THERMAL EFFECTS
ON WEAK INTERACTION MEDIATED PROCESSES
IN STELLAR ENVIRONMENT

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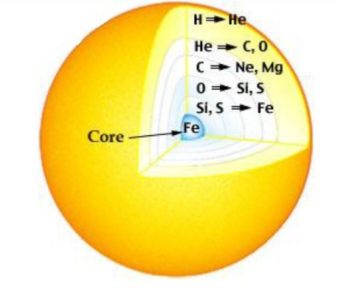
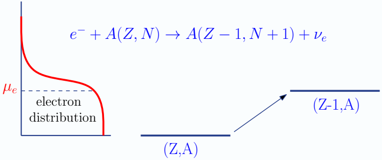
Outline

- Massive star: pre-collapse stage
- Processes involving neutrino during collapse
- Nuclear structure inputs:
  - Spin-isospin resonances
  - Temperature effects
- Theoretical approaches:
  - Large Scale Shell Model (LSSM) calculations
  - Thermo Field Dynamics $\Rightarrow$ Thermal QRPA (TQRPA)
- Calculation results:
  - Inelastic neutrino scattering (INNS) off a hot nucleus
  - Neutrino pair emission by a hot nucleus
- Conclusions

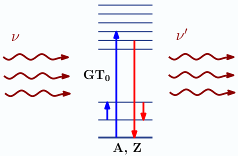


- At the end of its life a massive star ($M \geq 10 M_{\odot}$) has the structure similar to that of an onion.
- Just before the core-collapse all reactions mediated by the electromagnetic and strong interactions (but not weak interaction!) are in Nuclear Statistical Equilibrium.
- Electrons form a degenerate gas (keep a pressure due to the Pauli principle).
- Until $M_{\text{core}} < M_{\text{Ch}} = 1.44 (2Y_e)^2 M_{\odot}$, the gravitation is balanced by the pressure of the degenerate relativistic gas of electrons (Y_e is the number of electrons per one baryon in the star).
- The equilibrium is unstable since
  1. The silicon burning increases the iron core of the star.
  2. The electron captures by protons and nuclei (at $\rho \gtrsim 10^9 \text{g/cm}^3$) decrease the pressure of the degenerate electronic gas.
- When the iron core mass M_{core} exceeds M_{Ch} it collapses during $\sim 1\,\text{s}$.

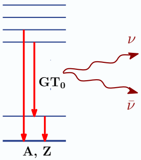


- Electron capture on nuclei is the main neutrino source during collapse.
- For $E_\nu < 20$ MeV the Gamow-Teller transitions ($\mathbf{GT_0 : \Delta J^\pi = 1^+, \sigma t_0}$ operator) dominate neutral-current neutrino-nucleus reactions.
- **Reactions involving neutrinos in collapsing star**
  - $\nu_e + (A, Z) \leftrightarrows \nu_e + (A, Z)$ – neutrino trapping
  - $(A, Z)^* + \nu_e \leftrightarrows (A, Z)^* + \nu_e'$ – neutrino thermalizing
  - $\nu_e + e^\mp \leftrightarrows \nu_e + e^\mp$ (inelastic process) – neutrino thermalizing
  - $(A, Z)^* \rightarrow (A, Z) + \nu_e + \bar{\nu}_e$
- At densities $\rho \lesssim 10^{11} \mathrm{g/cm}^3$ the low-energy neutrinos can leave the star unhindered carrying away energy. This is a very efficient cooling mechanism which keeps the entropy of the matter low. As a consequence heavy nuclei survive during the collapse.
- At densities $\rho \approx 4 \cdot 10^{11} \mathrm{g/cm}^3$ neutrinos start to get trapped in the core due to elastic scattering on nuclei (diffusion).
- At densities $\rho \gtrsim 10^{12}$ g/cm^3 neutrino inelastic scattering off electrons (mainly) and nuclei become important. Neutrino thermalization takes place.

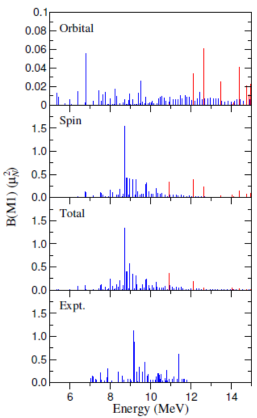
Neutrino inelastic scattering



Neutral-current β-decay



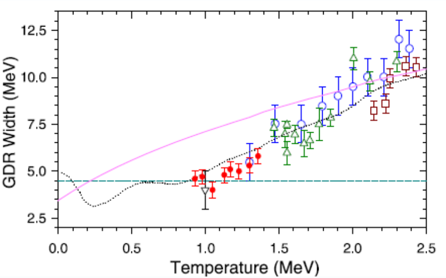
To calculate the ν-A cross sections one needs to know the nuclear response to $\boldsymbol{\sigma t_0}$ operator or, in other words, the distributions of the $\mathbf{GT_0}$ strength over a nuclear spectrum. The position and structure of the corresponding $\mathbf{GT_0}$ resonance is most important.



$M1$ strength distribution in ^{52}**Cr**.
Shell-model - upper panels; Experiment – bottom panel. Spin response $\Rightarrow$ $\mathbf{GT}_0$

For temperatures $T = 1.0 \div 2.0$ MeV the mean excitation energy for iron-group nuclei is $\langle E \rangle = 10 \div 30$ MeV.

Experimental information as well as theoretical predictions on nuclear resonance characteristics in highly excited nuclei are poor. Only giant $E1$ resonance in hot heavy nuclei was explored experimentally. Most of calculations for GT resonances were done in the thermal RPA approach and Shell-Model Monte Carlo (SMMC) method.



Experimental data on the width of giant $E1$ resonance as a function of temperature T in ^{120}Sn collected from different experiments.
The green dashed line: the width at $T = 0$. Continuous pink line: theory (the thermal shape fluctuation model)

Conclusions: the centroid is shifted downward, the width increases.

Large Scale Shell Model **(LSSM)** *(K. Langanke, G. Martínez-Pinedo et al.)*

Calculations were performed for $s-d$ and $p-f$ nuclei with $A \lesssim 65$.
The spin-isospin strength distributions in low-lying part of nuclear spectra are well described. Thermal effects are taken into account by state-by-state evaluation of the reaction rate and summing over Boltzmann-weighted, individually determined strengths for the various nuclear states. Moreover, some additional simplifications were introduced as well (e.g. Axel-Brink hypothesis).

Calculation of inelastic neutrino-nucleus cross section

$$\sigma_{\nu A}(E_\nu, T) = \sigma^{d}_{\nu A}(E_\nu, T) + \sigma^{up}_{\nu A}(E_\nu, T)$$

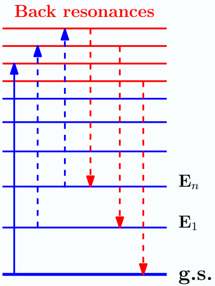
$$\sigma^{d}_{\nu A}(E_\nu, T) = \frac{G_F^2}{\pi W} \sum_{E_i < E_f} (E_\nu + E_i - E_f)^2 |\langle f|\sigma t_0|i\rangle|^2 \exp(-\frac{E_i}{T})$$

$$\sigma^{up}_{\nu A}(E_\nu, T) = \frac{G_F^2}{\pi W} \sum_{E_i > E_f} (E_\nu + E_i - E_f)^2 |\langle f|\sigma t_0|i\rangle|^2 \exp(-\frac{E_i}{T}).$$

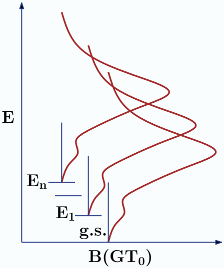
$W = \sum_i \exp(-E_i/T)$.
If, in accordance with Axel-Brink hypothesis, one assumes $\langle f|\sigma t_0|i\rangle = \langle f|\sigma t_0|g.s.\rangle$ then the value of $\sigma^{d}_{\nu A}(E_\nu)$ is independent of T.

Limitations of LSSM:

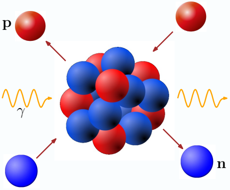
- The approach cannot be used for massive neutron-rich nuclei with $A > 65$.
- The Axel-Brink hypothesis is used to calculate $\sigma^{d}_{\nu A}(E_\nu)$.
- First-forbidden transitions cannot be calculated directly (additional simplifications are introduced).
- Detailed balance principle is violated $\boldsymbol{S(T,-E) \neq S(T,E)\exp\left(-\frac{E}{T}\right)}$



Axel-Brink hypothesis



Basics of Thermo Field Dynamics



- Thermal vacuum $|0(T)\rangle$:

$$\langle 0(T)|A|0(T)\rangle = \sum_i \frac{e^{-E_i/T}}{W}\langle i|A|i\rangle = \langle\langle A\rangle\rangle$$

- Thermal Hamiltonian

$$\mathcal{H} = H(a^\dagger, a) - \widetilde{H}(\tilde{a}^\dagger, \tilde{a}), \quad \mathcal{H}|0(T)\rangle = 0.$$

- Thermal state condition:

$$A|0(T)\rangle = \exp\left(-\frac{\mathcal{H}}{2T}\right)\widetilde{A}^\dagger|0(T)\rangle$$

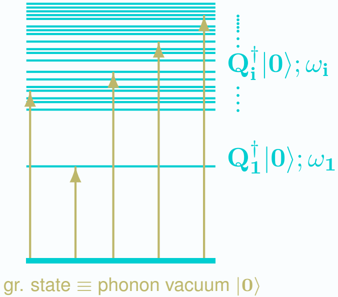
Thermal quasiparticle RPA

- The QPM Hamiltonian: $H = H_{WS} + H_{BCS} + H_{ph}$;
- Thermal quasiparticles: $\mathcal{H}_{WS+BCS} \approx \sum_j \varepsilon_j(\beta^\dagger_j\beta_j - \widetilde{\beta}^\dagger_j\widetilde{\beta}_j)$;
- Thermal phonons: $\mathcal{H} \approx \sum_k \omega_k(Q^\dagger_k Q_k - \widetilde{Q}^\dagger_k\widetilde{Q}_k)$.

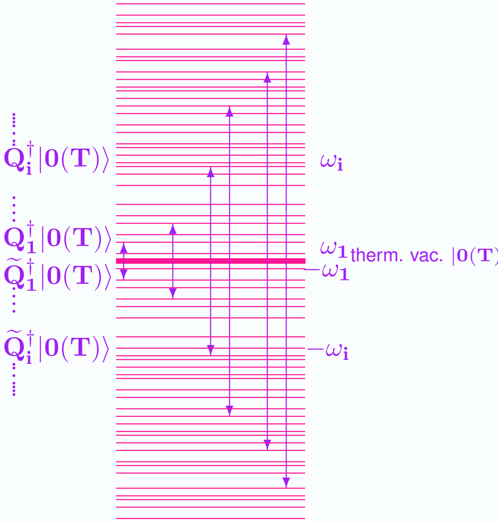
Finite temperature strength distributions

- $|0(T)\rangle \to Q^\dagger_k|0(T)\rangle$ – excitation process, $|0(T)\rangle \to \widetilde{Q}^\dagger_k|0(T)\rangle$ – de-excitation process;
- Transition strengths $\Phi_k = |\langle Q_k|\boldsymbol{\sigma t_0}|0(T)\rangle|^2$ and $\widetilde{\Phi}_k = |\langle\widetilde{Q}_k|\boldsymbol{\sigma t_0}|0(T)\rangle|^2$ obey the detailed balance principle $\widetilde{\Phi}_k = \exp\Big(-\omega_k/T\Big)\Phi_k$.

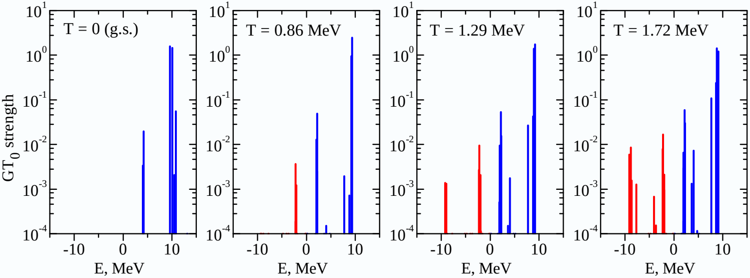
QRPA $(\mathrm{T}=0)$



TQRPA $(\mathrm{T}\neq 0)$

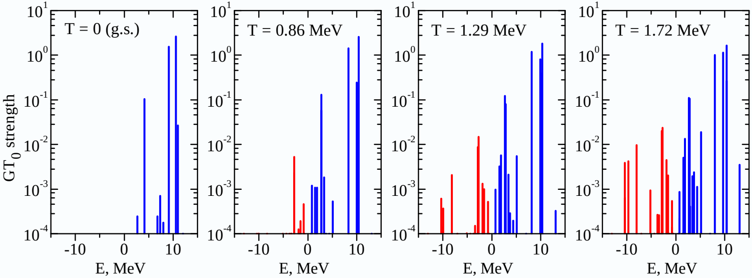


$\mathbf{GT_0}$ strength distributions in $^{56}\mathbf{Fe}$ at different temperatures:



$T = 0.86\ \mathrm{MeV}$ (1 GK) corresponds to the condition of a presupernova model for a $15M_\odot$ star; $T = 1.29\ \mathrm{MeV}$ (1.5 GK) - relates to neutrino trapping, $T = 1.72\ \mathrm{MeV}$ (2 GK) - to neutrino thermalization.

$\mathbf{GT_0}$ strength distributions in $^{82}\mathbf{Ge}$ at different temperatures:



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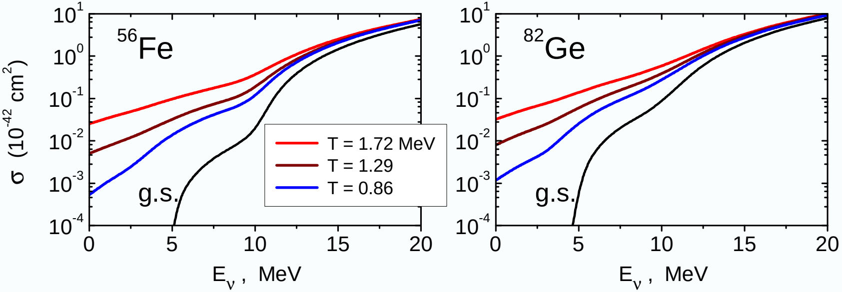
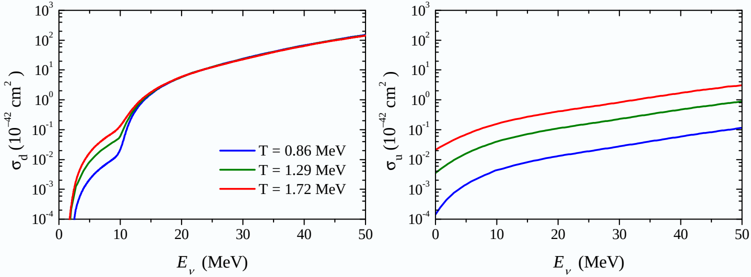
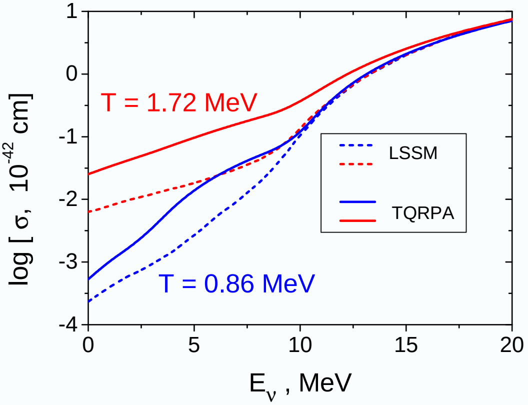
$$\begin{aligned}\sigma_{\nu A}(E_\nu,T) &= \sigma^{d}_{\nu A}(E_\nu,T) + \sigma^{up}_{\nu A}(E_\nu,T)\\ &= \frac{G_F^2}{\pi}\left\{\sum_k (E_\nu-\omega_k)^2\Phi_k + \sum_k (E_\nu+\omega_k)^2\widetilde{\Phi}_k\right\}\end{aligned}$$

- $\Phi_k = |\langle Q_k|\boldsymbol{\sigma} t_0|0(T)\rangle|^2$ and $E'_\nu = E_\nu - \omega_k$ for down-scattering;
- $\widetilde{\Phi}_k = |\langle \widetilde{Q}_k|\boldsymbol{\sigma} t_0|0(T)\rangle|^2$ and $E'_\nu = E_\nu + \omega_k$ for up-scattering.

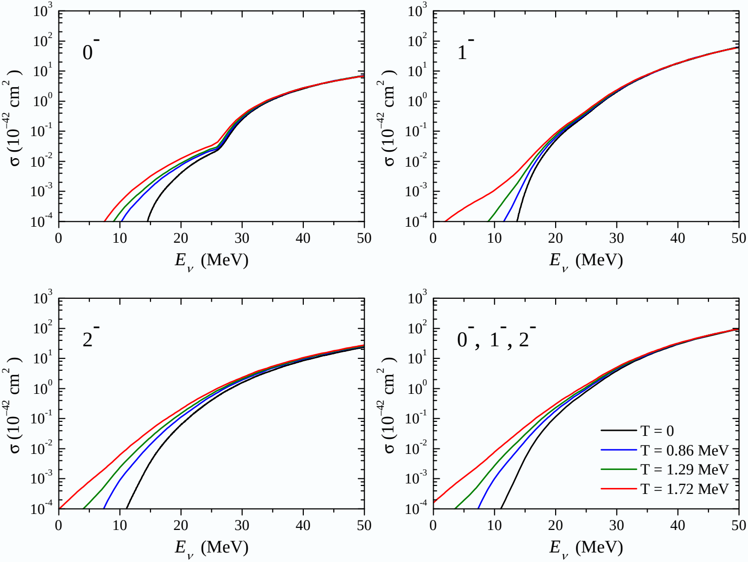




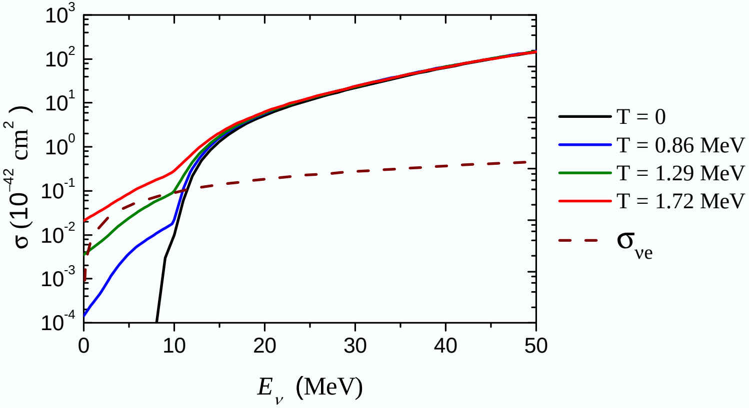
The down-scattering $\sigma^d_{\nu A}(E_\nu, T)$ (left panel) and the up-scattering $\sigma^{up}_{\nu A}(E_\nu, T)$ (right panel) parts of the INNS cross section for $^{54}\mathbf{Fe}$ at different T.



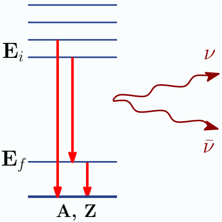
Comparison of the TFD-QRPA cross-sections for $\nu + {}^{56}\mathrm{Fe} \to \nu' + {}^{56}\mathrm{Fe}$ with those obtained within the LSSM calculations (K. Langanke, G. Martínez-Pinedo et al, Nucl. Phys. A 747 (2005) 87).



Contributions of first-forbidden transitions to the INNS cross section for $^{54}\mathrm{Fe}$ at different T



The INNS cross section at different T in comparison with the neutrino-electron inelastic scattering cross section $\sigma_{\nu e}(E_\nu) \approx \frac{G_F m_e}{\pi} E_\nu$



Decay probability:

$$\lambda_{if} = 3\frac{G_F^2 g_A^2}{60\pi^2\hbar^7 c^6}(E_i - E_f)^5 B(GT_0)_{if}$$
$$= 3\lambda_0 (E_i - E_f)^5 B(GT_0)_{if}, \quad \lambda_0 \approx 1.72\ \mathrm{s}^{-1}\ \mathrm{MeV}^{-5}$$

Total decay rate: $\Lambda = \sum_{if} \lambda_{if}\ g_i$, where $g_i \sim \exp(-E_i/T)$.

Partial decay rates within the TQRPA approach

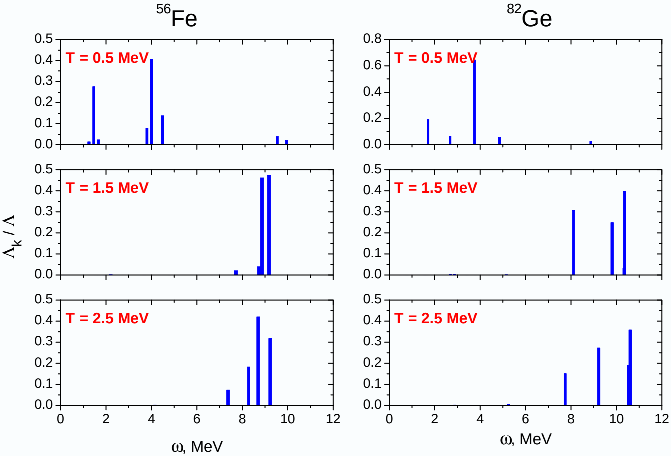
$$\Lambda_k = 3\lambda_0 \omega_k^5 \widetilde{\Phi}_k \quad \text{where} \quad \widetilde{\Phi}_k = |\langle 0(T)|\boldsymbol{\sigma} t_0|\widetilde{Q}_k^\dagger\rangle|^2, \quad \omega_k = E_\nu + E_{\bar{\nu}}.$$

Total decay rate:

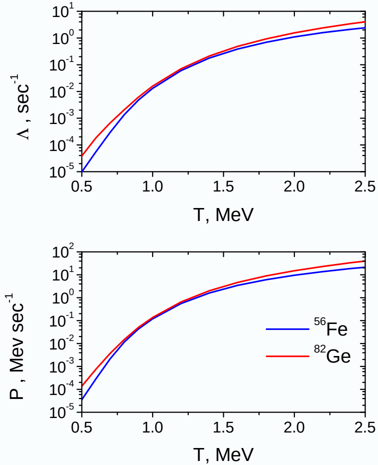
$$\Lambda = \sum\nolimits_k \Lambda_k.$$

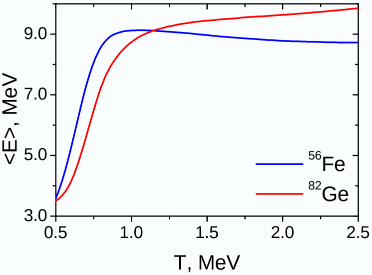
Energy emission rate:

$$P = \sum\nolimits_k \omega_k \Lambda_k.$$



Relative contribution to $\nu\bar{\nu}$-decay rate of different ranges of hot nuclear spectrum. Calculations are performed within the TQRPA approach for $^{56}\mathbf{Fe}$ and $^{82}\mathbf{Ge}$





The mean energy of $\nu\bar{\nu}$-pairs: $\langle E\rangle = \dfrac{P}{\Lambda}$

- The novel approach to study thermal effects on neutrino-nucleus reactions in supernova environment was presented. The approach is based on QRPA extended to finite temperatures within Thermo Field Dynamics formalism.

- In contrast to the LSSM calculations our method does not rely on the Brink hypothesis and it can be applied to massive neutron-rich nuclei. Moreover, the corresponding calculations are much less time consuming.

- Our calculations confirm results based on the LSSM calculations (by K. Langanke, G. Martínez-Pinedo et al.) about the thermal enhancement of the low-energy neutrino-nucleus cross sections.

- To improve the predictive power of the approach we are working to combine our TFD-based method with self-consistent QRPA calculations based on more realistic effective interactions (e.g. the Skyrme ones).

Credits: J. Wambach, V. Ponomarev.

THANK YOU FOR ATTENTION !