"Энергия Казимира в компактифицированной квантовой электродинамике на решетке"

arXiv:0901.1960

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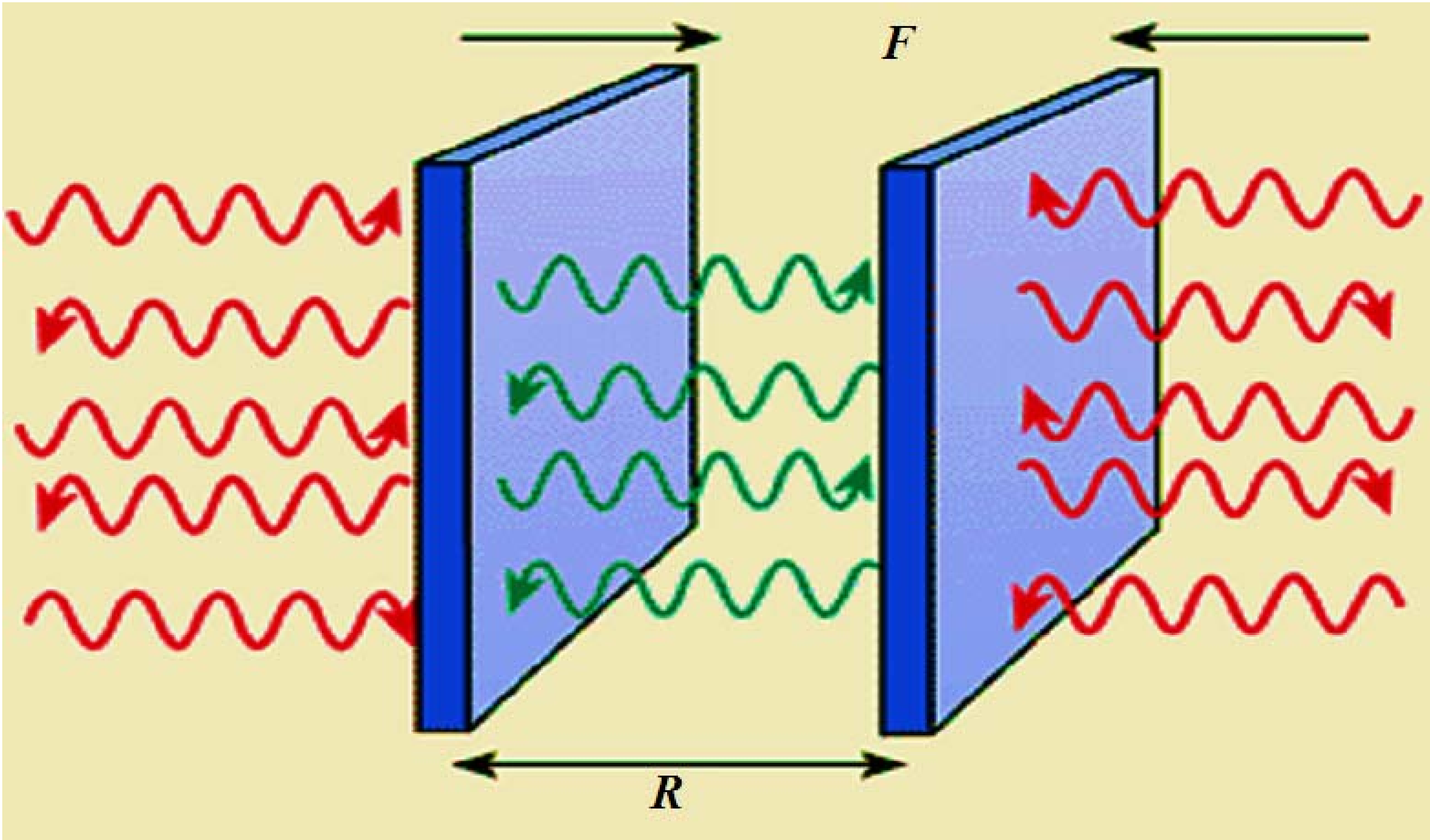
Casimir effect

$$E_{Cas} = -\frac{\pi^2}{720R^3} \quad (\hbar = c = 1)$$

$$E_{Cas} = -\frac{\pi^2 \hbar c}{720R^3}$$



Chern-Simons action and Casimir Effect

M. Bordag and D. V. Vassilevich, Phys. Lett. A 268 (2000) 75.
V. N. Markov and Yu. M. Pis'mak, J. Phys. A 39 (2006) 6525.

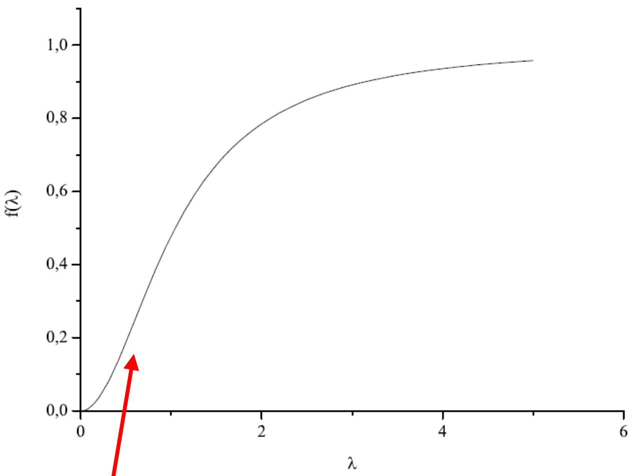
$$S = -\frac{1}{4}\int d^4x\ F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}\oint d^3s\, \varepsilon^{\sigma\mu\nu\rho} n_\sigma A_\mu(x) F_{\nu\rho}(x)$$

$$S_{CS} = \frac{\lambda}{2}\int (\delta(x_3) - \delta(x_3 - R))\varepsilon^{3\mu\nu\rho} A_\mu(x) F_{\nu\rho}(x) d^4x$$

$$\Box A^\mu + \lambda(\delta(x_3) - \delta(x_3 - R))\varepsilon^{3\sigma\nu\rho} A_\sigma \partial_\nu A_\rho = 0.$$

$$E_{\parallel}|_S = 0, \qquad H_n|_S = 0$$

$$E_{Cas} = -\frac{\pi^2}{720R^3} f(\lambda) = -\frac{\lambda^2}{8\pi^2 R^3} + O(\lambda^4)$$



$$f(\lambda) = \frac{90}{\pi^4}\mathrm{Li}_4\left(\frac{\lambda^2}{\lambda^2+1}\right)$$

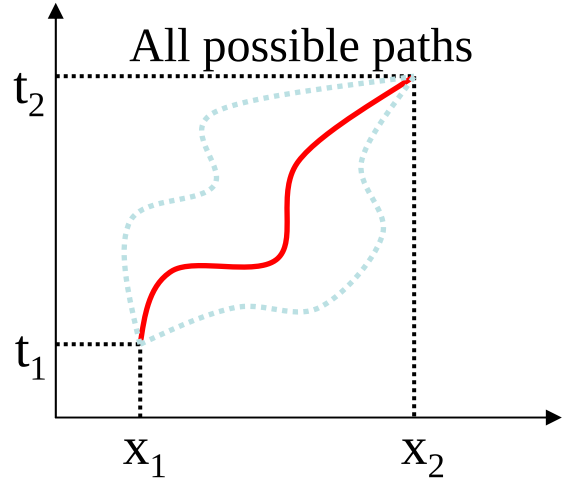
$$\mathrm{Li}_4(x) = \sum_{k=1}^{\infty}\frac{x^k}{k^4} = -\frac{1}{2}\int_0^\infty k^2 \ln(1 - xe^{-k}) dk.$$

Quantum Theory on Computer

FEYNMAN (1948): Quantum Theory is equivalent to Integration

$$\langle x_2 | e^{-iH(t_2-t_1)} | x_1 \rangle = \int Dx\, e^{iS(x)}$$

Action $S = \int_{t_1}^{t_2} dt \{K(x,t) - V(x,t)]\}$



- Integration is **'over the paths'** (infinitely many variables)
- Even the ordinary integration $\int$ is just a symbol representing the limiting procedure

$$\int_a^b f(x)dx \rightarrow \lim_{N\to\infty} \sum_{n=0}^{N-1} f(x_n)\delta; \qquad x_n = a + nb, \ \delta = \frac{b-a}{N}$$

$$\int Dx\, e^{iS(x)}$$

* **Paths are weighted with an oscillating function and so is not suitable for numerical calculation!**

* **Change real time to imaginary time (Minkowski space to Euclidean space)**

$$t \longrightarrow -i\tau,$$

$$< x_f, \tau_f \,|\, x_i, \tau_i > \;=\; \int_{x_i}^{x_f} \mathcal{D}x\, e^{-S_E[x(\tau)]}$$

What is Monte-Carlo Method ?

The Monte Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer.

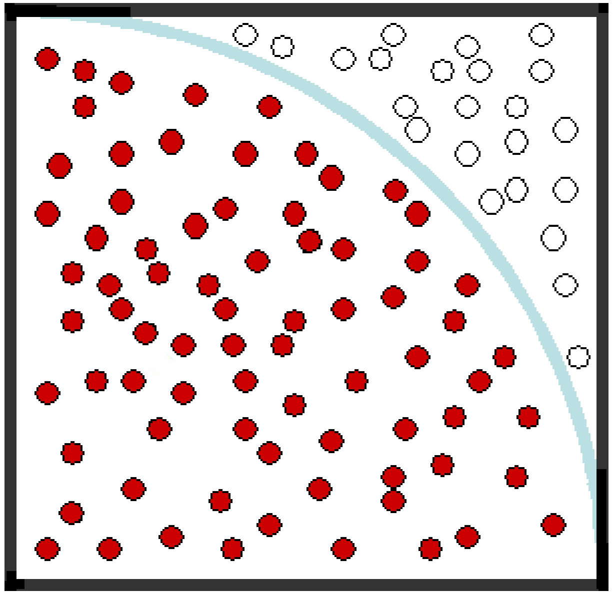
It deals with complex problems ranging from

QCD to economics to regulating the flow of traffic.

Stochastic method for calculating Pi

M : Total number of points

N : Points within circle



Area of a square $S = r^2$

Area of a quarter circle $C = \pi r^2/4$

$\pi = 4C/S$ $= 4N/M$

$$\frac{1}{V}\int p(x) f(\vec{x}) d^n x = \langle f \rangle \pm \sqrt{\frac{\langle f^2 \rangle - \langle f \rangle}{N}}$$

$$\langle f \rangle = \frac{1}{N}\sum_{i=1}^{N} f(\vec{x}_i)$$

Statistical Evaluation — Monte-Carlo Method

Ui distribution

$$I = \int dU O[U] e^{-S_E[U]}$$

Large number of integrations then reduces to an ensemble average:

$$I = \langle O \rangle \approx \frac{1}{N} \sum_{i=1}^{N} O\{U_i\}$$

{Ui}'s are the configurations generated in the stochastic process called Monte-Carlo Method.

Quantum theory and statistical systems

$$I = \langle O \rangle \approx \frac{1}{N} \sum_{i=1}^{N} O\{U_i\}$$

$$\sum_n e^{-\beta E_n}$$

This averaging is similar to a statistical ensemble average, with a Boltzmann distribution given by $e^{-S[U]}$

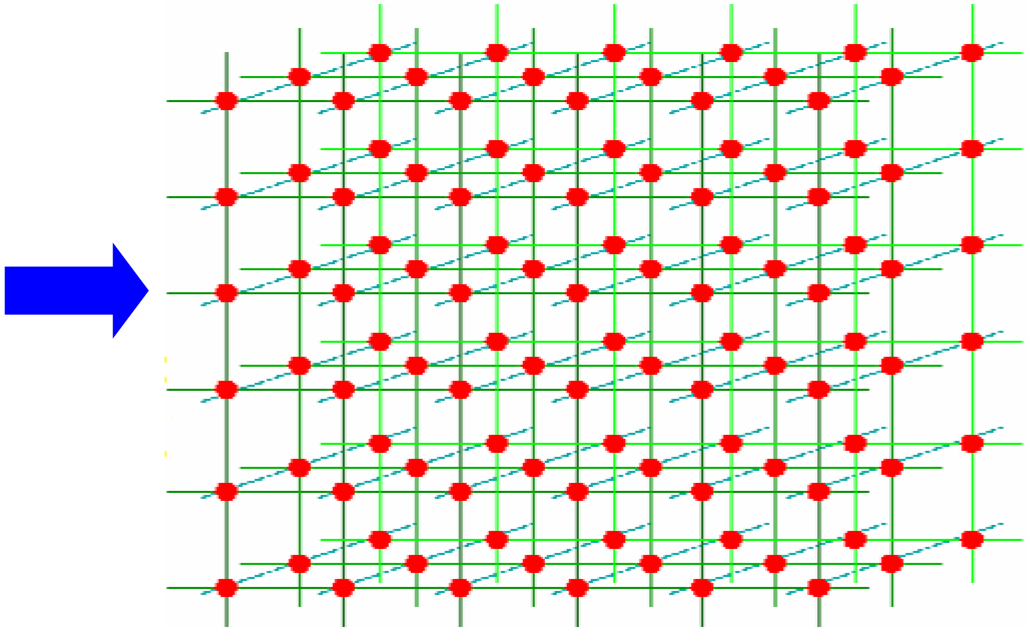
Green function of a Field Theory → ***Correlation function*** of the corresponding Statistical System

Paths → ***Statistical configurations***

From continuous space-time to lattice

Continuous space-time

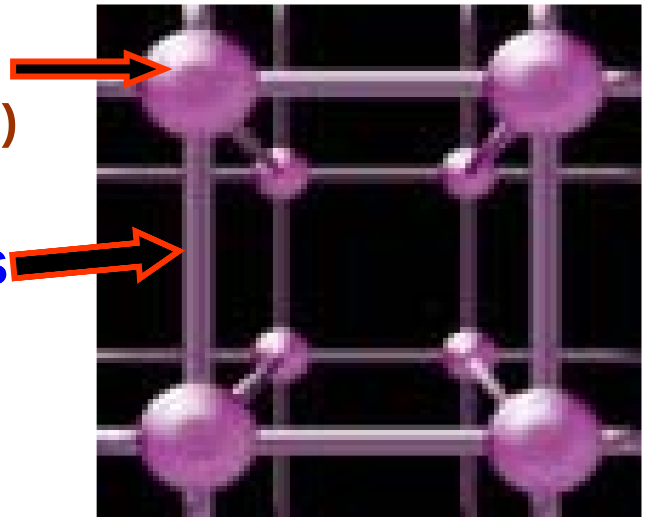
Discrete Euclidean space-time



a

Fermions
(on Lattice sites)

Photons
(on Links)

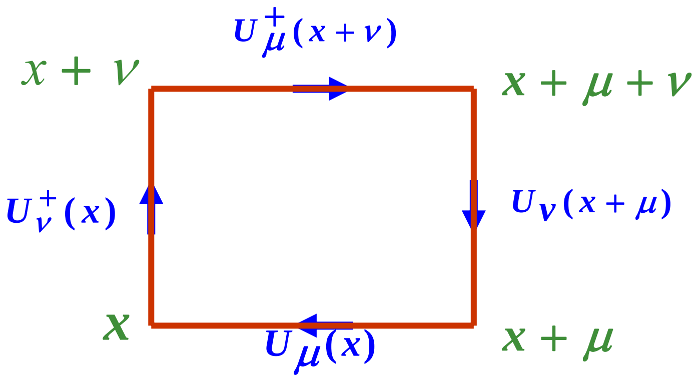


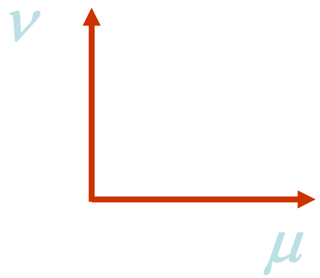
Lattice Formulation of QED

- Preserve the symmetry on the lattice **(QED has gauge symmetry)**
  - Gauge Symmetry —Wilson, 1974 ($A_\mu \rightarrow U_\mu$)

U : Link Variable

$$U_\mu(x) = e^{igaA_\mu}$$





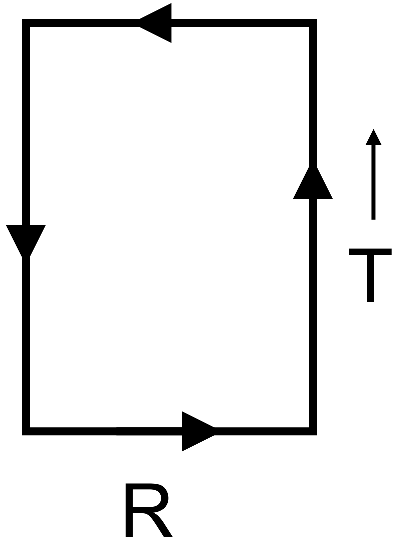
Violation of Lorentz invariance
is controllable and removable

- Wilson gauge action:

$$S_W = \frac{1}{2}\beta\sum_P\left(1-\mathrm{Re}\,trU_p\right), \quad \beta = \frac{1}{g^2} \quad \rightarrow \quad L = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$U_p = U_\mu(x)U_\nu(x+\mu)U^+_\mu(x+\nu)U^+_\nu(x)$$

Wilson loop



$$W_C = e^{ig\oint_C A_\mu dx^\mu} = e^{i\int J_\mu A^\mu dx^4}$$

$$J_\mu(x) = g\oint_C \delta(x-\xi)d\xi_\mu$$

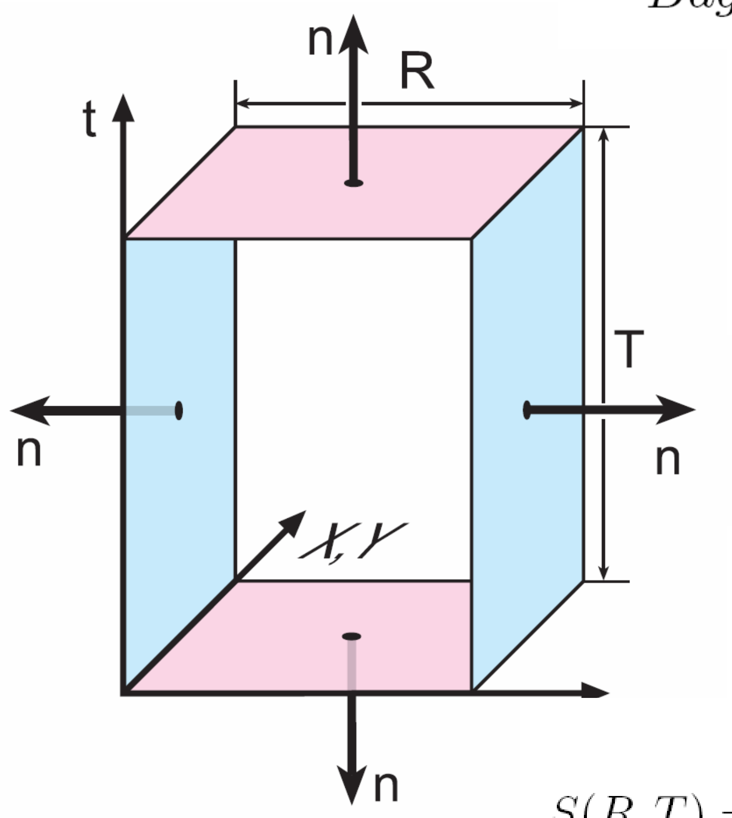
$$\langle W(R,T)\rangle \to Ce^{-V(R)T}$$

$$V(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C(R,T)\rangle \approx C_1 + \frac{C_2}{R}$$

Wilson "bag"

$$W_{Bag}(R,T) = e^{i\lambda S(R,T)}$$

$$= e^{i\lambda \oint\limits_{\Sigma} \varepsilon_{\mu\nu\rho\sigma} A^{\nu} F^{\rho\sigma} dS^{\mu}}$$



$$\langle W_{Bag}(R,T)\rangle \underset{T\to\infty}{\longrightarrow} Ce^{-E_{cas}(R)T}$$

$$S(R,T) = \int\limits_0^T dt \iiint dx dy dz (\delta(z-R) - \delta(z)) \varepsilon_{3\nu\rho\sigma} A^{\nu} F^{\rho\sigma} +$$

$$+ \int\limits_0^R dz \iiint dx dy dt (\delta(t-T) - \delta(t)) \varepsilon_{0\nu\rho\sigma} A^{\nu} F^{\rho\sigma}.$$

Chern-Simons action on the lattice

$$U_{l,x,\mu} \approx 1 + igA_\mu(x)a - \frac{1}{2}g^2a^2A_\mu^2 + O(g^3)$$

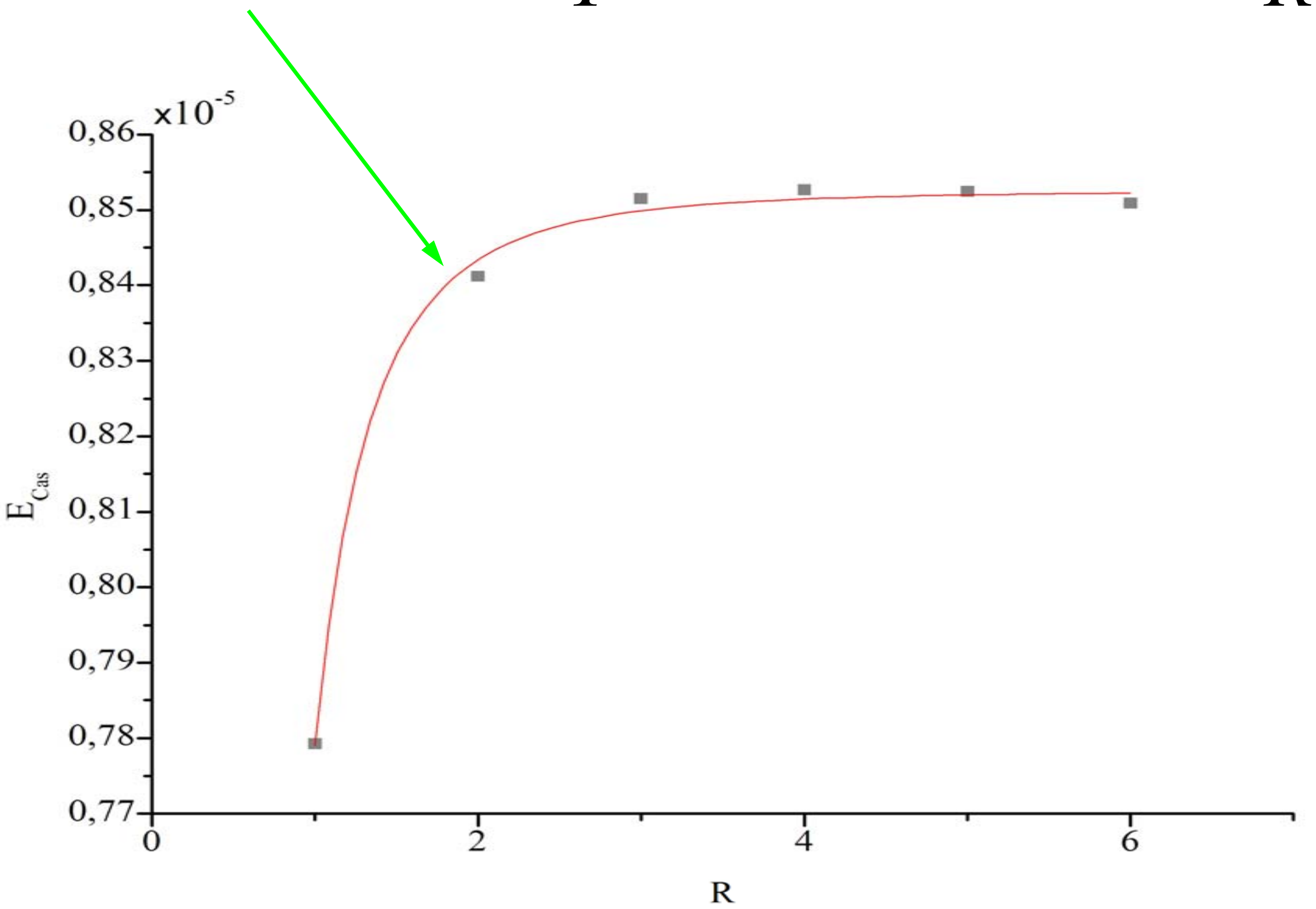
$$(1 - \mathrm{Re}\,(U_{p,x,\mu\nu}U_{l,x,\rho})) - (1 - \mathrm{Re}\,U_{p,x,\mu\nu}) - (1 - \mathrm{Re}\,U_{l,x,\rho})$$

$$U_{p,x,\mu\nu} \approx 1 + igF_{\mu\nu}(x)a^2 - \frac{1}{2}g^2a^4F_{\mu\nu}^2 + O(g^3)$$

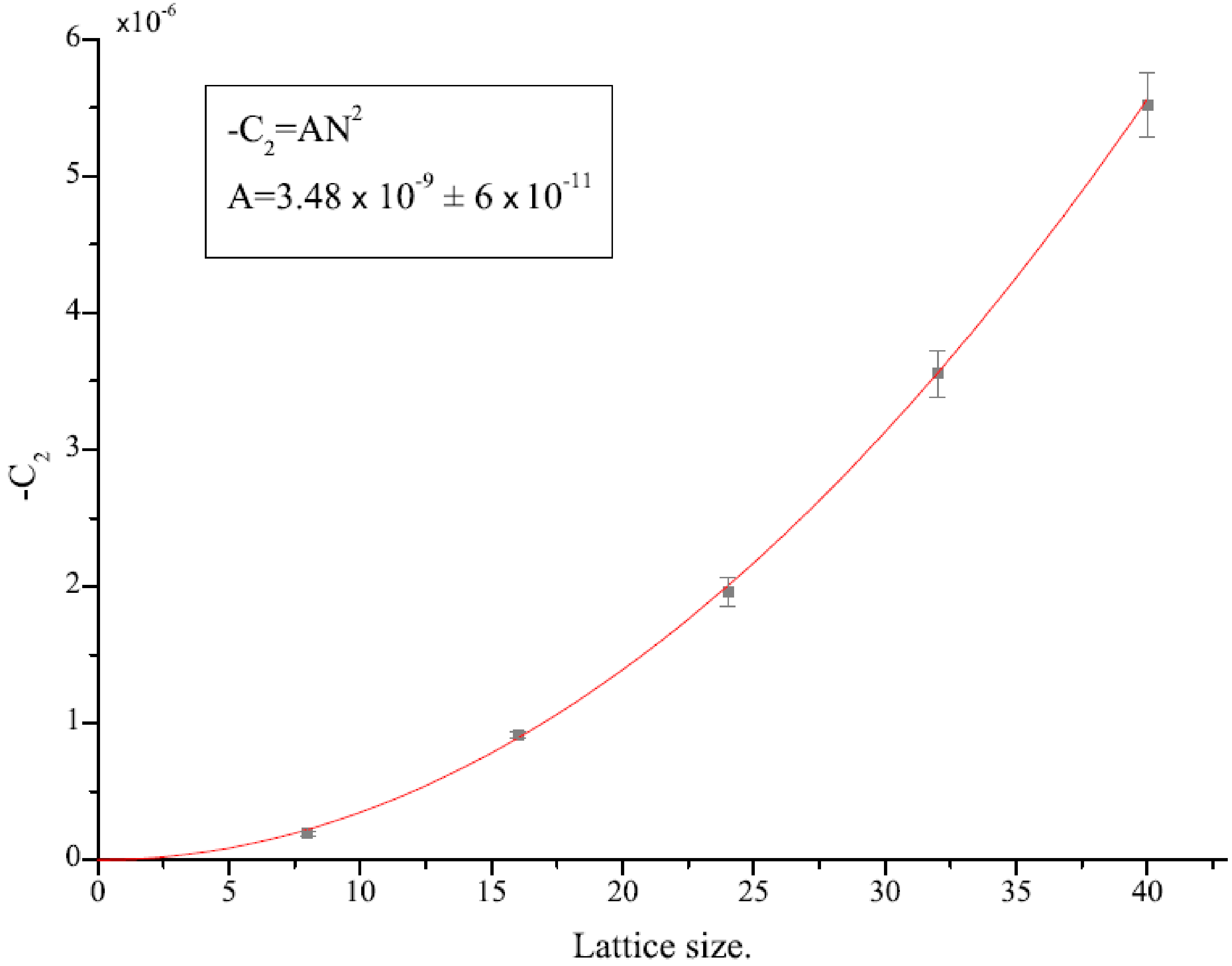
$$g^2a^3F_{\mu\nu}A_\rho + O(g^4)$$

$$\langle W_{Bag}(R,T)\rangle \longrightarrow Ce^{-E_{cas}(R)T}$$

$$E_{cas}(R) = -\lim_{T\to\infty}\frac{1}{T}\langle W_{Bag}(R,T)\rangle \approx C_1 + \frac{C_2}{R^3}$$

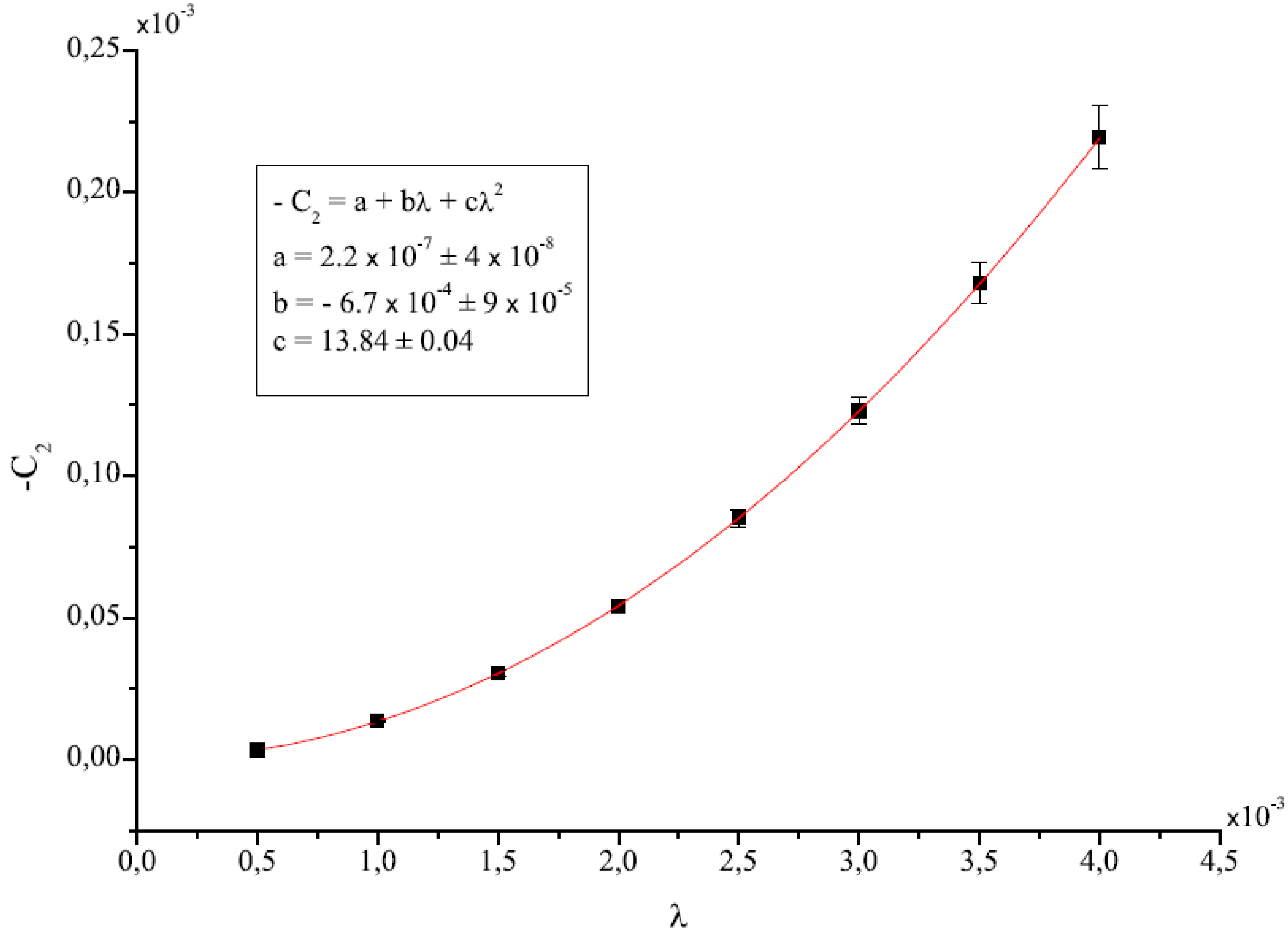


$$E_{Cas.phys} = \frac{1}{a}\frac{C_2}{R^3}\frac{1}{(aN)^2} = \frac{C_2 N^{-2}}{(Ra)^3}$$



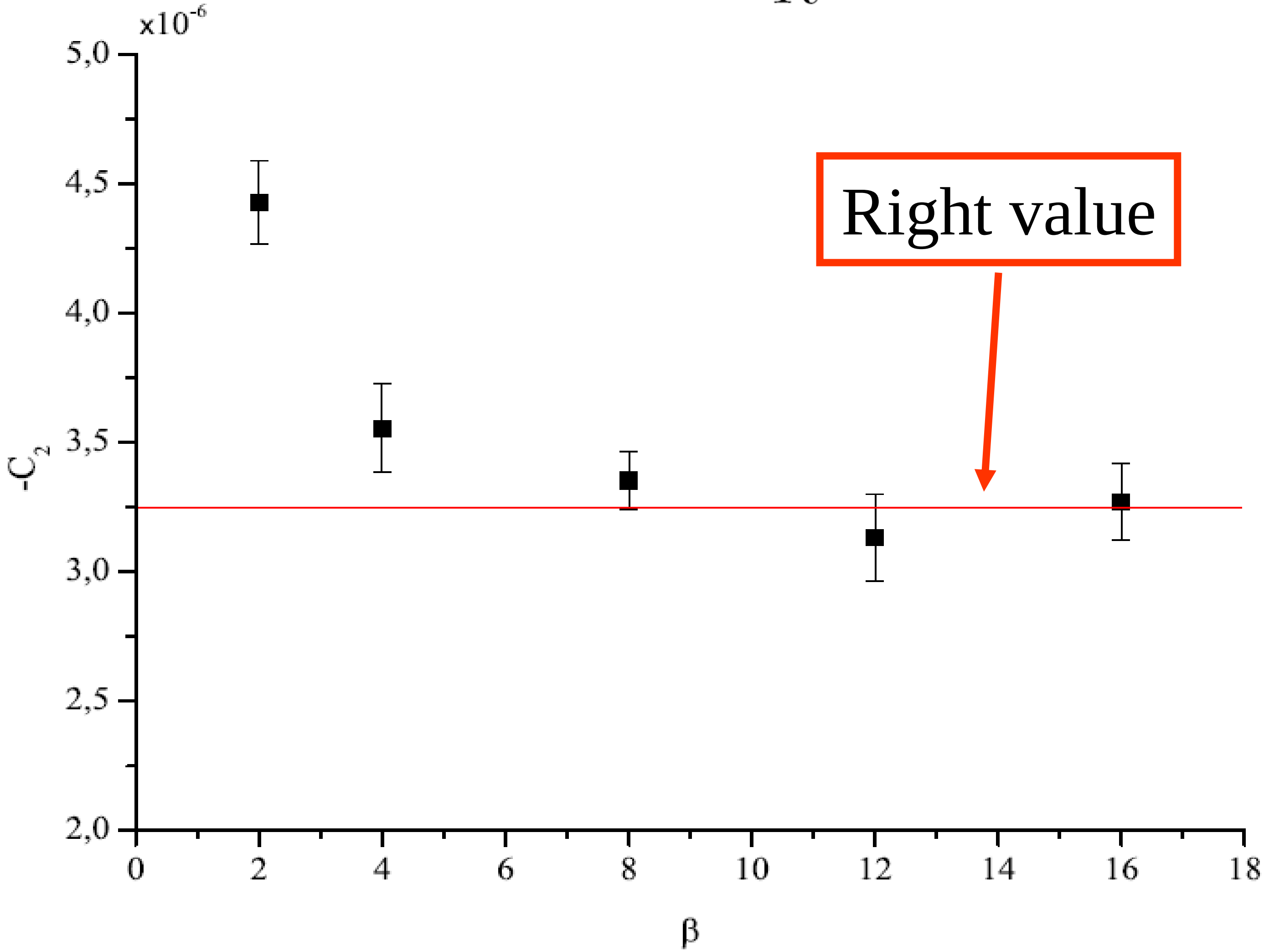
λ - dependence

$$E_{Cas} = -\frac{\lambda^2}{8\pi^2 R^3} + O(\lambda^4)$$



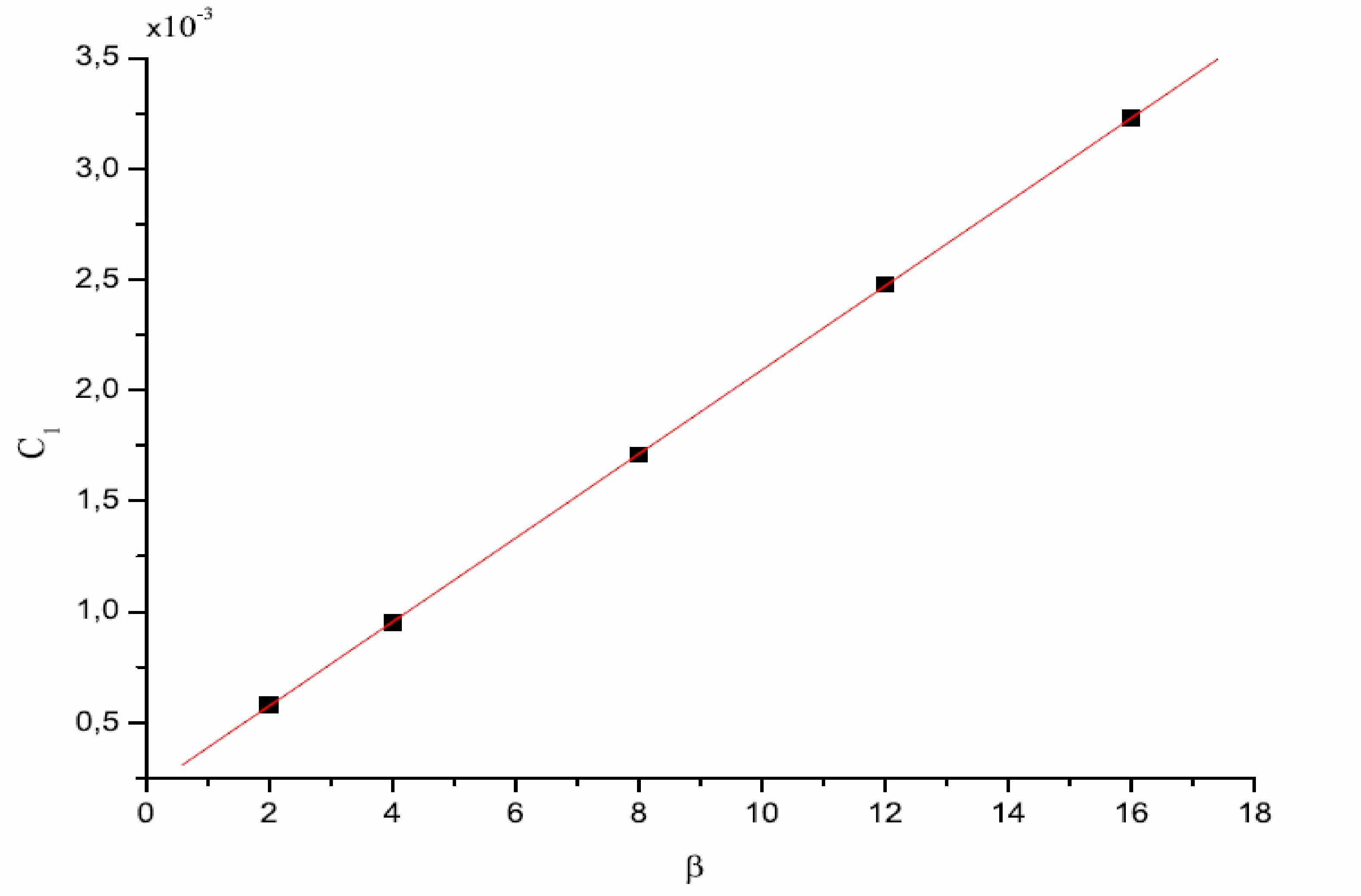
Continuum limit: physical part

$$E_{cas} = C_1 + \frac{C_2}{R^3}$$



Continuum limit: the artifact

$$E_{cas} = C_1 + \frac{C_2}{R^3}$$



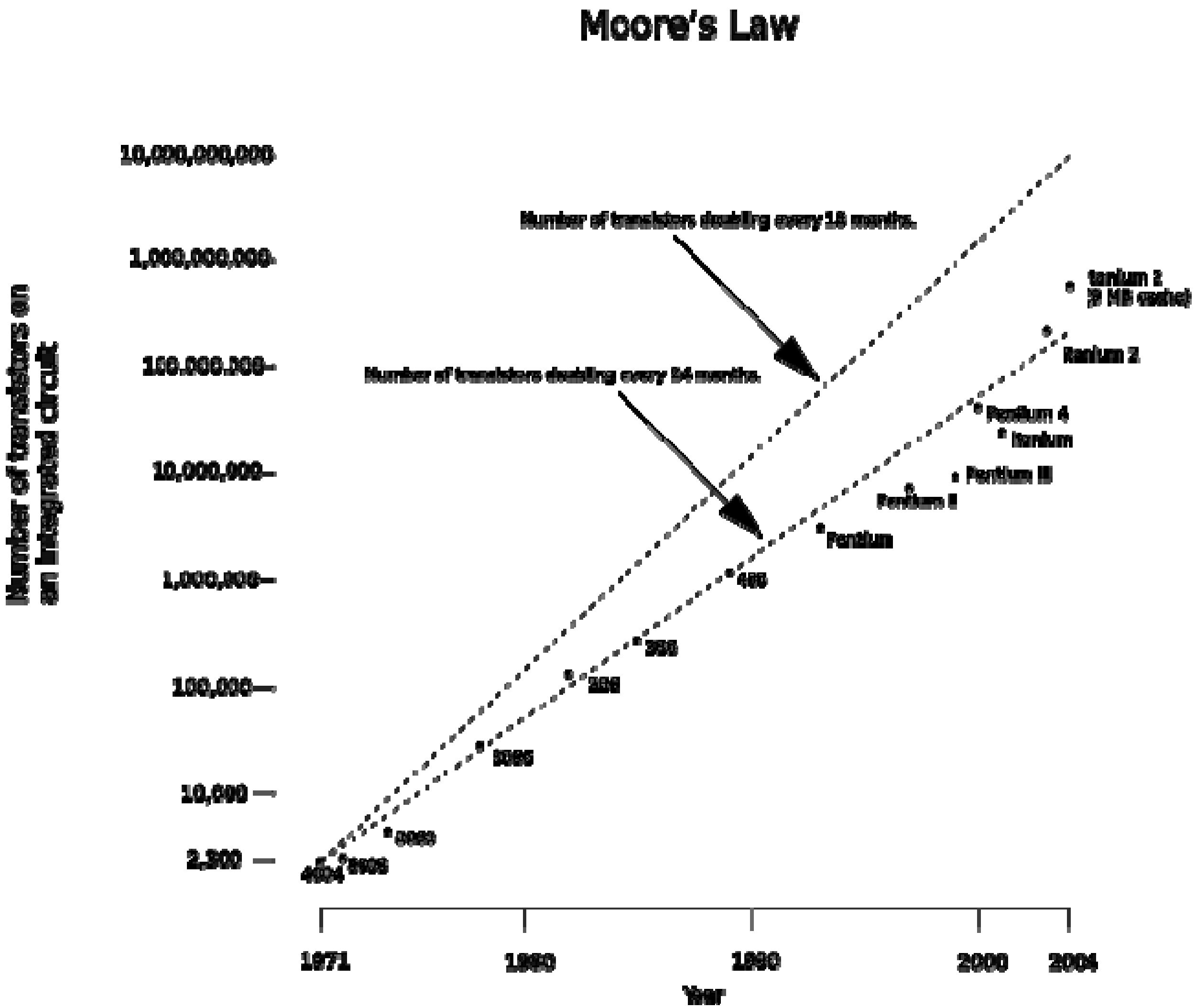
Conclusions:

We have proposed the numerical method for the Casimir energy calculation based on the lattice simulations of QED.

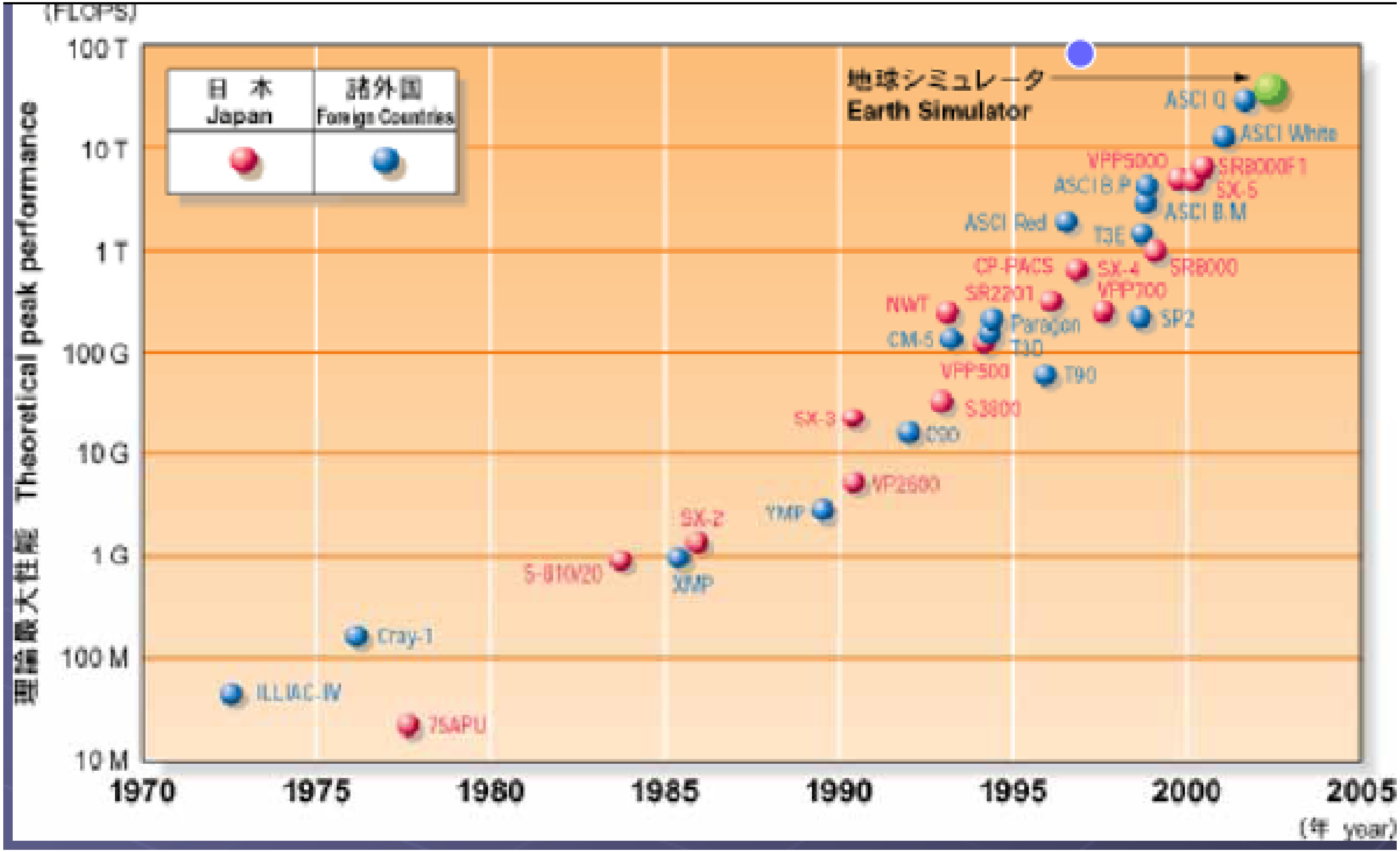
This method is the combination of two ideas: the generation of the boundary conditions by the additional Chern-Simons boundary action and the lattice "Wilson bag" concept. This combination is in fact a lattice definition of the quantum observable for the Casimir energy.

We have tested our method in the simplest case of the Casimir interaction between two plane surfaces and have achieved a good agreement with the analytical results for this problem.

Computers


Moore's Law
Number of transistors on an integrated circuit
10,000,000,000
1,000,000,000
100,000,000
10,000,000
1,000,000
100,000
10,000
2,300
Number of transistors doubling every 18 months.
Number of transistors doubling every 24 months.
Itanium 2 (9 MB cache)
Itanium 2
Pentium 4
Itanium
Pentium III
Pentium II
Pentium
486
386
286
8086
8080
8008
4004
1971
1980
1990
2000
2004
Year

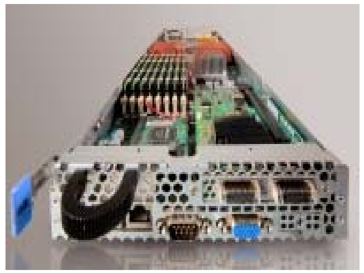
Computers


(FLOPS)
100 T
10 T
1 T
100 G
10 G
1 G
100 M
10 M
理論最大性能 Theoretical peak performance
日 本
Japan
諸外国
Foreign Countries
地球シミュレータ
Earth Simulator
ASCI Q
ASCI White
VPP5000
SR8000F1
SX-5
ASCI B.P
ASCI B.M
ASCI Red
T3E
SR8000
CP-PACS
SX-4
VPP700
SR2201
NWT
Paragon
SP2
CM-5
T3D
VPP500
T90
S3800
SX-3
C90
VP2600
YMP
SX-2
S-810/20
XMP
Cray-1
ILLIAC-IV
75APU
1970
1975
1980
1985
1990
1995
2000
2005
(年 year)

IBM Roadrunner Takes the Gold in the Petaflop Race

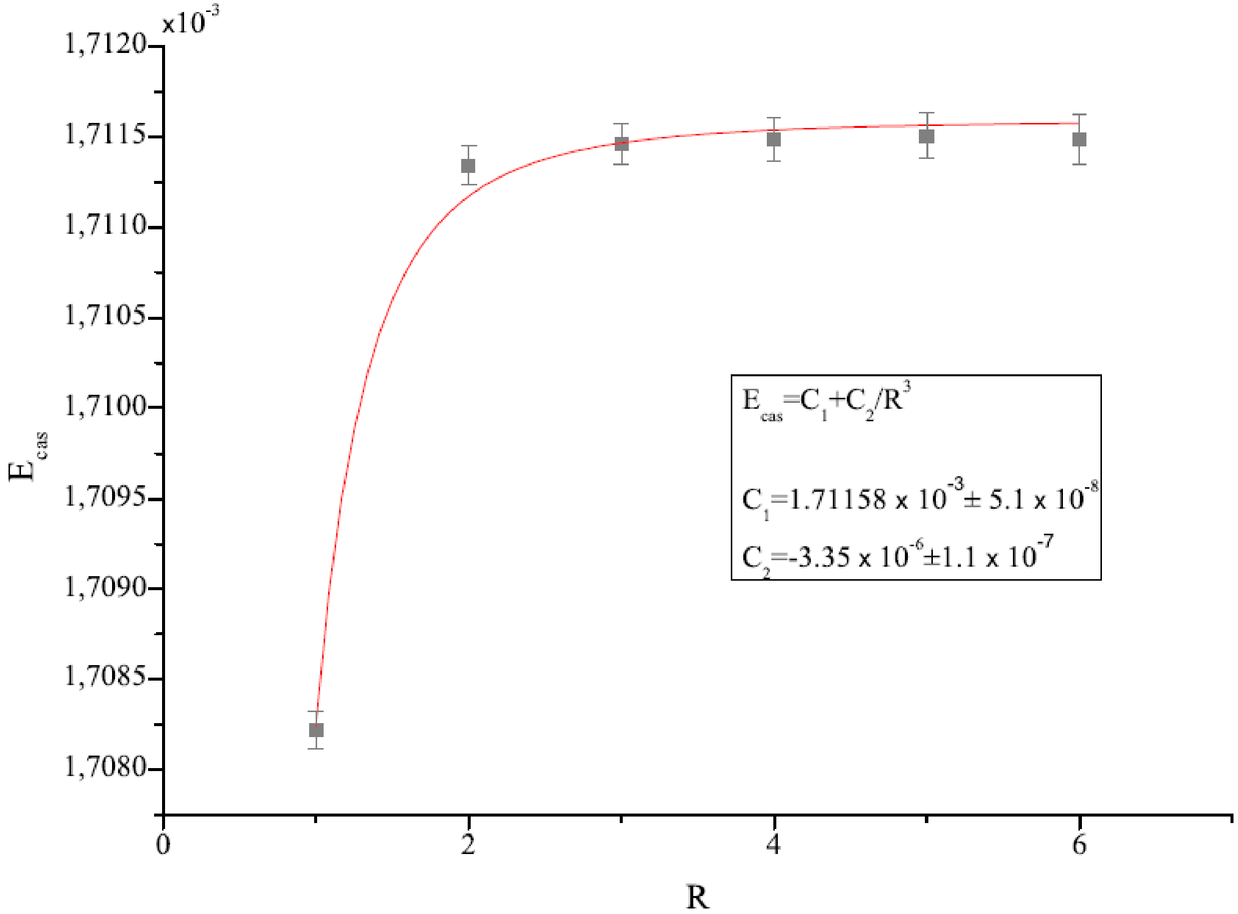
World's fastest computer, the US$133-million Roadrunner is designed for a peak performance of 1.5 petaflops!!!

СКИФ МГУ - 54-е место в мире и 2-е в России



60 TFlops

1250 Intel® Xeon® E5472, 3,0 Ггц



x10^-3
1,7120
1,7115
1,7110
1,7105
1,7100
1,7095
1,7090
1,7085
1,7080
E_cas
0
2
4
6
R
E_cas=C_1+C_2/R^3
C_1=1.71158 x 10^-3 ± 5.1 x 10^-8
C_2=-3.35 x 10^-6 ±1.1 x 10^-7