Structure of the Low-Lying States in the Odd-Mass Nuclei with $Z \sim 100$

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Introduction

In recent years an experimental program has been started aimed to extend the region where detailed spectroscopic information is available. In parallel, the theoretical efforts are necessary to extract from these data an information about the single particle levels lying near the Fermi surface and about an evolution of the single particle level scheme with Z and A. It is also important to extract from the data on the collective excitations the parameters of the residual forces which play the most important role at low excitation energies.

Basic concepts

- Nuclear mean field.
- Fluctuations of the nuclear mean field $\rightarrow$ Phonons and Particle-vibration coupling.

We consider the calculations of the energies and the two-quasiparticle structure of the collective low-lying states as the important task for nuclei with Z $\sim$ 100 since RPA type calculations for these nuclei are practically absent.

Selfconsistent mean feald or the Woods-Saxon potential

In recent years many calculations of the selfconsistent nuclear mean feeld have been performed using different choices of the Energy Density Functional. For this reason it is necessary to justify a possibility to use the Woods-Saxon form of the single-particle potential.
The experimentally investigated nuclei with Z $\sim$ 100 are deformed. The deformation leads to a more uniform distribution of the single-particle states emerjing from the high-j and low-j spherical subshells. For this reason a density profile of a deformed nucleus is relatively flat inside nucleus. This resemble the use of the Woods-Saxon potential for these nuclei.

The quasiparticle phonon model

The Hamiltonian of the QPM [7] contains the mean fields for protons and neutrons, monopole pairing and the multipole–multipole interaction, both isoscalar and isovector, acting in the particle–hole and the particle–particle channels

$$H = H_{sp} + V_{\rm pair} + V_M^{ph} + V_M^{pp}. \tag{1}$$

Here, H_{sp} is a one-body Hamiltonian, $V_{\rm pair}$ describes the monopole pairing, V_M^{ph} and V_M^{pp} are particle–hole separable multipole interaction and particle–particle multipole pairing interaction, respectively.

As the mean field term, we have taken the Woods–Saxon potential

$$\begin{aligned} V_{sp}(\vec{r}) &= V_{WS}(\vec{r}) + V_{so}(\vec{r}), \\ V_{WS}(\vec{r}) &= -V_0(1+\exp[(r-R(\theta,\varphi))/a])^{-1} \\ V_{so}(\vec{r}) &= -\kappa(\vec{p}\times\sigma)\nabla V_{WS}(\vec{r}) \end{aligned} \tag{2}$$

The quasiparticle phonon model

The Hamiltonian expressed in terms of the quasiparticle and phonon creation and annihilation operators has the form

$$H = \sum_q \varepsilon_q \alpha_q^+ \alpha_q + \sum_\nu \hbar\omega_\nu Q_\nu^+ Q_\nu + \sum_{q,q',\nu} f_{qq'}^{(\nu)} \alpha_q^+ \alpha_{q'} (Q_\nu^+ + Q_\nu). \quad (3)$$

Here the operator α_q^+ is a quasiparticle creation operator with quantum number q, and ν stands for the phonon quantum number $\nu = \lambda\mu i$, where $i = 1,2,\ldots$ label RPA phonons with given multipolarity $\lambda\mu$ in accordance with the phonon excitation energy; i.e., the one-phonon state $Q_{\lambda\mu 1}^+ |0\rangle_{RPA}$ has the lowest energy. The vector $|0\rangle_{RPA}$ is a phonon vacuum state.

The quasiparticle phonon model

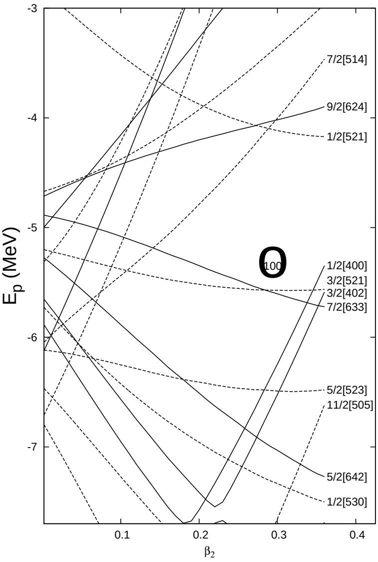
This Hamiltonian is diagonalized in a configurational space which includes one-quasiparticle and one-quasiparticle⊗phonon states. Correspondingly, the wave function of the state of the odd-proton nucleus takes the form

$$|\Psi_n\rangle = \left(\sum_q C_q^n \alpha_q^+ + \sum_{q\nu} D_{q\nu}^n \alpha_q^+ Q_\nu^+\right) |0\rangle_{RPA}. \qquad (4)$$

We have taken into account the phonons with the quantum numbers $\lambda\mu = 20$, 22, 30, 31, 32, and 44 and several values of i. The single-particle spectrum was taken from the bottom of the mean field potential up to $+5$ MeV.

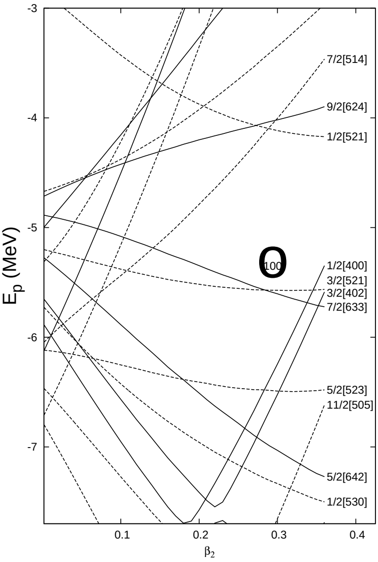
A comparison of the results of the selfconsistent calculations with the single-particle spectra obtained with Woods-Saxon potential demonstrate a good agreement for neutrons in nuclei with Z $\sim$ 100. Woods-Saxon proton singleparticle scheme confirms an existance of the gap at Z=96 and 100. In addition the experimental results indicate on existance of the proton 1/2[521] and 7/2[514] near the Fermi level at Z=103 in agreement with Woods-Saxon scheme.

Protons

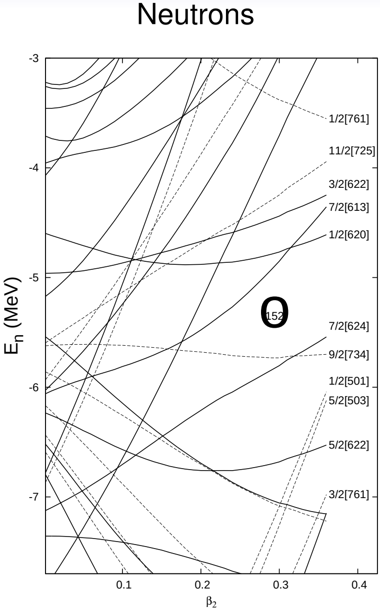


Analysis of the proton single-particle states extracted from the experimental data shows that the lowest states just above the gap at Z=96 are 7/2[633] and 3/2[521]. The next three orbitals are 7/2[514], 1/2[521] and 9/2[624]. This is in a correspondence with the Woods-Saxon level scheme calculated at the beta=0.26.

Protons



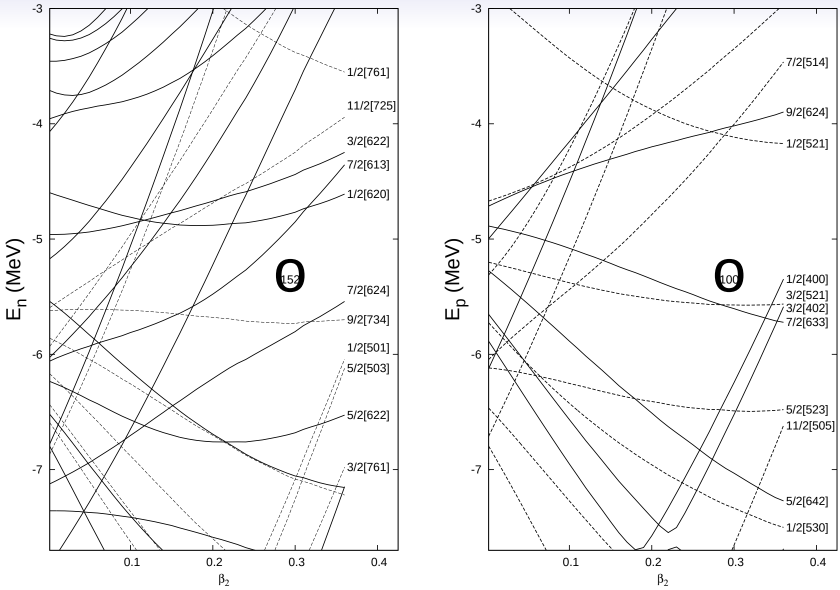
Comparison of the neutron W-S single-particle scheme with the single-particle energies extracted from the experimental data shows that in both cases just above N=152 the same closely lying single-particle levels, namely, 7/2[613], 1/2[620], 3/2[622] and 11/2[725] are located



Parameters of the Woods-Saxon potential used in the calculations.

A	isospin	V_0, MeV	r_0, fm	a, fm	κ, fm^2
243	n	46.0	1.26	0.72	0.430
243	p	62.0	1.24	0.65	0.370

$\beta_2 = 0.26$ $\qquad$ $\beta_4 = 0.06$



-3
-4
-5
-6
-7
0.1
0.2
0.3
0.4
E_n (MeV)
β2
O
152
1/2[761]
11/2[725]
3/2[622]
7/2[613]
1/2[620]
7/2[624]
9/2[734]
1/2[501]
5/2[503]
5/2[622]
3/2[761]
E_p (MeV)
O
100
7/2[514]
9/2[624]
1/2[521]
1/2[400]
3/2[521]
3/2[402]
7/2[633]
5/2[523]
11/2[505]
5/2[642]
1/2[530]

Residual forces

The factorized multipole-multipole forces used in our calculations are rather schematic. However, these forces have been tested in the known regions of the nuclide chart. As a result we know the values of the interaction constants determined for the rare earth and actinide regions. At the same time we know from the selfconsistent estimates of the interaction constants that they are smooth functions of the mass number. Therefore we can extrapolate the interaction constants from the known regions of Z and A to nuclei with Z $\sim$ 100.

Residual forces

$$V_{pair} = -\sum_{\tau=p,n} G_\tau \sum_{q,q'\in\tau} a^+_{q+} a^+_{q-} a_{q'-} a_{q'+},$$

$$V_M^{ph} = -\frac{1}{2} \sum_{\tau} \sum_{\lambda\mu\sigma} \left(\kappa_0^{\lambda\mu} + \rho(\tau)\kappa_1^{\lambda\mu}\right) M^+_{\lambda\mu\sigma}(\tau) M_{\lambda\mu\sigma}(\tau), \quad \rho(\tau) = \pm 1,$$

$$V_M^{pp} = -\frac{1}{2} \sum_{\tau} \sum_{\lambda\mu\sigma} G_\tau^{\lambda\mu} P^+_{\lambda\mu\sigma}(\tau) P_{\lambda\mu\sigma}(\tau),$$

$$M^+_{\lambda\mu\sigma}(\tau) = \sum_{q_1,q_2,\sigma_1,\sigma_2\in\tau} \langle q_1\sigma_1 | R_\lambda(r) Y_{\lambda\mu\sigma} | q_2\sigma_2\rangle a^+_{q_1\sigma_1} a_{q_2\sigma_2},$$

$$P^+_{\lambda\mu\sigma}(\tau) = \sum_{q_1,q_2,\sigma_1,\sigma_2\in\tau} \langle q_1\sigma_1 | R_\lambda(r) Y_{\lambda\mu\sigma} | q_2\sigma_2\rangle a^+_{q_1\sigma_1} \sigma_2 a^+_{q_2-\sigma_2},$$

$$R_\lambda(r) = \frac{d}{dr} V_{WS}(r),$$

Structure of some low-lying two- quasiparticle and collective octupole states in nuclei with Z $\sim$ 100

The structure of nuclei which have not been investigated yet or whose investigations are just started is interesting because we do not exclude a manifestation of the features which have not been observed in the well-investigated regions of the nuclide chart.

$K^\pi = 8^-$ states

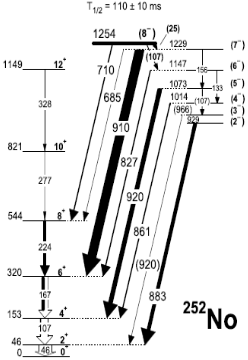
Experiment

Eur. Phys. J. A **33**, 327–331 (2007)
DOI 10.1140/epja/i2007-10469-3

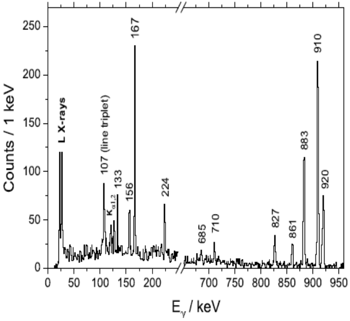
THE EUROPEAN PHYSICAL JOURNAL A

Identification of a K isomer in ^{252}No

B. Sulignano[1], S. Heinz[1], F.P. Heßberger[1,a], S. Hofmann[1,2], D. Ackermann[1], S. Antalic[3], B. Kindler[1], I. Kojouharov[1], P. Kuusiniemi[4], B. Lommel[1], R. Mann[1], K. Nishio[5], A.G. Popeko[6], S. Saro[3], B. Streicher[3], M. Venhart[3], and A.V. Yeremin[6]

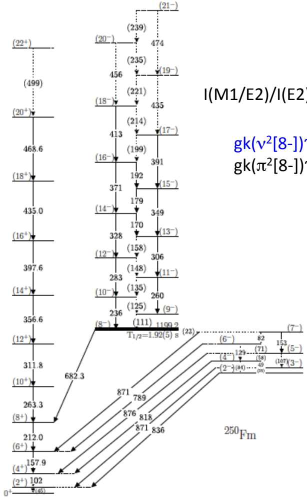


8^-:ν^2(7/2+[624],9/2-[734])



Experiment

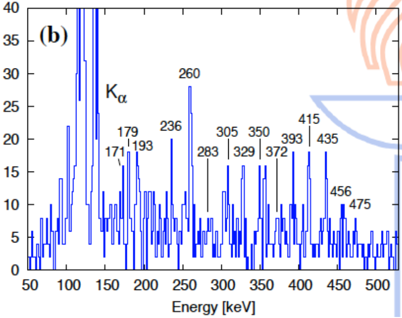
P.T. Greenlees et al., Phys. Rev. C78 (2008) 021303



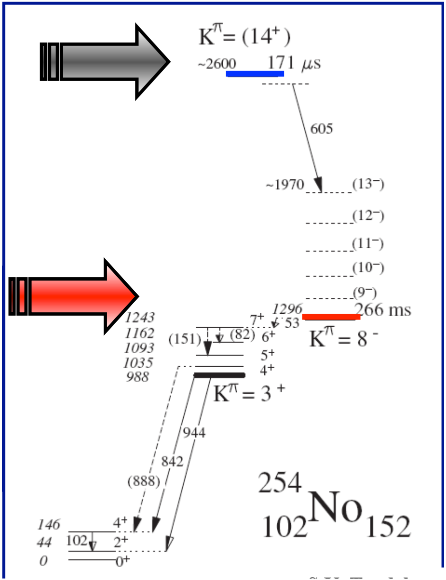
I(M1/E2)/I(E2)~(gk-gr)^2

gk(ν^2[8-])~0

gk(π^2[8-])~1



Experiment

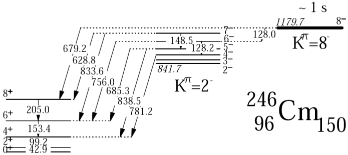


Kπ=(14+)

✓ assignment is tentative, but the isomer is firmly placed above the 8- one

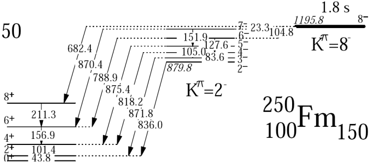
S.K. Tandel et al., Phys. Rev. Lett. 97 (2006)
R.D. Herzberg et al., Nature 442 (2006)

Experiment

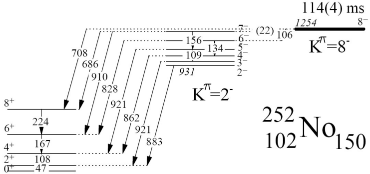


U. Tandel, Uni. Mass. Lowell, PhD Thesis
ANL (^{244}Pu target, MNT & pulsed beam)

N=150 Isotones
Z=96, 100 & 102



P.T. Greenlees et al., Phys. Rev. C78 (2008)



A. Robinson et al., Phys. Rev. C78 (2008)

$K^{\pi}=8^{-}$, 2-qp state

ν^2(7/2+[624],9/2-[734])

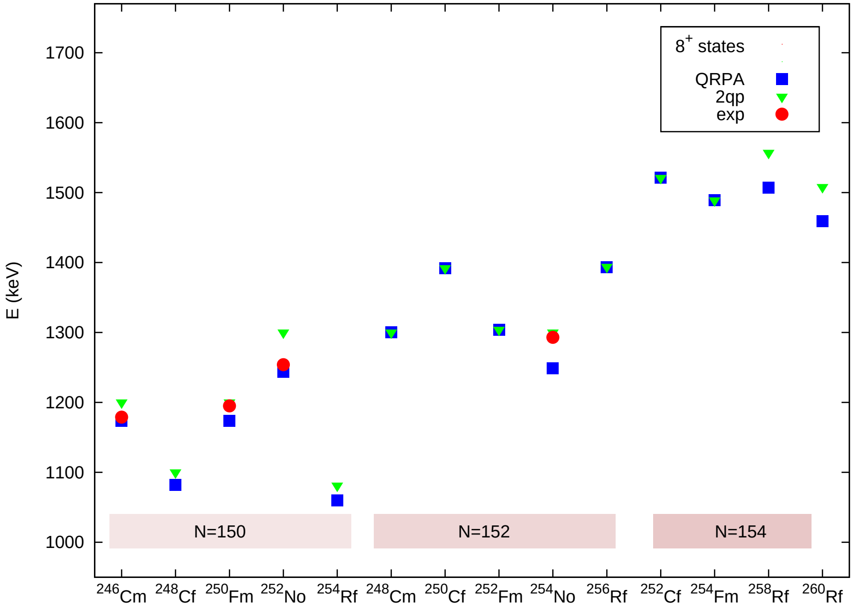
$K^\pi = 8^-$ states

Nucleus	$E(K^\pi = 8^-)$ exp, keV	$E(K^\pi = 8^-)$ 2qp, keV	$K^\pi = 8^-$ Structure	$E(K^\pi = 8^-)$ κ =0.021
^{246}Cm	1179	1200	nn 7/2[624]⊗9/2[734]	1174
^{248}Cf	–	1100	nn 7/2[624]⊗9/2[734]	1082
^{250}Fm	1195	1200	nn 7/2[624]⊗9/2[734]	1174
^{252}No	1254	1300	nn 7/2[624]⊗9/2[734]	1244
–		1336	pp 7/2[514]⊗9/2[624]	
^{254}Rf	–	1081	nn 7/2[624]⊗9/2[734]	1060
^{248}Cm	–	1300	nn 7/2[613]⊗9/2[734]	1300
^{250}Cf	–	1392	nn 7/2[613]⊗9/2[734]	1392
^{252}Fm	–	1304	nn 7/2[613]⊗9/2[734]	1304
^{254}No	1293	1300	pp 7/2[514]⊗9/2[624]	1249
–		1400	nn 7/2[613]⊗9/2[734]	
^{256}Rf	–	1394	nn 7/2[613]⊗9/2[734]	1393

$K^\pi = 8^-$ states

Nucleus	$E(K^\pi = 8^-)$ exp, keV	$E(K^\pi = 8^-)$ 2qp, keV	$K^\pi = 8^-$ Structure	$E(K^\pi = 8^-)$ κ =0.021
^{252}Cf	–	1521	nn 7/2[613]$\otimes$9/2[734]	1521
^{254}Fm	–	1489	nn 7/2[613]$\otimes$9/2[734]	1489
^{258}Rf	–	1557	pp 7/2[514]$\otimes$9/2[734]	1507
^{260}Rf	–	1508	pp 7/2[514]$\otimes$9/2[624]	1459

$K^\pi = 8^-$ states



8+ states
QRPA
2qp
exp
E (keV)
1700
1600
1500
1400
1300
1200
1100
1000
N=150
N=152
N=154
246Cm 248Cf 250Fm 252No 254Rf 248Cm 250Cf 252Fm 254No 256Rf 252Cf 254Fm 258Rf 260Rf

In the majority of cases, the lowest 8^- states are the neutron two-quasiparticle states. Exceptions are the cases of 254 No and 260Rf where the lowest 8- states are the proton two-quasiparticle states. However, in the cases of 252No and 258Rf, the proton two-quasiparticle states are located very near to the neutron two- quasiparticle states. The N=152 gap in the neutron s.p. spectrum shifts the neutron two- quasiparticle states to higher energies in 254 No and 260Rf and favours low energy proton two- quasiparticle states.

Hexadecupole excitations with $K^\pi = 3^+$

Nucleus	$E(K^\pi = 3^+)$ exp, keV	cal, keV	Structure (in %)	
^{246}Cm	–	1422	nn 7/2[624]⊗1/2[631]	21
			nn 7/2[624]⊗1/2[620]	18
			nn 7/2[613]⊗1/2[620]	13
^{248}Cf	–	1622	nn 7/2[624]⊗1/2[620]	53
			nn 7/2[624]⊗1/2[631]	20
^{250}Fm	–	1604	nn 7/2[624]⊗1/2[620]	24
			nn 7/2[624]⊗1/2[631]	22
			pp 5/2[512]⊗1/2[521]	14
			nn 7/2[613]⊗1/2[620]	11
^{252}No	–	1094	pp 7/2[514]⊗1/2[521]	75
			pp 5/2[512]⊗1/2[521]	5
^{254}Rf	–	1072	pp 7/2[514]⊗1/2[521]	41
			pp 5/2[512]⊗1/2[521]	32

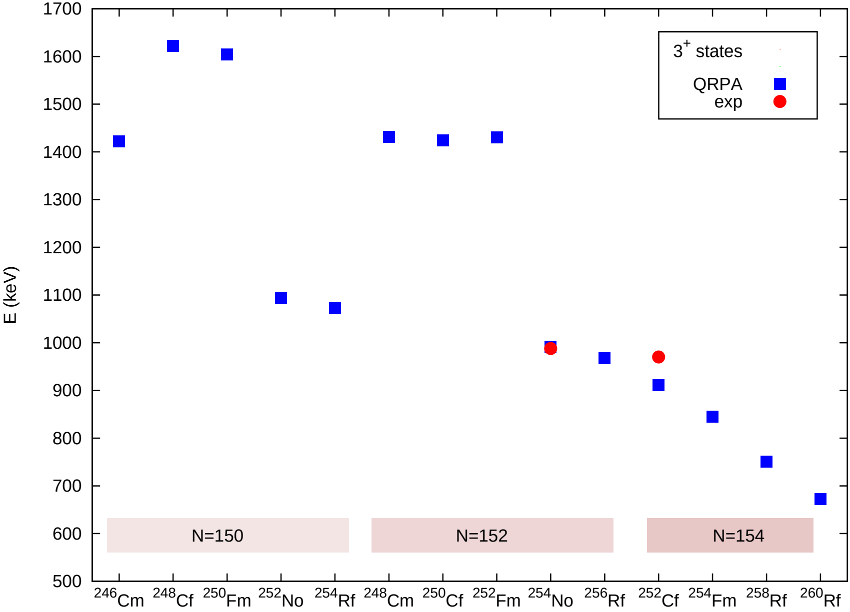
$K^\pi = 3^+$ states

Nucleus	$E(K^\pi = 3^+)$ exp, keV	cal, keV	Structure (in %)	
^{248}Cm	–	1431	nn 7/2[613]$\otimes$1/2[620]	57
			nn 7/2[624]$\otimes$1/2[620]	20
^{250}Cf	–	1424	nn 7/2[613]$\otimes$1/2[620]	57
			nn 7/2[624]$\otimes$1/2[620]	20
^{252}Fm	–	1430	nn 7/2[613]$\otimes$1/2[620]	51
			nn 7/2[624]$\otimes$1/2[620]	14
			pp 7/2[514]$\otimes$1/2[521]	7
^{254}No	988	992	nn 7/2[613]$\otimes$1/2[620]	7
			pp 7/2[514]$\otimes$1/2[521]	71
			pp 5/2[512]$\otimes$1/2[521]	6
^{256}Rf	–	968	nn 7/2[613]$\otimes$1/2[620]	13
			pp 7/2[514]$\otimes$1/2[521]	37
			pp 5/2[512]$\otimes$1/2[521]	29

$K^\pi = 3^+$ states

Nucleus	$E(K^\pi = 3^+)$ exp, keV	cal, keV	Structure (in %)	
^{252}Cf	970	911	nn 7/2[613]$\otimes$1/2[620]	84
^{254}Fm	–	845	nn 7/2[613]$\otimes$1/2[620]	83
			nn 9/2[615]$\otimes$3/2[622]	3
^{258}Rf	–	751	nn 7/2[613]$\otimes$1/2[620]	56
			pp 7/2[514]$\otimes$1/2[521]	16
			pp 5/2[512]$\otimes$1/2[521]	7
^{260}Rf	–	672	nn 7/2[613]$\otimes$1/2[620]	35
			nn 11/2[615]$\otimes$5/2[622]	15
			pp 7/2[514]$\otimes$1/2[521]	19

$K^{\pi} = 3^{+}$ states



E (keV)
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
3+ states
QRPA
exp
N=150
N=152
N=154
246Cm
248Cf
250Fm
252No
254Rf
248Cm
250Cf
252Fm
254No
256Rf
252Cf
254Fm
258Rf
260Rf

Collective states

- Especially interesting are the low-lying nuclear collective excitations which tell us about the softness of a nucleus with respect to some collective modes. There is already experimental information on some of these states. Study of the collective states gives an information about residual forces acting among nucleons.
- The low-lying octupole states play an important role in the structure of the well deformed axially symmetric nuclei. The interesting examples can be found among the rare earth nuclei and the actinides. The appearance of the low-lying octupole excitations can be related to the alpha-clustering of nuclei.

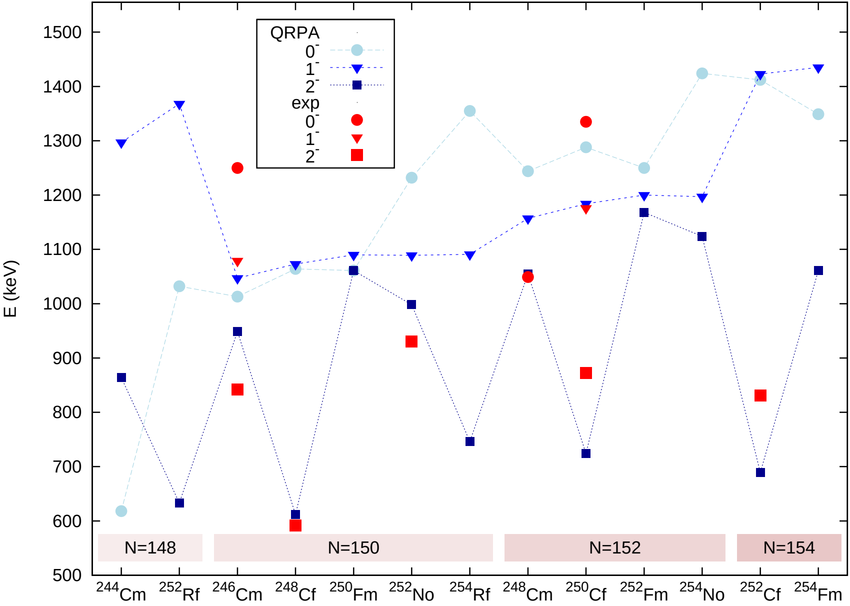
Octupole vibrations
$K^\pi = 0^-, 1^-, 2^-$ states

Nucleus	$E(K^\pi = 2^-)$ exp	$E(K^\pi = 2^-)$ cal	$E(K^\pi = 1^-)$ exp	$E(K^\pi = 1^-)$ cal	$E(K^\pi = 0^-)$ exp	$E(K^\pi = 0^-)$ cal
$^{244}Cm_{148}$	–	864	–	1297	–	618
$^{252}Rf_{148}$	–	633	–	1368	–	1032
$^{246}Cm_{150}$	842	949	1079	1047	1250	1013
$^{248}Cf_{150}$	592	612	–	1073	–	1064
$^{250}Fm_{150}$	–	1061	–	1090	–	1061
$^{252}No_{150}$	930	998	–	1089	–	1232
$^{254}Rf_{150}$	–	746	–	1091	–	1355

$K^\pi = 0^-, 1^-, 2^-$ states

Nucleus	$E(K^\pi = 2^-)$ exp	$E(K^\pi = 2^-)$ cal	$E(K^\pi = 1^-)$ exp	$E(K^\pi = 1^-)$ cal	$E(K^\pi = 0^-)$ exp	$E(K^\pi = 0^-)$ cal
$^{248}Cm_{152}$	–	1055	–	1157	1049	1244
$^{250}Cf_{152}$	872	724	1176	1184	1335	1288
$^{252}Fm_{152}$	–	1168	–	1200	–	1250
$^{254}No_{152}$	–	1124	–	1197	–	1424
$^{252}Cf_{154}$	831	689	–	1423	–	1412
$^{254}Fm_{154}$	–	1061	–	1435	–	1349

$K^\pi = 0^-, 1^-, 2^-$ states



Summary

We have investigated excited states with
$K^\pi = 8^-, 3^+, 0^-, 1^-, 2^-$.

- The states with $K^\pi = 8^-$ are the pure two–quasiparticle states. It means that their energies are very sensitive to the single particle level scheme near the Fermi surface. However the blocking effect is very important for the calculations of the energies of these states.
- The experimental energies of the $K^\pi = 3^+$ states cannot be described without inclusion into consideration of the hexadecapole residual forces. To describe the energies of the $K^\pi = 3^+$ states we have taken practically the same interaction constant as was used before for the rare earths nuclei and the light actinides.

Summary

- The energies of the collective $K^\pi = 0^-, 1^-, 2^-$ states have been calculated taking into account octupole residual forces with the interaction constant which is the same for all considered K^π. The value of this constant is close to the value used before for the rare earth and the light actinide nuclei. For the majority of the considered nuclei the lowest ones among the octupole excitations are the 2^- states. These states are especially low in the isotopes of Cf and Rf . In the other considered nuclei, the energies of the 2^- states are close to 1 MeV.

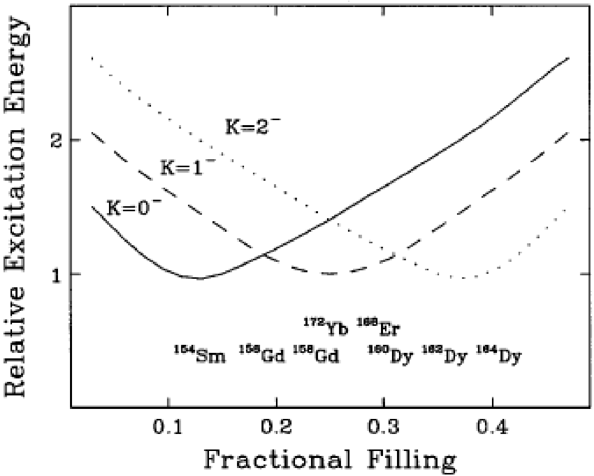
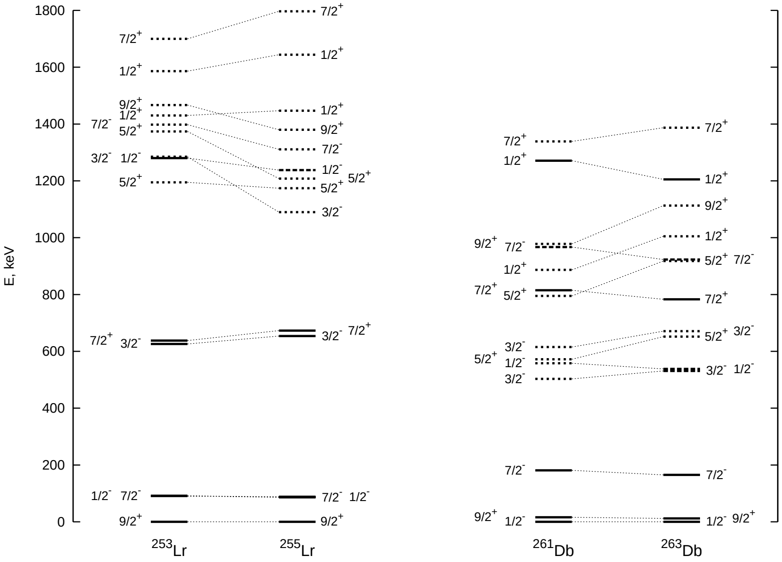
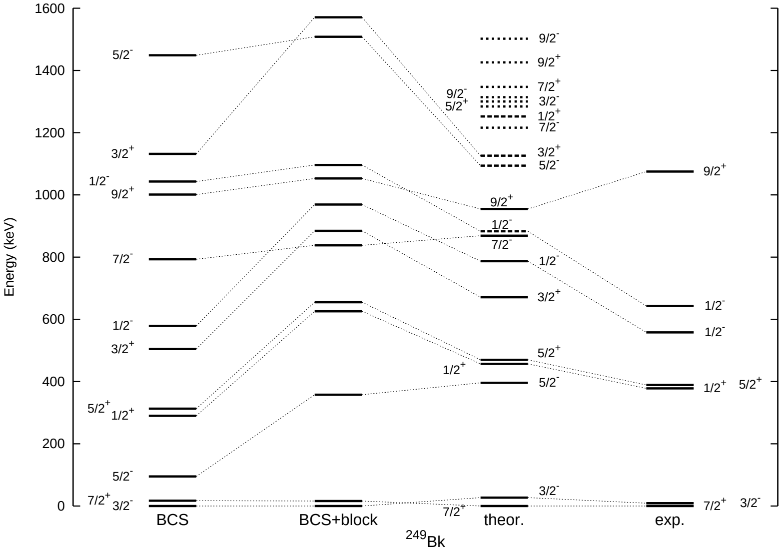


FIG. 2. Schematic illustration of the energies of the $K^{\pi}=0^{-}$, 1^{-}, 2^{-} octupole vibrational bands as a function of the fractional filling of the proton and neutron valence shells. The approximate locations of the nuclei which are shown here generally reproduce the observed ordering.

Odd-proton nuclei with Z ~ 100





Energy (keV)
1600
1400
1200
1000
800
600
400
200
0
BCS
BCS+block
theor.
exp.
249Bk
5/2-
3/2+
1/2-
9/2+
7/2-
1/2-
3/2+
5/2+
1/2+
5/2-
7/2+
3/2-
9/2-
9/2+
7/2+
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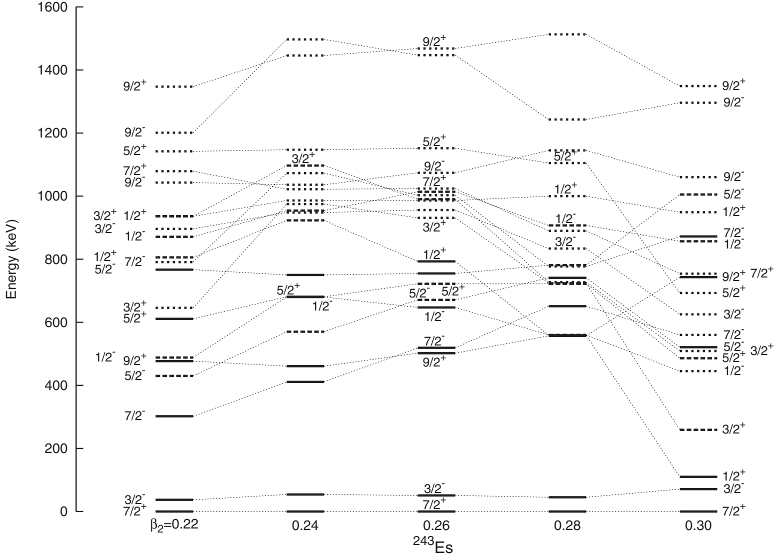
7/2-
1/2-
3/2+
5/2+
1/2+
5/2-
3/2-
7/2+
9/2+
1/2-
1/2-
1/2+
5/2+
7/2+
3/2-

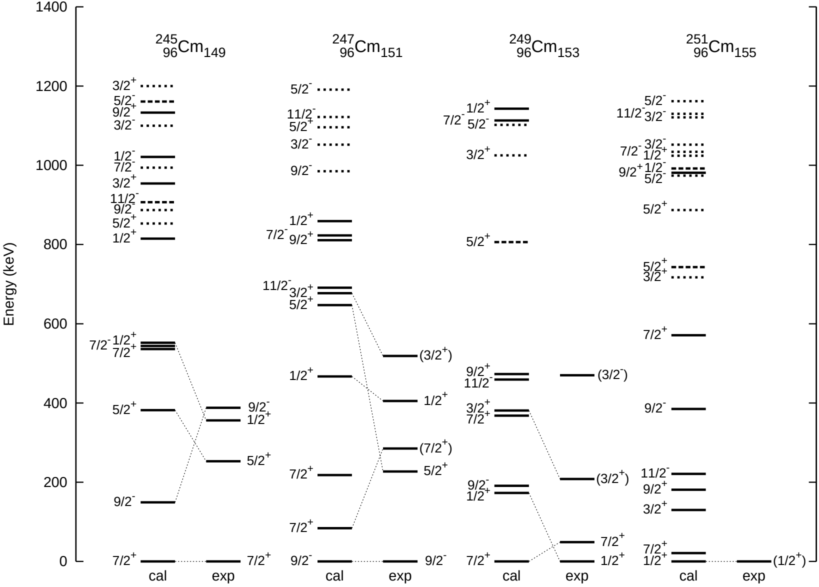
TABLE III. Experimental $E^*_{\rm exp}$ and calculated $E^*_{\rm cal}$ excitation energies and the structure of the excited states of ^{249}Bk. The energies are given in keV. The experimental data are taken from [32]. $Q_{\lambda\mu}$ denotes a phonon of multipolarity $\lambda\mu$ and $i = 1$.

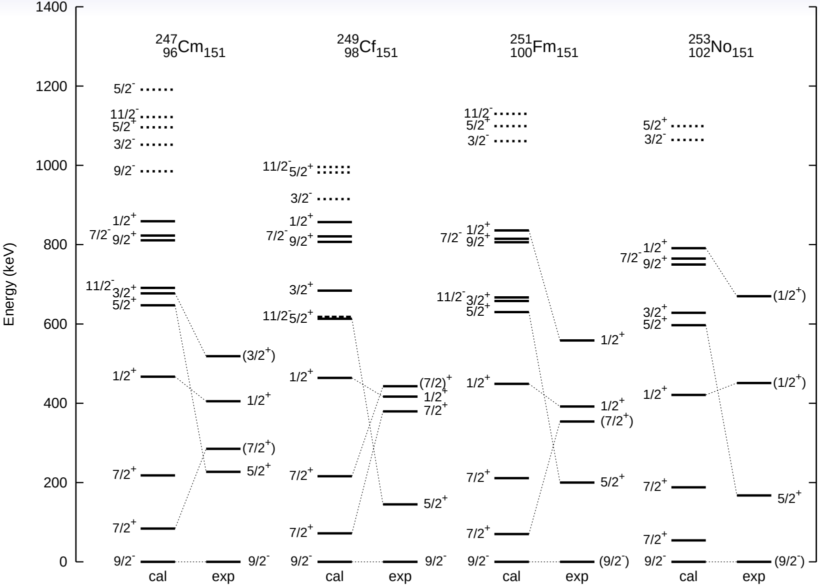
J^π	$E^*_{\rm exp}$	$E^*_{\rm cal}$	Structure
$7/2^+$	0	0	7/2[633] 94%
$3/2^-$	9	27	3/2[521] 95%
$5/2^-$		396	5/2[523] 95%
$1/2^+$	378	457	1/2[660] 85% + 3/2[651]$\times Q_{22}$ 10%
$5/2^+$	389	470	5/2[642] 823% + 5/2[642]$\times Q_{20}$ 10%
$3/2^+$		671	3/2[651] 75% + 1/2[660]$\times Q_{22}$ 20 %
$1/2^-$		787	1/2[530] 79% + 5/2[642]$\times Q_{32}$ 12%
$7/2^-$		879	7/2[514] 96%
$1/2^-$		883	1/2[521] 52% + 3/2[521]$\times Q_{22}$ 43%
$9/2^+$		955	9/2[624] 90% + 5/2[512]$\times Q_{32}$ 8%
$5/2^-$		1094	5/2[512] 65% + 7/2[633]$\times Q_{31}$ 15% + 9/2[624]$\times Q_{32}$ 13%
$3/2^+$		1126	3/2[402] 26% + 7/2[633]$\times Q_{22}$ 64%
$7/2^-$		1216	3/2[521]$\times Q_{22}$ 100%
$1/2^+$		1252	3/2[521]$\times Q_{32}$ 51% + 5/2[642]$\times Q_{22}$ 19% + 1/2[400] 14% + 5/2[523]$\times Q_{32}$ 12%
$5/2^+$		1284	1/2[660]$\times Q_{22}$ 82% + 5/2[402] 14%
$3/2^-$		1290	7/2[633]$\times Q_{32}$ 94%



Energy (keV)
1600
1400
1200
1000
800
600
400
200
0
β2=0.22
0.24
0.26
0.28
0.30
243Es
9/2+
9/2-
5/2+
7/2+
9/2-
3/2+
1/2+
3/2-
1/2-
1/2+
7/2-
5/2-
3/2+
5/2+
1/2-
9/2+
5/2-
7/2-
3/2-
7/2+
3/2+
5/2+
1/2-
9/2+
5/2+
9/2-
7/2+
3/2+
1/2+
5/2-
5/2+
1/2-
7/2-
9/2+
3/2-
7/2+
5/2+
1/2+
1/2-
3/2-
9/2+
9/2-
9/2-
5/2-
1/2+
7/2-
1/2-
9/2+
7/2+
5/2+
3/2-
7/2-
5/2-
3/2+
5/2+
1/2-
3/2+
1/2+
3/2-
7/2+

Odd-neutron nuclei with Z $\sim$ 100





Energy (keV)
0
200
400
600
800
1000
1200
1400
$^{247}_{96}Cm_{151}$
$^{249}_{98}Cf_{151}$
$^{251}_{100}Fm_{151}$
$^{253}_{102}No_{151}$
cal
exp
cal
exp
cal
exp
cal
exp

Summary

We have presented the results of calculations of the excitation energies and the structure of the low-lying states of the odd-mass nuclei with Z $\sim$ 100. The quasiparticle-phonon interaction and the blocking effect have been taken into account in the calculations. It is shown that below 400 keV the main component in the structure of the states is a one-quasiparticle component. However, above 600 keV the excitation of the phonons and the quasiparticle-phonon interaction begin to play an important role in a description of the properties of the excited states of nuclei with Z $\sim$ 100.

Thank you