Drag force and superfluidity in the one-dimensional Bose gas

Alexander Yu. Cherny

Bogoliubov Laboratory of Theoretical Physics,

Joint Institute for Nuclear Research, Dubna

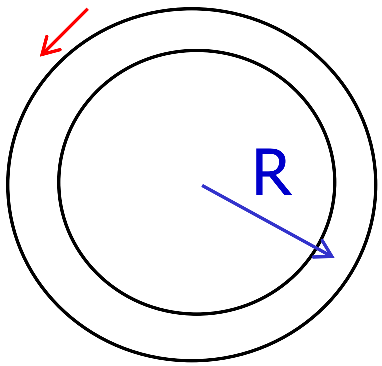
Properties of superfluid system

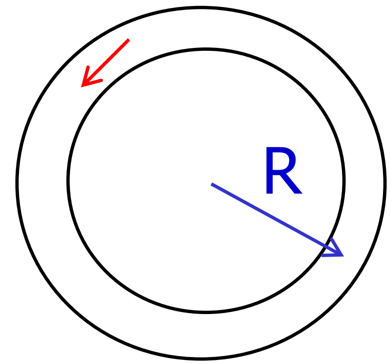
- Hess-Fairbank effect (analog of Meissner effect)
- Quantized circulation (vortices)
- Frictionless flow through capillaries
- Metastable currents
- Second sound
- Josephson effect
- …

Hess-Fairbank effect and metastable currents

Torus or annulus geometry ⇒ momentum is quantized $\Delta p = \dfrac{2\pi\hbar}{L}$

$\omega_c = \Delta p / (2mR) = \hbar / (2mR^2)$; $2\omega_c$ – quantum unit of rotation





Hess-Fairbank effect	Persistent current
$\omega \le \omega_c$	$\omega \gg \omega_c$
walls rotate	walls at rest
liquid at rest	liquid rotates
equilibrium effect	metastable effect

Predictions for superfluidity in 1D

1D Bose gas with the short-range repulsive interaction at zero temperature:

- Hess-Fairbank effect ✓
- Quantized circulation ✓
- Metastability of currens ✗

ACh, J.-S. Caux, and J. Brand, PRA **80**, 043604 (2009);
J. Sibirean Fed. Univ. Math. & Phys. **3**, 289 (2010)

Bose-Einstein condensation and superfluidity

If the Bose condensate exists ⟹ macroscopic occupation of the single-state with wave function

$$\psi = \sqrt{\rho_0} e^{i\chi}$$

Order parameter obeying Gross-Pitaevskii equation

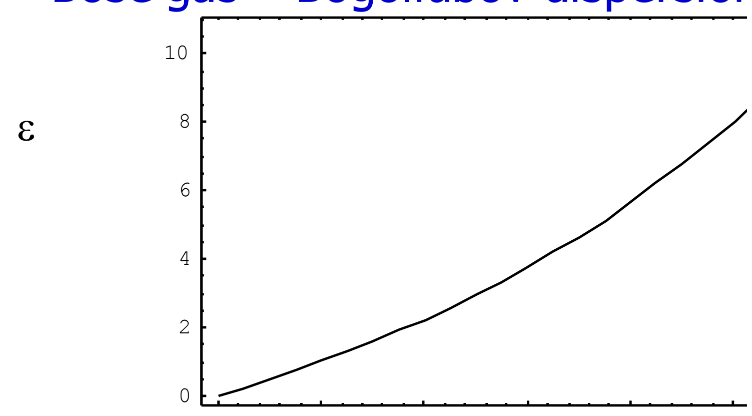
$$\vec{v}_s \equiv \hbar \nabla \chi / m$$

Superfluid velocity $\nabla \times \vec{v}_s = 0$ ⟹

$$\oint \vec{v}_s \mathrm{d}\vec{l} = 2\pi\hbar n / m$$

Quantized circulation of superfluid current

Bose gas – Bogoliubov dispersion



For weakly interacting gas with BEC one can explain the properties of superfuid system Bogoliubov (1947)

But in 1D there is no BEC even at T=0!

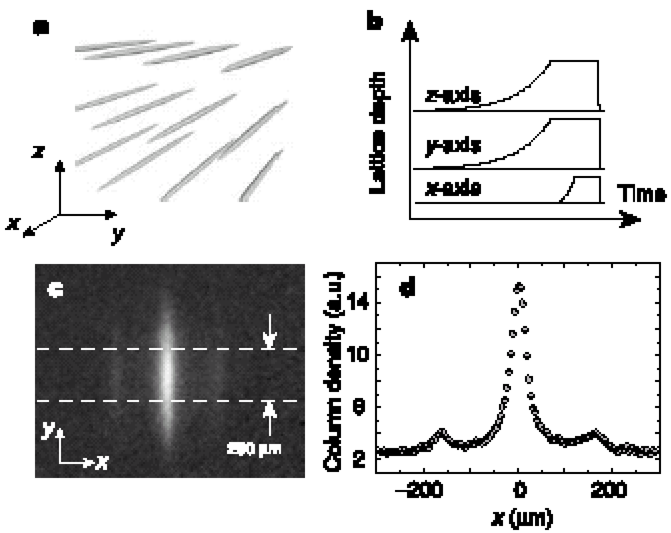
Tonks-gas – Experiments

letters to nature

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

MPQ Garching



up to $\gamma_{\mathrm{eff}} \approx 200$

other experiments:

T. Esslinger (Zürich)

W. Phillips (NIST)

D. Weiss (PSU), $\gamma \approx 5.5$

M. Köhl (Cambridge), $\gamma \approx 7$

$$\gamma \approx \frac{\text{interaction energy}}{\text{kinetic energy}}$$

$$\gamma \simeq \frac{m}{\hbar}\frac{\omega_\rho}{n_{1\mathrm{D}}} a_{3\mathrm{D}}$$

1D Bose Gas – Lieb-Liniger model

$$H = \sum_i \left[\frac{p_i^2}{2m} + V_{\text{ext}}(x_i)\right] + \frac{\hbar^2}{m} c \sum_{i<j} \delta(x_i - x_j)$$

- 1D Bosons with repulsive δ interactions
- Ground- and excited-state wavefunctions of homogeneous system (V_{ext}=0) are exactly known from Bethe ansatz [Lieb, Liniger 1963]
- Interaction parameter $\gamma = c / n_{1D}$
- Quasicondensate, GP+Bogoliubov for $\gamma \ll 1$
- For $\gamma \to \infty$, problem is mapped exactly to free Fermi gas (Tonks-Girardeau gas) [Girardeau 1960]

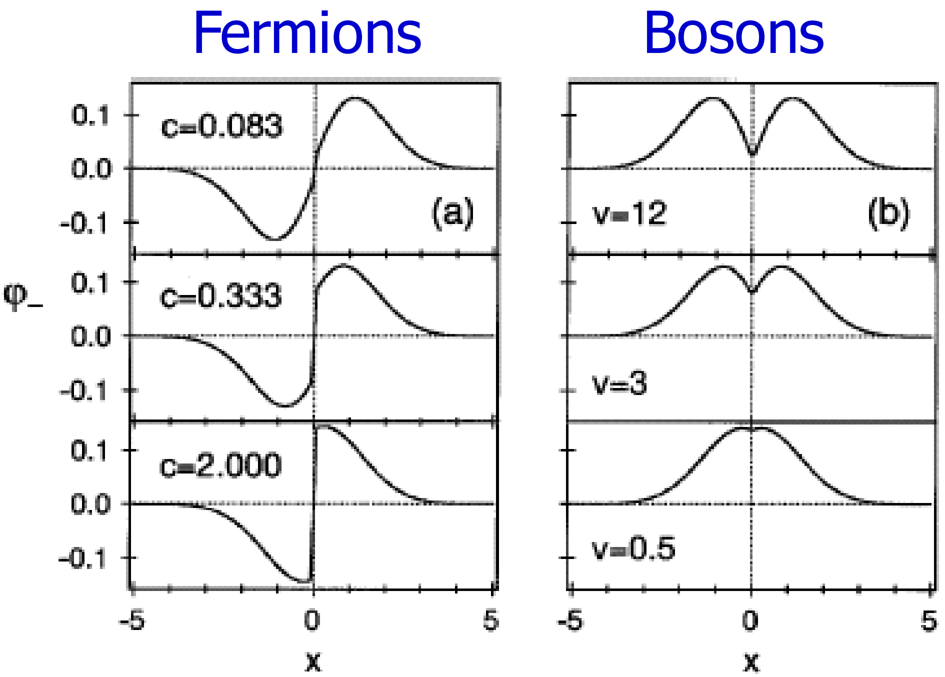
Bose-Fermi mapping

"In 1D, there is no distinction between Bosons and Fermions"

Strong repulsive interactions for bosons have the same effect as the Pauli exlusion principle for fermions.

$$\phi^{B} = |\phi^{F}|$$

Bosons with strong but finite interactions map to spinless (spin-polarized) fermions with weak short-range interactions

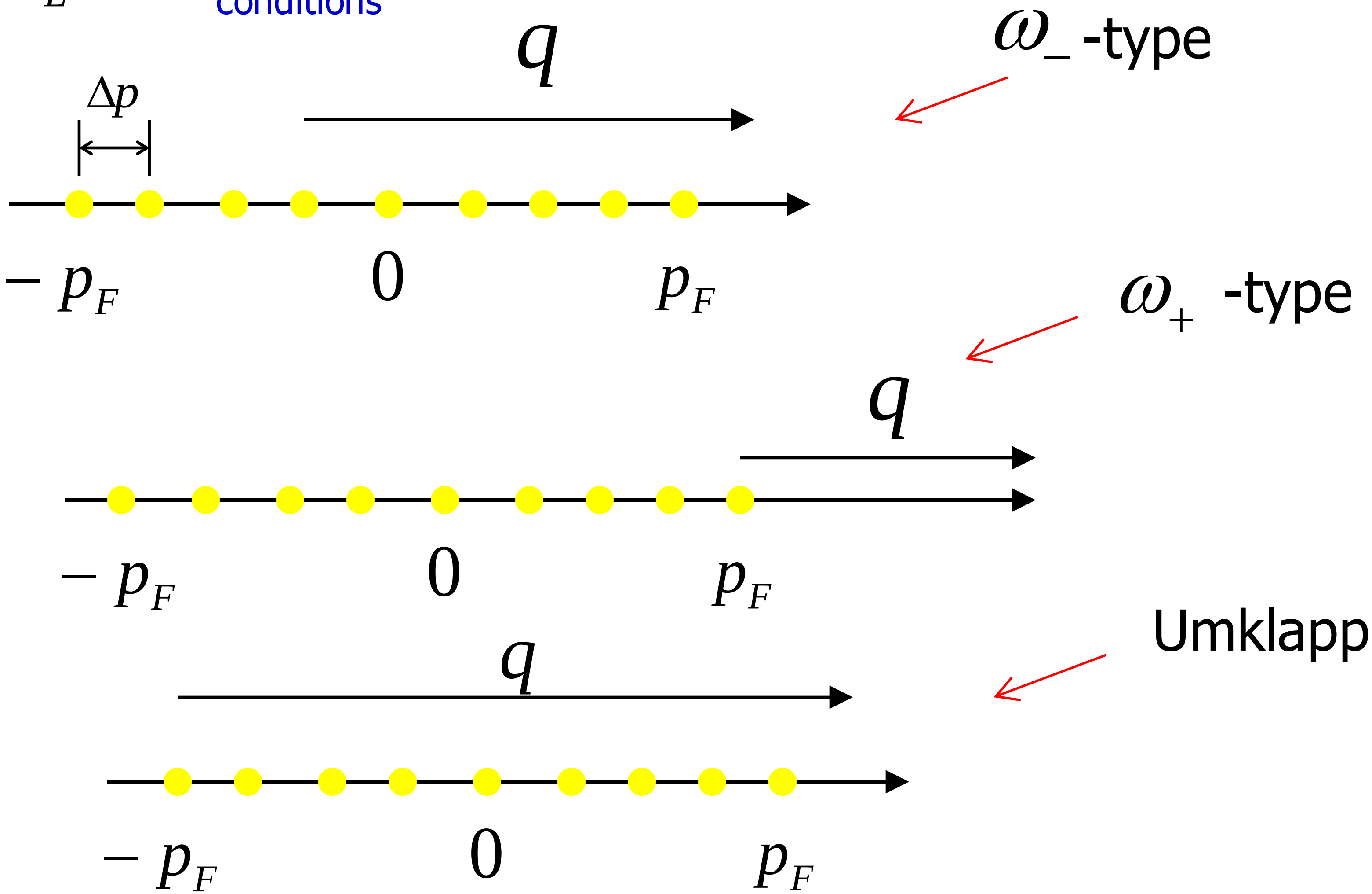


Cheon and Shigehara PRL 1999

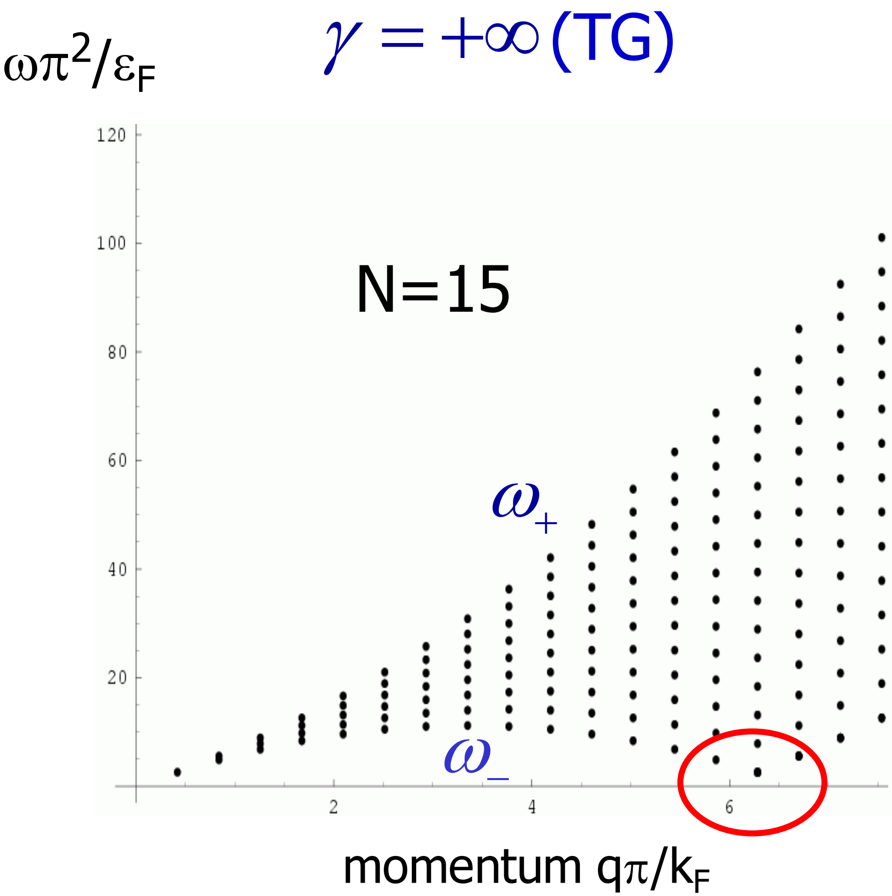
Particle-hole excitations in LL model

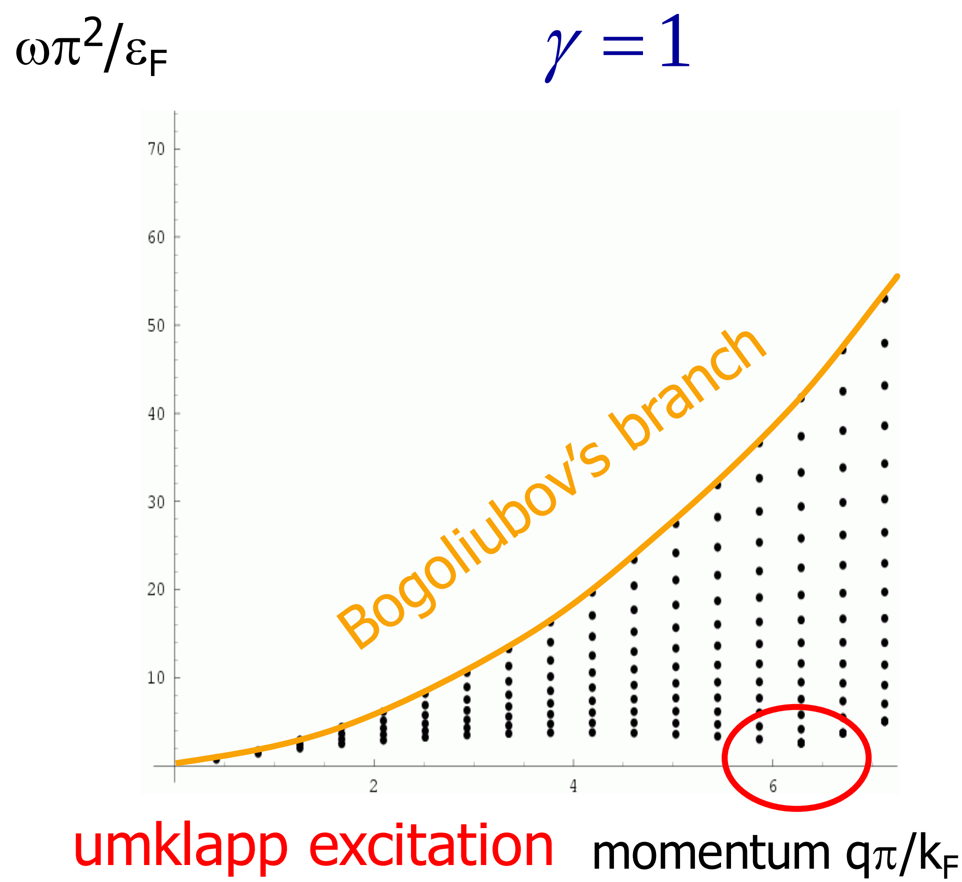
$$\Delta p = \frac{2\pi\hbar}{L}$$

Due to the periodic boundary conditions



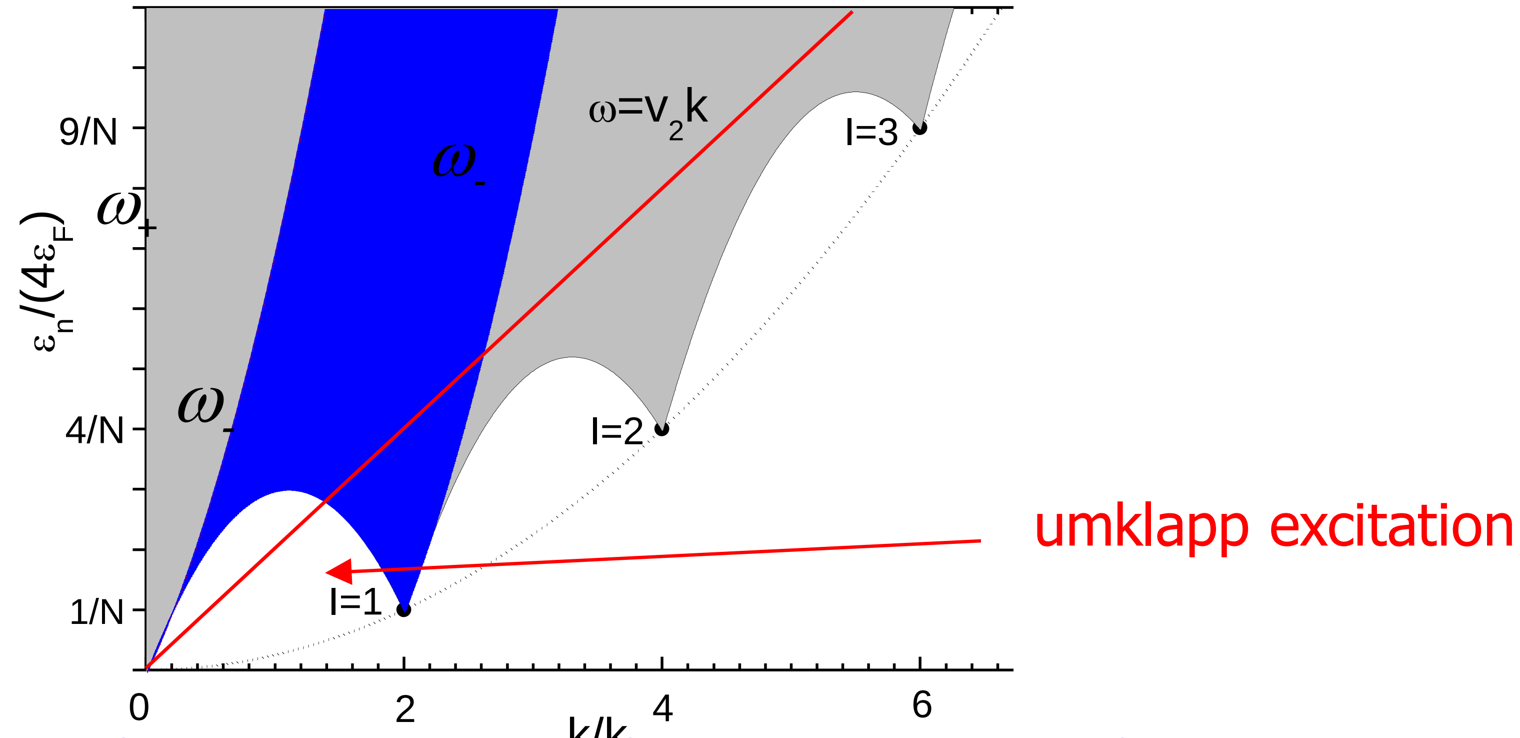
Excitation spectrum for the Lieb-Liniger model





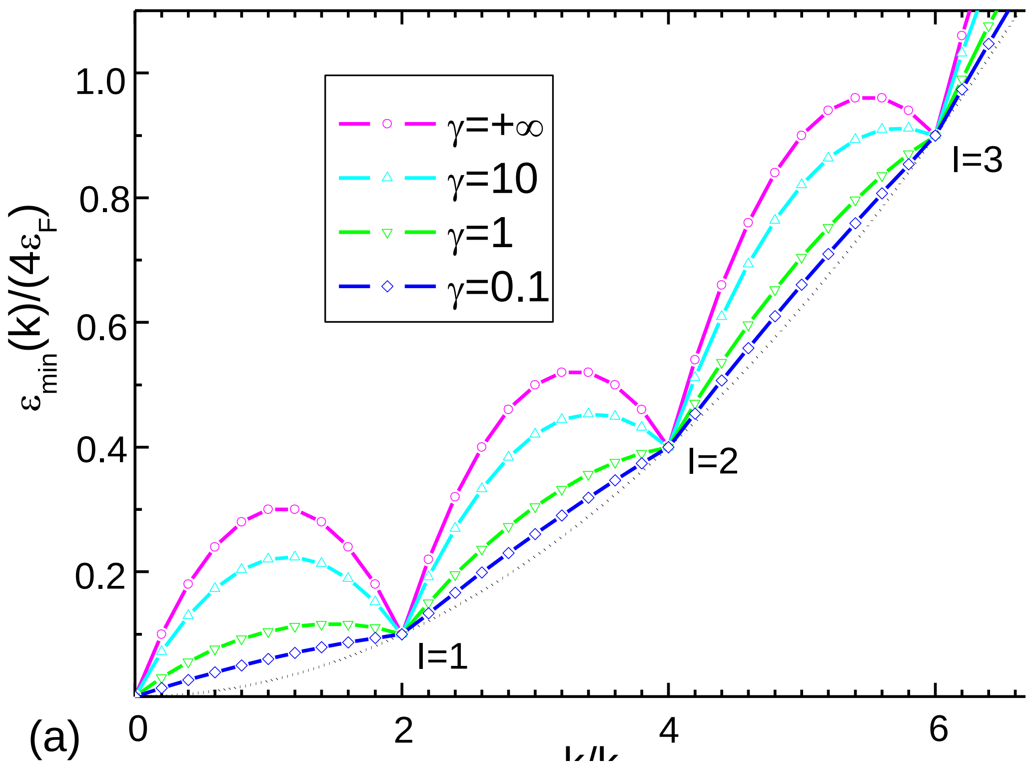
$k_F \equiv \pi n_{1D}$; $\varepsilon_F \equiv k_F^2/(2m)$

Multiparticle excitations in LL model



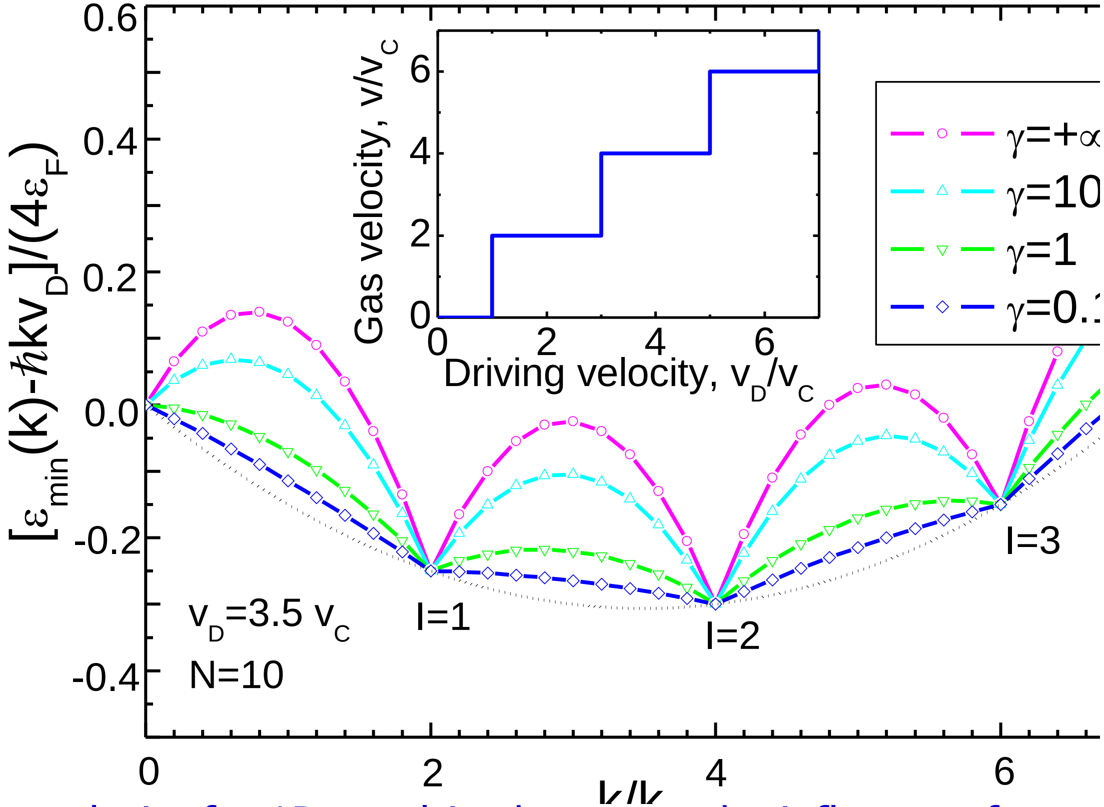
Schematic of the excitation spectrum of the 1D Bose gas in a perfectly isotropic ring. The supercurrent states I lie on the parabola (dotted line). Excitations occur in the shaded area; the discrete structure of the spectrum is not shown for simplicity. The blue area represents particle-hole excitations. Motion of the impurity with respect to the gas causes transitions from the ground state to the states lying on the straight red line.

Low-lying excitations in the Lieb-Liniger model



Low-lying excitation $\varepsilon_{\min}(k)$ spectrum for N = 10 particles for 1D repulsive bosons. At I=1, the low-lying spectrum verges towards that of the ideal Bose gas, which lies on the straight segments between points I = 1; 2; 3; ...

Hess-Fairbank effect



$v_c = \pi\hbar/(Lm)$

Quantization of current velocity for 1D repulsive bosons under influence of a moving trap. Shown are the low-energy excitations of the 1D Bose gas in the moving frame $\varepsilon_{\min}(k) - v_D \hbar k$ calculated from the Bethe-ansatz equations for different values of the coupling strength. Inset: The velocity of the gas at equilibrium changes abruptly at integer values of driving velocity, since the gas occupies the state with lowest energy. In particular, the system is at rest when the driving velocity is less than v_c (Hess-Fairbank effect). Here, $k_F = \pi n$ and $\varepsilon_F = \hbar^2 k^2/(2m)$.

Dinamic Structure Factor (DSF) definition

DSF is defined as the Fourier transform of the density-density correlations

$$S(k,\omega) = L \int \frac{\mathrm{d}t\mathrm{d}x}{2\pi\hbar}\, e^{i(\omega t - kx)} \langle 0|\delta\hat{\rho}(x,t)\delta\hat{\rho}(0,0)|0\rangle$$

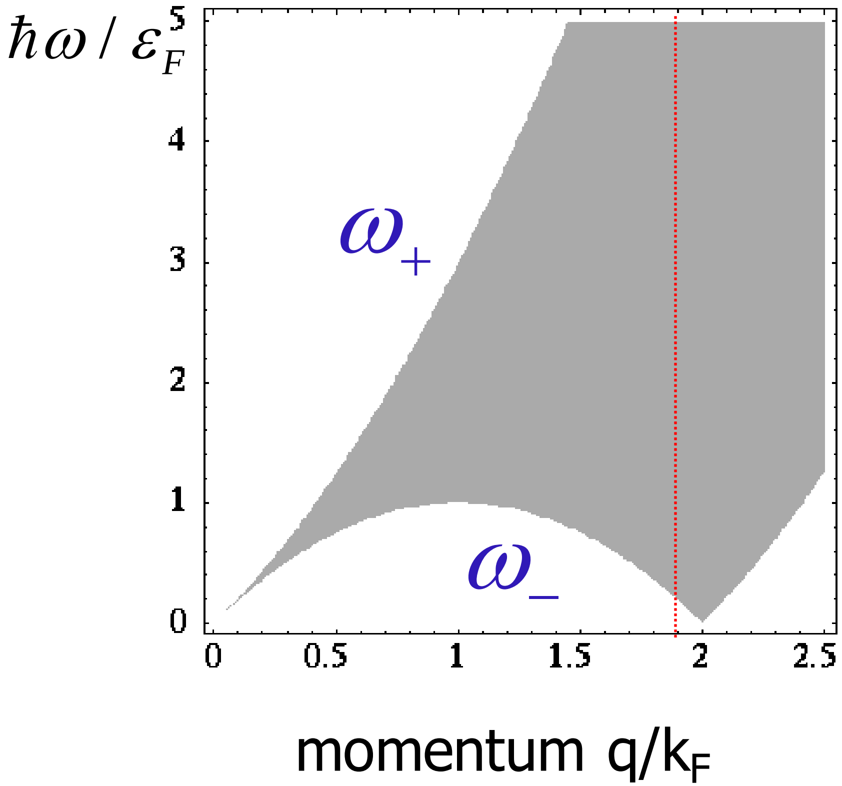
$$\delta\hat{\rho}(x,t) \equiv \hat{\rho}(x,t) - n$$

$$S(k,\omega) = \sum_m |\langle 0|\delta\hat{\rho}_k|m\rangle|^2 \delta(\hbar\omega - E_m + E_0)$$

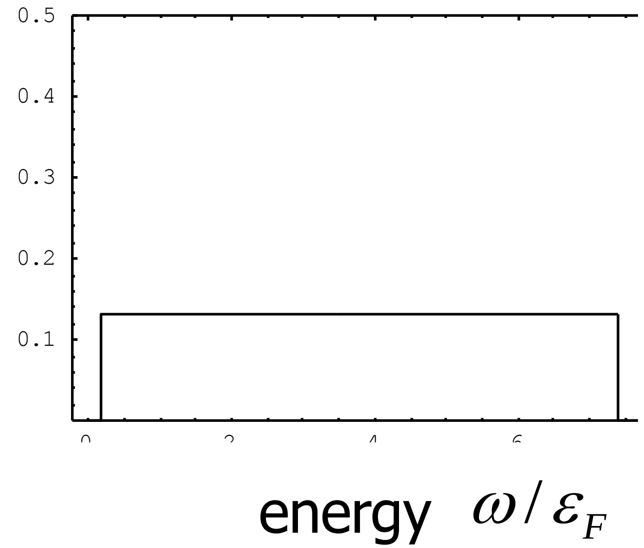
Dynamic structure factor contains information about <u>excitation probabilities</u>

Dynamic Structure Factor for the Tonk's gas $\gamma = +\infty$

$(q/k_F)S(q,\omega)\varepsilon_F/N$



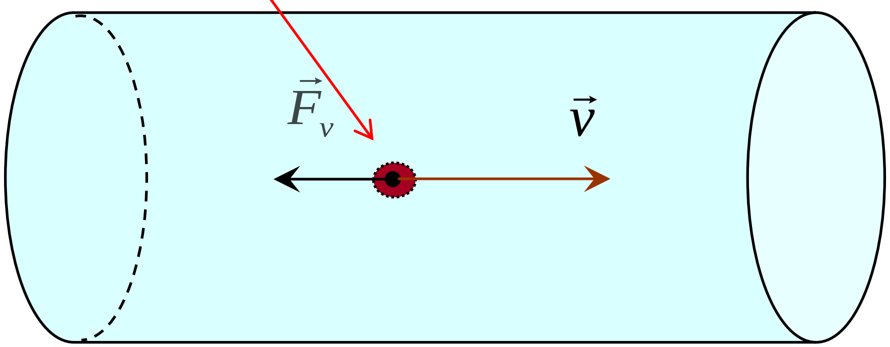
$(q/k_F)S(q,\omega)\varepsilon_F/(2N)$



Drag Force

Consider a heavy impurity, moving with constant velocity in the 1D medium of particles. By definition

$$\dot{E} = -\mathbf{F}_{\mathrm{v}} \cdot \mathbf{v}$$



Drag Force as a generalization of **the Landau criterion for superfluidity:** it should be zero to prevent the energy dissipation!

$$F_v = 0$$

Superfluidity in 1D Bose gas

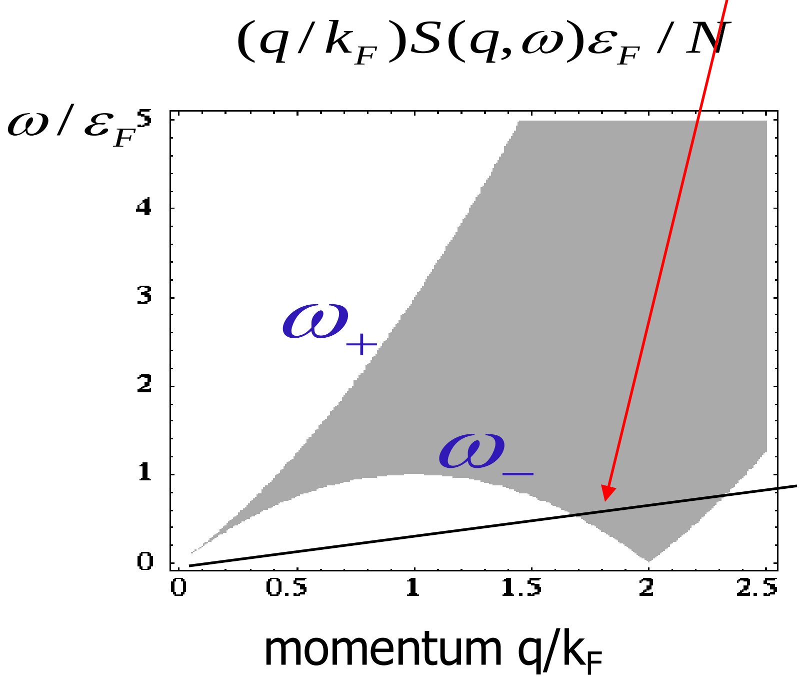
Drag Force

Impurity and particles interaction $V_{\mathrm{i}}(x)$ creates the perturbation $V_{\mathrm{i}}(x-vt)$

The linear response theory yields for the resulting drag force:

$$F_{\mathrm{v}}(v)=\int_0^{+\infty}\mathrm{d}k\,k\,|V_{\mathrm{i}}(k)|^2\,S(k,kv)/L$$

Example: TG gas



Superfluidity in 1D Bose gas

Dynamic Structure Factor in RPA

$$\chi(q,\omega+i\varepsilon) = \frac{\chi^{(0)}(q,\omega+i\varepsilon)}{(1-4\gamma^{-1})[B - D\chi^{(0)}(q,\omega+i\varepsilon)]}$$

$$S(q,\omega) = \frac{-\chi_2^{(0)}(q,\omega)B}{\pi(1-4\,\gamma^{-1})\left[\left(B - D\chi_1^{(0)}\right)^2 + \left(D\chi_2^{(0)}\right)^2\right]}$$

$$+\delta[\omega - \omega_0(q)]A(q)$$

$$B = 1 - 4\,(3\,\gamma - 16)/(\gamma - 4)^3$$

$$D = \frac{4\varepsilon_F}{N\gamma}\left\{\frac{q^2}{k_F^2}\frac{56 - 23\,\gamma + 2\,\gamma^2}{2\,(\gamma-4)^2} - \frac{2\,(\gamma-8)}{(\gamma-4)^2} - \frac{1}{4}\left[\frac{\hbar(\omega+i\varepsilon)\,k_F^2}{\varepsilon_F q}\right]^2 \frac{2\,\gamma^2\,(3\,\gamma-16)}{(\gamma-4)^4}\right\}$$

J. Brand and ACh, PRA **72**, 033619 (2005);

ACh and J. Brand, PRA **73** 023612 (2006)

First-order expansion of DSF

$$\varepsilon_F S(q,\omega) = \frac{k_F}{4q} + \frac{2k_F}{\gamma q} + \frac{1}{2\gamma}\ln f(q,\omega) + \mathcal{O}(\gamma^{-2})$$

$$f(q,\omega) = |(\omega^2 - \omega_-^2)/(\omega_+^2 - \omega^2)|$$

$$\omega_\pm(q) = \frac{\hbar|2k_F q \pm q^2|}{2m^*}; \qquad m^* = \frac{m}{1-4\gamma^{-1}}$$

$$k_F = \pi n_{1D}; \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

J. Brand and ACh, PRA **72**, 033619 (2005);
ACh and J. Brand, PRA **73** 023612 (2006)

Link to the Luttinger Liquid Theory

The Luttiger liquid theory yields for the DSF of spinless repulsive bosons in vicinity of the umklapp excitation $q = 2k_F$ and $\omega = 0$

Castro Neto et al. (1994); Astrakharchik and Pitaevskii (2004)

$$\frac{S(q,\omega)}{N} = \frac{nc}{\hbar\omega^2}\left(\frac{\hbar\omega}{mc^2}\right)^{2K} A(K)\left(1 - \frac{c^2(q-2k_\mathrm{F})}{\omega^2}\right)^{K-1}$$

for $c|q - 2k_\mathrm{F}| \leqslant \omega$ and 0 otherwise

$$K \equiv \hbar\pi n/(mc)$$

$$1 \leq K < \infty$$

TG Bogoliubov

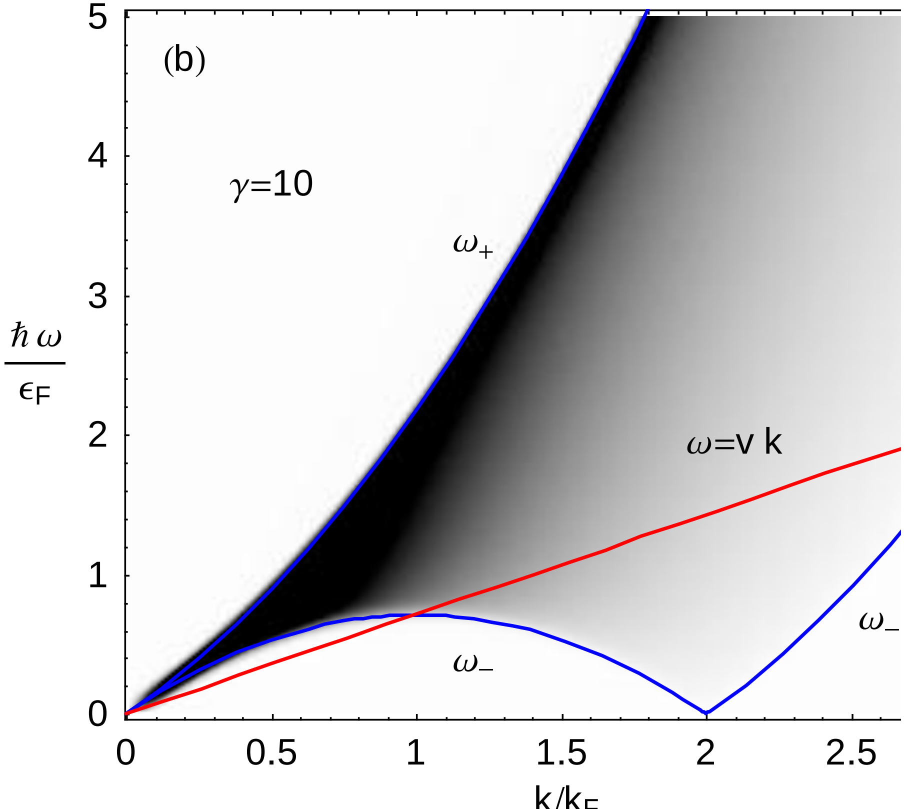
Model-dependent prefactor A(K)

$$A(K) = \pi / 4$$

$$A(K) \simeq 8^{1-2K}\exp(-2K\gamma_c)\pi^2/\Gamma^2(K)$$

First order expansion → $A(K) = \frac{\pi}{4}[1 - (1 + 4\ln 2)(K-1)] + O\left((K-1)^2\right)$

DSF with ABACUS



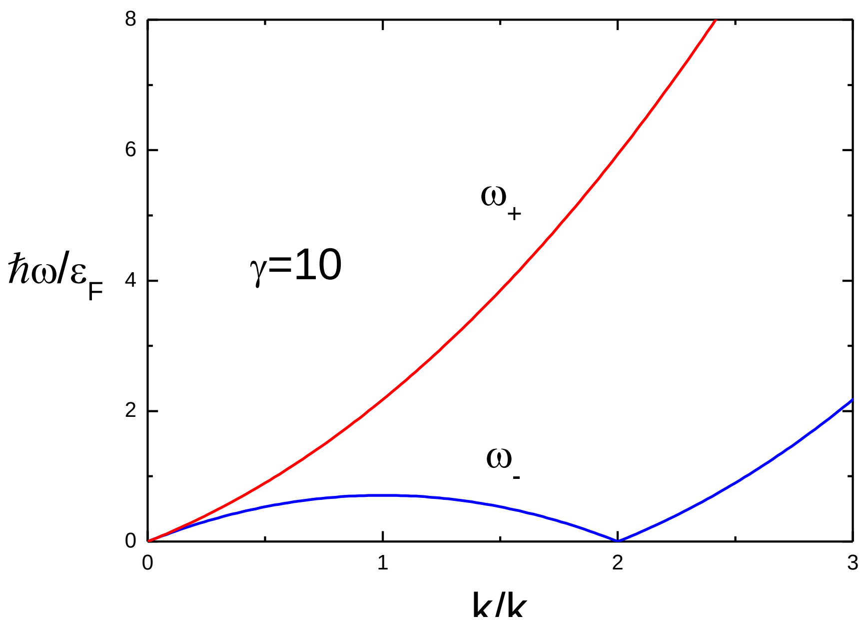
Caux and Calabrese, PRA **74**, 031605 (2006)

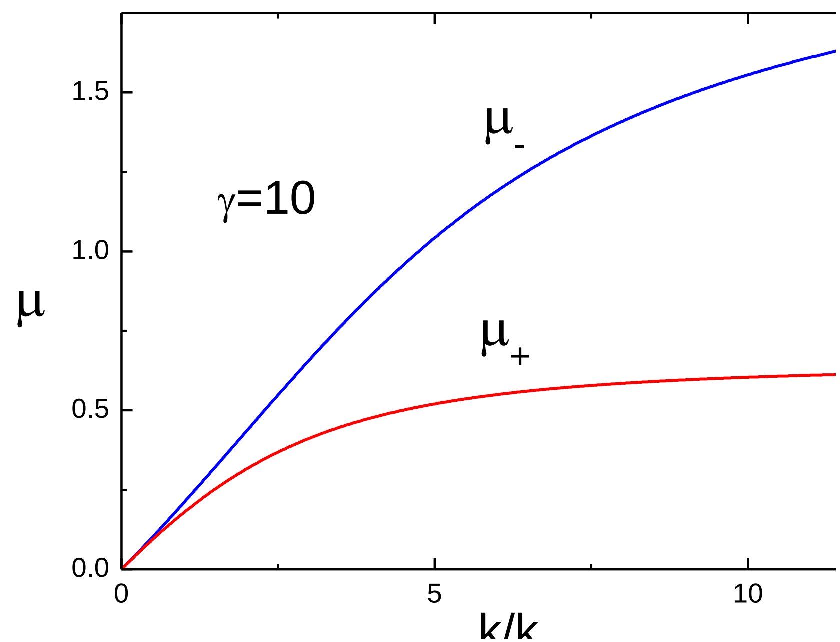
DSF of the 1D Bose gas for $N = 100$. Dimensionless values of $S(k,\ \omega)\varepsilon_F\ /\ N$ are shown in shades of gray between 0 (white) and 0.7 (black). The full (blue) lines represent the limiting dispersion relations and the straight (red) line is the line of integration in equation for the drag force.

The edge exponents

Within the Lieb-Liniger model the DSF exhibits the following powerlaw behavior near the borders of the spectrum $\omega_\pm(k)$

$$S(k,\omega) \sim \left|\omega - \omega_\pm(k)\right|^{\mp\mu_\pm(k)}$$





$$\lim_{k\to 2\pi n^-} \mu_-(k) = 2\sqrt{K}(\sqrt{K}-1)$$

$$K \equiv \hbar\pi n/(mc)$$

Imambekov and Glazman, PRL **100**, 206805 (2008); Science **323**, 228 (2009)

Interpolation formula for DSF

$$S(k,\omega) = C\frac{(\omega^\alpha - \omega_-^\alpha)^{\mu_-}}{(\omega_+^\alpha - \omega^\alpha)^{\mu_+}}$$

$$\alpha \equiv 1 + 1/\sqrt{K}.$$

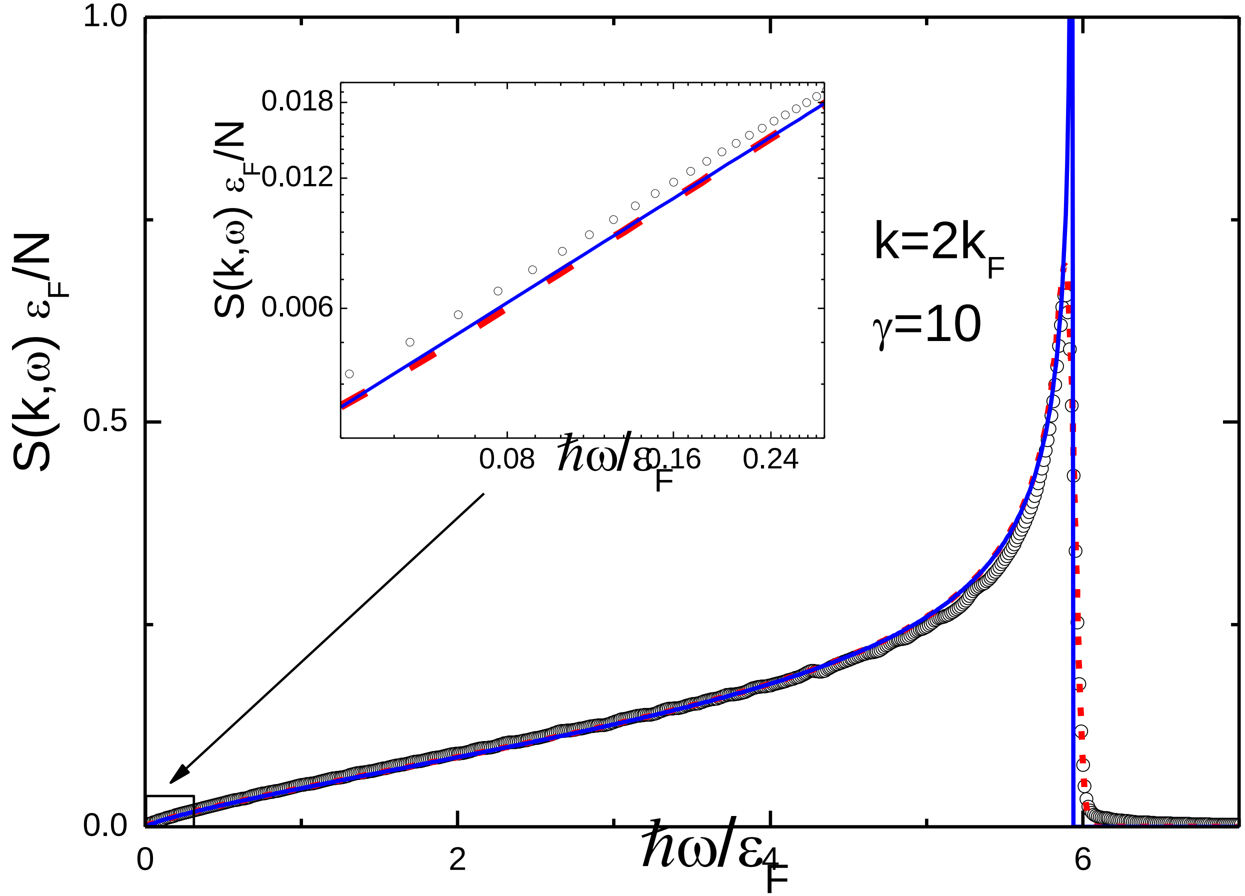
for $\omega_-(k) \leqslant \omega \leqslant \omega_+(k)$, and zero otherwise.

f-sum rule

$$\int_0^{+\infty} \mathrm{d}\omega\, \omega S(k,\omega) = N\frac{k^2}{2m}$$

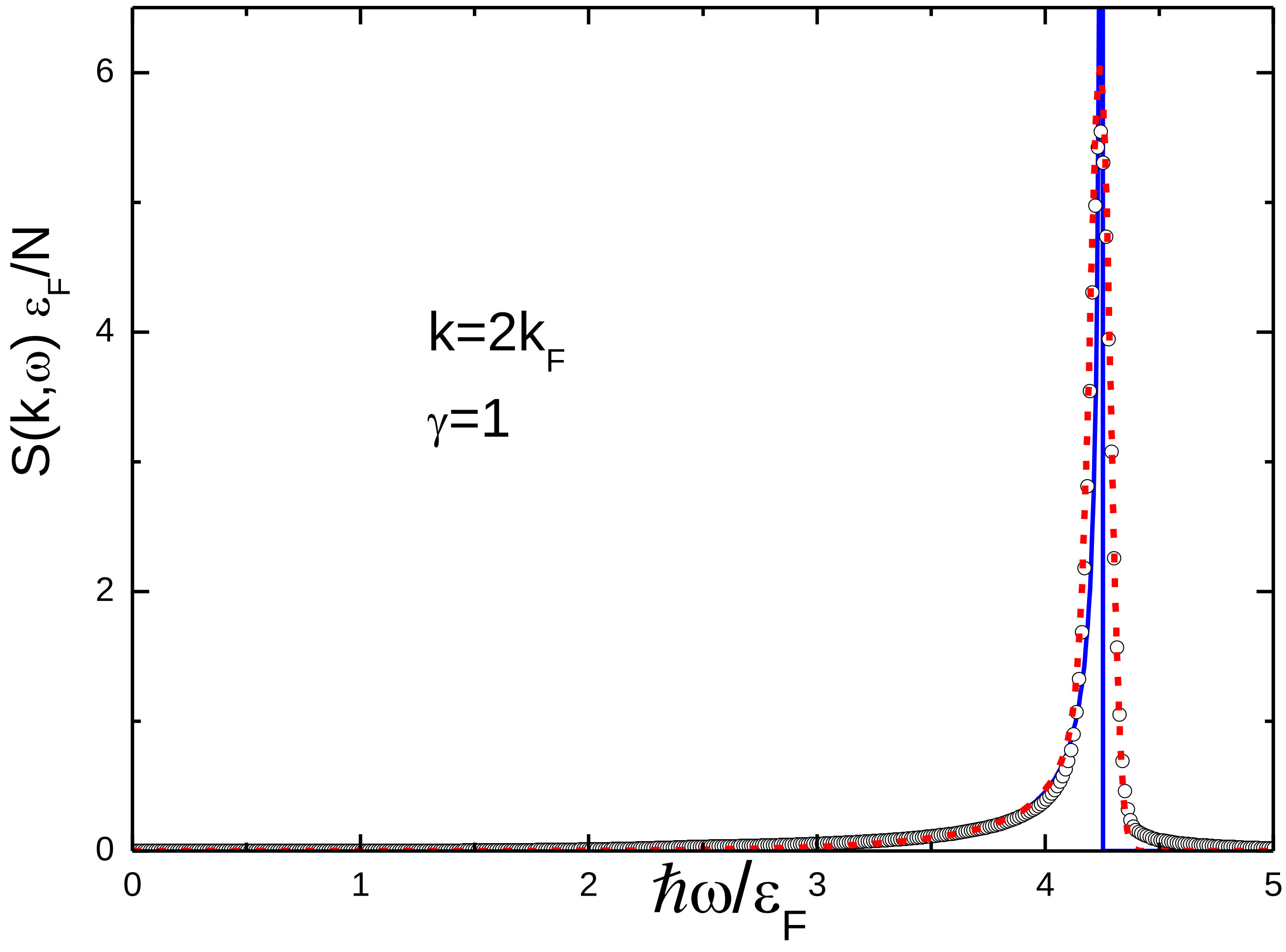
ACh and J. Brand, PRA **79**, 043607 (2009)

Interpolation formula for DSF and ABACUS

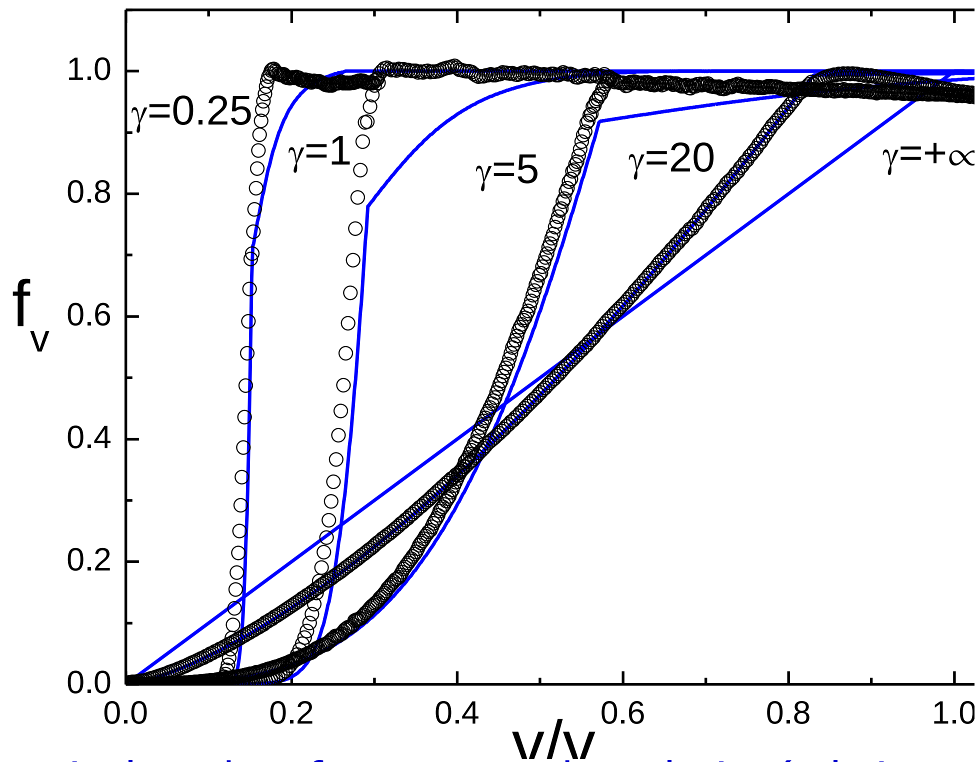


The proposed approximation (blue line) is compared to numerical data from Caux and Calabrese (open dots). The dashed red line shows the data convoluted in frequency with a Gaussian of width $\sigma = 0.042\varepsilon_F / \hbar$ in order to simulate smearing that was used in generating the numerical results by Caux and Calabrese.

Interpolation formula for DSF and ABACUS



Dimensionless drag force



Impurity's point interaction

$$V_i(x) = g_i \delta(x)$$

$$f_v \equiv \frac{F_v \pi \varepsilon_F}{g_i^2 k_F^3}$$

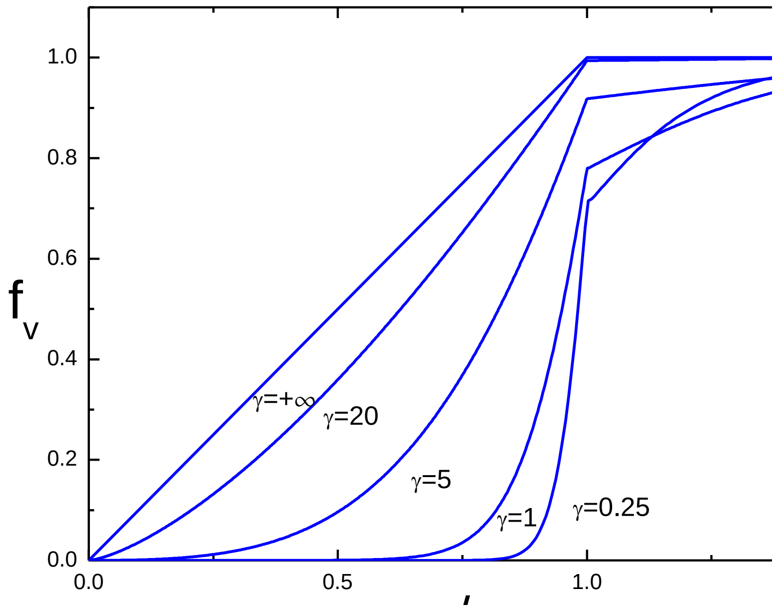
The dimensionless drag force versus the velocity (relative to $v_F = \hbar \pi n / m$) of the impurity at various values of the coupling parameter. The solid (blue) lines represent the force obtained with the approximation formula (open circles) are the numerical data obtained using ABACUS.

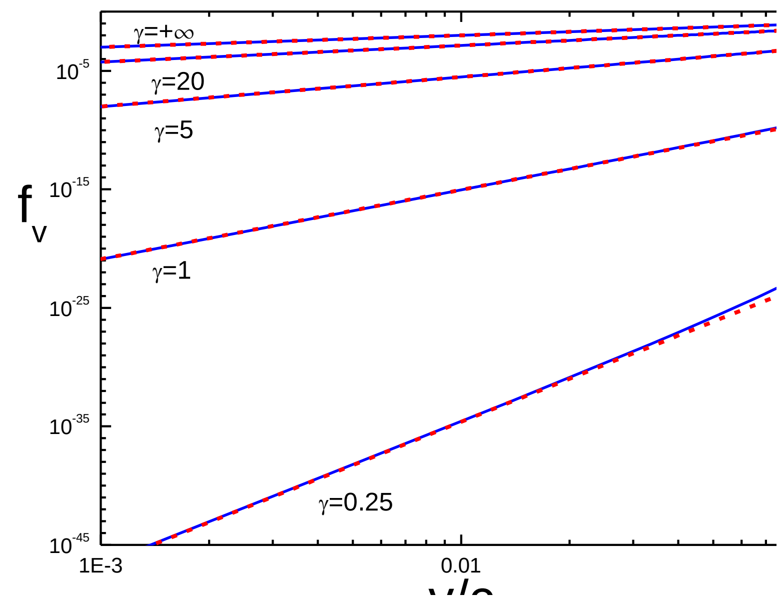
Dimensionless drag force

Impurity's point interaction

$$V_{\mathrm{i}}(x) = g_{\mathrm{i}}\delta(x)$$

$$f_{\mathrm{v}} \simeq \mathrm{const}\, v^{2K-1}, \quad \mathrm{const}\ \text{- ?}$$



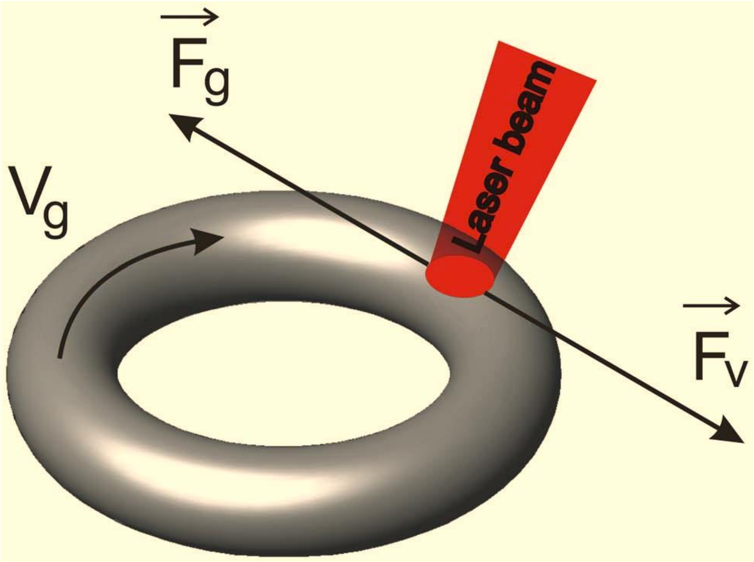


$$f_{\mathrm{v}} \equiv \frac{F_{\mathrm{v}}\pi\varepsilon_{\mathrm{F}}}{g_{\mathrm{i}}^2 k_{\mathrm{F}}^3} \simeq 2K\left(\frac{v}{v_{\mathrm{F}}}\right)^{2K-1}\left(\frac{4\varepsilon_{\mathrm{F}}}{\hbar\omega_+(2k_{\mathrm{F}})}\right)^{2K}$$

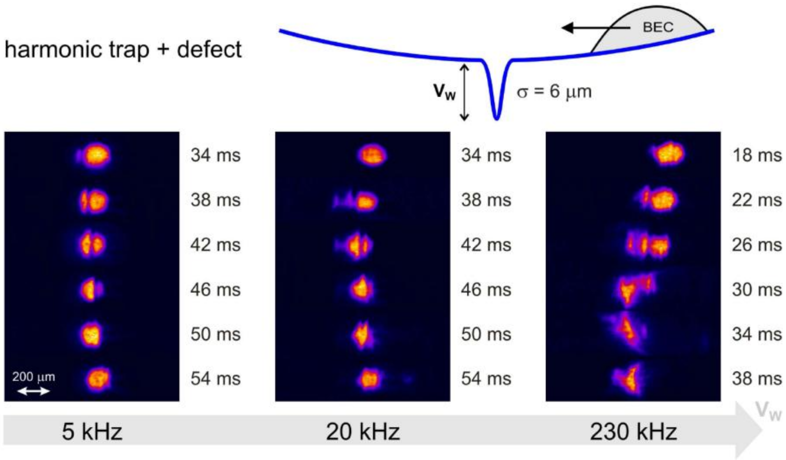
$$\times\frac{\Gamma\left(1+\frac{2K}{\alpha}-\mu_+(2k_{\mathrm{F}})\right)}{\Gamma\left(\frac{2K}{\alpha}\right)\Gamma\left(1-\mu_+(2k_{\mathrm{F}})\right)}\frac{\Gamma\left(1+\mu_-(2k_{\mathrm{F}})\right)\Gamma\left(1+\frac{1}{\alpha}\right)}{\Gamma\left(1+\mu_-(2k_{\mathrm{F}})+\frac{1}{\alpha}\right)}$$

Current decay

1) Consider a ring of 1D Bose gas moving with initial velocity v_{g0}. Under the influence of an obstacle ("impurity") with the effective strength g_i, the gas slows down as

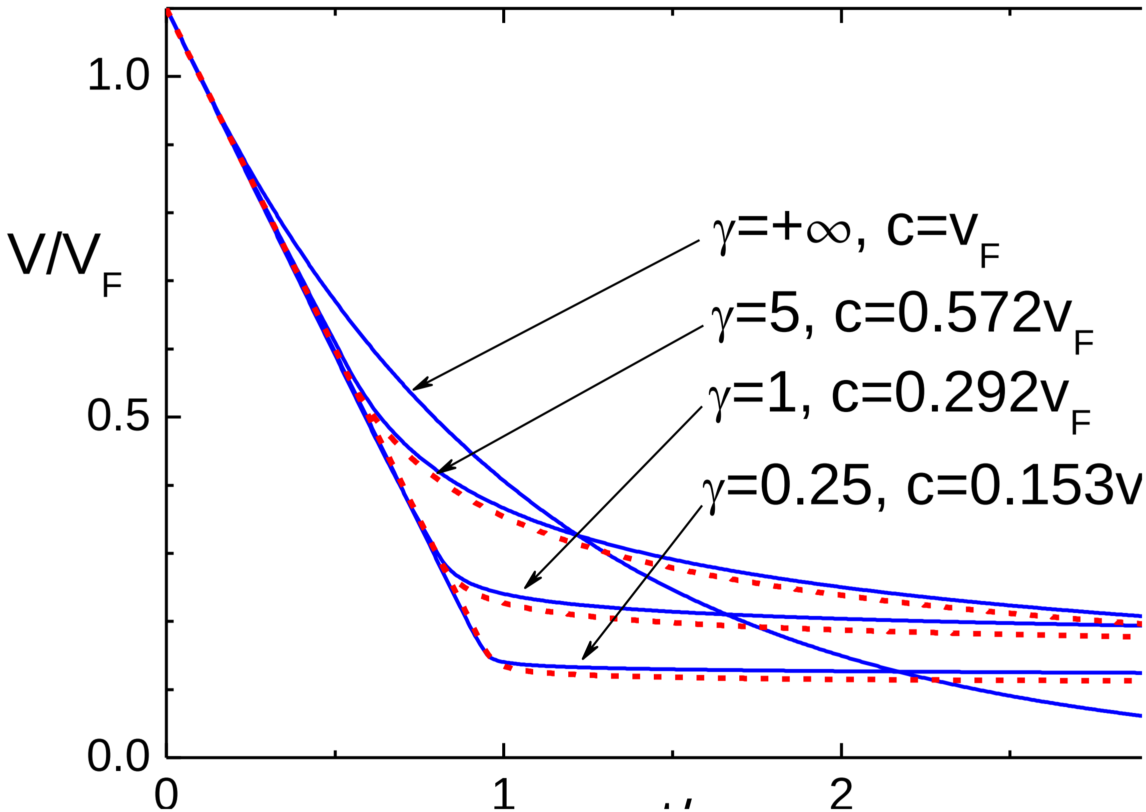


2) By observing damped oscillations of the Bose gas



Superfluidity in 1D Bose gas

Current decay



Velocity damping of the 1D ring of rotating bosons for various values of the coupling parameter

Impurity's point interaction

$$V_{\mathrm{i}}(x) = g_{\mathrm{i}}\delta(x)$$

$$\tau \equiv \frac{N\pi\hbar^3}{2mg_{\mathrm{i}}^2}$$

1D Bosons in moving shallow lattice

$$F_v(v) = \int_0^{+\infty} dk\, k\, |V_i(k)|^2\, S(k,kv)/L$$

An external potential, created by the optical methods laser beams, has only one Fourier componet

$$V_i(x) = g_i \cos(4\pi x/\lambda)$$

$k_G \equiv 4\pi/\lambda$ - reciprocal lattice vector

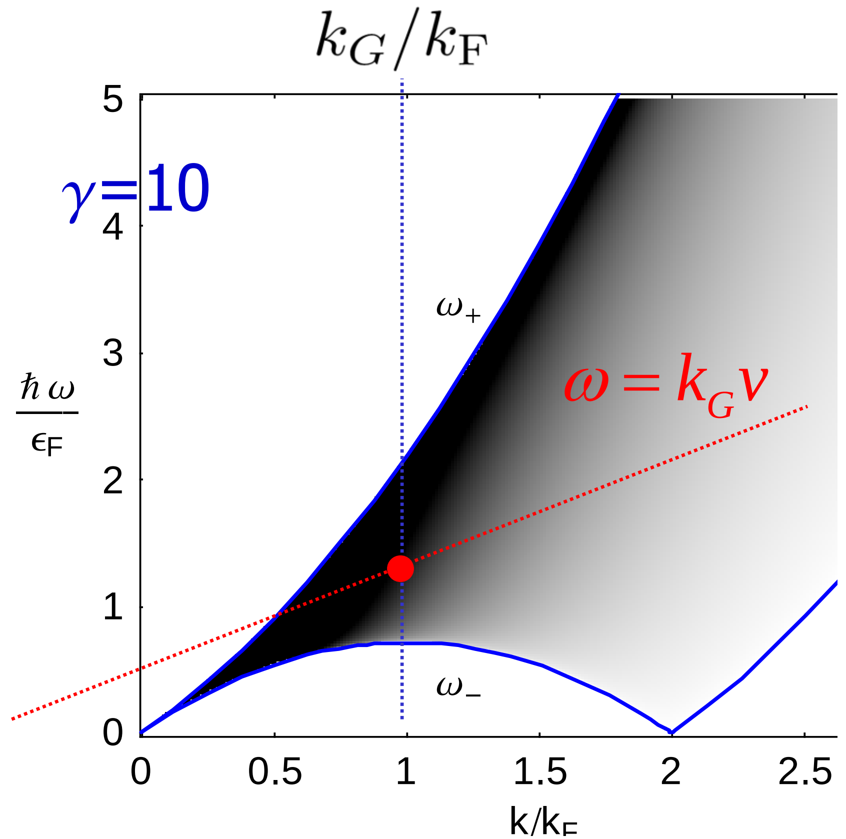
$$F_v = g_i^2 k_G S(k_G, k_G v)/4$$

The filling factor of the lattice:

$$\alpha = 2\pi n/k_G$$

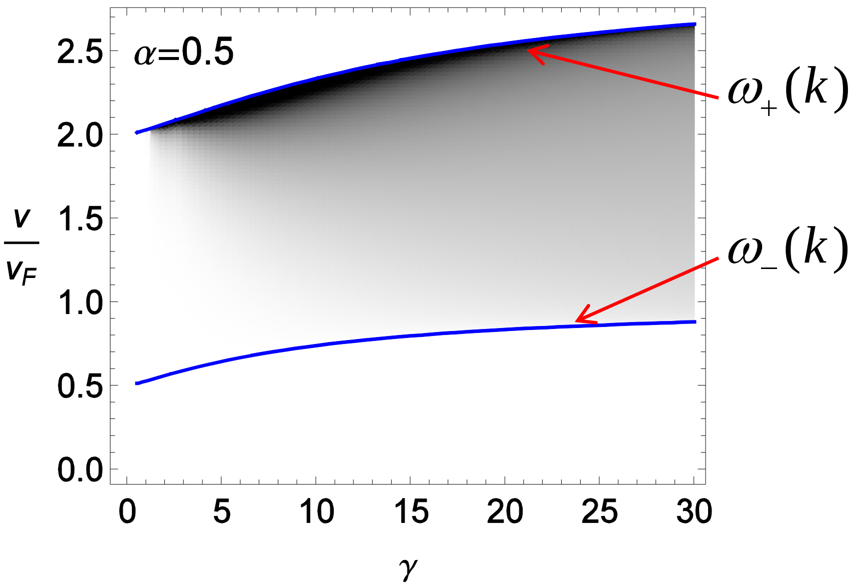
The lattice potential as a perturbation

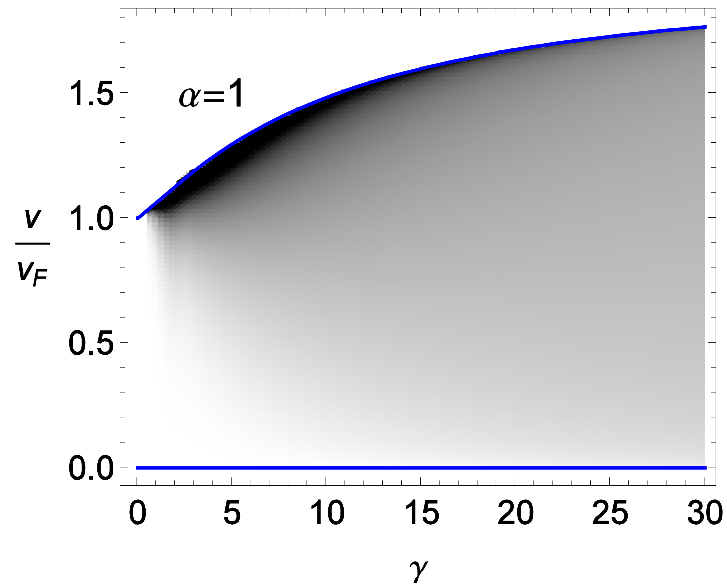
(a shallow lattice)!

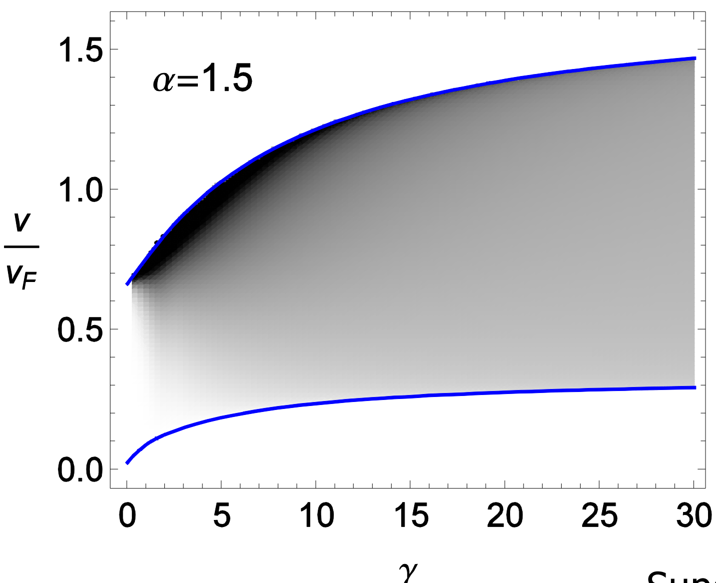


$k_F = \pi n$

Phase diagrams for shallow lattices





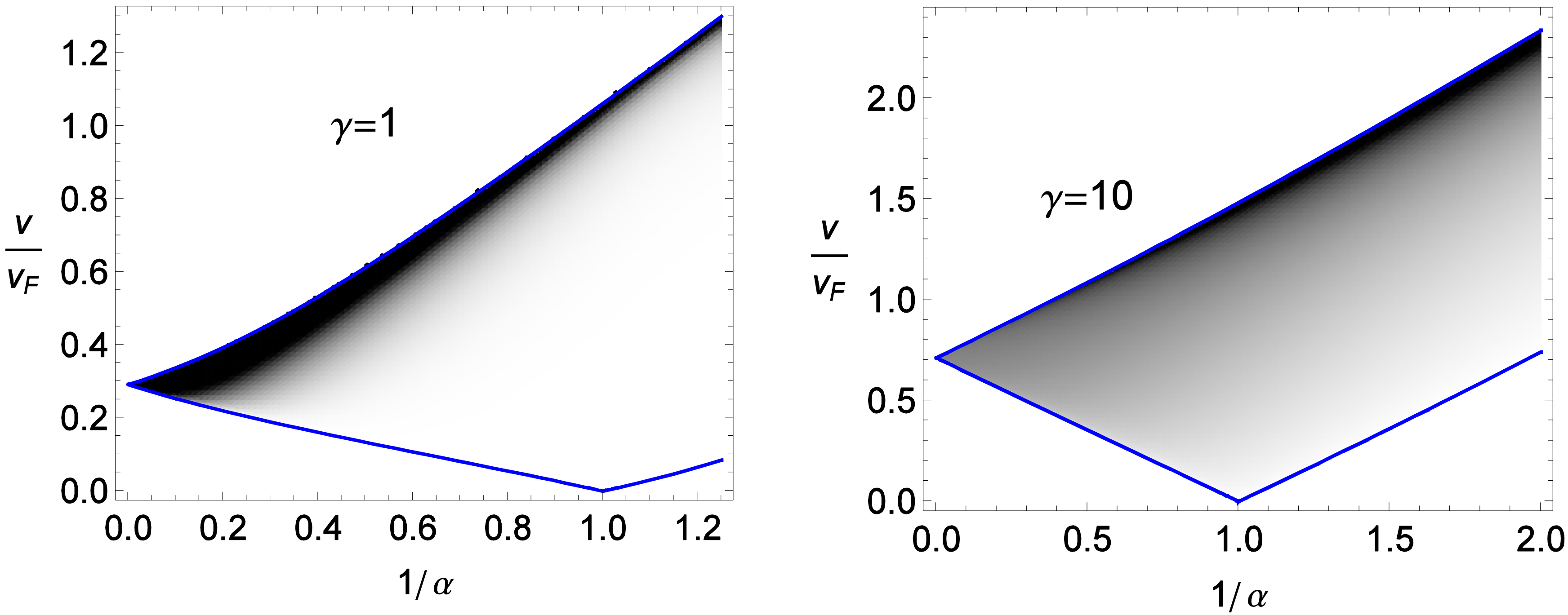


Zero temperature phase diagram for superfluid-isolator transition of the Bose gas in a moving shallow lattice: dimensionless drag force

$$F_{\mathrm{v}} 2\varepsilon_F / (\pi g_{\mathrm{L}}^2 k_F N)$$

versus the lattice velocity (in units v_{F}) and the interaction strength. The dimensionless values are represented in shades of gray between zero (white) and 1.0 (black). The solid (blue) lines correspond to the DSF borders

Phase diagrams for shallow lattices



The same diagram, but here the drag force is represented as a function of velocity and inverse filling factor.

Predictions for superfluidity in 1D

1D Bose gas with the short-range repulsive interaction at zero temperature:

- Hess-Fairbank effect ✓
- Quantized circulation ✓
- Metastability of currens ✗

The last statement means that we have **no qualitative criterion** of metastability in 1D and have to use the quantitative criterion (drag force)

ACh, J.-S. Caux, and J. Brand, PRA **80**, 043604 (2009)