Skyrme-RPA analysis of multipole giant resonances

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TU Dresden, Institute of nalysis, Dresden, Germany

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Motivation and Content:

PRC'02,06

- Self-consistent Skyrme RPA for spherical (1D) and deformed (2D) nuclei:
  - SRPA : fully self-consistent separable RPA - since 2002
  - fRPA : full RPA (without separable ansatz) - since 2014

- Investigation of multipole giant resonances (GR):
  - E1(T=1) giant resonance (GR) in rare-earth, actinide and superheavy deformed nuclei <u>: model test + equil. def. + systematic study</u> PRC'08
  - E1 PDR in Sn isotopes IJMPE'11
  - deformation effect on E1 strength near the threshold IJMPE'09
  - E0(T=0) GR in deformed nuclei: <u>E0-E2 coupling</u> EPJA15
  - spin-flip M1 GR:, tensor contribution, failure of Skyrme forces to descsribe both one- and two-bump structures PRC'09

- <u>In the present talk:</u> PRC'11,13,14
  - exotic E1 GR: PDR, toroidal (TR), compression (CR):
    - TR as a measure of nuclear vorticity
    - PDR as a local peripheral manifestation of TR

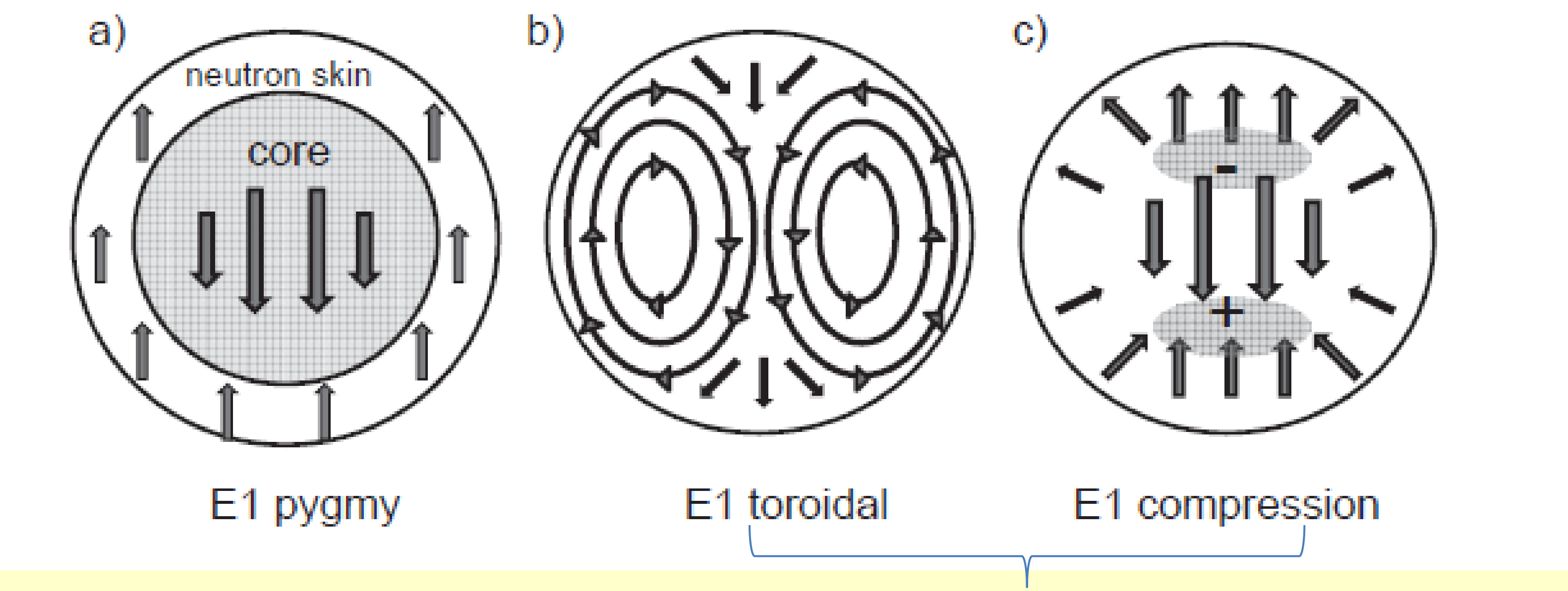
Exotic dipole resonances

[1] V.M. Dubovik and A.A. Cheshkov, Sov. J. Part. Nucl. v.5, 318 (1975).
[2] S.F. Semenko, Sov. J. Nucl. Phys. v. 34, 356 (1981).

R. Mohan et al (1971),

V.M. Dubovik (1975)
S.F. Semenko (1981)

M.N. Harakeh (1977)
S. Stringari (1982)



E1 pygmy

E1 toroidal

E1 compression

Dominate in E1(T=0) channel
(after exclusion of spurious E1(T=0) c.m. motion)

	$E = 68\,A^{-1/3} MeV$	$E = 132\,A^{-1/3} MeV$
irrotationalal	vortical	irrotational
HD	Elastic, no restoring force in HD	HD

Experiment:
TR and CR consitute low- and high-energy ISGDR branches

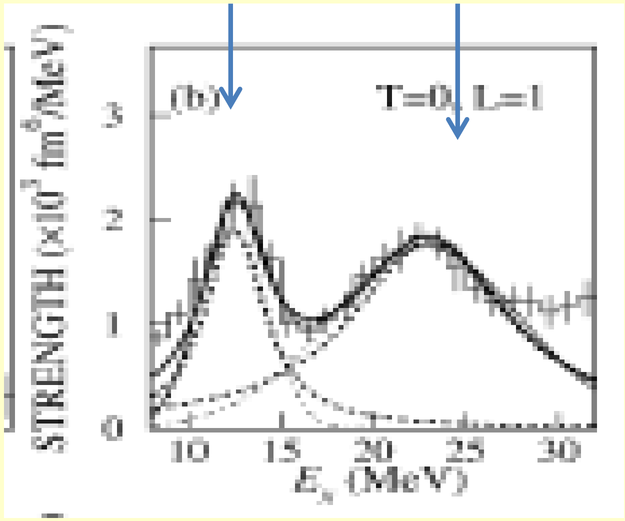
(α, α')

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

LE (toroidal)

HE (compression)



Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes and manifestations of vorticity:

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).

M.N. Harakeh et al, PRL 38, 676 (1977).

S.F. Semenko, SJNP 34 356 (1981).

J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).

S. Stringari, PLB 108, 232 (1982).

E. Wust et al, NPA 406, 285 (1983).

E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).

S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).

I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).

E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).

N.Ryezayeva et al, PRL 89, 272502 (2002).

G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).

D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).

V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).

J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).

A. Richter, NPA 731, 59 (2004).

X. Roca-Maza et al, PRC 85, 024601 (2012).

……….

N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007). review

Our results have been published in:

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely,
"General treatment of vortical, toroidal, and compression modes",
Phys. Rev. C84, n.3, 034303 (2011)

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
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Eur. Phys. J. A, v.49, 119 (2013).

J. Kvasil, V.O. Nesterenko, A. Repko, W. Kleinig, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in 124Sn",
Phys. Scr., T154, 014019 (2013).

P.-G. Reinhard, V.O. Nesterenko, A. Repko, and J. Kvasil,
"Nuclear vorticity in isoscalar E1 modes: Skyrme-RPA analysis",
Phys. Rev. C89, 024321 (2014).

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
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V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
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Strength functions SLy6

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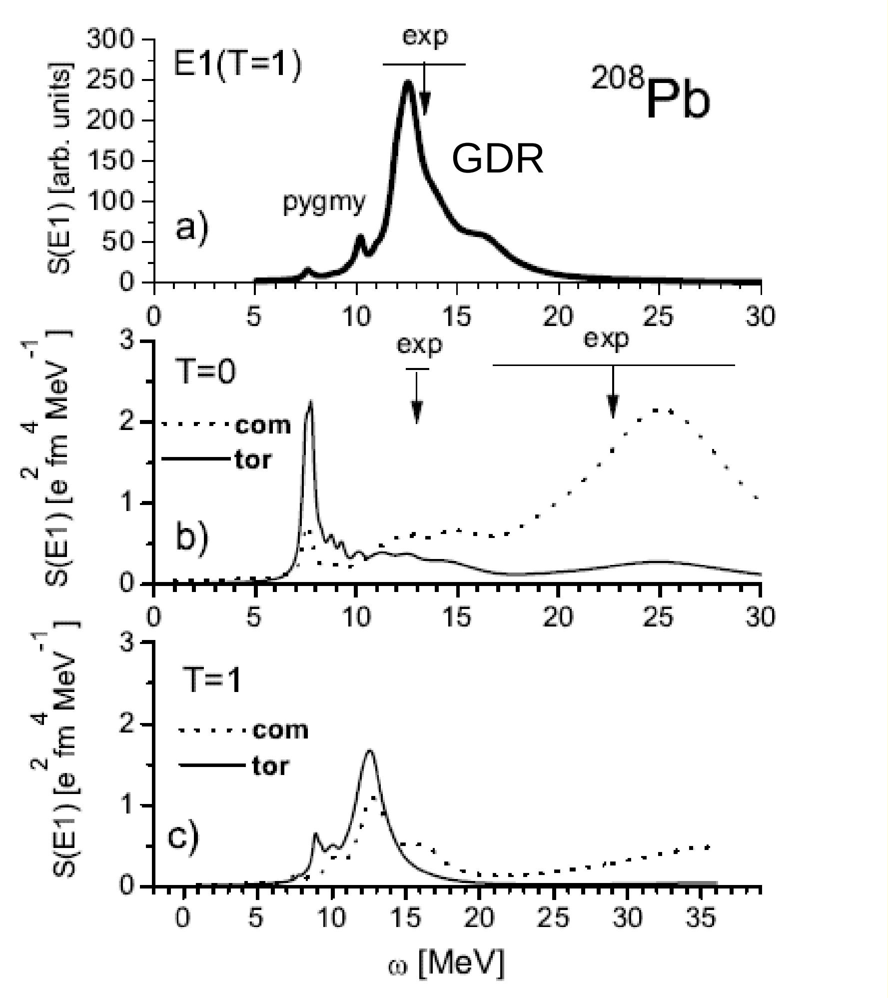
PRC, 87, 024305 (2013)



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(α,α') experiment
of Uchida et al (2003)

PDR region may host TR and CR!

Review:
D. Savran, T. Aumann, and A. Zilges,
“Experimental studies of the Pygmy Dipole Resonance”
Prog. Part. Nucl. Phys. 70, 210 (2013).

Toroidal and compression operators

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard,
P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \frac{5}{3} r < r^2 >_0]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot\underbrace{[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]}_{\text{vortical flow}\quad \vec{\nabla}\times\vec{j}(\vec{r})\neq 0}$$

- second-order part of the electric operator

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi + \vec{\nabla}\times(\vec{r}\upsilon) + \vec{\nabla}\times\vec{\nabla}\times(\vec{r}\chi)$$

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})\,]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}[1-\frac{(kr)^2}{2(2\lambda+3)}+\ldots]$$

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^\lambda Y_{\lambda\mu}$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3 - \frac{5}{3} r < r^2 >_0]Y_{1\mu}\,\underbrace{[\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})]}_{\text{irrotational flow}}$$

- probe operator of the compression mode
- c.m. corrections, r^3-dependence
- relation of TR and CR
- main IS-E1 vortical and irrotational flow

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3} r < r^2 >_0]Y_{1\mu}$$

$$\hat{M}_{com}(E1\mu) = -k\hat{M}'_{com}(E1\mu)$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

P.-G. Reinhard, V.O. Nesterenko, A. Repko, and J. Kvasil,
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Toroidal motion as the measure of the nuclear vorticity

Introduction of voricity:

Nuclei demonstrate both

- **irrotational flow** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$

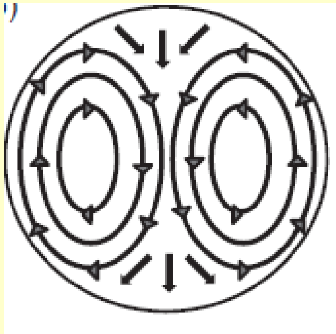
examples: most of electric giant resonances (GR)

- **vortical flow** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$

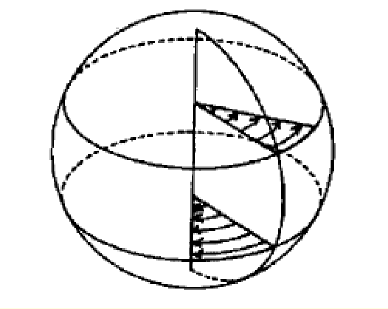
examples:

- **nuclear rotation of deformed nuclei,**
- **s-p excitations,**
- **toroidal E1 GR**
- **twist M2 GR**

- rotation-like oscillations



Toroidal E1
(to be considered)



Twist M2

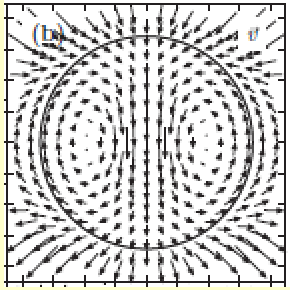
Vortical current:

- does not contribute to the continuity equation (CE) $\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$
- is necessary to get the complete current distribution

Two+one conceptions of nuclear vorticity : HD, RW + toroidal

1. Hydrodynamical vorticity:

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$



$$(\vec{\nabla} \times \delta\vec{j}_{nuc}) \rightarrow \rho_0(\vec{r})(\vec{\nabla} \times \delta\vec{v}) \rightarrow \rho_0(\vec{r})\,\vec{w}(\vec{r})$$

2. Wambach vorticity ⟷ j_+ vorticity

D.G.Raventhall, J.Wambach,
NPA <u>475</u>, 468 (1987).

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$ **- continuity equation**

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{*}_{\lambda\lambda+1\mu}]$$

$$\delta\vec{j}^{\nu}_{1\mu}(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, 0 \right\rangle = -\frac{i}{\sqrt{3}} [\underbrace{j^{\nu}_{10}}_{j_-}(r)\vec{Y}^{*}_{10\mu} + \underbrace{j^{\nu}_{12}}_{j_+}(r)\vec{Y}^{*}_{12\mu}]$$

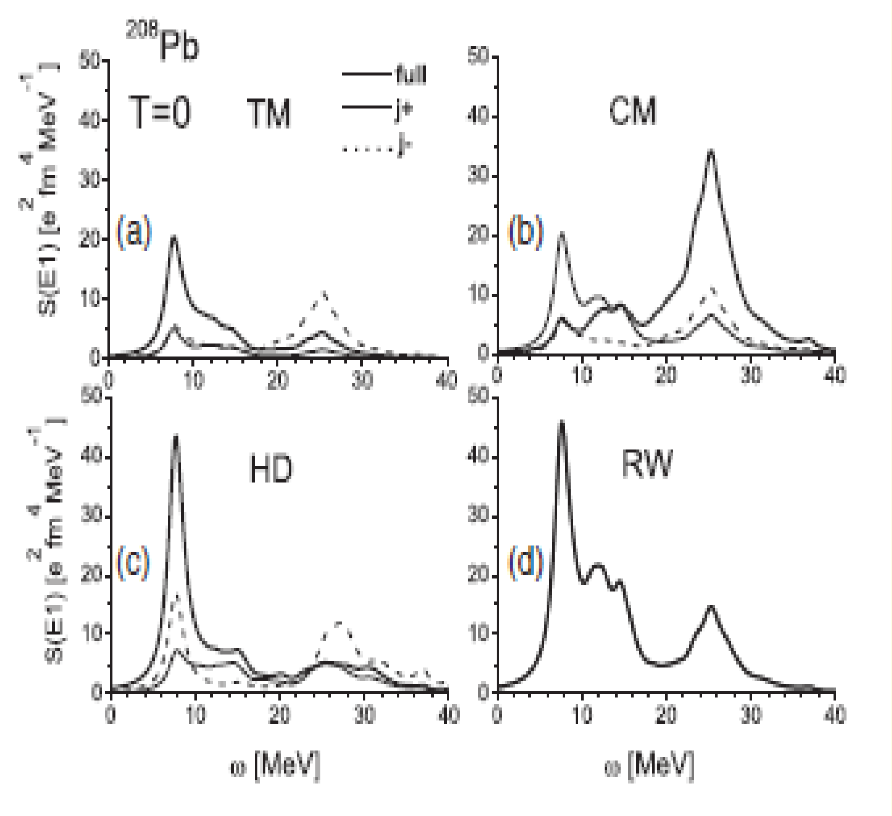
- current transition density

$$\boxed{j^{\nu}_{+}(r)}$$

- independent part of charge-current distribution, decoupled to CE
- may be the measure of the vorticity

HD and j+ prescriptions give opposite conclusions on CM vorticity!

j+ and j- contributions



-Both current components are peaked at low-energy and high-energy regions

-They serve as building blocks of TM, CM, HD, RW.

-TM, CM and HD are formed by constructive interference of the current components while in other regions there is the destructive interference .

-Both j+ and j- are equally active in vortical TM and irrotational CM.

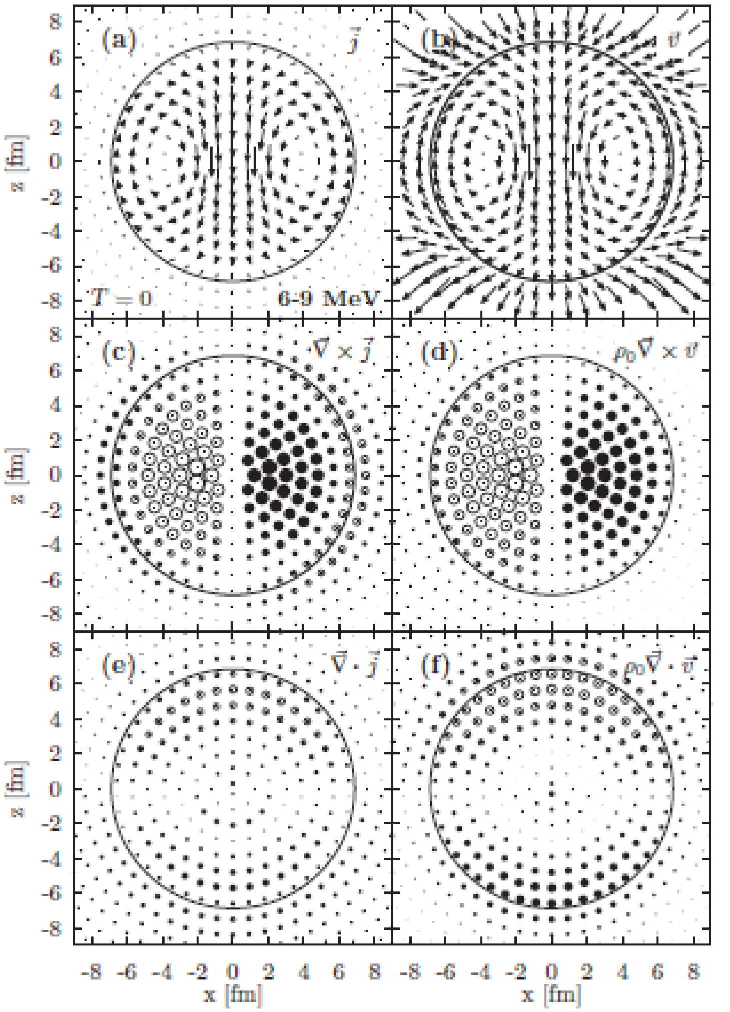
-j+ has no any strong advantage to be a vortical descriptor!

$$\langle \nu / \hat{M}_{tor}(E1\mu)/0\rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{\sqrt{2}}{5} r^2 j_+^{\nu}(r) + (r^2 - \langle r^2\rangle_0) j_-^{\nu}(r)]$$

$$\langle \nu / \hat{M}_{com}(E1\mu)/0\rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{2\sqrt{2}}{5} r^2 j_+^{\nu}(r) - (r^2 - \langle r^2\rangle_0) j_-^{\nu}(r)]$$

The vortical or irrotational character of the flow is provided not by j+ or j- components separately but by their proper superposition

j v

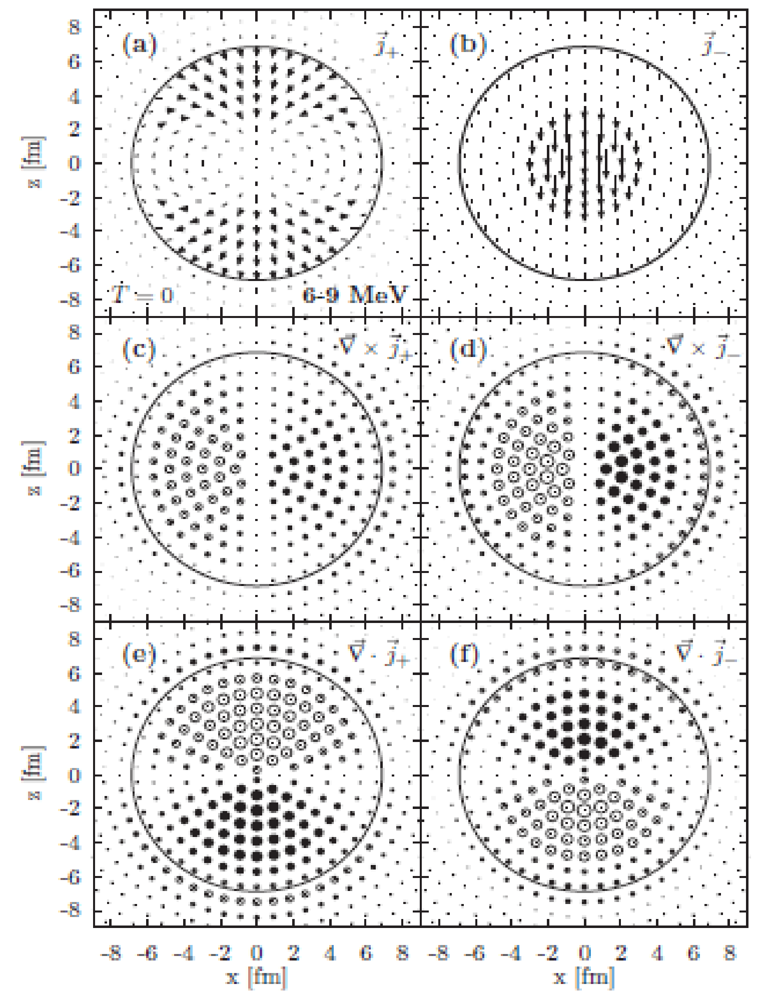


- toroidal motion
- j-v difference at the surface
- curls are stronger than divs

all RPA states
at E=6-9 MeV

only w. f.,
not toroidal
external field

j+ j-



j+, j-:
- different fields
- both have strong curls and divs
- Both locally vortical and irrotational
- no any curl-advantge of j+ over j-

Finally:

- **RW conception of the vorticity is not relevant:**
  - **CE-unrestricted in integral sense,**
  - **failure for CM,**
  - **j+ has no advantages over j-.**

- **TR conception is more correct:**
  - **vortical by construction,**
  - **locally CE-unrestricted,**
  - **close to HD conception,**
  - **gives visually vortical image,**
  - **correct for both TR and CR.**

So just the toroidal current and strength functions are the best fingerprint and measure of the nuclear vorticity .

Relation of E1 toroidal and pygmy resonances

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
Phys. Rev. C87, 024305 (2013).

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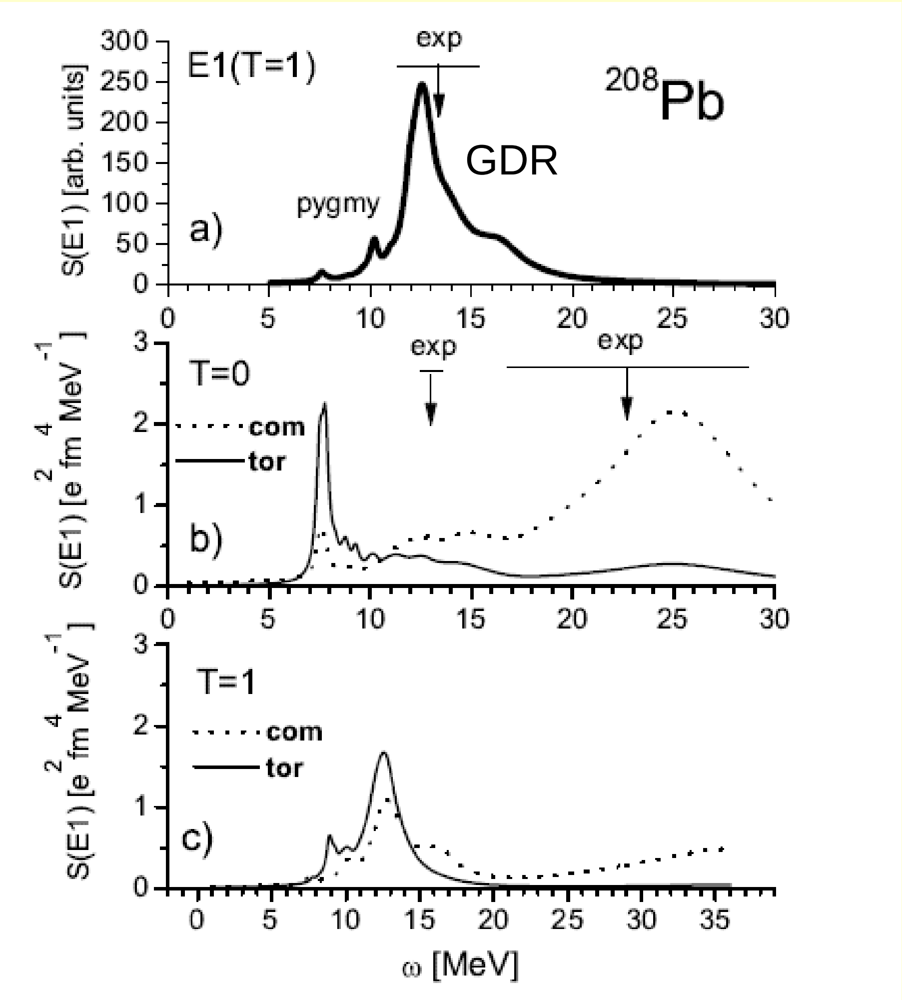

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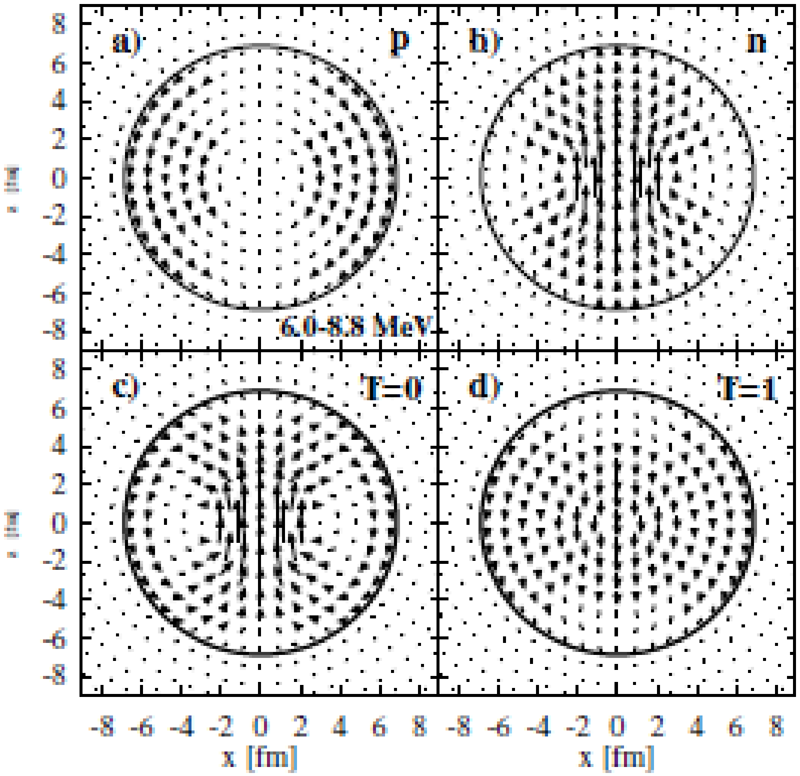
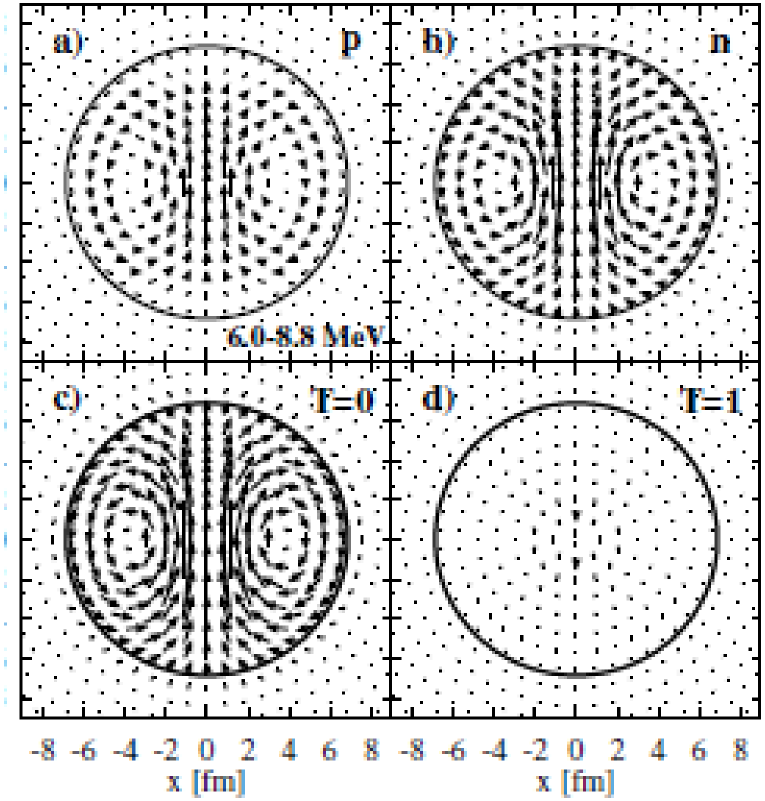
RPA vs 1ph

1ph



- both isoscalar and isovector
- toroidal flow mainly fom neutrons

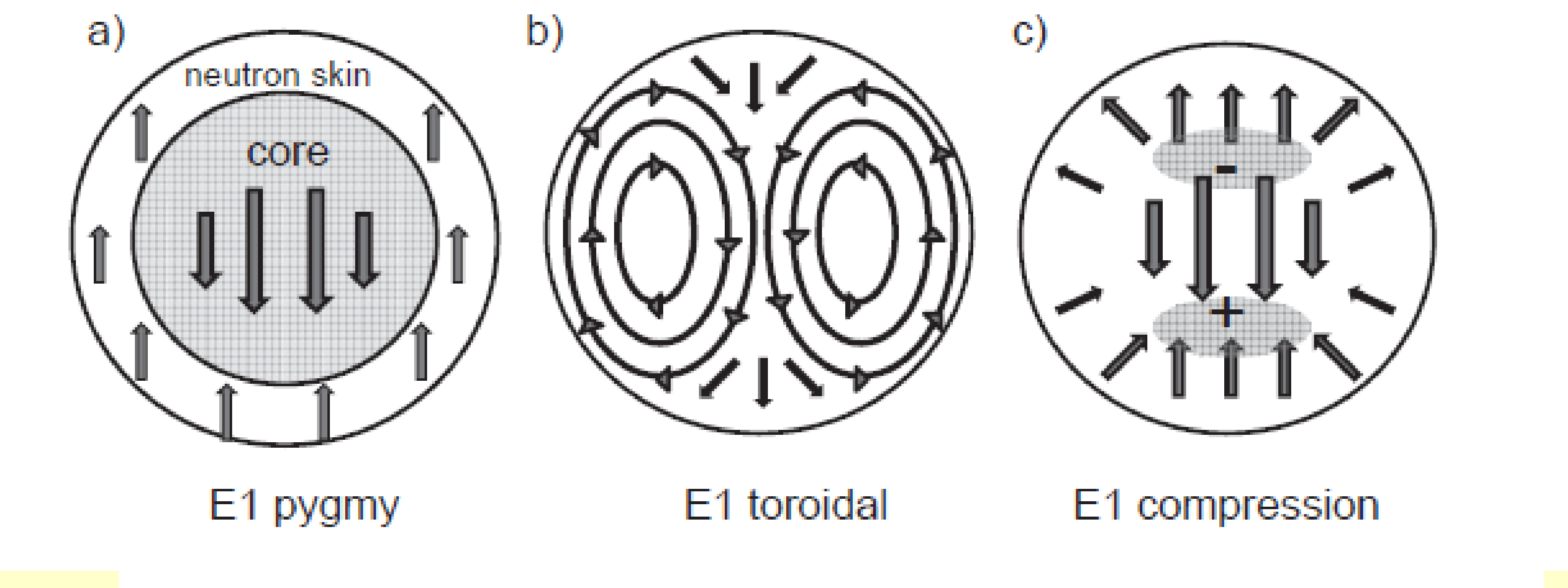
RPA

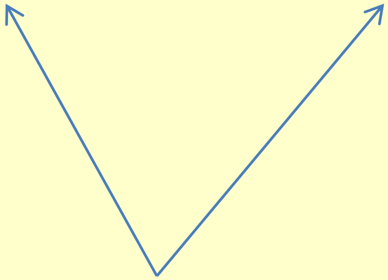


- mainly isoscalar
- toroidal flow from both n/p

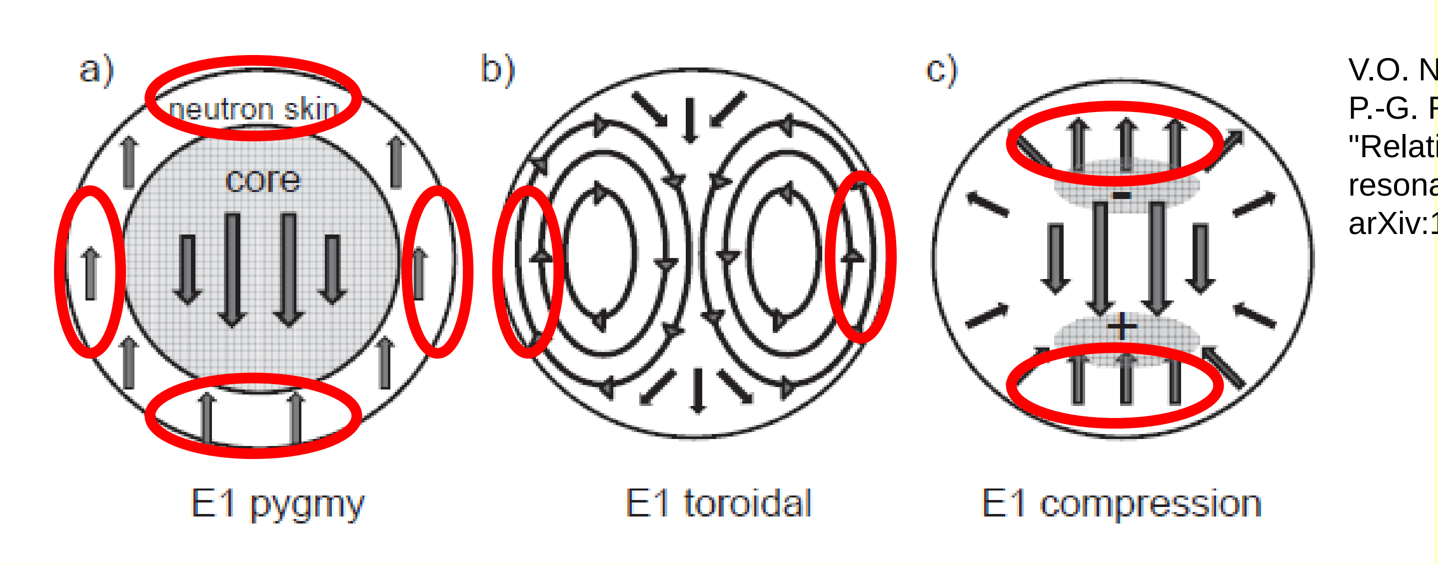
So the toroidal flow is basically formed already by the mean-field. But residual interaction makes it collective and more impressive.

Does the toroidal flow contradicts the familiar PRD picture?





?



E1 pygmy E1 toroidal E1 compression

V.O. Nesterenko, A. Repko,
P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal
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arXiv:1410.5634[nucl-th],

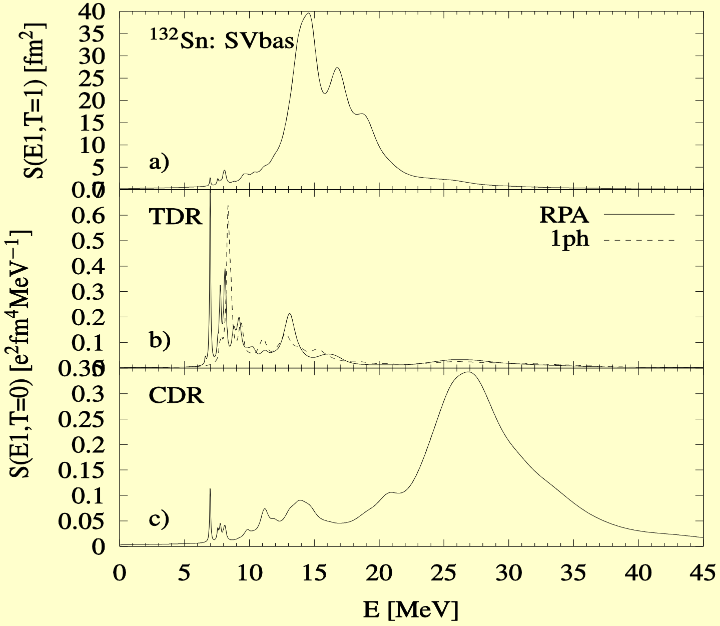
So:

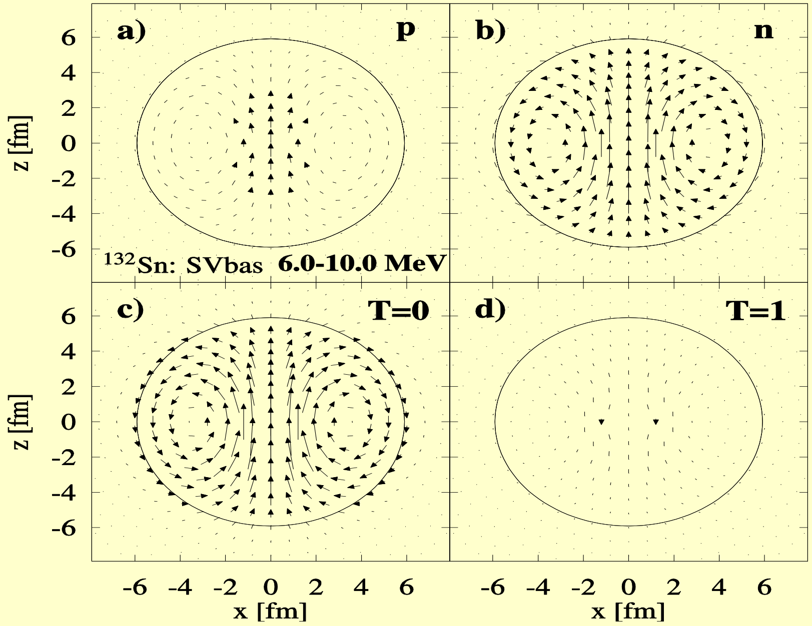
- **PDR can be viewed as a local peripheral part of the toroidal mode**

- **Our calculations demonstrate the TR motion in PDR energy region for other nuclei as well**

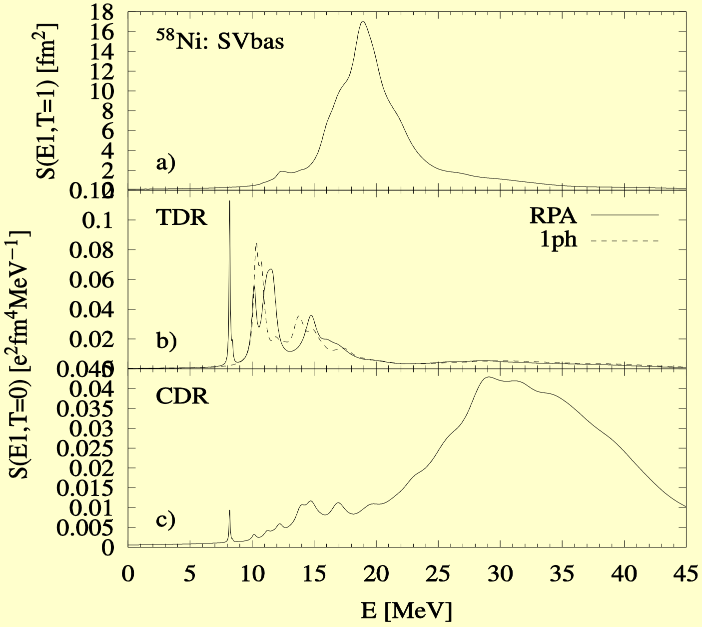
- **It is not yet clear how to check the PDR-TR relation experimentally**

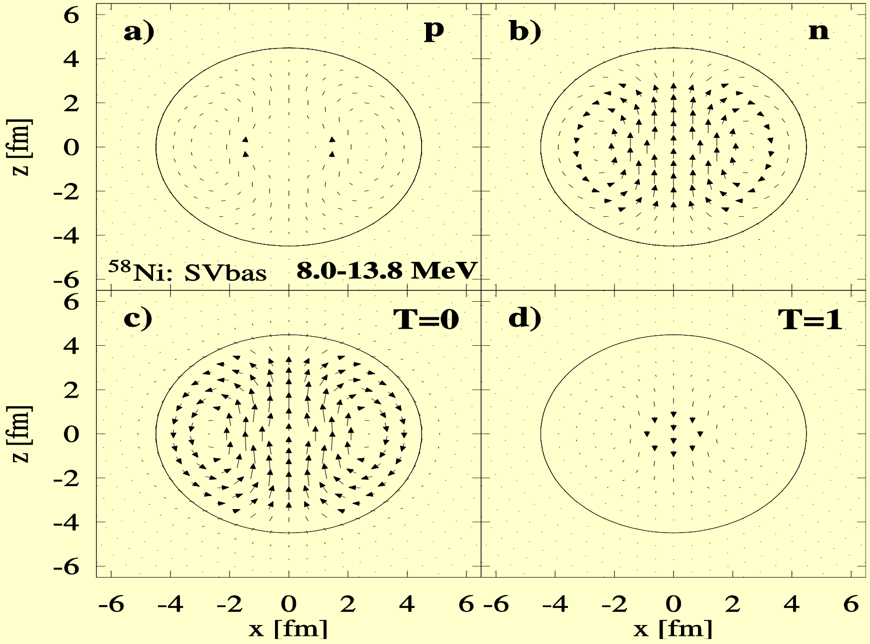
132Sn, SVbas, with PDR



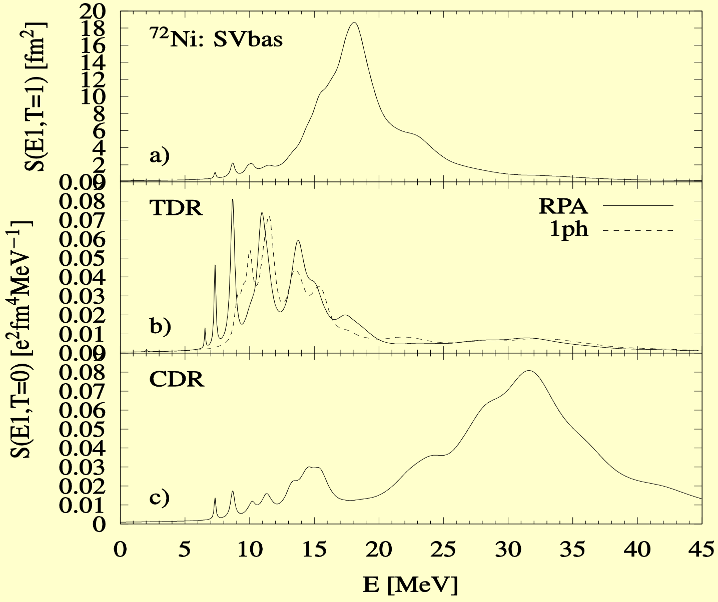


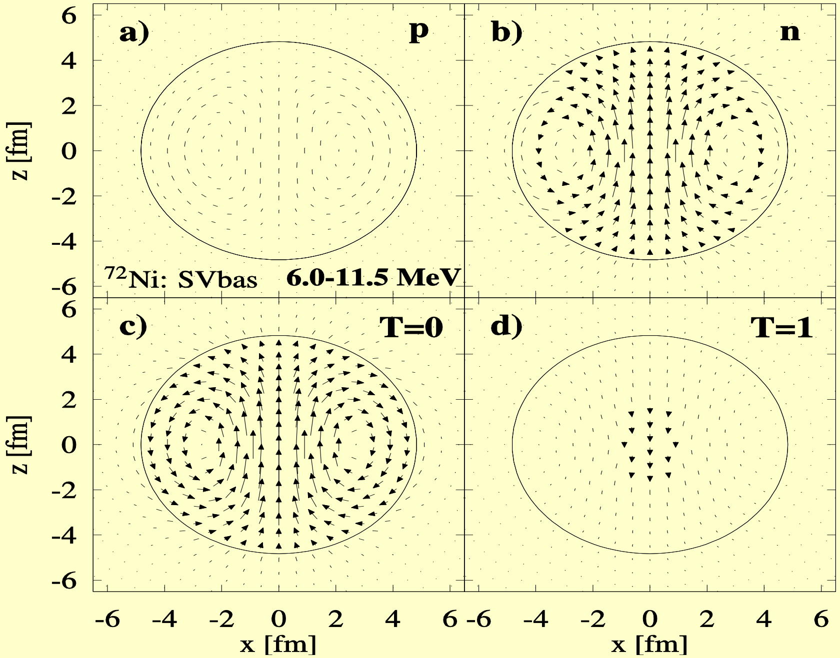
58Ni, SVbas



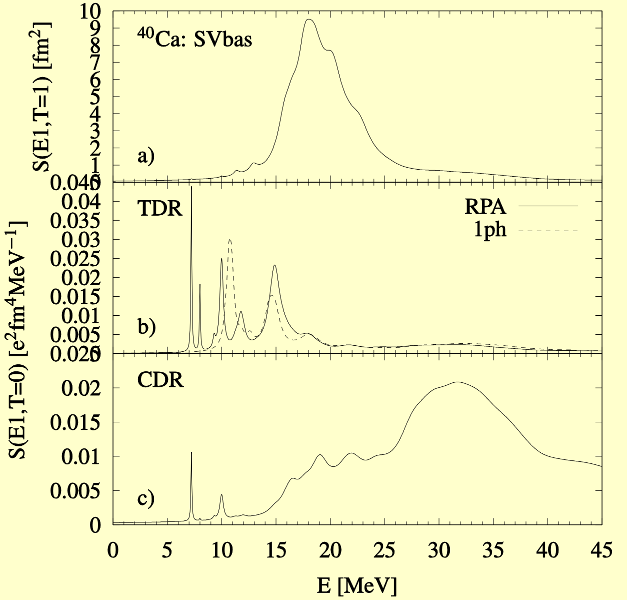


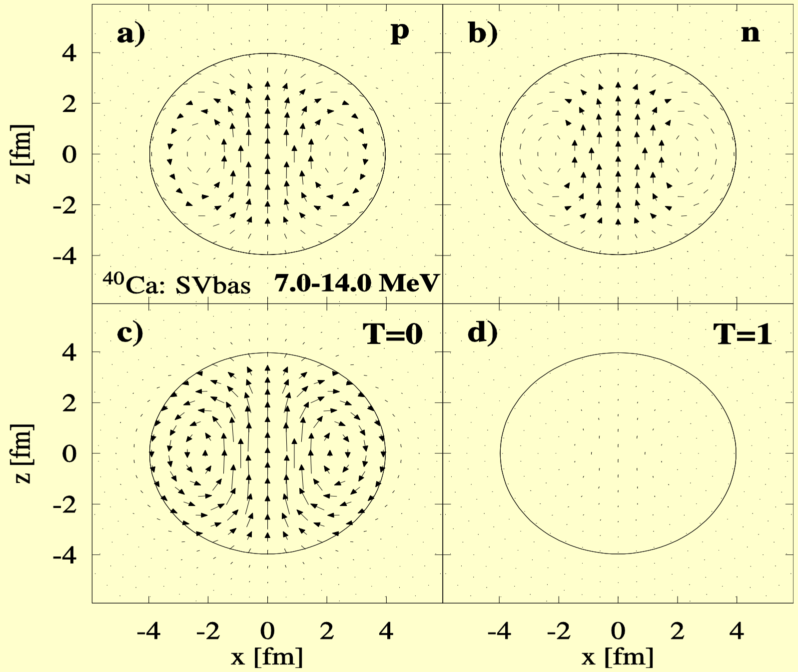
72Ni, SVbas



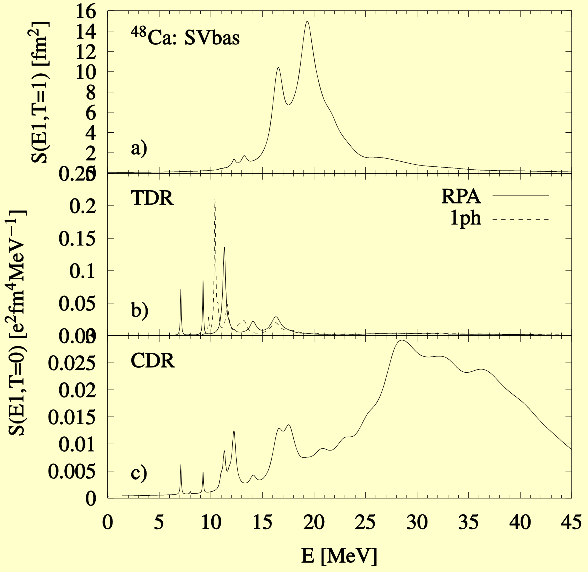


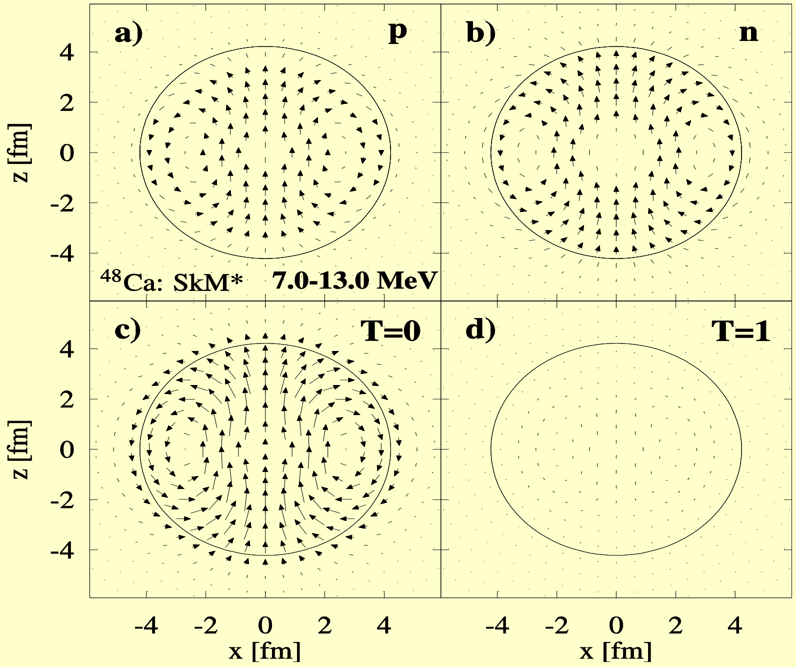
40Ca, SVbas





48Ca, SVbas





Deformation effects in the toroidal resonance

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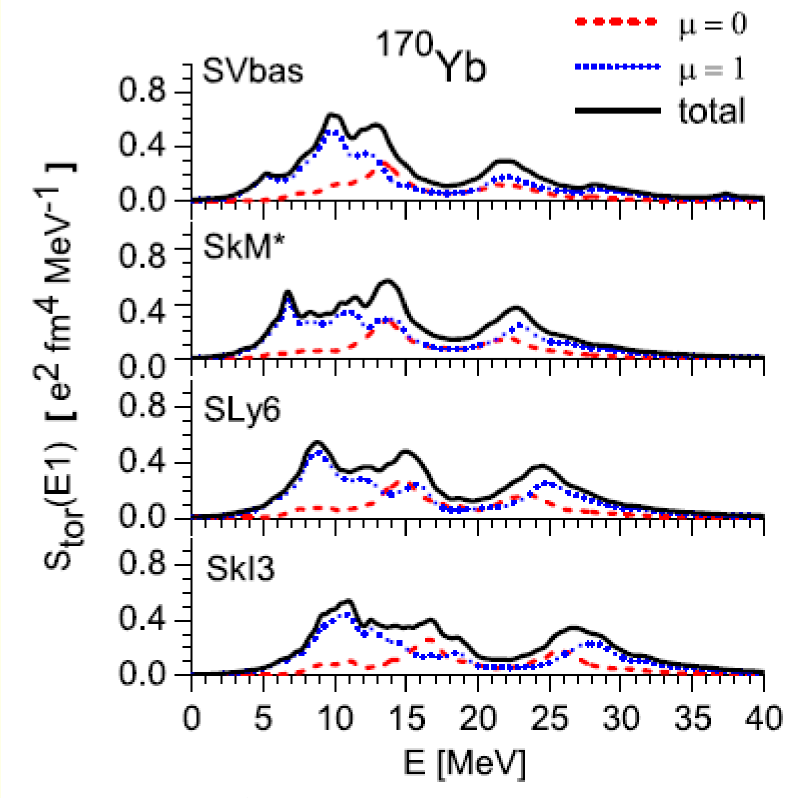
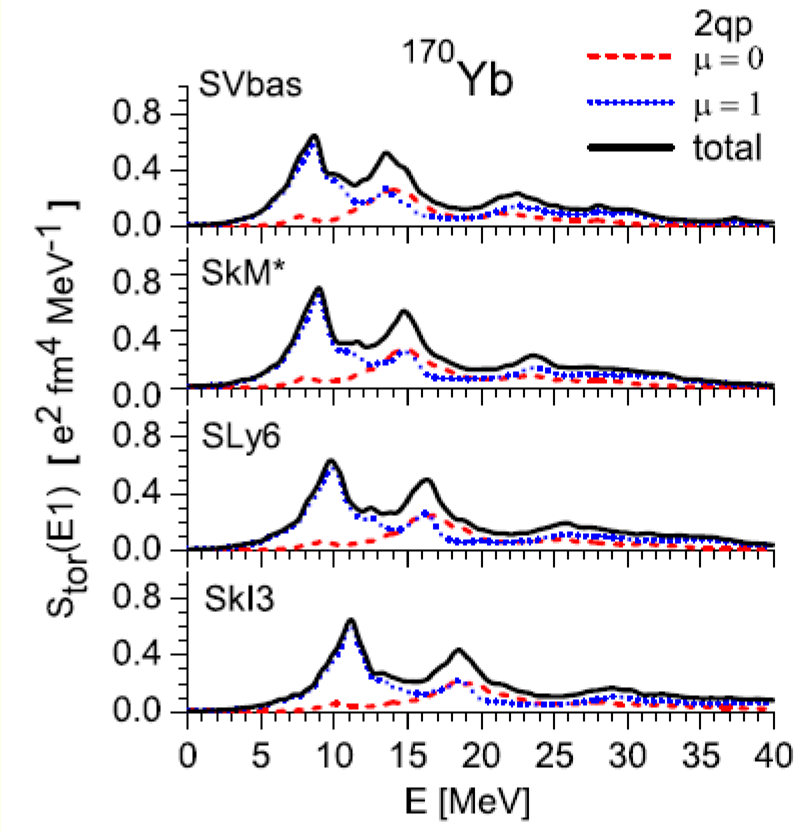
Deformation effects in the toroidal mode

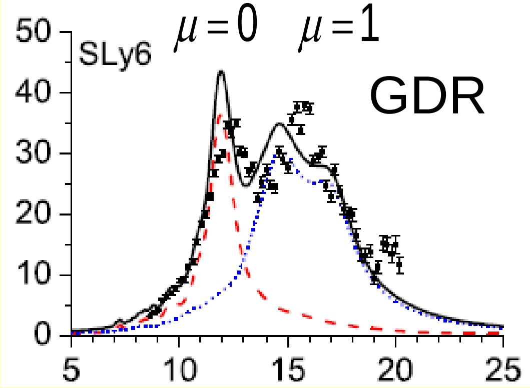
J. Kvasil, VON, W. Kleinig and P.-G. Reinhard,
Phys. Scr. 89, 054023 (2014)

RPA



2qp





GDR: $E(\mu=0) < E(\mu=1)$

TM: $E(\mu=0) > E(\mu=1)$

Unusual sequence of $\mu=0$ and $\mu=1$ branches

Deformation (not resid. Interaction) effect

Non-Tassie mode!

Should affect PDR properties

Conclusions

- **Skyrme-RPA analysis of TM in terms of strength functions, transition densities, and current fields.**

- **Toroidal current (strength) is the most relevant fingerprint and measure of the nuclear vorticity. At least, it is more convenient and relevant than RW and HD prescriptions.**

- **TM is the only known example of the vortical collective electric motion.**

- **PDR could be a local surface part of the toroidal motion.**
  PDR energy region was cleaned by IV-E1 residual interaction from Tassie modes and thus is dominated by the vortical non-TassieTM.

Perspectives:

- Further inspection of exotic E1 modes (TR, CR),
  search of relevant reactions

- Wavelet analysis of E1(T=1) and E2(T=0) GR
  (in collaboration with Darmstadt TU and SA)

- Tensor forces in spin-flip M1 resonance

P. Vesely, J. Kvasil, V.O. Nesterenko, W. Kleinig,
P.-G. Reinhard, and V.Yu. Ponomarev
PRC 80, 031302(R) (2009)

- Deformation effects in E0(T=0) resonance

-Description of lowest vibrational states (β, γ, octupole)
 in axially deformed even-even nuclei

V.O. Nesterenko, V. G. Kartavenko, W. Kleinig, R.V.
Jolos, J. Kvasil, and P.-G. Reinhard
arXiv:1504.06492[nucl-th], submitted to phys. Rev. C.

Thank you for attention!