Interpretation of results

1. Conservative.

$\varepsilon_{DE} = \text{const} = \varepsilon_0$ agrees with all

SN + CMB data (inside 2σ or better)

Especially good with Ly-α data from SDSS added and for SNLS data.

Models.

a) Casimir energy or vacuum polarization from additional compact or curved non-compact spatial dimensions

$$\varepsilon_{DE} = \frac{C}{R_d^4}$$

$D = 4 + d$
$d = 2$ flat compact
$d = 1$ AdS

$R_d < 5 \cdot 10^{-3}$ cm $\rightarrow$ $0 < C < 0.1$ (D.J. Kapner et al. hep-ph/0611184)

Deviation from the Newton law at $R \lesssim R_d$ - the most crucial test for these class of models

b) String theory de Sitter vacua -

- have appeared in fantastically large amount recently (the "third" string revolution ?)

Models of dynamical dark energy

Practical use of the remarkable similarity between primordial DE (supporting inflation) and present DE: the same models may be used for description of both inflation and present dark energy

Single inflation	Quintessence
$R+R^2$ model	$F(R)$ model
Extended inflation	Scalar-tensor DE
k-inflation	k-essence
Brane inflation	Brane DE
String inflation	String DE

The most critical problem:

"Graceful exit" "Graceful entrance"

Model requirements

1. Stability of the Minkowski space-time with respect to perturbations with $\omega^2 \gg H_0^2$
   a) absence of tachyons,
   b) absence of ghosts
2. Solar System tests
3. Stability of the MD-stage

   Example: $\mathcal{L} = f(R) \Rightarrow \begin{cases} \frac{df}{dR} > 0 \\ \frac{d^2f}{dR^2} \geq 0 \end{cases}$
4. MD- and RD-stages should be generic

Physical DE models

1. Quintessence = minimally coupled scalar field with some potential ("inflaton today")

$$\varepsilon = \frac{\dot{\varphi}^2}{2} + V\,, \quad p = \frac{\dot{\varphi}^2}{2} - V$$

No crossing of $w = -1$ line

If $V \propto \varphi^{-n} \Longrightarrow n < 1$

Slow-roll regime: $w \simeq -1 + \frac{\dot{\varphi}^2}{V} \simeq -1$

2. The Chaplygin gas model

$$p = -\frac{\varepsilon_0^2}{\varepsilon}$$

$$c_s^2 \equiv \frac{dp}{d\varepsilon} = \frac{\varepsilon_0^2}{\varepsilon^2} > 0$$

$\varepsilon > \varepsilon_0$ for the "usual" model $\begin{pmatrix} \varepsilon + p > 0 \\ c_s^2 < 1 \end{pmatrix}$

$$\varepsilon = \sqrt{\varepsilon_0^2 + C(1+z)^6}$$

Unifies DM and DE : can describe both the MD stage in the past and the transition to the Λ-dominated stage today

Equivalent field-theoretical models

a) Quintessence with

$$V(\varphi) = \frac{\varepsilon_0}{2}\left(\cosh(2\sqrt{6\pi G}\,\varphi) + \frac{1}{\cosh(2\sqrt{6\pi G}\,\varphi)}\right)$$

Equivalence for some solutions (see V. Gorini et al., PRD 72, 103518 (2005); astro-ph/0504576)

b) k-essence $\mathcal{L} = -\varepsilon_0\sqrt{1 - T_{,\mu}T^{,\mu}}$

Equivalence for all solutions with $T_{,\mu}T^{,\mu} > 0$

However, λ_J is too large!

$\lambda_J \propto c_s t \propto t^3$ at the MD stage

Perturbations stop growing for

$z \sim 3$ for the present comoving scale 100 Mpc

$z \sim 14$ —— " —— " —— " —— 1 Mpc

Wrong $P(k)$ today!

DE models with "gravity leaking to higher dimensions"

The simplest model (Dvali et al., 2000)

Gravity in the $D=5$ bulk + induced gravity on the brane

$$H^2 = \left(\sqrt{\frac{8\pi G\rho}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2$$

$$H_0 r_c = \frac{1}{1-\Omega_m} \simeq 1.4 \quad (1.25 \text{ for } \Omega_m = 0.2)$$

$\Omega_m = 0.3$

Sahni & Shtanov (2002, 2004) – generalization admitting $w_{DE} < -1$ and/or $\varepsilon_{DE} < 0$

$$a = a_0 \sinh^{2/3} \varphi$$

$$\frac{3t}{2r_c} = \varphi - \frac{e^{-2\varphi}}{2} + \frac{1}{2}$$

$$\Omega_m(t) = e^{-2\varphi}$$

$$q(t) = \frac{2\Omega_m - 1}{1 + \Omega_m}$$

$$r(t) = 1 - \frac{9\Omega_m^2 (1-\Omega_m)}{(1+\Omega_m)^3}$$

$\Omega_m = 0.3$: $\quad q_0 = -0.31$, $r_0 = 0.74$

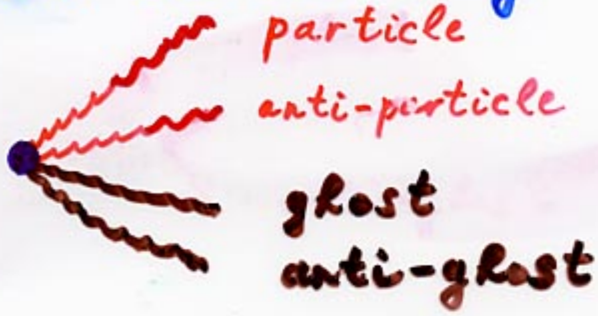
This model is on verge to be falsified, (using $\frac{\Delta T}{T}$ and $\frac{\delta\rho}{\rho}$) but still not!

Has a ghost (see, e.g., hep-th/0610282)

What if recent phantom behaviour of dark energy will be confirmed by observations ?

Ghost phantom models of dark energy are bad.

1. Quantum instability



2. At the classical level :
does not explain homogeneity and isotropy of the Universe

E.g. : for a given $\bar{H} = \frac{1}{3}\frac{d}{dt}\ln abc$, it is much more probable to have very different $\frac{\dot{a}}{a}, \frac{\dot{b}}{b}, \frac{\dot{c}}{c}$ compensated by the negative energy density of the ghost field.

Scalar-tensor models of dark energy do not have this problem.

Reconstruction of dark energy in scalar-tensor gravity

B. Boisseau, G. Esposito-Farese, D. Polarski, A.S.

Phys. Rev. Lett. 85, 2236 (2000)

$\varepsilon_{DE} + p_{DE} < 0$ is permitted

$$\mathcal{L} = \frac{1}{2}\left(F(\varphi)R + Z(\varphi)\varphi_{,\mu}\varphi^{,\mu}\right) - V(\varphi) + \mathcal{L}_m$$

Includes $R + f(R)$ theory for $Z(\varphi) = 0$.

$Z(\varphi) = 1$ $\qquad\qquad \omega^{-1}(\varphi) = F^{-1}\left(\frac{dF}{d\varphi}\right)^2$

Two independent observable cosmological functions are required for reconstruction of $F(\varphi)$ and $V(\varphi)$

$$D_L(z), \quad \delta(z)$$

$$\downarrow \qquad\qquad \downarrow$$

$$H(z) \longrightarrow F(z) \longrightarrow V(z)$$

Background equations

$$3FH^2 = \rho_m + \frac{\dot\varphi^2}{2} + V - 3H\dot F$$

$$-2F\dot H = \rho_m + \dot\varphi^2 + \ddot F - H\dot F \qquad \rho_m \propto a^{-3}$$

Their consequence:

$$\ddot\varphi + 3H\dot\varphi + \frac{dV}{d\varphi} - 3(\dot H + 2H^2)\frac{dF}{d\varphi} = 0$$

In terms of redshift:

$$F'' + \left[(\ln H)' - \frac{4}{1+z}\right]F' + \left[\frac{6}{(1+z)^2} - \frac{2(\ln H)'}{1+z}\right]F$$

$$= \frac{2V}{(1+z)^2 H^2} + 3(1+z)\left(\frac{H_0}{H}\right)^2 F_0\, \Omega_{m,0}$$

$$\varphi'^2 = -F'' - \left[(\ln H)' + \frac{2}{1+z}\right]F' + \frac{2(\ln H)'}{1+z}F$$

$$-3(1+z)\frac{H_0^2}{H^2} F_0\, \Omega_{m,0}$$

I. First step → as in GR

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}$$

II. Equation for sufficiently small-scale inhomogeneities

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m\,\delta = 0$$

where $\delta \equiv \left(\frac{\delta\rho}{\rho}\right)_{CDM+bar}$ at a fixed comoving scale $\lambda = a(t)/k$ and

$$\frac{k^2}{a^2} \gg \max\left(\frac{d^2V}{d\varphi^2},\, H^2\cdot\max\left(1, \frac{d^2F}{d\varphi^2}\right)\right);$$

$$G_{eff} = \frac{1}{8\pi F}\cdot\frac{F + 2\left(\frac{dF}{d\varphi}\right)^2}{F + \frac{3}{2}\left(\frac{dF}{d\varphi}\right)^2}$$

From this, excluding $d\varphi$:

a second-order differential equation for $F(z)$.

Properties of scalar-tensor models of dark energy

R. Gannouji, D. Polarski, A. Ranquet, A. S.
JCAP 09, 016 (2006) [astro-ph/0606287]

1. Temporary phantom behaviour and crossing of the phantom boundary $w = -1$ are possible for an open set of $F(\varphi)$ and non-zero and non-constant $V(\varphi)$.
"Curvature induced phantomness"

2. In the absence of dust-like matter ($\Omega_m = 0$), power-law solutions leading to the Big Rip singularity in future and to $w < -1$ exist if

$$F = \alpha \varphi^2 , \quad \varphi \to \infty$$

(Barrow & Maed[illegible] 1990)

$$V = V_0 |\varphi|^n , \quad 2 < n < 4$$

Then $a(t) \propto (t_0 - t)^q$ $\qquad q = \dfrac{2\left(n + 2 + \frac{1}{\alpha}\right)}{(n-2)(n-4)} < 0$

$\varphi(t) \propto (t_0 - t)^z$ $\qquad z = \dfrac{2}{2-n} < 0$

However, for these solutions $|w+1| \le \frac{\alpha}{3} \sim \frac{1}{\omega_B}$ and very small.

Present observational bounds:

$$\gamma_{PN} - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$$ Bertotti et al., 2003 → Cassini mission

$$\beta_{PN} - 1 = (0 \pm 1) \cdot 10^{-4}$$ Pitjeva, 2005 → ephemerides of planets

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = (-0.2 \pm 0.5) \cdot 10^{-13} y^{-1}$$

$$\beta_{PN} - 1 = (1.2 \pm 1.1) \cdot 10^{-4}$$ Williams et al., 2005 → lunar laser ranging

$$\omega_{BD,0} \equiv \left(\frac{F}{\left(\frac{dF}{d\varphi}\right)^2} \right)_0 > 4 \cdot 10^4$$

3. Small z expansion.

$$\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + \ldots$$

$$\frac{V(z)}{3 F_0 H_0^2} = \Omega_{V,0} + u_1 z + u_2 z^2 + \ldots$$

$$\frac{H^2(z)}{H_0^2} = 1 + h_1 z + h_2 z^2 + \ldots$$

$$F_0^{-1/2} \varphi'(z) = \varphi_0' + \varphi_1' z + \varphi_2' z^2 + \ldots$$

$$w_{DE}(z) = w_0 + w_1 z + w_2 z^2 + \ldots$$

$$|F_1| < 10^{-2}$$

$$\varphi_0'^2 = 6 (1 - \Omega_{m,0} - \Omega_{V,0} - F_1) \geq 0$$

What is required to get significant phantomness ($|w+1| \gg \frac{1}{\omega_{BD,0}}$) ?

$$F_2 < 0 \,, \quad |F_2| \sim 1 \gg |F_1| \qquad (|F_1| < 10^{-2})$$

$$|F_2| > 3\,(\underbrace{\Omega_{DE,0}}_{1-\Omega_{m,0}} - \Omega_{V,0}) > 0$$

$$w_0 + 1 = \frac{2F_2 + 6(\Omega_{DE,0} - \Omega_{V,0})}{3\Omega_{DE,0}} < 0$$

F_1 can be negative, too

4. Connection with post-Newtonian parameters in the significantly phantom case.

$$\gamma_{PN} - 1 = -\frac{F_1^2}{6(\Omega_{DE,0} - \Omega_{V,0})} < 0$$

$$\beta_{PN} - 1 = -\frac{F_1^2 F_2}{72(\Omega_{DE,0} - \Omega_{V,0})} > 0$$

$$-4 < \frac{\gamma_{PN} - 1}{\beta_{PN} - 1} = \frac{12(\Omega_{DE,0} - \Omega_{V,0})}{F_2} < 0$$

However, $|\gamma_{PN} - 1|$ and $|\beta_{PN} - 1|$ may be much smaller than $|1+w|$ if F_1 is very small

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = H_0 F_1 \left(1 - \frac{F_2}{3(\Omega_{DE,0} - \Omega_{V,0})} \right)$$

Positive detection of $\gamma_{PN} < 1$, $\beta_{PN} > 1$ may be a strong argument for significant phantomness of present DE.

Negative detection tells nothing.

5. Correct asymptotic behaviour for large z ($\varphi'^2 \geq 0$, $w_{DE} \leq 0$) requires non-zero and non-constant $V(\varphi)$

E.g. $F(\varphi) \to F_\infty < F_0$

$$V(\varphi) \propto \exp\left(\sqrt{\frac{3}{2F_0\Omega_{u,\infty}}}\,\varphi\right)$$

$$z, \varphi \to \infty$$

6. In the stable case $F > 0$, $\omega_{BD} > -\frac{3}{2}$, no possibility to construct a stable wormhole (even with an electromagnetic field)

Geometrical $f(R)$ model of DE

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R) \qquad R \equiv R_\mu^\mu$$

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = -8\pi G\left(T_{\mu(m)}^\nu + T_{\mu(DE)}^\nu\right)$$

$$8\pi G\, T_{\mu(DE)}^\nu \equiv F'(R) R_\mu^\nu - \frac{1}{2}F(R)\delta_\mu^\nu$$

$$+ \left(\nabla_\mu \nabla^\nu - \delta_\mu^\nu \nabla_\rho \nabla^\rho\right) F'(R)$$

Particle content: graviton + massive scalar particle $\left(M^2 = \frac{1}{3f''(R)}\right)$

(dubbed "scalaron" in A. S., 1980)

Stability conditions:

1. $f' > 0$ graviton is not a ghost
2. $f'' > 0$ scalaron is not a tachyon

imposed for $R \geq R_{now}$ at least

(i.e. during the whole evolution of the Universe)

Violation of these conditions is undesirable from the classical point of view, too!

$f'(R_0) = 0$ - instant loss of homogeneity and isotropy

$f''(R_0) = 0$ - weak singularity

$$R(t) = R_0 + O(\sqrt{t})$$
$$a(t) = a_0 + a_1 t + a_2 t^2 + O(t^{5/2})$$

③ Existense of the Newtonian regime

$$(\Delta\varphi = 4\pi G\rho)$$

$$|F| \ll R, \quad |F'(R)| \ll 1, \quad R|F''(R)| \ll 1$$

for $R_{now} \lll R$ (at up to some very large R)

$\longleftrightarrow$

De Sitter regime

$$Rf' = 2f$$

Stable if

$$f'(R_1) > R_1 f''(R_1)$$

$\longleftrightarrow$

Equivalent to $\omega_{BD} = 0$ scalar-tensor gravity

Use for inflation

$$f(R) = R + \frac{R^2}{6M^2}$$ (+ small non-local terms)

AS, 1980

Internally self-consistent inflationary model with slow-roll decay, a graceful exit to the subsequent RD FRW stage (through an indermediate matter-dominated stage) and sufficiently effective reheating

$\tau \sim M_{Pl}^2 / M^3$ $\quad N \sim 50$

Remains viable

$$M = 2.8 \times 10^{-6} (N/50)^{-1} M_{Pl}$$

$$n_s = 1 - \frac{2}{N} = 0.96 \quad \text{for } N = 50$$

$$r = \frac{12}{N^2} = 4.8 \times 10^{-3} (N/50)^2$$

Eksp.: $\overline{n_s} = 0.96 \pm 0.02$, $r < 0.3$

Use for DE

$F(R) \propto R^{-n}$ for $R \to 0$

Does not work for many reasons

Viable model – regular at $R=0$

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1+\frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

AS, JETP Lett. 86, 183 (2007)
arXiv: 0706.2041 [astro-ph]

or even

$$f(R) = R - \lambda R_0 \tanh^2 \frac{R}{R_0}$$

$f(0)=0$ – 'disappearing' cosmological constant in flat space-time

Induced Λ at high curvatures:

$$\Lambda_\infty \equiv -\frac{1}{2} F(\infty) = \frac{\lambda R_0}{2}$$

Observational restrictions

1. Cosmology

Anomalous growth of non-relativistic matter perturbations in the regime

$k \gg M(R)\, a$

$G_{eff} = 4G/3f'(R) \simeq \frac{4G}{3}$

$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}}$ (instead of $\propto t^{2/3}$)

Results in the apparent mismatch

$\Delta n_s = n_s^{(gal)} - n_s^{(CMB)} = \frac{\sqrt{33}-5}{2(3n+2)}$

$\Delta n_s < 0.05 \longrightarrow n \geq 2$

2. Laboratory and Solar system tests

$M(R)\, L \gg 1$ with $R = 8\pi G T_m = 8\pi G \rho_m$

Otherwise, $\gamma_{PN} = \frac{1}{2}$ and the 'fifth' force appears.

$M(R(\rho_m)) \propto \rho_m^{n+1}$

$n \geq 2$ is sufficient for all tests

CONCLUSIONS

1. Deviation of dynamical DE from an exact cosmological constant is $\lesssim 10\%$, but still may exist

2. The simplest DE model which can accomodate its possible recent phantom behaviour and crossing of the "phantom boundary" $w_{DE} = -1$ is based on scalar-tensor gravity and does not have ghosts or instabilities

3. Viable models in $f(R)$ gravity, though more restricted, are possible, too

4. However close the present DE may be to Λ, simply by analogy with primordial DE, one should <u>not</u> expect it to be absolutely stable and eternal