Lecture 4

Bose–Einstein condensate (BEC)
Optical lattices
Nano in Dubna and Russia
Conclusions

Bose–Einstein condensate (BEC)

- definition
- history
- main characteristics
- laser cooling

- role of interaction
- superfluidity
- rotation, vortices, vortex lattices
- BEC laser

Bose –Einstein condensate (BEC)

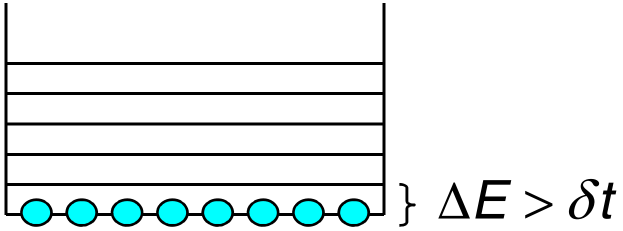
Fermi statistics $(I=\frac{2n+1}{2})$
allows to keep in one state only one fermion.

Bose statistics $(I=n)$
allows to put to one state as many bosons as we want.

If:
- to put many boson particles to the ground state,
- to have low temperature to prevent excitations
  due to temperature fluctuations,

then we can get BEC.

$$\boxed{{}^{A}_{Z}X_{N}} \qquad I=\sum_{k=1}^{Z} I_i^p+\sum_{k=1}^{Z} I_i^e+\sum_{k=1}^{N} I_i^n$$



N=2n ← Bose atoms $^{7}_{3}Li_4$

N=2n+1 ← Fermi atoms $^{6}_{3}Li_3$

Bose-Einstein condensate (BEC)

1924: S.N. Bose: paper on statistical description of the quanta of light.

1926: A. Einstein predicted condensate of Bose atoms in lowest ground state.

1938: Kapitza; Allan and Misener discovered superfluidity of liquid helium.

1938: London suggested superfluidity as a manifestation of BEC.

1941: Landau derived first self-consistent theory of superfluids in terms of elementary excitations.

1947: Bogoliubov developed first microscopic theory of interacting Bose gases.

1961: derivation of Gross-Pitaevskii equation

1995: E. Cornell and C. Wieman (JILA, US) cooled gas of ^{87}Rb atoms to 170 nK!

1995: Bradley et al first observed BEC for ^{7}Li atoms.

W. Ketterle (MIT,US) observed BEC for ^{23}Na atoms.

1997: Nobel prize of S. Chu, C. Cohen-Tannoudji and W.D. Phillips for laser cooling technique.

2001: Nobel prize of Cornell, Wieman and Ketterle for discovering BEC.

Strong-interacting
^{4}He

Is it possible to get BEC beyond He?

Weak-interacting Bose-gas

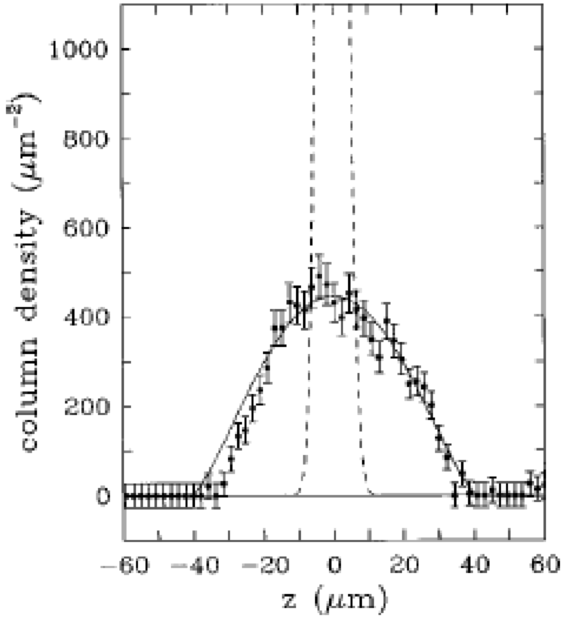
Optic-magnetic traps with harmonic confinement potential

$$V(\vec{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) = \frac{1}{2} m\omega_\perp^2 \rho^2 + \frac{1}{2} m\omega_z^2 z^2 = \frac{1}{2} m\omega_\perp^2 (\rho^2 + \lambda^2 z^2)$$

- spherical, $\omega_x = \omega_y = \omega_z$
- ellipsoidal, $\omega_x = \omega_y \neq \omega_z$
- disc-shaped, $\omega_\perp << \omega_z, \quad \lambda >> 1$
- cigar-shaped, $\omega_\perp >> \omega_z, \quad \lambda << 1$

Most of existing traps are anisotropic

Density distribution of BEC atoms at the trap



$N \approx 10^2$-10^9 atoms

$n \approx 10^{14} - 10^{15}\ cm^{-3}$

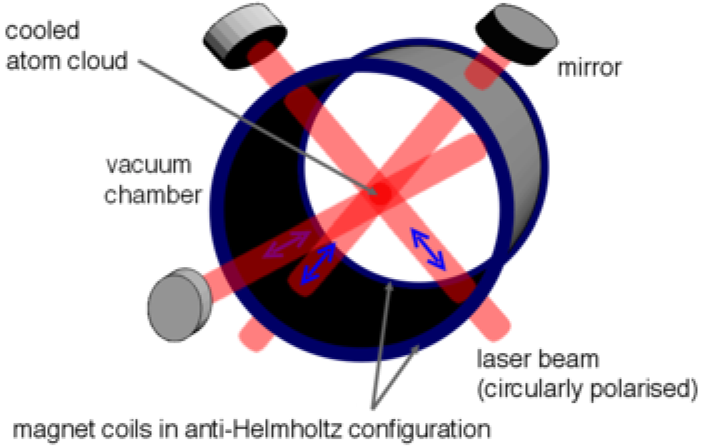
$L \approx\ 10 - 50\ \mu m, 15 \times 300\ \mu m$

$T_c \approx 500\ nK$ - $2\ \mu K$

$t_{cooling} \approx s - \min$

$\hbar\omega_0 \approx 5\ nK$

Magneto-optical traps



MOP:

- can be used to trap both charged and neutral particles, e.g. Boson atoms

- optical part (3 lasers with 3 mirrors)

used to cool the gas of atoms,

- magnetic part (coils in anti-Helmholz congiguration)

keeps the cool gas of atoms at the center of the trap

Basic properties:

- dilute gas of weakly interacting atoms ↔ strongly interacting ^{4}He condensate
- well known s-wave interaction determined by the scattering length a_s ,
- possibility to change magnitude and even sign of the interaction via Feshbach resonance, ↔ new kinds of BEC
- easy to generate different kinds of dynamics: (rotation, various vibrational modes, …), ↔ vortices, vortex lattice, scissors mode, …
- 1d, 2d, 3d forms,
- multi-component BEC, tunneling, ↔ Josephson effect
- superfluidity, ↔ various superfluid. eff.
- coherent phases, ↔ geom. (Berry) phases
- BEC with repulsive and attractive interaction, ↔ Super-Nova explosion
- BEC in optical lattice,
- atomic laser, …

BEC is a great and unique laboratory of new exciting physics!

Interesting crossings with other areas!

How to get condensate not from He?

How to avoid transition to liquid and solid at low temperature?

- Low density of atomic gas!
  prevents formation of liquid state (two-body collisions dominate over three-body ones) but lowers temperature requirement

- Big suspect still in 1994!
  - cooling up to nK!
  - avoiding liquid phase
  - free-well confinement

Steve Chu: "I'm betting on Nature to hide BEC from us. The last 15 years she's been doing a great job".

-Two-stage cooling:

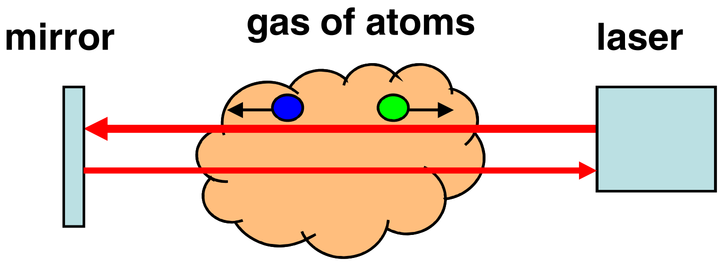
1) laser precooling → magn. trap,

2) forced evaporating cooling by reducing the trap depth

The trap depth is reduced, allowing the most energetic atoms to escape while the reminder rethermalize at lower temperature.

W. Ketterle, Nobel lecture, RMP, 74, 1131 (2002)

Doppler laser cooling: main principles



- Gas of atoms is irradiated by laser beam with a frequency slightly detuned below the resonance frequency of atoms.
- The mirror creates the beam of photons in the opposite direction.
- Atoms move and so their resonance frequencies exhibit the Doppler shift:

atoms moving against the laser beam have and so are in resonance with detuned laser $\omega_R' = \omega_R - \delta\omega_{Doppler} = \omega_{laser}$

atoms moving in direction of laser beam have $\omega_R' = \omega_R + \delta\omega_{Doppler}$ and so are off resonance with detuned laser

- Hence only atoms moving against the laser beam
  1) absorb the photons and acquire its momentum -q so as the atomic velocity decreases to

$$v \to v - q$$

  2) spontaneously emit the same photons and hence gain the recoil also in random direction

1) Laser beam slows down green atoms. Mirror beam slows down blue atoms.
2) Only cooling until Doppler temperature is possible.

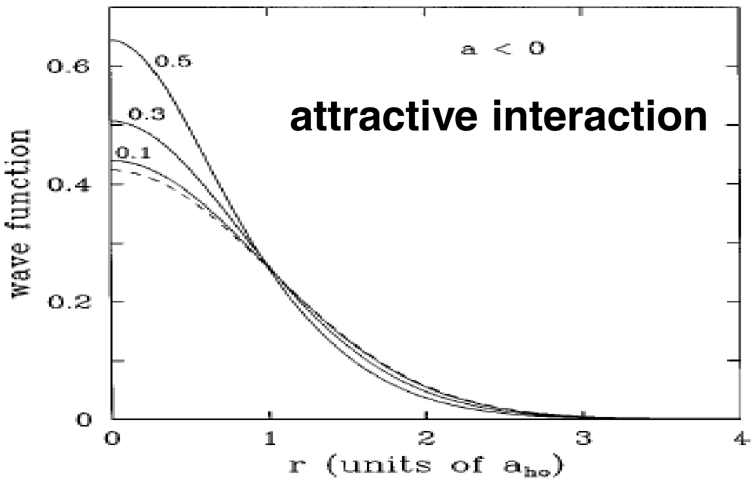
Finally we get in average decreasing atomic velocity and hence decreasing the kinetic energy and temperature.

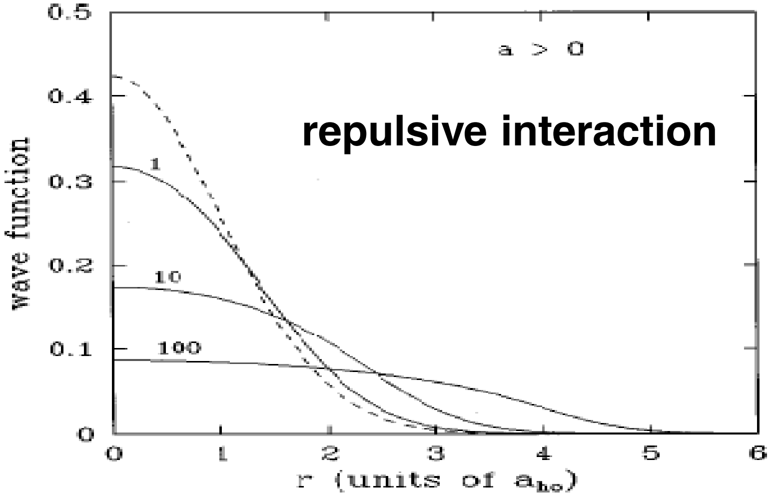
$$T_D \approx 10^{-4}\ \mathrm{K}$$

BEC: role of interaction

- **BEC can exist in non-interacting (ideal) gas.**
  Then order parameter has a Gauss profile.

- **Interaction between atoms results in change of Gauss profile:**





Without interaction Bose gas:

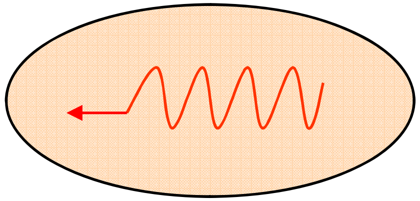
- **can form BEC (for $a_s < 0$)**
- **cannot be superfluid**

Bose dynamics: collective modes in traps

Uniform gas: traveling waves

$$\delta n(\vec{r},t) = \exp\{i\vec{q}\vec{r} - i\omega t\}$$

Propagation of waves via a volume.
Boundaries and shape are fixed.

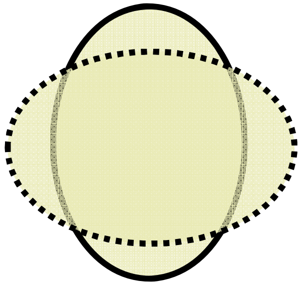


Non-uniform (trapped) gas: oscillations

$$\delta n(\vec{r},t) \propto r^{l} Y_{lm}(\theta,\varphi)\exp\{-i\omega t\}$$

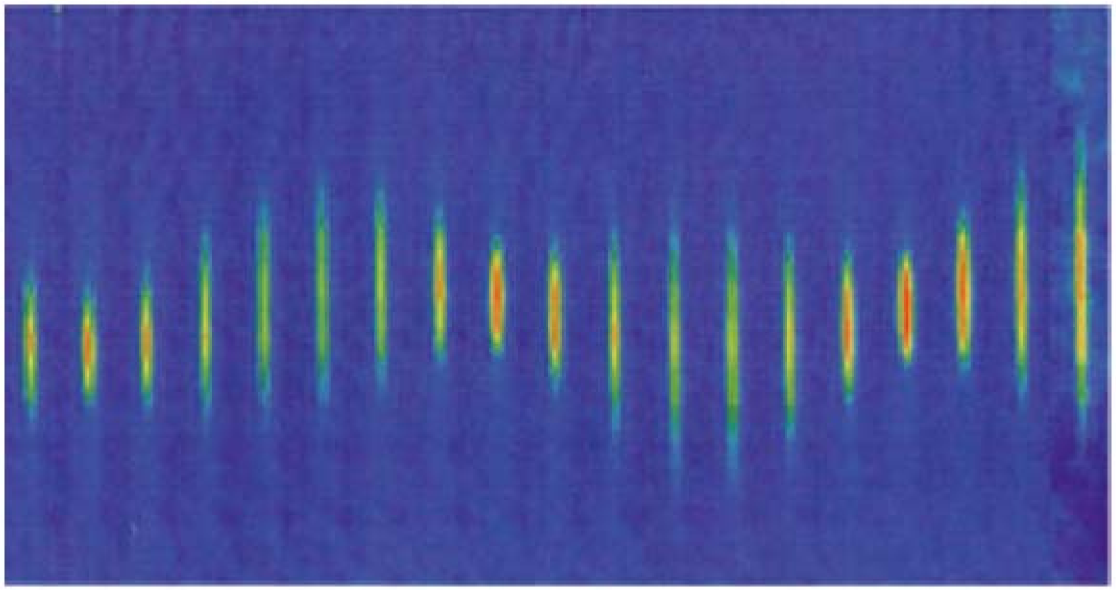
All the system participates in oscillation.
Shape oscillates as well.

$$n(\vec{r},t) = n_0 + \delta n(\vec{r},t)$$



BEC collective modes (4): experimental picture:

Excitations are produced by modulating the magnetic fields which confine BEC and then letting BEC evolve freely



time

- **cigar-shaped trap**
  $\omega_z / \omega_\perp = 17/230$
- **Thomas-Fermi regime**
- **in situ repeated phase-contrast images**
- **center of mass (l=1) and shape (l=2) collective oscillations**

MIT experiment (Stamper-Kurn, et al, 1998)

Scissors mode!

Energies of collective modes ~ 5 nK (~ 100 Hz) are safely below the critical temperature.

Superfluidity in BEC

BEC $\longleftrightarrow$ superfluidity

Are these two phenomena the same?

Superfluidity is already known in macroscopic system: superfluid ^{4}H

Signatures of superfluidity:

- absence of viscosity,
- reduction of moment of inertia,
- persistent currents,
- quantum vortices

However, superfluid ^{4}H is:

- macroscopic system,
- with strong correlations

-exceptional case:
very low $m^*/m=0.16$
and hence high zero-point energy

Which superfluid properties does take place BEC which is:

- microscopic (trap-limited),
- with weak correlations

?

Rotating BEC

The principle BEC property is its irrotational flow:

$$\boxed{\vec{v} = \frac{\hbar}{m}\vec{\nabla}\phi}$$

$$\boxed{\vec{\nabla}\times\vec{v} = 0}$$

No vorticity!

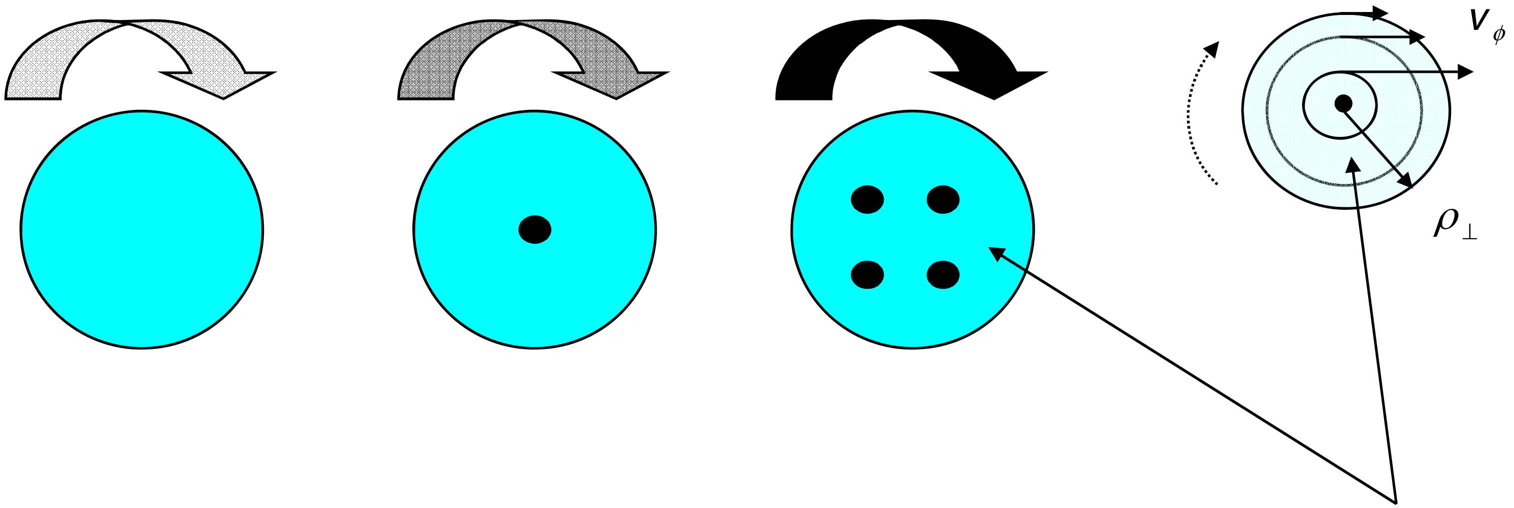
If to rotate a plate filled by water, the water will rotate together with the plate with velocity

$$\vec{\nabla}\times\vec{v} = \vec{\nabla}\times(\vec{\Omega}\times\vec{r}) \neq 0$$

BEC cannot rotate by this way!

Then how will BEC react to rotation of the trap?

Rotating BEC: reaction to rotation

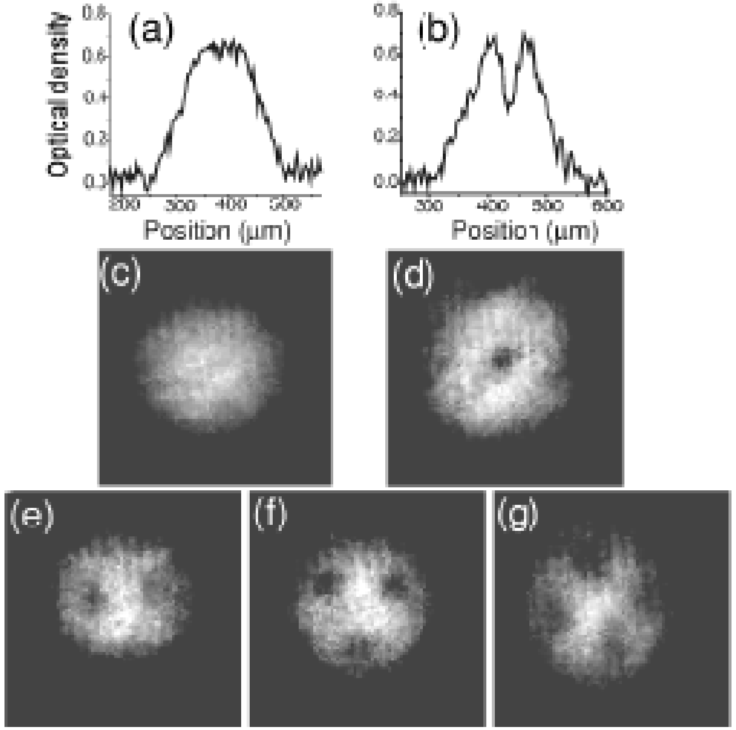


$$\vec{\nabla} \times \vec{v} = 0$$

BEC tries to keep as much as possible the irrotational flow

Rotating BEC: vortex core

K.W. Madison, et al,
PRL, 84, 806 (2000)



$$\Omega/(2\pi) = 145\,Hz\,(c), 152\,Hz\,(d), 159\,Hz\,(e), 163\,Hz\,(f), 168\,Hz\,(g)$$

$$\xi = \frac{1}{\sqrt{8\pi a_s \rho}}$$ **- vortex-core size (healing length)**

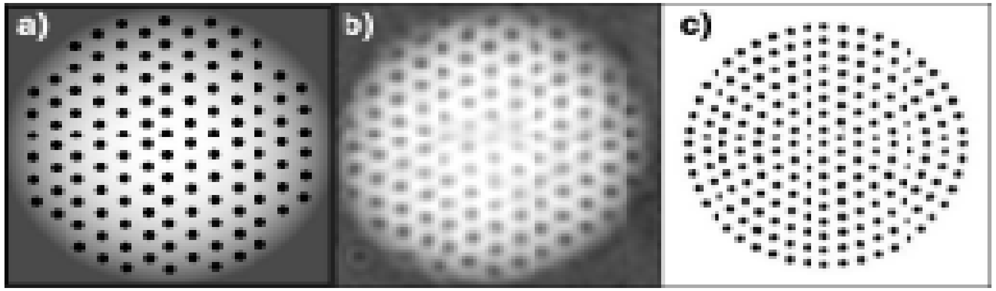
- **Since BEC has a weak interaction and low density, his vortex size is three order of magnitude larger than the vortex diameter in superfluid He II.**
- **The vortex core becomes even larger while BEC expansion at the trap switched off.**
- **Then vortices in BEC become large enough to be observed optically.**

Rotating BEC (2): vortex lattice

Abrikosov (1957): vortices in a type II superconductor create triangular crystalline lattice due to their mutual repulsion.

Vortex lattice in trapped BEC:

- surprising regularity in spite of BEC density varies greatly through the sample,
- in trapped (inhomogeneous) BEC the vortex lattice is more regular than in homogeneous BEC,



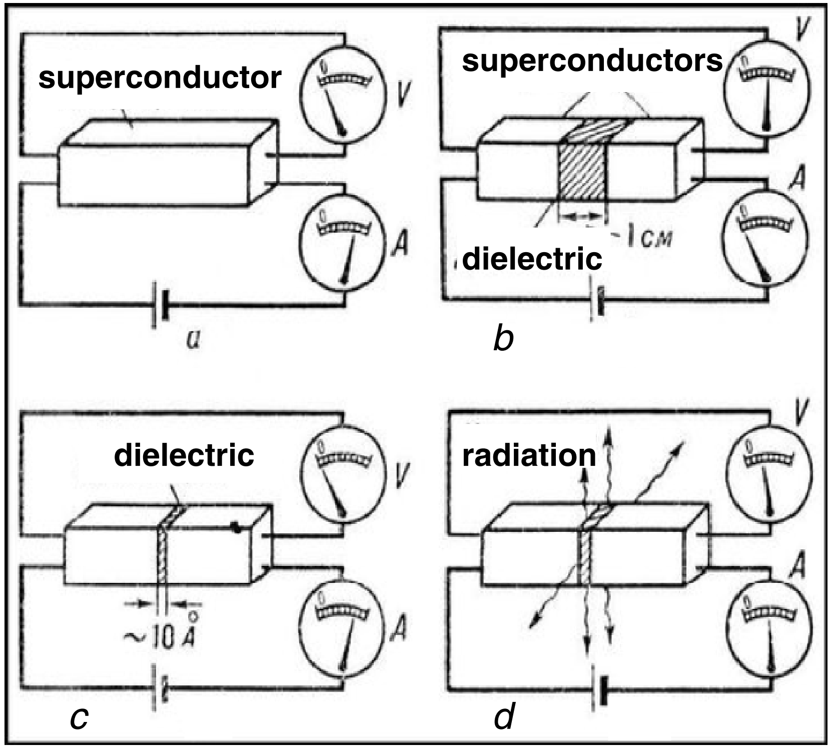
$\rho(\vec{r})$ **exper.** $\rho = const$

- The regularity of the observed BEC lattice is ultimately due to their triangular structure.
- Triangular symmetry removes most of damping terms from the equation of state.

J.R. Anglin and M. Crescimanno, ArXive:cond-mat/ 0210063, (2002)

Josephson effect

- **Predicted for superconductors by Josephson in 1962.**
- **Observed in 1963.**

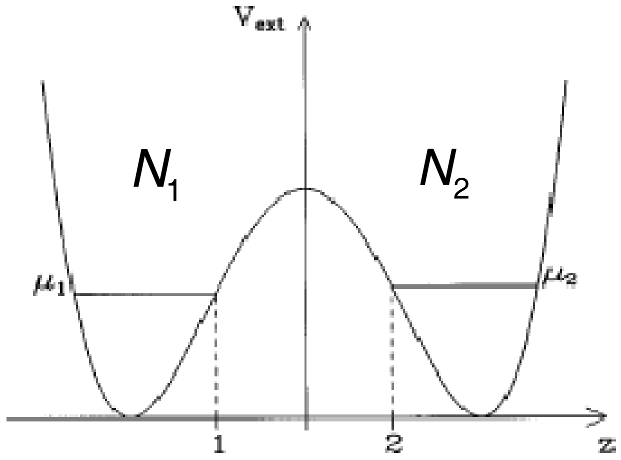


- **Tunneling transfer of electron pairs via thin dielectric separating two superconductors**

- **Stationary effect at small current** $I < I_c$ **: no voltage (V=0), no irradiation**

- **Non-stationary effect at large current** $I > I_c$ **:**
  there are voltage (V>0) and emission of photons

Coherent phenomena: Josephson effect in BEC



Tunneling between:
two spacially separated BECs,

Tunneling transfer of atoms between two traps separated by a barrier

In the overlap region:

$$\psi(\vec{r},t) = \psi_1(\vec{r})\exp\{-i\frac{\mu_1 t}{\hbar}\} + \psi_2(\vec{r})\exp\{-i\frac{\mu_2 t}{\hbar}\}$$

$$I(z,t) = \frac{i\hbar}{2m}\int dxdy(\psi(\vec{r},t)\frac{\partial}{\partial z}\psi^*(\vec{r},t) - \psi^*(\vec{r},t)\frac{\partial}{\partial z}\psi(\vec{r},t)) =$$

$$= I_0 \sin\{(\mu_1 - \mu_2)t/\hbar\} \longleftarrow$$ **typical Josephson current**

New effects: macroscopic quantum self-trapping, ….

Atomic laser

W. Guertin et al,
Arxiv: cond-mat/0607438

- Coherent beam of propagated atoms,
- created from Bose – Einstein condensate,
- first atomic laser was demonstrated at MIT by W. Ketterle in 1996,
- numerous attempts to create continuous BEC beam,
- perspectives: coherent atom sources, quantum transport phenomena,
- since atoms are massive particles, gravity plays an important role

- W. Guertin et al, Orsay, 2006:
  Quasicontinuous atom laser
  (joined magnetic trap and
  optical wave guide)

BEC: different kinds

2003: R. Grimm (Innsbruck Univ.), D. S. Jin (Univ. of Colorado), W. Ketterle (MIT): molecular BEC

Spin BECs

Mixed BECs

Dipolar BECs

BEC in optical lattices

BEC

New quantum system which is unique in the precision and flexibility with which it can be controlled and manipulated!

References:

Books:

L. P. Pitaevskii and S. Stringari,
"Bose-Einstein condensation"
Oxford University Press, Oxford, 2003.

C.J. Petrick and H. Smith,
"Bose-Einstein condensation in dilute gases",
Cambridge University Press, Cambridge, 2002.

Reviews:

F. Dalfovo, S. Glorgini, L.P. Pitaevskii, S. Stringari,
"Theory of Bose-Einstein condensate in trapped gases",
RMP, 71 (3), 463 (1999).

A.J. Leggett,
"Bose-Einstein condensate in the alkali gases: Some fundamental concepts"
RMP, 73 (2), 307 (2001).

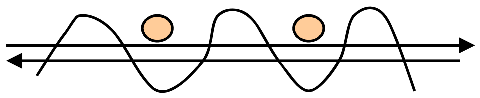
W. Ketterle, "Nobel lecture",
RMP, 74, 1131 (2002)

O. Morsch and M. Oberthaler,
"Dynamics of Bose-Einstein condensate in optical lattices",
RMP, 78 (1), 179 (2006).

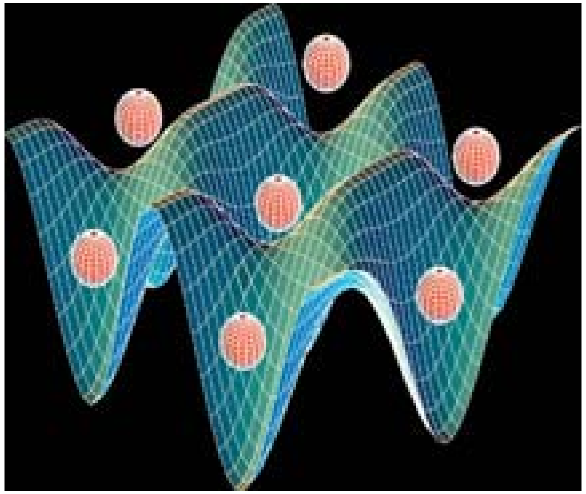
OPTICAL LATTICE

Optical lattice (1): general info

- OL appeared in late 90s,
  can be 1D, 2D, 3D.



- OL is formed by counter-propagating laser beams to create a potential periodic in space.



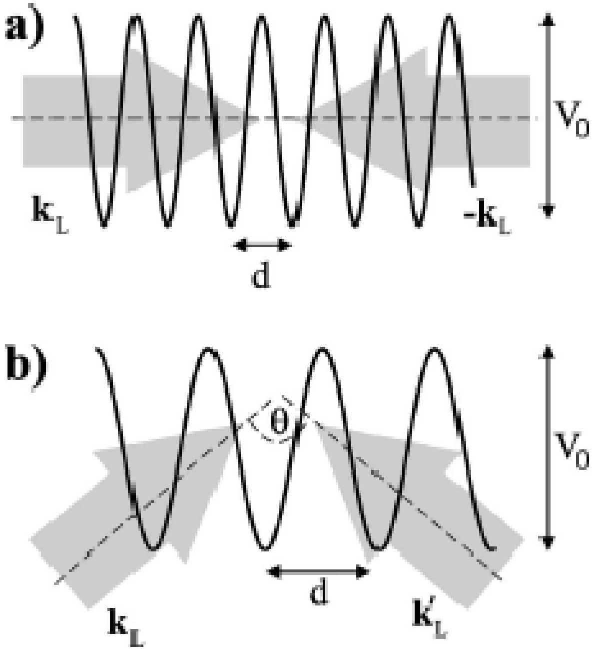
- Obvious connections with:
  - electrons in a crystal lattice,
  - nonlinear optics,
  - nonlinear physics in general.

- Great perspectives for investigation of BEC in periodic potentials!

Optical lattice (2): monitoring of trap

Potential depth V_0 is regulated via laser intensity I and detuning Δ

Lattice spacing d is regulated via angle θ between directions of laser beams



$$V(x) = V_0 \cos^2(\pi x/d)$$

$$V_0 \propto I/\Delta$$

References: optical lattice

1) O. Morsch and M. Oberthaler,
“Dynamics of Bose-Einstein condensate in optical lattices”,
Rev. Mod. Phys., 79, 179 (2006).

What we have learned from lecture 4:

Bose-Einstein condensate:
- definition
- history
- main characteristics
- laser cooling
- role of interaction
- superfluidity
- rotation, vortices, vortex lattices
- BEC laser

Optical lattices

Outlook of nanotechnologies in Russia

Minister of education Fursenko: 08.02.08

- **Program of development of nano-industry in Russia until 2015**
- **Russian corporation of nanotechnologies (budget of 130 billions rubles)**
- **150 research institutes**
- **special economical zones with low taxes** → **Dubna**
- **5 main directions:**
  - **new materials,**
  - **catalysis and refinement of carbohydrates,**
  - **bio-medical equipment for diagnostics and treatment,**
  - **new light sources,**
  - **technological and diagnostic equipment**

Dubna

JINR:

Flerov laboratory of nuclear reactions

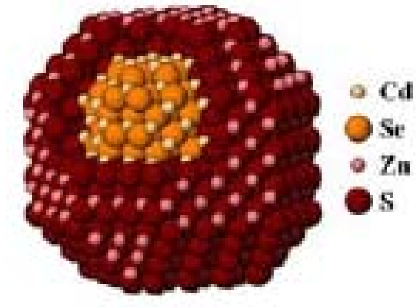
- nano-filters, …..

Frank laboratory of neutron physics:

small-angle neutron scattering to study nanodispersed objecs
(magnetic liquids, dendrimers, …)

Dubna

Nanotech-Dubna:
- colloidal production of high-quality quantum dots of different size



The outer layer allows to keep fluorescence for much longer time.

- selling 10 rub/1 mg
- to be used for markers, photo-detectors, photo-diods, …

Conclusions and outlook

Multidisciplinary area

- **condensed matter**
- **thermodynamics**
- **laser physics**
- **atomic, molecular physics**

- **superconductivity**
- **chemistry**
- **DFT, …**
- **quantum transport**

Promises new discoveries!

Fantastic perspectives for

fundamental physics

and **APPLICATIONS !!!**

Thank you for your attention!