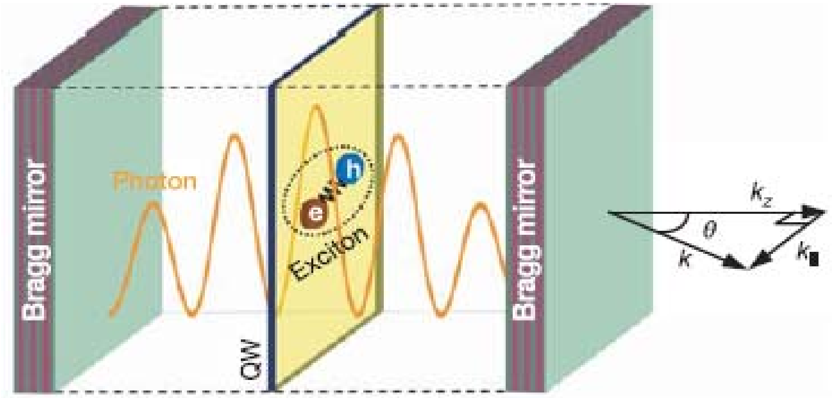
Some recent experiments

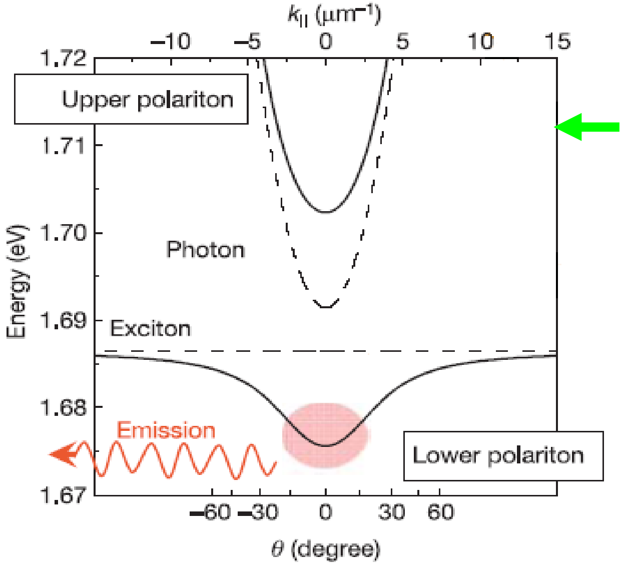
- J. Kasprzak, et al, *Nature,* 443, p. 409 (2006) – evidence for BEC of polaritons;
- R. Balili, et al, *Science* , 316, p.1007 (2007) – polaritons in a harmonic potential trap;
- A. Amo, et al, *Nature,* 457, p.291 (2009) - collective fluid dynamics of a polaritons.

Non-Equilibrium Low Threshold LB Polariton BEC

- ✓Strong matter-field coupling condition is achieved;
- ✓The thermalization time is ten of picoseconds and compatible with polariton lifetime

Polaritons in the semiconductor QW structures placed in microcavity

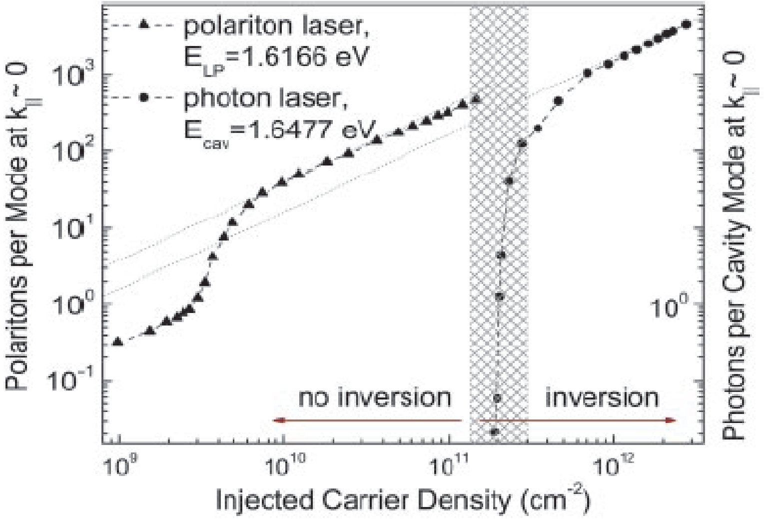




Polariton dispersion in the in CdTe/CdMgTe cavity

"BEC" of polaritons at 5 K

J. Kasprzak, et al, Nature, 2006



Number of LPs and cavity photons per mode vs. injected carrier density for a polariton laser in scheme I (triangles) and a photon laser in scheme II (circles), respectively. The gray zone marks the population inversion densities from band edge to 15 meV above the band edge.

Y.Yamamoto, et al, Ginzton Lab., Stanford Uni. (2003)

Optical Collisions (OC's); Some Remarks

First attempts to describe collisionally broadened spectral lines

V. Weisskopf, *Phys. Z.* 34, 1 (1933); **A. Jablonski**, *Phys. Rev.* 68, 78 (1945);

P. W. Anderson, *Phys.Rev.* 76, 647 (1949); **H. M. Foley**, *ibid. 69, 616 (1946).*

Definition of OC's

$$\boxed{A(a) + B + \hbar\omega_L \rightleftarrows A(b) + B}$$

where A is a two-level atom, B is a buffer gas atom, ω_L is a frequency of light.

L. I. Gudzenko and S. I. Yakovlenko, *Zh. Eksp. Teor. Fiz.* 62, 1686 (1972),

R. E. Hedges, D. L. Drummond, and A. Gallagher, *Phys. Rev. A* 6, 1519 (1972).

OCs properties at small Rabi frequencies $\hbar\Omega_R << k_B T$

C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, *Atom- Photon Interactions: Basic Processes and Applications* (Wiley, New York, 1998).

OC's at large Rabi frequencies and relevant atom-field detunings

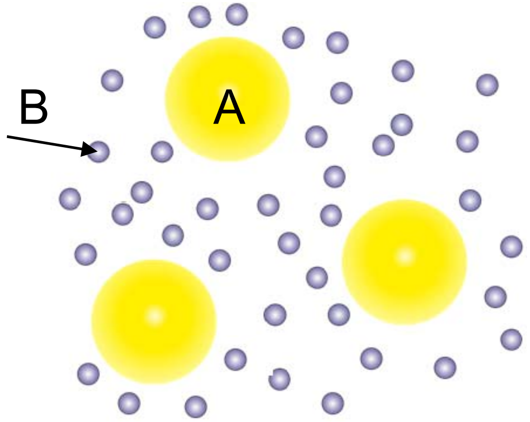
$$\hbar\Omega_R \simeq k_B T$$

R. V. Markov, A. I. Plekhanov, A. M. Shalagin, *Phys. Rev. Lett.* 88, 213601 (2002;

I. Yu. Chestnov, A. P. Alodjants, S. M. Arakelian, J. Nipper, U. Vogl, F. Vewinger, and M. Weitz, Phys. Rev. A 81, 053843 (2010).

Optical Collisions (OC's)

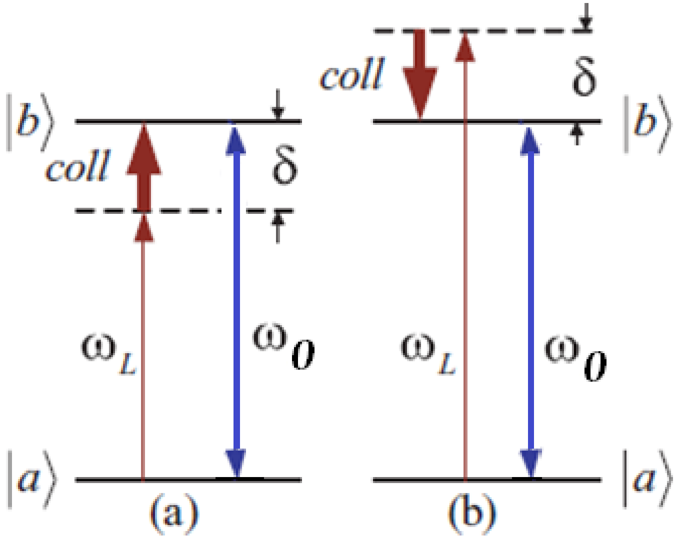
Non-resonant absorption in a two-level atom



Typical densities are

$$n_A \simeq 10^{16} cm^{-3},$$

$$n_B \simeq 10^{21} cm^{-3}.$$

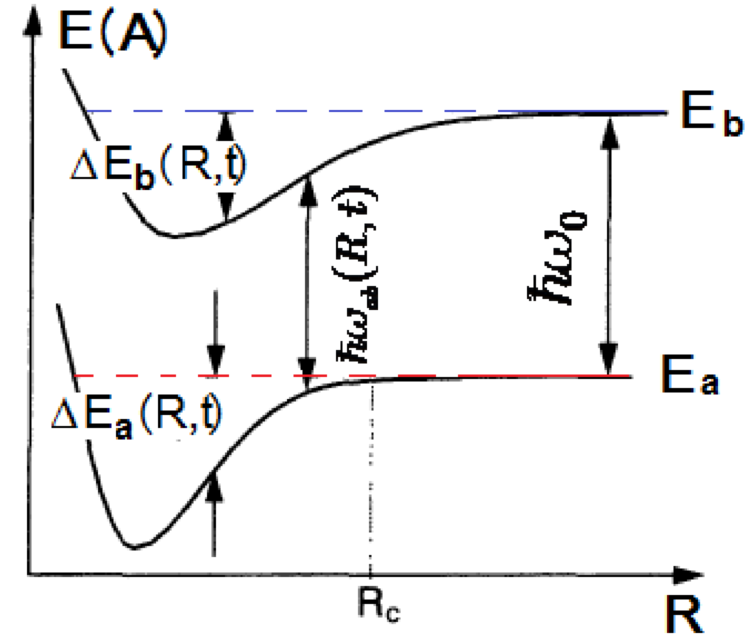


$\delta < 0$ $\delta > 0$

ω_0 is atomic transition frequency;

$\delta = \omega_L - \omega_0$ **- atom-field detuning**

Energy levels of the atom A in the presence of buffer gas atom at a distance R.



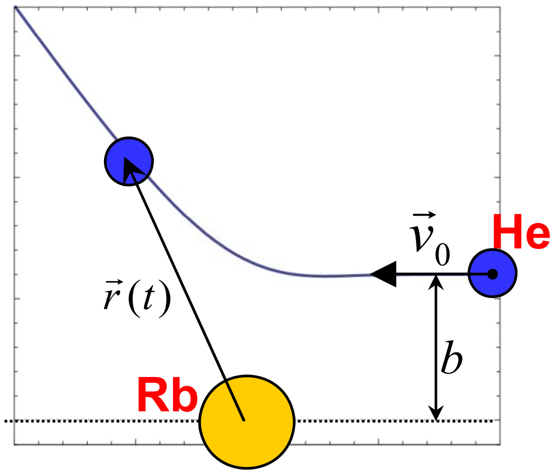
$\Delta E_{a,b}$ are energy shifts introduced by collision

$2R_c$ is collision diameter

$\tau_{coll} = R_c / v$ is collisional time

Основные подходы к ОС

Основные предположения (И. И. Собельман, *Введение в теорию спектров*, ФИЗМАТЛИТ, 1963г.)



- ✓ Относительное движение атома и возмущающей частицы квазиклассично, что позволяет пользоваться понятием траектории возмущающей частицы;
- ✓ Основную роль в уширении играют взаимодействия с ближайшей возмущающей частицей (бинарные взаимодействия),
- ✓ Возмущение адиабатично, т. е. т. е. не вызывает переходов между различными состояниями атома без испускания и поглощение фотонов, что справедливо при $k_B T << \hbar\omega_0$.

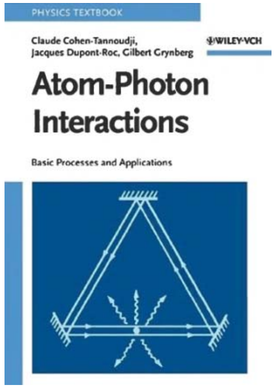
При пролете возмущающей частицы на атом накладывается внешнее поле

$$U(R) = U\sqrt{r^2 + v^2(t - t_0)^2}$$

R — расстояние до возмущающей частицы в данный момент времени t , r — прицельное расстояние, t_0 — момент наибольшего сближения и v — относительная скорость.

The Model of Optical Collisions

Equation for atomic coherence (C. Cohen-Tannoudji, et al, *Atom Photon Interactions: Basic Processes and Applications*, Wiley, 2004)



PHYSICS TEXTBOOK

Claude Cohen-Tannoudji, Jacques Dupont-Roc, Gilbert Grynberg

WILEY-VCH

Atom-Photon Interactions

Basic Processes and Applications

$$\frac{d\sigma_{ba}}{dt} = -(\gamma + i\cdot\eta)\sigma_{ba}$$

Collisional (dephasing) rate

$$\gamma = \langle 1-\cos\phi\rangle_{coll}$$

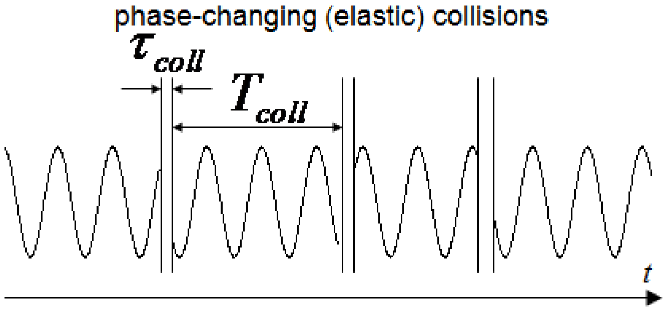
Collisional shift

$$\eta = \langle \sin\phi\rangle_{coll}$$

ϕ **is phase shift introduced by collision**

Main approximation

$$\tau_{coll} << \Delta t << T_{coll}$$



Δt **is the time intervals;**

τ_{coll} **is the duration of collision;**

T_{coll} **is the time interval separating two collisions;**

Master Equation for OC's

$$\left\{\frac{d\sigma}{dt}\right\}_{coll} = -\frac{\gamma}{2}\sigma + 2\gamma S_z \sigma S_z - i\eta\left[S_z, \sigma\right]$$

where $S_z = \frac{1}{2}\left(|b\rangle\langle b| - |a\rangle\langle a|\right)$ is atomic population inversion operator

For accessible experimental conditions with Rb^{87} atoms and Ar high pressure $(500\ \mathrm{bar})$ buffer gas there is

$$\boxed{\begin{aligned} \gamma/2\pi &= 3.6\,THz \\ \eta/2\pi &= -3\,THz \end{aligned}}$$

Theoretical Approach for Optical Collisions

Master equation for density matrix σ

$$\frac{d}{dt}\sigma = -\frac{i}{\hbar}\left[H_{AL},\sigma\right] + \left\{\frac{d}{dt}\sigma\right\}_{rad} + \left\{\frac{d}{dt}\sigma\right\}_{coll}$$

Atom-optical interaction characterizing by resonant Rabi frequency Ω_0

Spontaneous emission contribution characterizing by rate Γ

Collisional processes characterizing by rate γ

$\Omega_0 / 2\pi = 0.1 THz$

$\Gamma = 2\pi * 6\, MHz$, **or**
$\tau_{spont} = 27\, ns$

$\gamma / 2\pi = 3.6\, THz$

-for accessible experimental conditions with Rb^{87} atoms and Ar high pressure $(500\ \mathrm{bar})$ buffer gas.

Radiative (spontaneous emission) processes

Master equation term for spontaneous emission

$$\left\{\frac{d\sigma}{dt}\right\}_{rad} = -\frac{\Gamma}{2}\left(S_+S_-\sigma + \sigma S_+S_-\right) + \Gamma S_-\sigma S_+$$

where $\Gamma = 1/\tau_{spont}$ is spontaneous emission rate.

In the current experiment with Rb^{87} atoms the numbers are

$\Gamma = 2\pi * 6\, MHz$, **or**

$\tau_{spont} = 27\, ns$

✓**About 10^4 collisions happen during the natural lifetime of rubidium atoms.**

✓**Frequent collisions with buffer gas atoms lead to thermalization of coupled atom-light states.**

Uncoupled Atom+Field System Properties

Uncoupled Hamiltonian is

$$H_0 = \hbar\omega_L c^\dagger c + \hbar\omega_0 |b\rangle\langle b|$$

H_L H_{at}

Free quantum photonic field with frequency ω_L

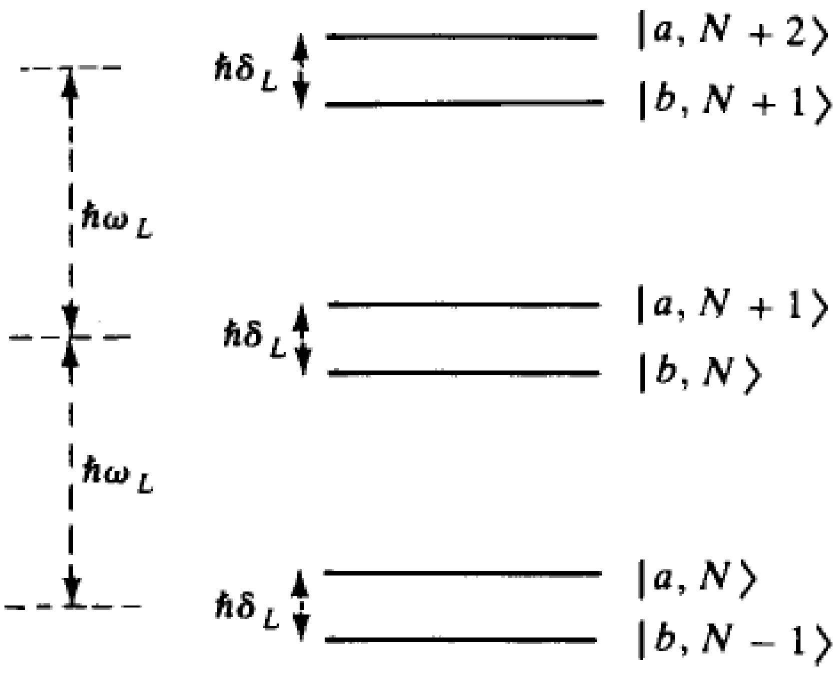
Atomic Hamiltonian with transition frequency ω_0

where c $(c^\dagger)$ is annihilation (creation) operator for the photons,

$\sigma_b = |b\rangle\langle b|$ is upper level population operator.

$\delta_L = \omega_L - \omega_0$ is atom-light detuning

Manifolds of uncoupled states of the atom + laser photons system



Atom-Field Interaction

The Hamiltonian is

$$H = \underbrace{\hbar\omega_L c^\dagger c}_{H_L} + \underbrace{\hbar\omega_0 |b\rangle\langle b|}_{H_{at}} + \underbrace{\hbar g\left(|b\rangle\langle a|c^\dagger + |a\rangle\langle b|c\right)}_{H_{\text{int}}}$$

Free quantum photonic field with frequency ω_L

Atomic Hamiltonian with transition frequency ω_0

Atom-field interaction term ; g is coupling constant

where $c \ (c^\dagger)$ **is annihilation (creation) operator for the photons,**

$S_+ = |b\rangle\langle a|, \ \ S_- = |a\rangle\langle b|$ **are operators of atomic transitions**

In the current experiment with Rb^{87} atoms the numbers are

$$\omega_0 / 2\pi = 282 \ \mathrm{THz}, \ \ \omega_L / 2\pi = 293 \ \mathrm{THz},$$

$\Omega_0 \simeq g\sqrt{N} = 2\pi \times 0.1 \ \mathrm{THz}$ **is resonant Rabi frequency,**

$N = \langle c^\dagger c \rangle$ **is average photon number.**

The Dressed States (DS's)

Definition of DS's (C. Cohen-Tannoudji and S. Reynaud, *J. Phys. B: At. Mol.. Phys.*, **Vol. 10, No. 3, (1977)**)

$$\left|1(N)\right\rangle = \sin\theta \boxed{\left|a, N+1\right\rangle} + \cos\theta \boxed{\left|b, N\right\rangle}$$

$$\left|2(N)\right\rangle = \cos\theta \boxed{\left|a, N+1\right\rangle} - \sin\theta \boxed{\left|b, N\right\rangle}$$

Eigenstates of $H_L + H_{at}$

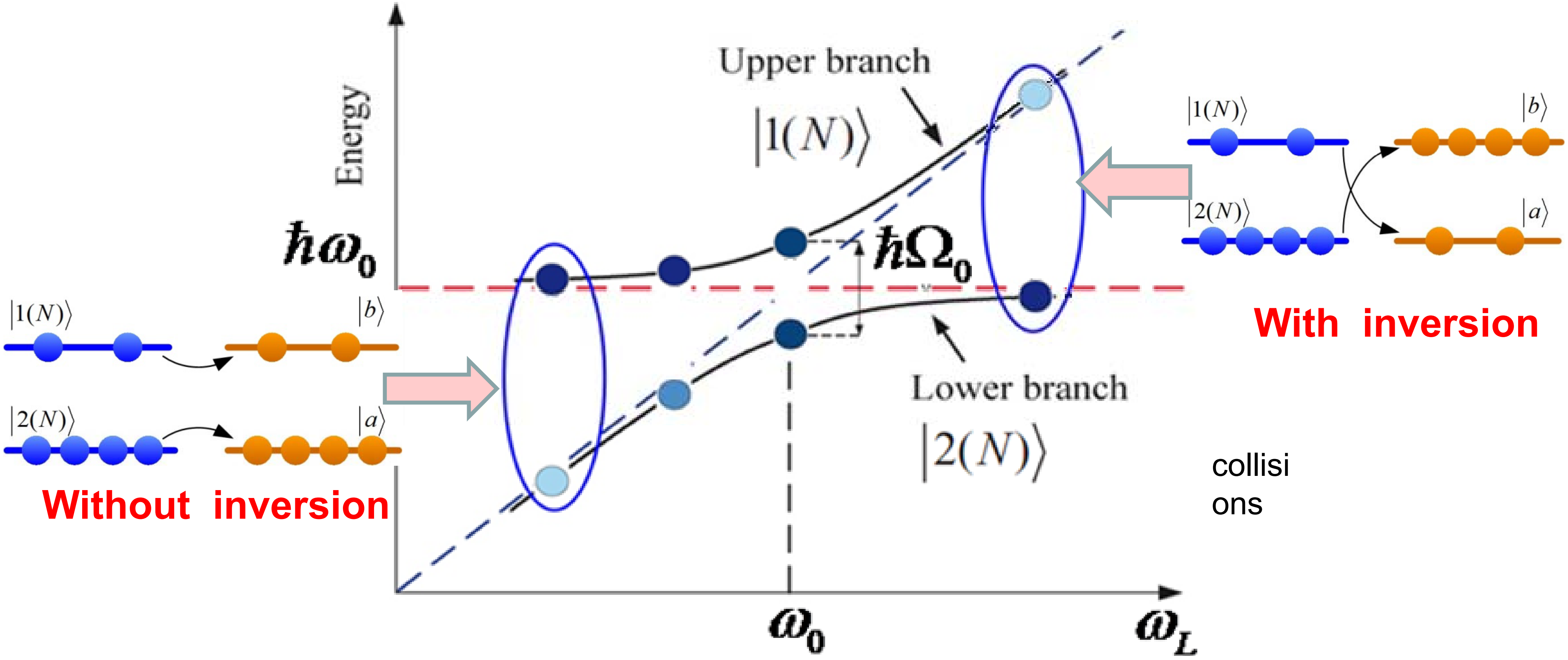
where $\left\{\begin{matrix}\sin\theta \\ \cos\theta\end{matrix}\right. = \dfrac{1}{\sqrt{2}}\sqrt{1 \pm \dfrac{\delta}{\Omega_R}},$

$\Omega_R = \sqrt{\Omega_0^2 + \delta^2}$ **is Rabi splitting frequency**

DS's $\left|1(N)\right\rangle$ and $\left|2(N)\right\rangle$ are eigenstates of total H

Properties of Dressed States

Energies of dressed levels



Without inversion

With inversion

collisions

Perturbative limit: $\Omega_0 << k_B T / \hbar \leq |\delta|$

Bloch Parameters in the DS Basis

Definition of density matrix elements in the DS basis

$$\sigma_{ij} = \sum_N \langle i(N) | \sigma | j(N) \rangle \quad i,j = 1,2, \quad N$$ **is average number of photons.**

Pseudo-spin Bloch vector components

$$S_x = \sigma_{12} \underbrace{+ \sigma_{21}, \; S_y = i(\sigma_{12}}_{\text{Defines coherent properties of DS's}} - \sigma_{21}), \quad S_z = \sigma_{11} - \sigma_{22}$$

Defines population imbalance for DS's

Main approximations are

$$\Omega_R \tau_{\text{coll}} << 1$$

Impact limit

$$\Omega_R >> \gamma >> \Gamma$$

Secular limit

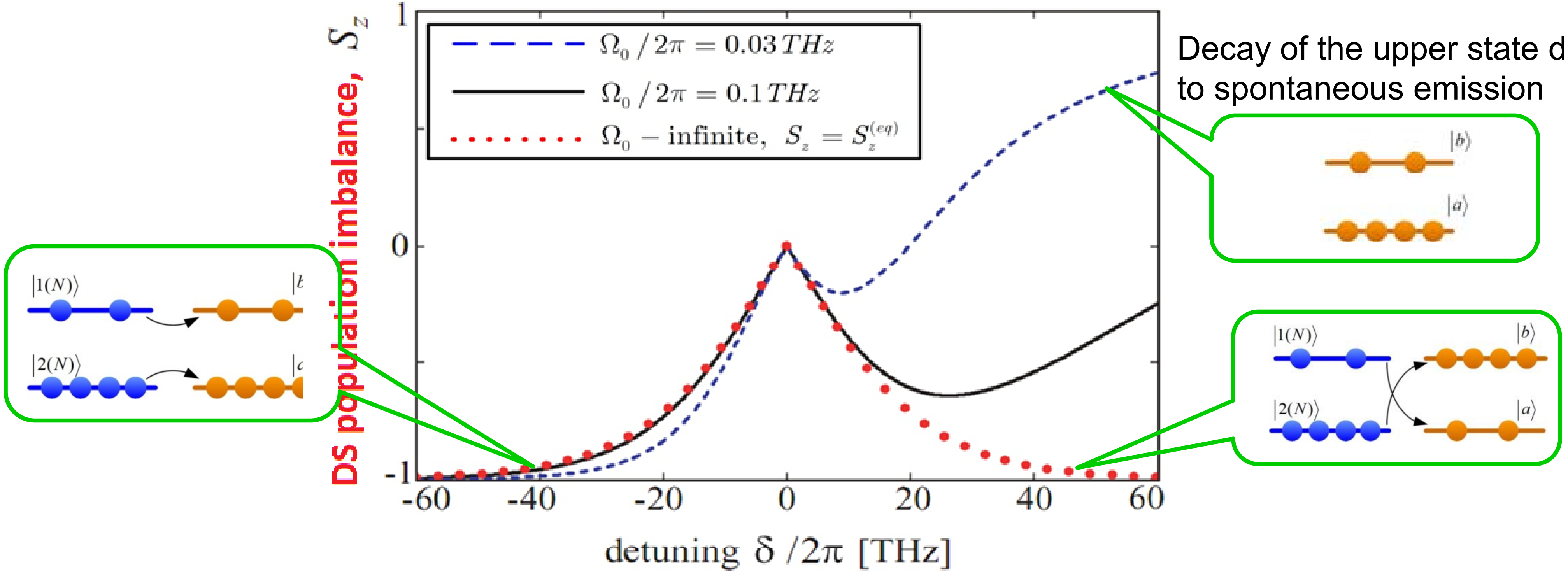
$$\Omega_0 << |\delta| \simeq k_B T / \hbar$$

Perturbative limit

τ_{coll} **is duration of elementary act of collision,** Ω_R **is Rabi frequency**

DS Population Imbalance

Dependence of dressed-state population imbalance $S_z = \sigma_{11} - \sigma_{22}$ **for argon gas collisional broadening**



Decay of the upper state due to spontaneous emission

Equilibrium population imbalance

$$S_z^{(eq)} = -\tanh\left(\hbar\Omega_R/2k_BT\right) \approx -\tanh\left(\hbar\left|\delta\right|/2k_BT\right)$$

Can be obtained only at $\hbar\Omega_R >> k_BT$ i.e. at $\hbar\left|\delta\right| >> \hbar\Omega_0 >> k_BT$

Thermalization Conditions

⚠ **Thermalization occurs if thermalization rate is smaller than effective spontaneous emission rate** $\Gamma << w$

where $w = \gamma\Omega_0^2 / 2\delta^2$ **is thermalization rate.**

Rigorous condition of thermalization is

$$\boxed{\Gamma/\gamma << \Omega_0^2/\delta^2 << 1}$$

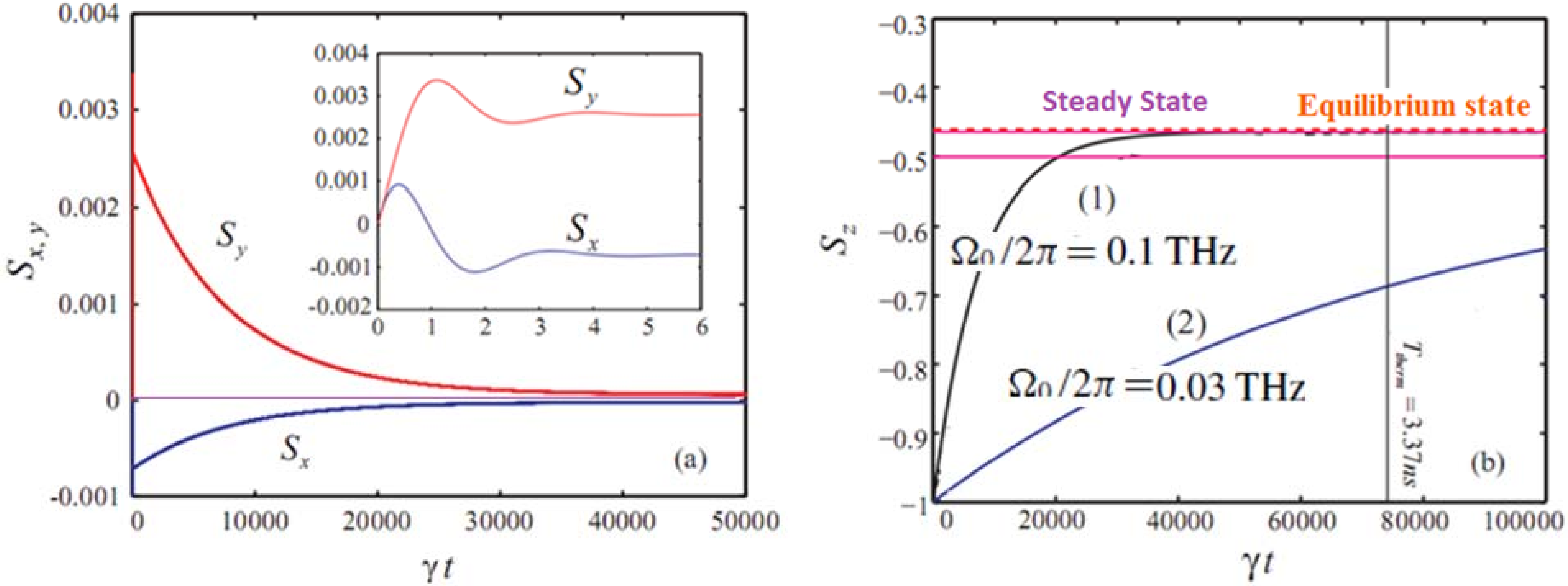
Or, in the time domain it is looks like

$$\boxed{T_{therm} << \tau_{spont}}$$

where $T_{therm} = \dfrac{2\pi}{2w} \simeq \dfrac{2\pi\delta^2}{\gamma\Omega_0^2}$ **is the time of thermalization;**

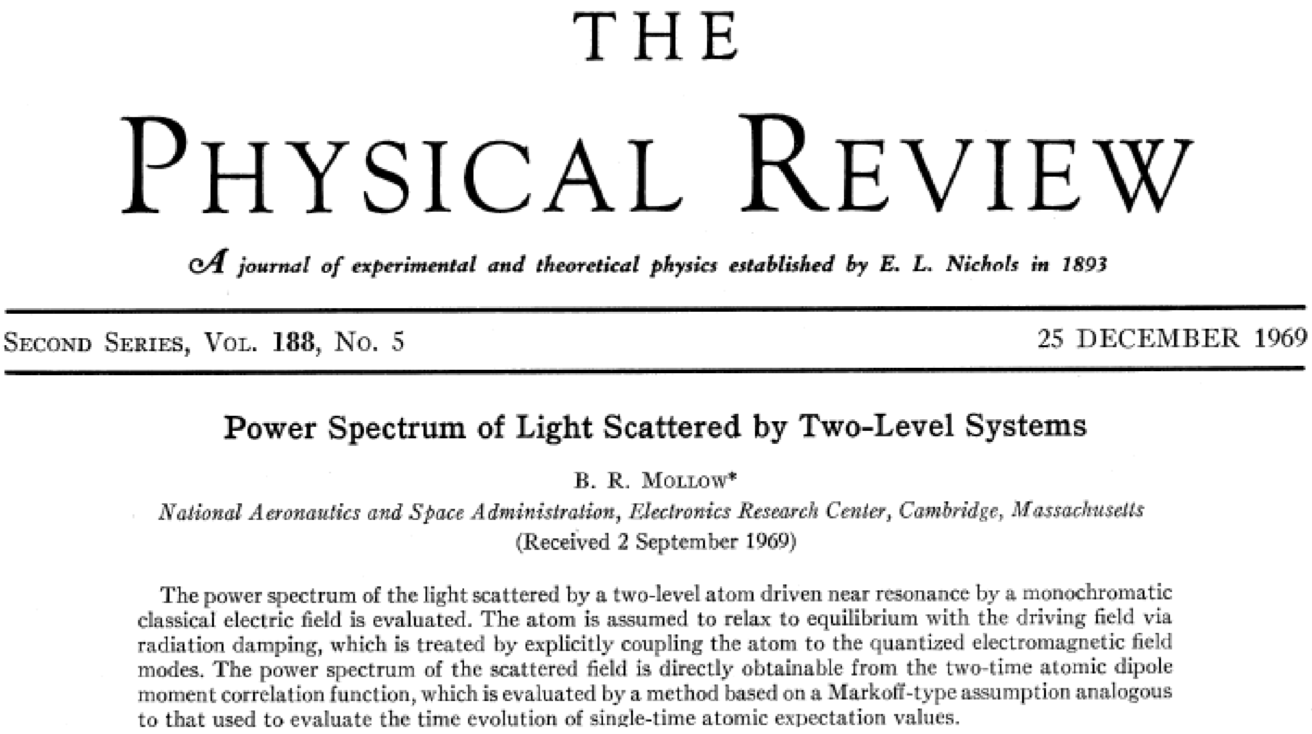
τ_{pol} **is spontaneous emission lifetime**

Pseudo-Spin Components



Dependences of (a) dressed-state pseudospin components $S_{x,y}$ and (b) population imbalance S_z as a function of reduced time γt. Parameters are $\gamma/2\pi = 3.6$ THz, $\eta/2\pi = -3$ THz, $\Gamma \simeq 37$ MHz, $\delta/2\pi = -11$ THz, and $T = 530$ K. The resonant Rabi splitting frequency $\Omega_0/2\pi$ is 0.1 THz for curves (1) and 0.03 THz for curves (2). Initial conditions are $S_{x,y}(0) = 0$ and $S_z(0) = -1$.

Fluorescence (Mollow) Triplet



THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 188, No. 5 25 DECEMBER 1969

Power Spectrum of Light Scattered by Two-Level Systems

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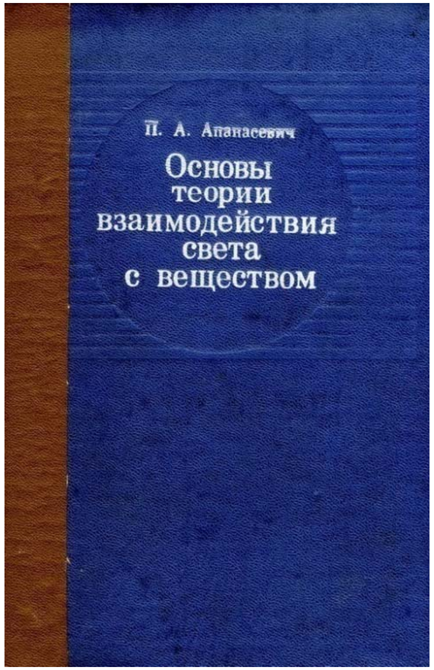
National Aeronautics and Space Administration, Electronics Research Center, Cambridge, Massachusetts

(Received 2 September 1969)

The power spectrum of the light scattered by a two-level atom driven near resonance by a monochromatic classical electric field is evaluated. The atom is assumed to relax to equilibrium with the driving field via radiation damping, which is treated by explicitly coupling the atom to the quantized electromagnetic field modes. The power spectrum of the scattered field is directly obtainable from the two-time atomic dipole moment correlation function, which is evaluated by a method based on a Markoff-type assumption analogous to that used to evaluate the time evolution of single-time atomic expectation values.



P.A. Apanasevich, 1974

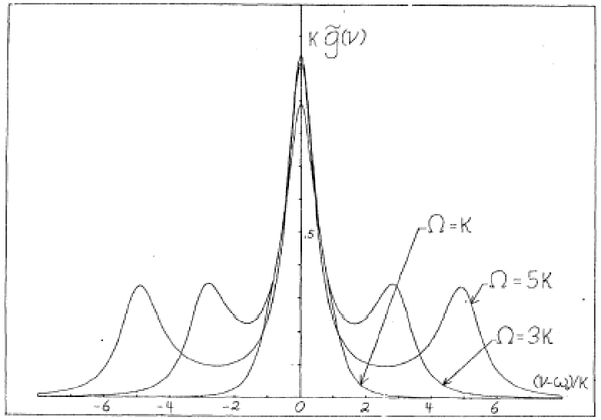
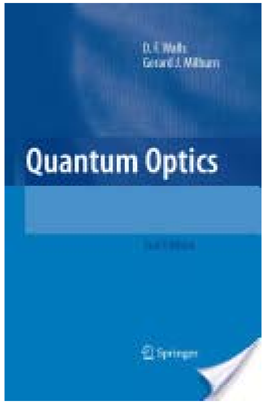


FIG. 1. Spectral density $\tilde{g}(\nu)$ for a two-level atom driven exactly on resonance.

> *"The power spectrum of the scattered field has peaks centered at the incident field frequency* ω_L *and the displaced frequencies* $\omega_L \pm \Omega_R$*, with widths proportional to the atomic relaxation rate"*

Fluorescence Spectrum

Definition (D.F. Walls, G. Milburn, *Quantum Optics*, Springer, 2008)



$$S(\omega) = \frac{\Gamma}{2\pi} \int_{-\infty}^{+\infty} \langle \sigma_+(\tau) \sigma_-(0) \rangle_{ss} e^{-i\omega\tau} d\tau$$

$\sigma_+ = |b\rangle\langle a|$ is pseudospin operator.

Normalization condition – total intensity

$$I = \int S(\omega) d\omega = \Gamma \langle \sigma_+(0) \sigma_-(0) \rangle_{ss} = \Gamma \sigma_{bb}$$

Intensity of fluorescence triplet in dressed state picture

$$I = I_0 + I_{11}(T) + I_{22}(T) = \sigma_{11}\Gamma_{1\to 2} + \sigma_{22}\Gamma_{2\to 1} + \Gamma_{1\to 1}$$

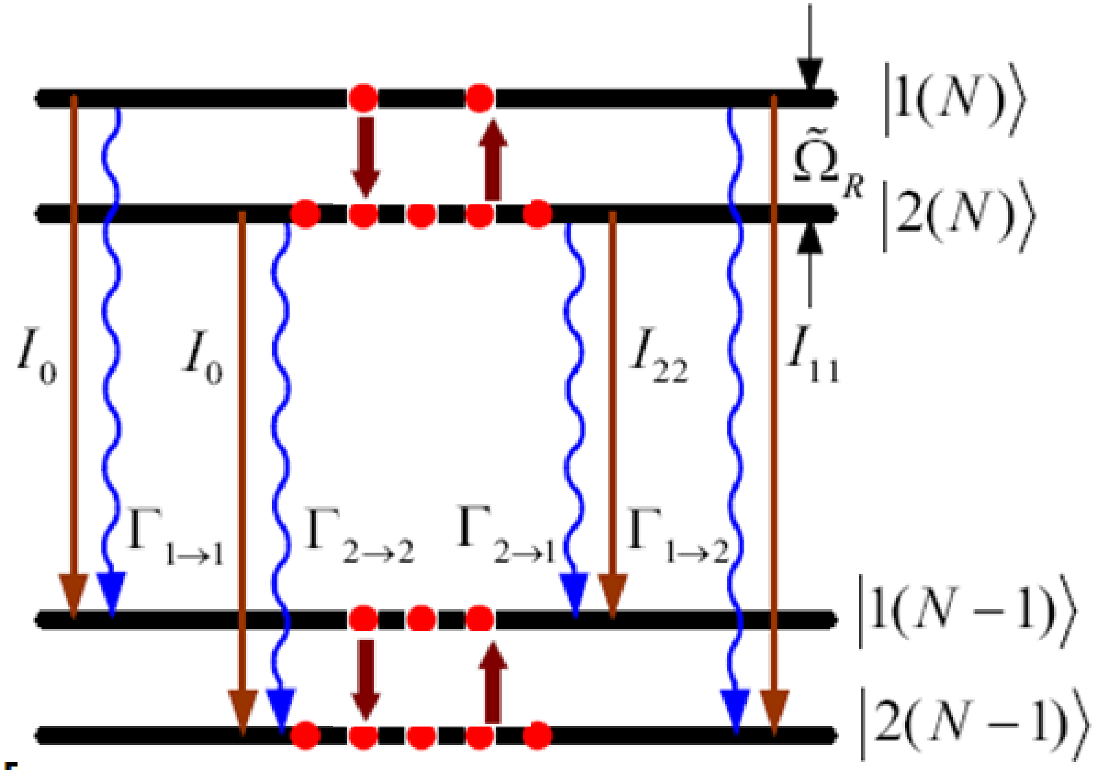
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\omega_L, \qquad \omega_L + \Omega_R, \qquad \omega_L - \Omega_R$$

Manifolds of DS's in The Presence of Atom-Light Interaction

Allowed spontaneous transitions for DS's

OC's lead to transitions between dressed states of the same manifold



Transitions between DS's are

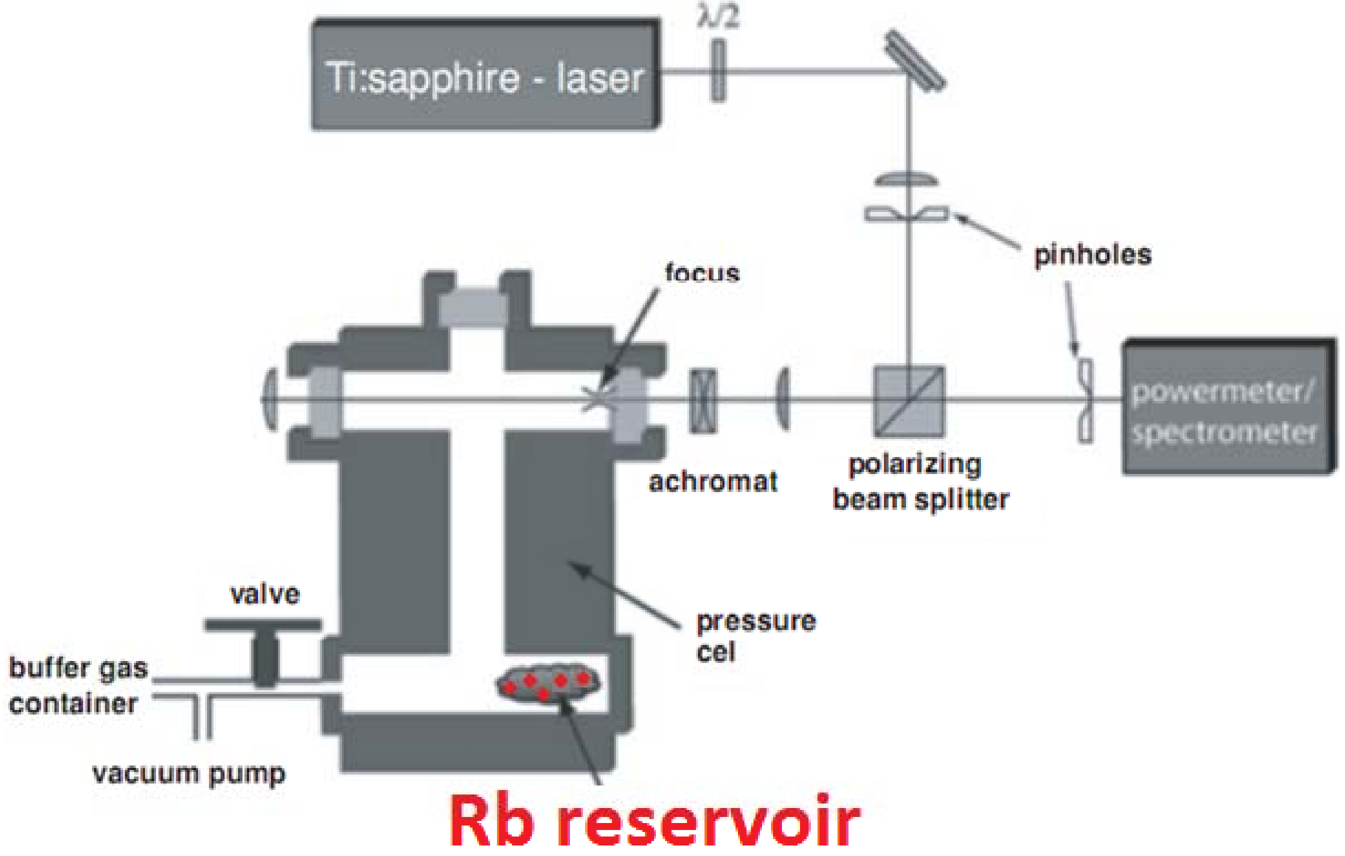
$|1(N)\rangle \rightarrow |2(N-1)\rangle$: frequency $\omega_L + \Omega_R$

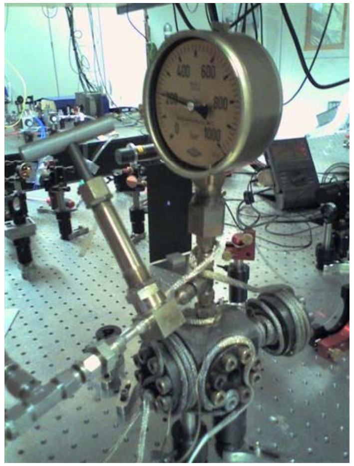
$|2(N)\rangle \rightarrow |1(N-1)\rangle$: frequency $\omega_L - \Omega_R$

$|i(N)\rangle \rightarrow |i(N-1)\rangle$: frequency $\omega_L \quad (i = 1, 2)$.

Experiment

Experimental set up (Bonn University, Germany)





Temperature in the chamber

$T=530\,K$

Maximal power of the laser irradiation is

$300\,mW$

(**resonant Rabi splitting is** $\Omega_0/2\pi = 0.1THz$)

As a buffer gas have been explored

Ar (**under the 500 bar pressure)**

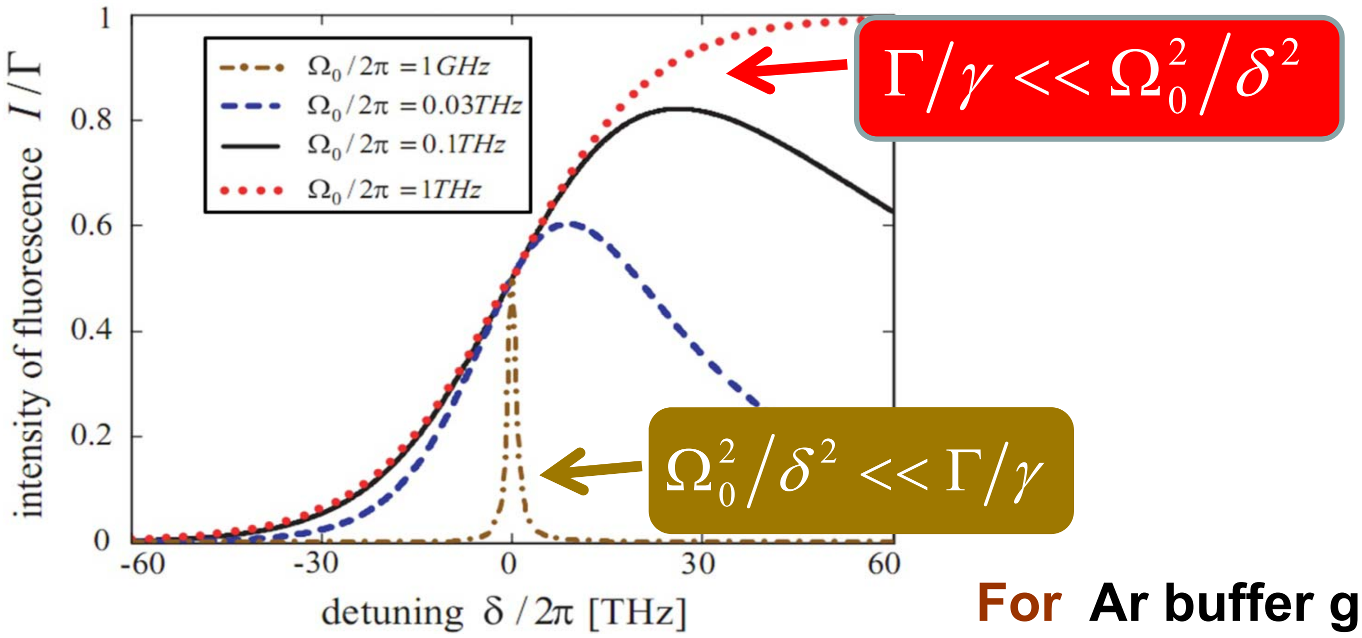
He (**under the 400 pressure)**

Rb atom density is $10^{16}\,cm^{-3}$

Thermalization: Theory and Experiment

Intensity of spectral components

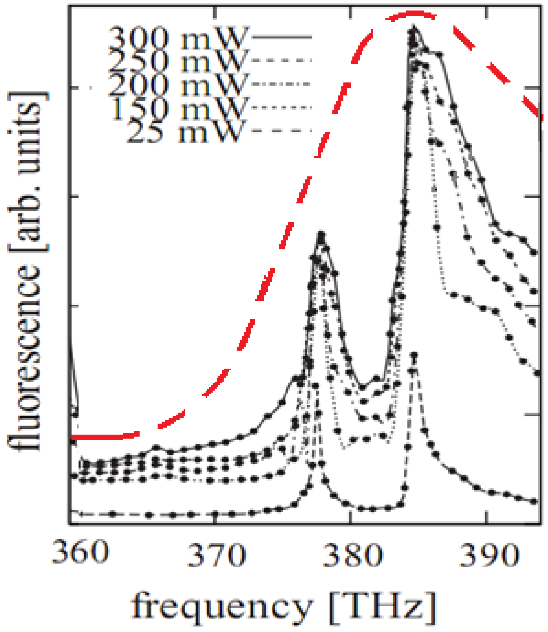
Theory



$\Gamma/\gamma << \Omega_0^2/\delta^2$

$\Omega_0^2/\delta^2 << \Gamma/\gamma$

Experiment, Bonn Uni.



For Ar buffer gas

At full thermal equilibrium we obtain

$$I = \underset{\omega_L,}{\underset{\downarrow}{I_0}} + \underset{\omega_L + \Omega_R,}{\underset{\downarrow}{I_{11}}}(T) + \underset{\omega_L - \Omega_R}{\underset{\downarrow}{I_{22}}}(T) \simeq \frac{1}{1+e^{-\hbar\delta/k_B T}}$$

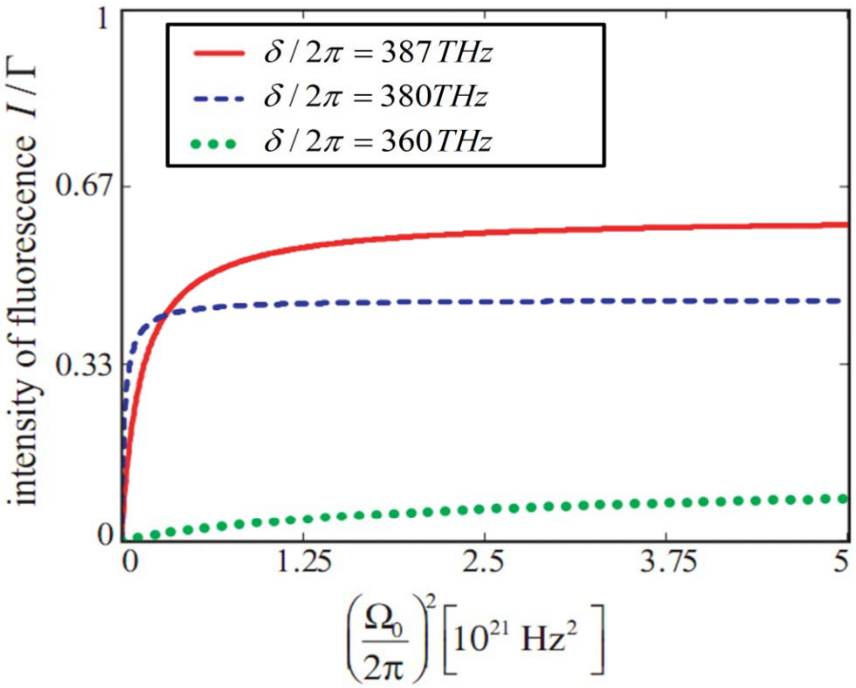
For OC experiment we have obtained

$$T_{therm} \simeq 3.3ns, \quad \tau_{spont} \simeq 27ns$$

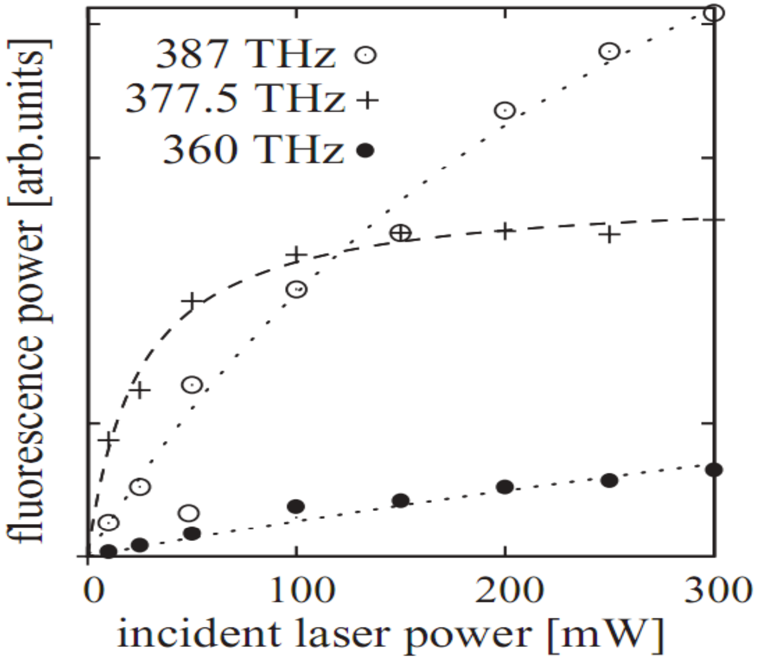
Intensity of Spectral Components

The reduced intensity of fluorescence versus resonant Rabi frequency (incident laser power)

Theory



Experiment, Bonn Uni.



Saturation parameter $K = \frac{\Omega_0^2\gamma}{\delta^2\Gamma}$ **should be** $K >> 1$

Full thermalization occurs at infinite resonant Rabi frequency

Outlook

Проблема фазовых переходов для связанных состояний среды и квантованного поля неразрывно связана с физическими механизмами термализации в системе, которые могут быть совершенно разными для различных систем.

Когерентное состояние физической системы может быть получено совершенно различными способами: физические процессы, приводящие к его формированию могут иметь разную физику.

Термализация для связанной атомно-оптической системы может быть достигнута с помощью оптических столкновений, происходящих при высоких температурах с участием атомов буферного газа. Данный процесс существенно ограничивается временем спонтанных переходов двухуровневых атомов.

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1. A. P. Alodjants, I. Yu. Chestnov, and S. M. Arakelian, *High temperature phase transition in the coupled atom-light system in the presence of optical collisions*, Phys. Rev. A 83, 053802 (2011).
2. I. Yu. Chestnov, A. P. Alodjants, S. M. Arakelian, J. Nipper, U. Vogl, F. Vewinger, and M. Weitz, *Thermalization of coupled atom-light states in the presence of optical collisions*, Phys. Rev. A 81, 053843 (2010).

Thank you for attention !