Study of statistical properties of scattered photons from a driven three-level emitter embedded in 1D open space waveguide

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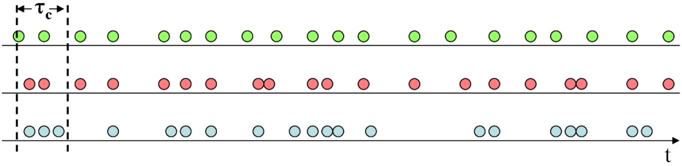
D. Roy and N. Bondyopadhaya, Phys. Rev. A 89, 043806 (2014)

Outline

1. Introduction
   - Statistical property of scattered photons
   - Realization of strong interaction between matter and photon field in 1D
   - Open space formalism for photon transport
2. Description of the Model
   - Hamiltonian of the system
3. Techniques:
   - How to find single, two or multiple photons scattering state in the full system
4. Results
   - Strongly correlated photons: Electromagnetically Induced Transparency (EIT)
   - Behaviour of second order correlation with system parameters
5. Conclusion

1. Photons are neutral particle, do not interact with each other. Photons should arrive independently of one another from a source (an ideal laser or a single frequency).
2. Non-classical correlations e.g. bunching (spatial attraction) and antibunching (spatial repulsion).



Photon detections as a function of time for a) antibunched, b) random, and c) bunched light

3. Second order correlation (Intensity-intensity correlation).

$$g^2(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

For, light from ideal coherent laser $g^2(\tau) = 1$, for bunched light $g^2(\tau) \leq g^2(\tau = 0)$, and for anti-bunched light $g^2(\tau) \geq g^2(\tau = 0) = 0$.

4. To study statistics in few photon level, one need strong light-matter interaction

Efficient strong coupling between matter and photon field in 1D open space:

- Highly confined propagating microwave photon modes in a $1D$ open superconducting transmission line and a large dipole moment of an artificial atom such as a superconducting qubit,
- Line defects in photonic crystals coupled to quantum dots and surface plasmons of a metallic nanowire coupled to quantum dots or nanocrystals.

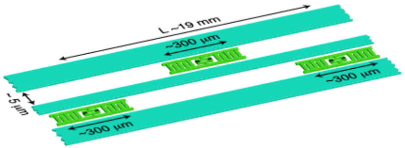


Figure: Transmon qubits acting as artificial atoms (in Green) coupled to a 1D superconducting transmission line (in Blue).

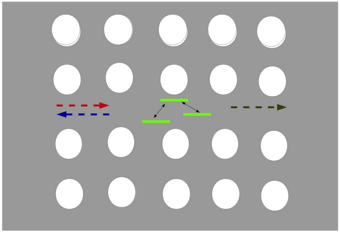
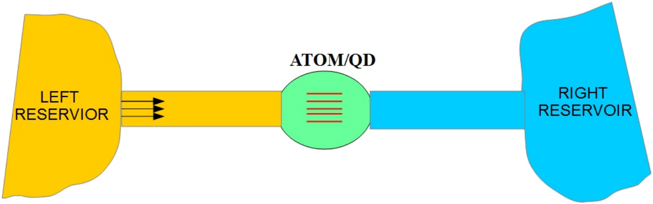


Figure: Line defects in photonic crystal coupled to quantum dots

Open system description



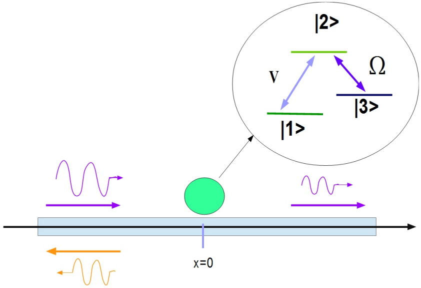
- Scattering of probe photons by three level emitter (3LE) $\Rightarrow$ Study of one and two photon transport incident on a single three-level emitter(atom), when the photons are restricted to a one-dimensional system.
- Exact theoretical approach, based upon real-space equations of motion and the Bethe ansatz. (Ref. Shen and Fan. PRL 98, 153003 (2007))

Model Hamiltonian

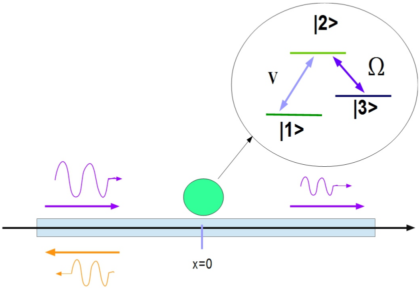
The full Hamiltonian describing the scattering of photons from a driven 3LE embedded in a 1D photonic waveguide:

$$\mathcal{H} = \mathcal{H}_{wg} + \mathcal{H}_{3LE} + \mathcal{H}_c$$

1. Hamiltonian for Free probe photons in wave-guide: $\mathcal{H}_{wg}$
2. Hamiltonian for a driven 3LE: $\mathcal{H}_{3LE}$
3. Hamiltonian describing the interaction between probe photons and 3LE: $\mathcal{H}_c$



- We consider a linear energy-momentum dispersion ($E_k = v_g k$) for the free probe photons in waveguide $\rightarrow$ Time evolution $\equiv$ Space evolution.
- Divide the positive and negative momentum photons as right-moving modes and left-moving modes. $a_R(x)$ $[a_L(x)]$ is the annihilation operator of a right-(left-) moving photon at position x.



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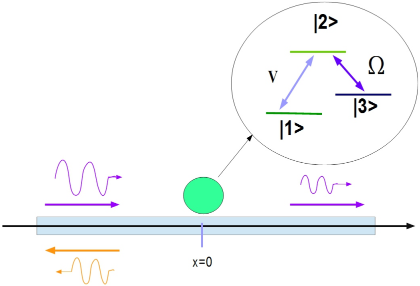
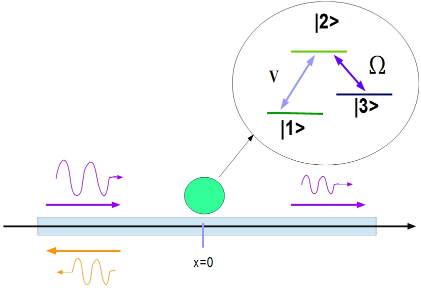


Wire Hamiltonian

Hamiltonian for Free probe photons in waveguide (Linear dispersion $E_k = v_g k$)

$$\mathcal{H}_{wg} = -iv_g \int dx[a_R^\dagger(x)\partial_x a_R(x) - a_L^\dagger(x)\partial_x a_L(x)],$$

where v_g is the group velocity of the photons.



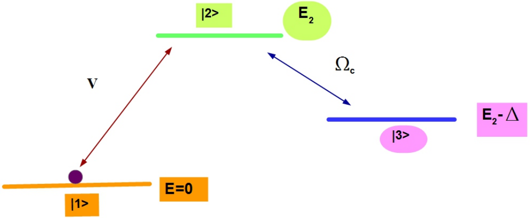
Hamiltonian for driven 3LE

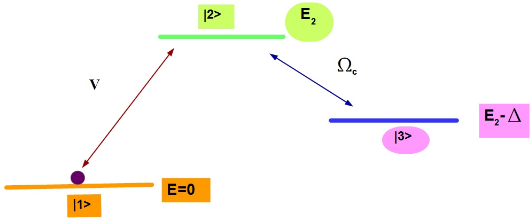
The Hamiltonian of a driven 3LE embedded in a 1D open waveguide:

$$\mathcal{H}_{3LE} = (E_2 - i\gamma_2/2)|2\rangle\langle 2| + (E_2 - \Delta - i\gamma_3/2)|3\rangle\langle 3| + (\Omega_c/2)(|3\rangle\langle 2| + |2\rangle\langle 3|),$$

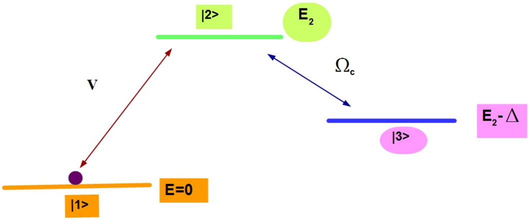
where spontaneous emission loss: $-i\gamma_2/2$ and $-i\gamma_3/2$ to the energy of the respective states $|2\rangle$ and $|3\rangle$.

The excited state $|2\rangle$ of the emitter is connected to the state $|3\rangle$ by a classical laser beam (Control beam) with Rabi frequency Ω_c(Proportional to the amplitude of control beam and dipole moment of 3LE),





1. Transitions $|1\rangle - |2\rangle$ and $|2\rangle - |3\rangle$ would couple to different polarizations of light by selection rule.
2. A probe beam in the waveguide is sent near resonant to the transition $|1\rangle - |2\rangle$.
3. We also consider that there is no direct transition between the states $|1\rangle$ and $|3\rangle$ by dipole selection rule.



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Light-matter interaction Hamiltonian

- Within Rotating-wave approximation (RWA)
  the Hamiltonian for the 3LE side-coupled to the propagating light fields locally at $x = 0$:

  $$\mathcal{H}_c = V|2\rangle\langle 1|(a_R(0) + a_L(0)) + h.c.,$$

  where V is coupling strength between emitter and probe photons.

Introduce a new basis, even-odd basis of probe photons, defined by

$$a_e(x) = (a_R(x) + a_L(-x))/\sqrt{2}, \;\; a_o(x) = (a_R(x) - a_L(-x))/\sqrt{2}$$

In even-odd mode, photons at $x < 0$ represent an event before scattering, and photons at $x > 0$ after scattering.

In the even-odd basis the Hamiltonian can be decoupled as $\mathcal{H} = \mathcal{H}_e + \mathcal{H}_o$, where

$$\begin{aligned}
\mathcal{H}_e &= -iv_g \int dx\, a_e^\dagger(x)\partial_x a_e(x) + \mathcal{H}_{3LE} \\
&+ \bar{V}\left(a_e^\dagger(0)|1\rangle\langle 2| + |2\rangle\langle 1|a_e(0)\right), \text{ and} \\
\mathcal{H}_o &= -iv_g \int dx\, a_o^\dagger(x)\partial_x a_o(x),
\end{aligned}$$

where $\bar{V} = \sqrt{2}V$.

Single photon state

$$|k\rangle = \int dx\{A_1[g_k(x)a_e^\dagger(x)|0,1\rangle + e_k|0,2\rangle + f_k|0,3\rangle] + B_1 h_k(x)a_o^\dagger(x)|0,1\rangle\},$$

where the constants $A_1 = B_1 = 1/\sqrt{2}$ for a right-moving photon with the incoming state $|k\rangle_{\text{in}} = (1/\sqrt{2\pi})\int dx e^{ikx} a_R^\dagger(x)|0,1\rangle$.

- Here $g_k(x)$ and $h_k(x)$ are the amplitude of a single photon in the even and odd field modes when the emitter in the ground state.
- For an incident photon coming from the left, $g_k(x<0) = h_k(x<0) = e^{ikx}/\sqrt{2\pi}$.
- The amplitude of the excited states $|2\rangle$ and $|3\rangle$ are respectively given by e_k and f_k.
- The basis state $|0,i\rangle$ denotes zero photon in the waveguide and the emitter in the i^{th} state.

Solution of stationary Schrödinger equation

Stationary Schrödinger equations, $\mathcal{H}|k\rangle = E_k|k\rangle$ with $E_k = v_g k$ gives

$$-iv_g\partial_x g_k(x) - E_k g_k(x) + \bar{V} e_k \delta(x) = 0 \Rightarrow g_k(0^+) = g_k(0^-) - i\frac{\bar{V}}{v_g} e_k,$$

$$(E_2 - i\gamma_2/2 - E_k)e_k + \bar{V} g_k(x)\delta(x) + \frac{\Omega_c}{2} f_k = 0,$$

$$(E_2 - \Delta - i\gamma_3/2 - E_k) f_k + \frac{\Omega_c}{2} e_k = 0,$$

$$-iv_g\partial_x h_k(x) - E_k h_k(x) = 0.$$

Regularization of amplitudes across the emitter position, $g_k(0) = [g_k(0-) + g_k(0+)]/2$ and the initial boundary conditions to solve the above differential equations of the amplitudes.

Single photon state

$$g_k(x) = h_k(x)\Big[\theta(-x) + t_k\theta(x)\Big], \;\; h_k(x) = e^{ikx}/\sqrt{2\pi}\,,$$

$t_k = (\chi - i\Gamma/2)/(\chi + i\Gamma/2)$ (Single photon transmission amplitude),

$$e_k = \bar{V}/(\sqrt{2\pi}(\chi + i\Gamma/2)), \;\; f_k = 0.5\Omega_c e_k/(E_k - E_2 + \Delta + i\gamma_3/2)$$

Here $\theta(x)$ is the step function, $\Gamma = \bar{V}^2/v_g = 2V^2/v_g$ and

$$\chi = E_k - E_2 + i\gamma_2/2 - \frac{\Omega_c^2}{4(E_k - E_2 + \Delta + i\gamma_3/2)}\,.$$

Now onwards we set $v_g = 1$.

For an incident photon from the left,

- Single-photon transmission amplitude of right moving photon : $\tilde{t}_k = (1 + t_k)/2 = \chi/(\chi + i\Gamma/2)$
- Single photon reflection amplitude left moving photon: $\tilde{r}_k = (t_k - 1)/2 = -0.5i\Gamma/(\chi + i\Gamma/2)$

Two-photon state

Two-photon Initial state

$$|k_1,k_2\rangle_{in} = \int dx_1 dx_2 \phi_{\mathbf{k}}(x_1,x_2)\frac{1}{\sqrt{2}}a_R^{\dagger}(x_1)a_R^{\dagger}(x_2)|0,1\rangle,$$

where $\phi_{\mathbf{k}}(x_1,x_2) = (e^{ik_1x_1+ik_2x_2} + e^{ik_1x_2+ik_2x_1})/(2\sqrt{2}\pi)$ with $\mathbf{k} = (k_1,k_2)$.

Two-photon Scattering state:

$$\begin{aligned}
|k_1,k_2\rangle &= \int dx_1 dx_2 \Big[A_2\{g(x_1,x_2)\frac{1}{\sqrt{2}}a_e^{\dagger}(x_1)a_e^{\dagger}(x_2)|0,1\rangle + (e(x_1)a_e^{\dagger}(x_1)|0,2\rangle + f(x_1)a_e^{\dagger}(x_1)|0,3\rangle)\delta(x_2)\} \\
&\quad +B_2\{j(x_1;x_2)a_e^{\dagger}(x_1)a_o^{\dagger}(x_2)|0,1\rangle + (v(x_1)a_o^{\dagger}(x_1)|0,2\rangle + w(x_1)a_o^{\dagger}(x_1)|0,3\rangle)\delta(x_2)\} \\
&\quad +C_2 h(x_1,x_2)\frac{1}{\sqrt{2}}a_o^{\dagger}(x_1)a_o^{\dagger}(x_2)|0,1\rangle\Big], \\
g(x_1,x_2) &= \frac{1}{\sqrt{2!}}\Big[\sum_P g_{k_{P_1}}(x_1)g_{k_{P_2}}(x_2) + \sum_{PQ} B^{(2)}_{k_{P_1},k_{P_2}}(x_{Q_1},x_{Q_2})\theta(x_{Q_2})\Big], \\
j(x_1;x_2) &= \sum_P g_{k_{P_1}}(x_1)h_{k_{P_2}}(x_2), \quad h(x_1,x_2) = \frac{1}{\sqrt{2!}}\sum_P h_{k_{P_1}}(x_1)h_{k_{P_2}}(x_2), \\
\Gamma_k(x-y) &= (d_-\varepsilon_k e^{i(s-t)|x-y|} + d_+\varsigma_k e^{i(s+t)|x-y|}), d_\pm = (\frac{1}{2\beta} \pm \frac{\epsilon}{2\Omega_c}), \\
& s = -E_2 + \Delta/2 + i(\tilde{\gamma}_2+\gamma_3)/4, \ t = \sqrt{\epsilon^2 + 4\Omega_c^2}/4 \\
B^{(2)}_{k_{P_1},k_{P_2}}(x_{Q_1},x_{Q_2}) &= -i\bar{V}\beta\,(1-t_{k_{P_1}})\Gamma_{k_{P_2}}(x_{Q_{12}})\,h_{k_{P_1}}(x_{Q_1})h_{k_{P_2}}(x_{Q_1})\theta(x_{Q_{12}})
\end{aligned}$$

← Two-photon bound state

Here $P = (P_1,P_2)$ and $Q = (Q_1,Q_2)$ are permutation of $(1,2)$, and $x_{Q_{12}} = x_{Q_1} - x_{Q_2}$.

Electromagnetically Induced Transparency

Transmission coefficient for right moving photon: $T_k = |\tilde{t}_k|^2$, when $g(x_1 > 0, x_2 < 0)$.

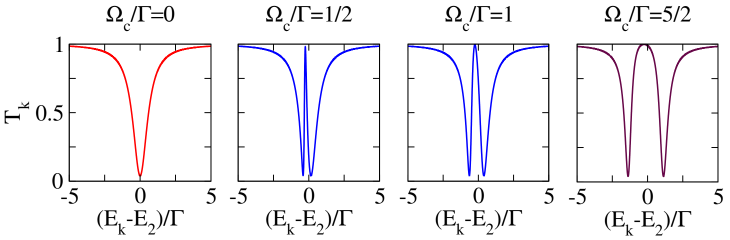

Figure: T_k vs detuning $(E_k - E_2)$; Parameters: $\Delta/\Gamma = \gamma_2/\Gamma = 1/4, \gamma_3/\Gamma = 1/40$

- Appearance of EIT at two-photon resonance, $E_k - E_2 = -\Delta$ at weak control beam ($\Omega_c < \Gamma$).
- Destructive quantum interference between the two paths that lead to a transition to $|2>$ $\Rightarrow$ Cancellation of the population of the state $|2>$ ("dark state").

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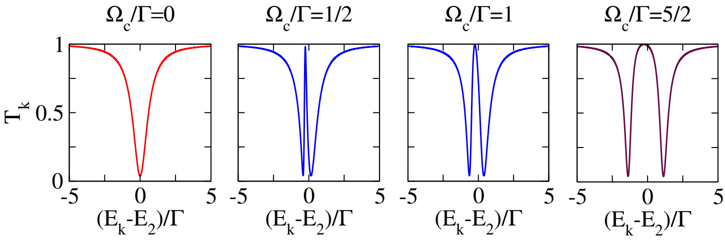

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2nd order correlation (Intensity-Intensity correlation)

- Study of photon statistics by measuring second-order spatial coherence of the scattered photons
- $$g^2(x_2 - x_1) = \frac{\langle\psi|a_m^\dagger(x_1)a_m^\dagger(x_2)a_m(x_2)a_m(x_1)|\psi\rangle}{\langle\psi|a_m^\dagger(x_1)a_m(x_1)|\psi\rangle\langle\psi|a_m^\dagger(x_2)a_m(x_2)|\psi\rangle},$$
  where $m = R(L)$ for the transmitted (reflected) photons for an incident probe beam from the left.
- $|\psi\rangle$ is a N-photon scattering Fock state with incident momenta $k_1, k_2..k_N$.

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Keeping higher order contributions in the numerator and denominator: 2nd order correlation for two photon state

$$g^2(x_2 - x_1) = \frac{1}{|(t_{k_1} \pm 1)(t_{k_2} \pm 1)|^2} \Bigg(|\sum_P (t_{k_{P_1}} \pm 1)(t_{k_{P_2}} \pm 1)\tilde{h}_{k_{P_1}}(x_1)\tilde{h}_{k_{P_2}}(x_2) + 2i \sum_{PQ} V\beta\,(t_{k_{P_1}} - 1)\Xi_{k_{P_2}}(x_{Q_{12}})\,\tilde{h}_{k_{P_1}}(x_{Q_1})\tilde{h}_{k_{P_2}}(x_{Q_1})\theta(x_{Q_{12}})|^2 \Bigg).$$

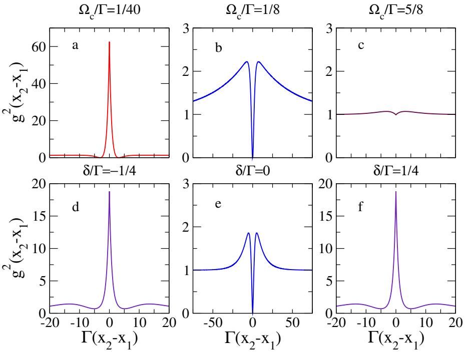
$+(-)$ sign for the transmitted (reflected) probe beam.

$$\tilde{h}_k(x) = e^{ikx}\theta(x)/\sqrt{2},\ \Xi_k(x_1 - x_2) = \sum_{j=\pm} d_j \varepsilon_j(k) e^{i(s + j\Omega_c/4\beta)|x_1 - x_2|}.$$

$$\varepsilon_\pm(k) = V/(E_k + s \pm \Omega_c/4\beta),\ s = -(E_2 - \Delta/2) + i(\gamma_2 + \gamma_3 + \Gamma)/4$$

$$\beta = \Omega_c/\sqrt{\epsilon^2 + 4\Omega_c^2},\ d_\pm = (1/(2\beta) \pm \epsilon/(2\Omega_c)), \epsilon = -2\Delta + i(\gamma_2 + \Gamma - \gamma_3).$$

We use $P = (P_1, P_2)$ and $Q = (Q_1, Q_2)$ for permutation of $(1, 2)$ and $x_{Q_{12}} = x_{Q_1} - x_{Q_2}$.



- Two photon resonance: $E_{k_1} = E_{k_2} = E_2 - \Delta$.
- First Row: g^2 vs Ω_c: two-photon resonance $\delta = (E_k - (E_2 - \Delta)) = 0$ and $\gamma_3/\Gamma = 1/40$
- Second Row: g^2 vs δ : $\Omega_c/\Gamma = 3/10$ and $\gamma_3/\Gamma = 1/8$.
- The other parameters are $\Delta = 0,\ \gamma_2/\Gamma = 0.31$.

- g^2 shows antibunching of the transmitted probe photons $\Rightarrow$ Two probe photons cannot transmit through the emitter simultaneously.
- This happens for a Rabi frequency when a complete dark state is not yet formed.
- Further increase Ω_c a dark state is formed $\Rightarrow g^2(x_2 - x_1) = 1$.

Conclusion

1. We calculate the exact one photon and two-photon wave functions for this model.
2. Appearance of EIT at two-photon resonance, $E_k - E_2 = -\Delta$ when a weak control beam is switched on.
3. Second-order coherence of the scattered probe photons from Λ or ladder-type 3LE can be tuned by changing Rabi frequency of the control beam. It can be measured experimentally using a Hanbury Brown and Twiss measurement setup .
4. Generalization :To derive multiphoton scattering states and second-order coherence of the scattered probe photons in case of multiple interacting multilevel emitters.

THANK YOU !